Putting the Practices Into Action

Implementing the **Common Core Standards** for Mathematical Practice **K–8**

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For the newest additions to our family,
Blake Olivia and Molly Kathleen, with love
S.O.

To my parents for
encouragement, love, and support
J.S.

Contents

CHAPTER 2

Exploring Standard 1: *Make Sense of Problems and Persevere in Solving Them*

WHY PROBLEM SOLVING?

As we examine the first Standard for Mathematical Practice, we may find ourselves wondering, "Why problem solving?" In looking back on our own experiences as math students, we may recall computation as the main focus. A page in our math textbook probably contained 20–30 computations, with maybe one or two word problems at the bottom of the page. Many teachers report that they received As and Bs in math despite never understanding *how* to solve problems. And now the ability to solve math problems is the first standard! This attests to our belief that being able to do computations alone does not equate to math proficiency. Our new definition of proficiency includes knowing when, why, and how to apply calculations to situations. The practice of solving problems is critical to math success.

UNDERSTANDING THE STANDARD

The ability to solve problems by applying varied math skills is what makes our students effective mathematicians. This standard focuses on the development of essential skills and dispositions for becoming an effective problem solver, including:

1. An understanding of the problem-solving process and how to navigate through the process from start to finish
2. A repertoire of strategies for solving problems and the ability to select a strategy that makes sense for a given problem
3. The disposition to deal with confusion and persevere until a problem is solved.

UNDERSTANDING THE PROBLEM-SOLVING PROCESS

Problem solving is not an algorithm to be practiced or a fact to be memorized. Effective problem solvers decide—for each unique problem—what is being asked, what is important to consider, an appropriate path to the solution, and the reasonableness of their actions. Problem solvers think about their own thinking (metacognition) so they are better able to regulate and modify their thinking. They plan ways to approach a task, select appropriate tools, evaluate their own progress, and revise their actions.

The work of George Polya (1957) identified critical steps in this process, which have since been restated and refined in a variety of ways. The following questions highlight important steps in the thinking process:

What is the problem asking?

How should I begin?

Where is the necessary data?

What should I do with that data?

Did my plan work?

Does my answer make sense?

Do I need to go back and try a different strategy?

To support our students, we identify, discuss, and move toward making this process automatic, so they are then able to focus on other aspects of the complex task of solving problems.

DEVELOPING STRATEGIES

When it is time to devise a plan for solving problems, what do your students do? Do they have a repertoire of strategies from which to choose? Have they explored varied problems to see the many ways they might be solved? Have they discussed alternate ways to solve problems, to expand their thinking? Do they recognize familiar problems and determine how they might solve a given problem?

Simply knowing how to proceed through the stages of solving a problem is not enough. Our students must determine a strategy to help them solve problems, and that strategy will vary with each problem. Once our students identify the question and pinpoint the necessary information, they must decide on a plan that will lead them to the answer. While all textbooks mention and provide a few problems related to strategies such as Choosing Operations, Drawing Pictures, Making Tables, or Working Backward, the underlying skills are actually quite complex thinking skills (see Figure 2.1) that require more attention and discussion (O'Connell 2007b).

Strategy teaching is a blend of student exploration and direct teaching. Highlight and discuss students' approaches at every opportunity, especially when they lead to insights about effective problem solving. But it is also okay to introduce students to strategies (e.g., constructing tables as a way of organizing data). Many students would never think of these approaches on their own, but they come to understand and use them effectively. Our students need ongoing experience in applying these strategies, with particular emphasis on how and when to apply them and why they work for a particular problem.

BUILDING A PROBLEM-SOLVING DISPOSITION

Many students become easily frustrated when solving math problems. Anxiety, stemming from self-doubt about their abilities or a fear of failure, blocks their progress.

Am I able to do this?

What if I get stuck?

What if it takes me too long to get the answer?

What if my idea doesn't work?

What if my answer is wrong?

Believing it's possible to solve a problem, recognizing that confusion is part of the process, and discovering that persistence pays off are all components of a positive problem-solving disposition. This disposition allows our students to self-monitor, checking the reasonableness of their approaches and solutions and modifying their course of action, without becoming frustrated, anxious, or discouraged.

Figure 2.1 *This chart identifies some of the underlying thinking skills for common problem-solving strategies.*

STRATEGY	UNDERLYING THINKING SKILL(S) What students really need to know!
Choose an Operation	Understand the meaning of operations Build appropriate equations to represent problem situations
Draw a Picture	Visualize problem situations Use models to represent problems Analyze models to gain insight into problems
Find a Pattern	Recognize the importance of looking for connections between numbers Identify patterns that lead to solutions
Make a Table	Organize data to more easily identify patterns and functions that lead to solutions Continue a pattern or apply a function to find a solution
Guess and Check	Employ trial-and-error thinking Utilize number sense to move closer to an answer Make adjustments during the problem-solving process, based on mathematical reasoning
Make an Organized List	Identify a starting point and move systematically toward a solution Create a model (e.g., organized list, tree diagram) to organize confusing information and simplify a problem
Use Logical Reasoning	Organize confusing information to simplify a problem Make inferences to solve a problem Draw conclusions based on clues Identify if/then or cause/effect relationships
Work Backward	Identify known data and find missing data, regardless of where it might appear in a problem Use inverse operations to work backward to find solutions

HOW DO WE GET THERE?

CCSS Practice Standard 1 provides us with a clear vision of the knowledge and skills that make our students effective problem solvers. But how do we help our students develop these important practices? Think about your own classroom as you read the following questions. You may already be able to answer "yes" to many of them!

What do we do each day in our classrooms to build mathematical thinkers?

- Do our classroom activities and discussions focus on students' thinking related to how and why they chose a particular strategy, rather than on just getting the answer?
- Do our classroom discussions move beyond oversimplified, and sometimes unreliable, methods like key words (e.g., "I saw the word *altogether* so I just added.")?
- Do we pose one or two problems for students to solve and discuss thoroughly, rather than supply a list of problems to be solved as quickly as possible?
- Do we often replace easier, more direct problems with problems that push students to apply their understanding of math content?

What do we do to keep our students actively engaged in solving problems?

- Do we routinely ask students to talk and write as they solve problems? Do we provide ongoing opportunities for them to talk about both process and solution, to identify their own thinking, and to discuss alternate ways to approach a problem?
- Are our questions frequent, purposeful, and high level?
- Do we maintain students' interest and expand their insights by asking them to share their ideas and actively solve problems with partners and groups?

How do we help our students develop positive attitudes and demonstrate perseverance during problem solving?

- Do we provide opportunities for students to explore complex problems that may include multiple approaches or answers that are not immediately apparent?
- Do we praise their efforts, with value placed on persistence and process rather than on the answer?

- Is our classroom environment supportive and nonthreatening? Is speed de-emphasized and is confusion openly discussed, including insights on ways to simplify problems and move through confusion?
- Do we acknowledge the efficiency of particular strategies but still celebrate individual, reasonable approaches?

What does problem solving look like and sound like? You probably see it more often in your students than you realize! As you read the classroom vignettes throughout this chapter and the rest of the book, think about when you've seen your own students using similar skills. In the pages that follow, we'll discuss ways you can help build students' proficiency with these math practices.

CLASSROOM-TESTED TECHNIQUES

Following are some valuable, classroom-tested techniques that work across grade levels and with varied math content to expand students' thinking and support the development of their problem-solving skills.

Focus on the Question

It can be daunting to think about the many skills we want to develop within our students and frustrating to think that each skill needs to be developed separately. Consider the following student expectations:

- Determine and articulate what the problem is asking
- Find a starting point by understanding mathematical situations
- Identify relevant data for solving the problem
- Identify an appropriate way to solve the problem
- Connect problem situations to abstract representations of the problem (e.g., equations, visuals) in order to clarify the task
- Identify or understand different ways to solve the problem

Focus on the Question is a simple classroom technique that addresses multiple problem-solving skills with little teacher planning, minimal class time, and no additional paperwork. In addition, it provides ongoing problem-solving experiences for students at all grade levels and can be easily integrated with various math content

Connections
to other practice standards

Students convert the problem situations to abstract equations in order to find solutions (Standard 2).

Students explain their choice of problem-solving strategies, construct viable arguments to justify their strategies, and precisely communicate their mathematical thinking (Standards 3 and 6).

(e.g., place value, fractions, measurement). This allows students to explore or review specific content skills through the problem situations. Here is how it works:

1. Post a set of data in the classroom on Monday (e.g., a picture graph of the colors of jellybeans in a bag, a list of the cost of snacks at a fair, a circle graph of favorite sports, or a list of the heights of students' bean plants in science). That data remains posted for the week.
2. Each day, pose a different problem in which students must use some or all of the data.
3. Each day, students talk to partners about what they are being asked to solve, what data will help them solve it, and how they would solve it.
4. Facilitate a class discussion in which students share their ideas about how the problem could be solved.
5. No answers are needed and written work is not required. The activity generally takes no more than 10 minutes.

In the Classroom

On Monday Mrs. Alexander posted the following situation and data on the board:

The Holiday Tree

The Partin family counted the different types of ornaments on the town's holiday tree. Here is the list of what they saw.

Stars – 24

Gingerbread men – 14

Snowflakes – 12

Reindeer – 18

Candy canes – 6

She asked her third-grade students to read the data with their partner, and then she posted the following problem:

Six of the reindeer had red noses. What fraction of the reindeer had red noses? Tell how you would get the answer.

The students dove right into their discussions, talking about the question and debating how they would proceed to find the solution. They enthusiastically deliberated about the equation that might lead them to the answer, knowing that they did not have to find the actual answer. After several minutes, seeing that the students had discussed their plans of action, Mrs. Alexander asked them to share their thoughts with the class. She guided the discussion with questions that helped make their thinking visible to their classmates. The students, already having discussed their thinking with partners, were ready and willing to share their ideas.

Teacher: What is the question?

Liam: What fraction of reindeer ornaments had red noses?

Teacher: What does that mean? Can you say it another way?

Liam: Of all of the reindeer, what part had red noses?

Colin: It won't be a number. It will be a fraction.

Teacher: So, what data did you and your partner think might be important to solve the problem?

Molly: We need to know how many reindeer ornaments there are.

Teacher: Where would you find that data?

Molly: You just look at the list.

Blake: You use the number for the reindeer ornaments — 18.

Teacher: What about the rest of the data on the list? Do you need it?

Bailey: No.

Teacher: Why not?

Bailey: It just asked about reindeer ornaments, not the other ones.

Teacher: So we just ignore the other data?

Kate: Yes.

Allison: We don't need it.

Brendan: It doesn't matter for the problem.

Jason: But we also need to know 6 had red noses!

Teacher: I don't see that on the list.

Jason: No, it's in the problem.

Teacher: Oh, okay. So, what will you do with the data about the reindeer ornaments?

Patrick: We'll make a fraction with it.

Teacher: Why? Tell me what you mean.

Patrick: We know there were 6 reindeer with red noses and 18 altogether, so we make a fraction.

Kate: Yeah, so we know $\frac{6}{18}$ is the fraction.

Brendan: Yeah, $\frac{6}{18}$ have red noses.

Bailey: But you could say $\frac{1}{3}$.

Teacher: Could we say it that way?

Bailey: We could say both. They are the same thing.

Susan: Yeah, cause with fractions you can simplify them, but it means the same.

Patrick: Yeah, $6 + 6 + 6 = 18$, so 6 is one out of three.

Susan: It would be the same answer, but $\frac{1}{3}$ sounds easier.

As the week unfolded, students explored a different problem each day using the same set of data about the holiday tree. Since an understanding of fractions was a primary expectation for her students, many of the problems required them to use their understanding of fractions as they contemplated a method to solve the problem. She posed questions like:

What fraction of the ornaments were snowflakes?

What fraction of the ornaments were edible?

If 6 of the stars were silver, what fraction of the stars were not silver?

Mrs. Alexander noticed a new level of confidence in her students as they talked about solving these problems. Early in the year, it had taken more prompting to get students to discuss their thinking, and she had felt many moments of awkward silence as they considered her questions. Now, after many experiences with these types of discussions, they were relaxing, more easily identifying the question and the data that would help them solve it, and they appeared comfortable debating their decisions about ways to solve the problem.

By using the same data each day, but changing the question, students become skilled at pinpointing the question and identifying the specific data needed for that particular question. The data that is used one day is not the same data that is used on the following day. In the same way, the equation that solves the problem on Monday is different from the one that solves Tuesday's problem. Students begin to see that the question guides them to the relevant data and the appropriate strategy.

The key to effectively implementing this technique is the questions we pose during the class discussions. Asking open-ended questions, such as the ones in Figure 2.2, makes our students' thinking visible. Those students who struggle with problem solving hear how others approach problem tasks. Students hear alternate

Figure 2.2 *These questions prompt students to communicate about their thinking.*

QUESTIONS TO GUIDE STUDENT THINKING

- What is the question?
  - Can you restate it in your own words?
  - Can anyone say it a different way?
- What information will you need to solve it?
  - Where will you find the information?
  - Do you need all of the information on the list, table, graph, etc.? Why or why not?
  - How did you know which information is needed?
- What should you do with the information?
  - Should you add, subtract, multiply, or divide? Which one? Why?
  - What in the problem tells you to add, subtract, multiply, or divide?
  - What is the equation that matches this problem?
- Is there another way to solve it?
  - Could you have used a different operation? Explain which one and why.
  - Could you have made a diagram or used manipulatives to solve it? Explain how.
  - How are these different ways connected?

Later, we will explore modeling as a critical math practice (Standard 4). Notice how modeling is integrated into this problem-solving experience.

strategies chosen by their classmates and expand their repertoire of approaches. The activity also allows students to revisit various types of graphs and math concepts throughout the year and discuss solutions to many different types of math problems. In addition, students gain confidence and increase their skills in reading and analyzing word problems through ongoing modeling and think-alouds related to analyzing details, making inferences, and other comprehension skills.

Whether you use this activity with a small group of students or the whole class, you will enjoy the conversations that emerge during *Focus on the Question*, and these conversations will not only guide your students' thinking but will also provide you with a wealth of information about the current level of their problem-solving skills.

Give It a Try!

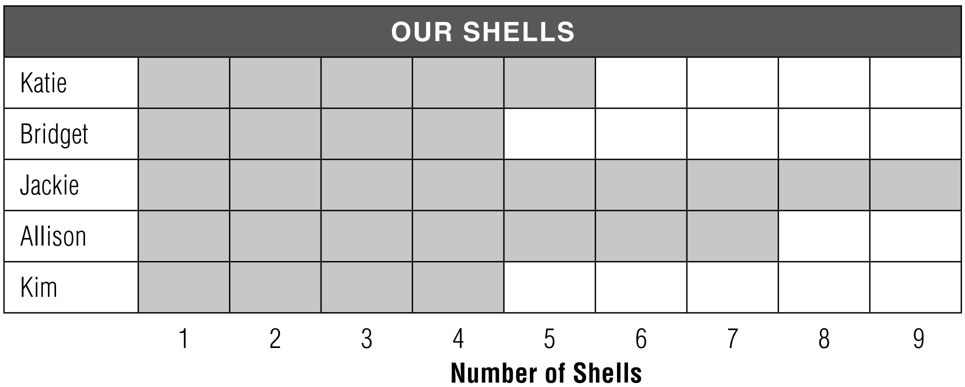
Below you will find three examples of *Focus on the Question* at three different levels (primary, intermediate, and middle grades), including a data set and five questions related to the data. Keep in mind that it is the questions you ask that get your students thinking and talking about their problem-solving efforts.

Primary Example

Gathering Seashells

The children were picking up seashells at the beach.

The graph shows the number of shells that each child picked up.



Day 1 Katie and Kim put their shells in a box. How many did they have altogether? Tell how you would find the answer.

Day 2 Allison wanted to collect 10 shells. How many more will she need to collect? Tell how you would find the answer.

Day 3 Jackie wanted to collect a dozen shells. How many more does she have to collect to have a dozen? Tell how you would find the answer.

Day 4 Allison said she collected more shells by herself than Katie and Kim collected together. Is she right? Tell how you would find the answer.

Day 5 How many shells did the children collect altogether? Tell how you know.

Intermediate Example

Shipley Aquarium

Admission Cost

Adults – $8.00

Children (ages 3 and over) – $6.50

Children (ages 2 and under) – Free

Day 1 How much more does an adult pay to get into the aquarium than a six-year-old child? Tell how you would get the answer.

Day 2 Mr. and Mrs. Jones brought their two-year-old son to the aquarium. How much did they pay for admission? Tell how you would get the answer.

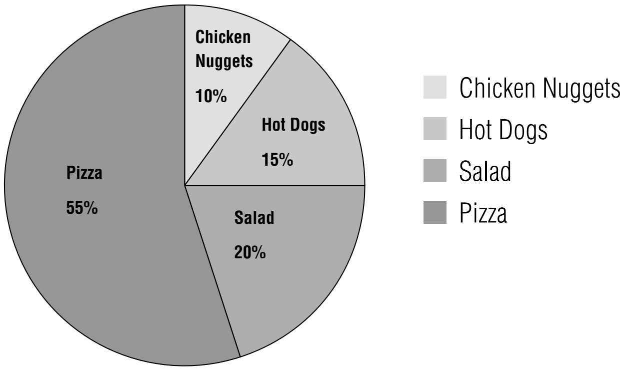
Day 3 Karen had her 10th birthday party at the aquarium. She invited 6 friends. What was the total admission cost for the seven children? Tell how you would get the answer.

Day 4 The third grade took a field trip to the aquarium. There were 20 children and 3 adults. How much was admission? Tell how you would get the answer.

Day 5 Some adults went to the aquarium. They paid $48.00 for admission. How many adults were there? Tell how you would get the answer.

Middle Grades Example

There were 420 students who ate lunch in the cafeteria. Following are their lunch choices.



Day 1 Each plate of chicken nuggets was served with three carrot sticks. How many carrot sticks were needed? Tell how you would get the answer.

Day 2 How many more students ordered salad than hot dogs? Tell how you would get the answer.

Day 3 One-third of the pizza orders were for pepperoni pizza and $\frac{2}{3}$ were for sausage pizza. How many orders were for each kind? Tell how you would get the answer.

Day 4 Two-thirds of the students who ordered a hot dog took a ketchup packet, $\frac{1}{7}$ took a mustard packet, and the rest took one of each. How many ketchup packets and how many mustard packets were taken that day? Tell how you would get the answer.

Day 5 The cafeteria orders packages of lettuce to make the salads. Each package holds 2 pounds of lettuce. Each salad was made with approximately 10 ounces of lettuce. How many packages did they need? Tell how you got your answer.

Posing Open-Ended Questions

The questions we ask during math class have so much power! We can ask questions that stifle learning by prompting a quick number response. We know these questions all too well from our own experiences.

What is the answer to number 3 on your worksheet?

How many degrees are in a right angle?

What is 4 × 5?

How many feet are in a yard?

We recognize that these questions do not require mathematical thinking, do not create proficient mathematicians, and do not promote the Practices we are focusing on in the CCSS Practice Standards. We have the power to select questions that build proficient mathematicians.

As we explore questioning, we recognize two distinct opportunities within our classrooms: posing rich problems for students to solve and posing thoughtful, open-ended questions as students explore mathematical ideas.

POSING RICH MATH PROBLEMS Rich problems are those that require students to think beyond a quick response. These problems promote discussion, thinking, and perseverance (see Figure 2.3). They may be able to be solved in multiple ways and may even have multiple solutions. Consider the modifications to the problems in Figure 2.4. The modified problems require more thinking, can be solved in a variety of ways, and may lead to varied solutions. Ongoing experience with these types of problems, along with a supportive classroom environment, helps build students' perseverance as it shows that quick answers are often not possible and that a bit of a struggle is a natural part of math problem solving.

Figure 2.3 *Students benefit from opportunities to work with partners to discuss rich problems.*

Connections
to other practice standards

Rich math problems require students to construct viable arguments as they explain their responses (Standard 3).

Students model with mathematics as they explain and justify their solutions to open-ended questions or tasks (Standard 4).

Figure 2.4 *Creating problems with more complexity challenges students to apply their mathematical thinking.*

TRADITIONAL PROBLEM	RICH PROBLEM
What is 6 + 4?	Ten children went to the movie. How many were girls? How many were boys? Explain your answer. Could there be other answers?
Molly has a quarter, 2 nickels, and a dime. How much money does she have?	Molly has 6 coins in her piggy bank. She has more than 85¢, but less than $1.10. What coins could she have? Explain your answers.
Three children shared a pizza. They each had the same amount. What fraction did each child have?	Three children are sharing a pizza. How might they share it? What fraction of the pizza could each child get? Justify your answers.
A rectangle has a length of 4 units and a width of 3 units. What is the area?	Create rectangles with an area of 24 square units. How are they alike and different? Are their perimeters the same? Explain your observations.
How many hours of TV do you watch in a week?	How many hours (or years) of TV would you have watched if you live to be 100 years old? Be ready to justify your answer.
What is the surface area of a box that is 8 inches tall, 6 inches wide, and 12 inches long?	(Provide students with several boxes and a roll of gift wrap with dimensions on the label.) How many rolls of gift wrap will be needed to cover all of the boxes? Justify your answer.

Connections
to other practice standards

Deep questions require students to construct viable arguments as they explain their thinking (Standard 3).

Deep questions challenge students to look for patterns or generalizations (Standards 7 and 8).

Rich tasks require our students to think and act like mathematicians. See Appendix D for sample rich tasks at primary, intermediate, and middle grades levels. (The tasks are also available at www.heinemann.com/putting-the-practices-into-action). These tasks challenge our students to use multiple math practices.

REFINING OUR GUIDING QUESTIONS At all grade levels, teacher questioning is a critical component of problem-solving instruction. Mrs. King's first-grade class gathered data and created a picture graph about their pets. The graph included dogs, cats, birds, and fish. Mrs. King paired the students and posed the following problem:

How many of our pets have four legs?

1.OA.1

Note that, rather than asking students to find the number of cats and dogs, she asked for the number of four-legged animals. This simple change in questioning required her primary students to employ reasoning skills to determine the appropriate data.

Mrs. King reminded her students that they would be sharing their ideas and that they should be ready to show how they had figured out the solution. She encouraged them to solve the problem in a way that made sense to them. The students worked with their partners, discussing the numbers of legs on various animals, reasoning about which data to use, and deciding how to approach the problem. Some students immediately grabbed counters to represent the animals, others grabbed paper and a pencil to record ideas with either numbers or pictures. Some used addition and created a number sentence to represent the total number of cats and dogs. After the students had explored the problem, Mrs. King began a sharing circle. Partners shared their work and Mrs. King probed with questions to make their thinking visible. She purposefully solicited ideas from partners who had solved the problem in different ways, wanting to highlight their different approaches.

The expectation is for more than an answer. Students are asked to explain how they got their answer. This shifts the focus from the answer to the process.

Teacher: How did you know which animals to count?

Emma: They needed 4 legs.

Teacher: Kristen and John, why didn't you use the data about fish or birds?

Kristen: Birds have 2 legs.

John: Fish don't have any legs!

Teacher: But they were on the graph, shouldn't we use them?

Kristen: No! They don't have 4 legs.

Teacher: Oscar and Deryn, why did you use counters?

Oscar: They were the cats and dogs.

Deryn: We put one for each cat and dog.

Teacher: After you used counters to show the number of cats and dogs, how did you get your answer?

Oscar: We counted all of the counters.

Deryn: We had to count them all 'cause they all had 4 legs. We started with the 5 dogs and then we counted on for the cats: 6, 7, 8, 9. (as she demonstrated her counting by pointing to each counter)

Teacher: Molly and Blake added. Why did you add?

Blake: We added 'cause we put them together. Adding is counting all of them.

Molly: We joined them together. That's what you do when you add.

Teacher: Why did you write 4 + 5?

Molly: There were 4 cats and 5 dogs and they all have 4 legs, so we knew it was 4 + 5.

Aidan: You could count too if you wanted.

Teacher: Would you get the same answer?

Aidan: Yes!

Madison: We all got the same even if we drew it.

Jordan: Counting and adding give you the same. It's the same. (she counted the marks on her diagram to show 9)

The students showed multiple ways to solve the problem, from drawing pictures to using counters to creating number sentences. They proudly talked about their mathematical thinking and were interested to see the various ways their classmates found the total number of four-legged animals.

Regardless of grade level, providing opportunities for partner and group discussions during problem solving allows students to investigate strategies together. Asking students to share two ways to solve a problem can stretch their thinking. Orchestrating a class sharing time, in which students show and talk about how they solved problems, allows them to pause and reflect on how and why they did what they did.

Give It a Try!

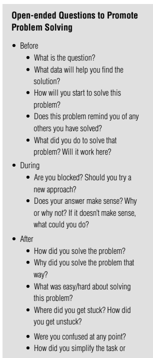
The questions in Figure 2.5 are particularly helpful as we focus on the development of problem-solving skills. Frequently asking our students to articulate, both orally and in writing, how they solved problems and why they chose their strategies is an

Figure 2.5 *These questions guide our students' thinking as they solve math problems.*

OPEN-ENDED QUESTIONS TO PROMOTE PROBLEM SOLVING

- Before
  - What is the question?
  - What data will help you find the solution?
  - How will you get started solving this problem?
  - Does this problem remind you of any others you have solved?
  - What did you do to solve that problem? Will it work here?
- During
  - Are you blocked? Should you try a new approach?
  - Does your answer make sense? Why or why not? If it doesn't make sense, what could you do?
- After
  - How did you solve the problem?
  - Why did you solve the problem that way?
  - What was easy/hard about solving this problem?
  - Where did you get stuck? How did you get unstuck?
  - Were you confused at any point? How did you simplify the task or clarify the problem?
  - Can you describe another way to solve the problem? Which way might be more efficient?
  - Is there another answer? Explain.

important part of problem-solving instruction. Our goal is for students to self-regulate their thinking, not needing as much support from us, so prompting should decrease as students become more adept at solving problems. See Appendix B for a convenient bookmark of these questions. Place it in your teacher's guide, on your desk, or by your daily lesson plans to guide you during your classroom questioning.



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  - Where did you get stuck? How did you get unstuck?
  - Were you confused at any point?
  - How did you simplify the task or

ADDITIONAL IDEAS FOR DEVELOPING THE PRACTICE

The chart in Figure 2.6 shares effective Student Practices related to Practice Standard 1 and offers suggestions for ways you might develop these Practices. This is not intended to be a checklist in which you attempt to complete each item on the list, but rather suggestions for teaching options that address essential Student Practices.

Figure 2.6 *This chart provides suggestions for ways a teacher might address specific problem-solving skills.*

Our students are better able to . . .	Because as teachers we . . .
Determine and articulate what the problem is asking.	Ask students to restate the problem in their own words. Have students turn to a partner to state the problem.
Find a starting point by understanding mathematical situations.	Use diagrams to model math situations. Frequently ask, "What should we do first?" or "How should we get started?"
Identify an appropriate way to solve the problem.	Discuss familiar problems (When have we seen something like this before? What did we do?). Discuss the efficiency of various strategies (Will it work? Why does the strategy make sense with this problem? Which strategy is more efficient?).
Connect problem situations to abstract representations of the problem (e.g., equations, visuals) in order to clarify the task.	Avoid simply circling key words to decide on the appropriate operation and instead focus on identifying the concepts or actions of the operations (i.e., Why should we add to solve this problem? What is the problem asking us to do? Why add, subtract, multiply, or divide?). Consistently discuss building appropriate equations to solve problems (What equation shows this situation?). Provide materials (e.g., manipulatives, paper/pencil) to allow students to visualize situations.
Self-monitor their progress and change directions when necessary; adjust strategies when having difficulty.	Think aloud to show students how we change course when needed during the problem-solving process. Have students talk or write about how they got stuck and then unstuck when solving a problem.
Demonstrate perseverance and make adjustments until a problem is solved.	Think aloud, acknowledging that everyone feels like giving up at times. Share ways we persevere or demonstrate patience when solving problems (e.g., "I'm getting frustrated. Let me try something else . . ."). Ask questions like, "Are you getting stuck? What else could you do?" Acknowledge those who modify their thinking and persevere to get to the solution rather than acknowledging the quickest to get to the answer.

Figure 2.6 *Continued*

Our students are better able to . . .	Because as teachers we . . .
Articulate the strategies they use to solve problems.	Provide opportunities for partner and group discussions while solving problems. Frequently ask students to articulate, both orally and in writing, how they solved problems and why they chose their strategy. Orchestrate a class sharing time so students can show and talk about how they solved problems.
Identify or understand different ways to solve a problem.	Make classroom lists of possible strategies. Share alternate strategies when discussing how a problem was solved. Encourage students to show two ways to solve a problem.
Articulate the reasonableness of strategies or solutions.	Frequently ask, "Does your answer make sense? Why?" Ask students to predict/estimate prior to computing the solution to a problem, and then compare their answer to their estimate to check for reasonableness. Ask students to go back to the question and restate the question followed by their answer. Does the answer work with the question?

ASSESSMENT TIP

PUT IT IN WRITING

Asking students to write about how they solve problems, why they choose a particular strategy, how they adjust their thinking during the problem-solving process, and what is easy or hard about the task all promote reflection and allow us to assess our students' problem-solving skills. The student in Figure 2.7 explains his approach to a problem, including the way he adjusted his approach after finding that his first strategy (folding the paper and simply counting the folds) was not going to work. By gathering some data and organizing it in a table, he was able to see and extend a pattern to find the solution.

Figure 2.7 *This student's writing shows his thinking as he solved the problem and highlights his perseverance as he adjusted his strategy when his first attempt was ineffective.*

Jim folded a piece of paper in half 8 times. How many sections were there when he unfolded it?

Tell how you found your answer.

there will be 256 sections. I started to fold paper but there were to many to count, so I made a table. I saw that the number of sections doubled each time

folds	1	2	3	4	5	6	7	8
Sections	2	4	8	16	32	64	128	256

Some key writing prompts include:

- How did you solve this problem?
- What strategy did you choose? Why?
- Was there another way to solve the problem? Explain.
- Did you get stuck at any point in this problem? How did you get unstuck?
- What was easy about solving this problem? What was hard?
- Did this problem remind you of any others you have solved? In what ways?

SUMMING IT UP

Teaching problem solving is more than assigning problems to our students. It is a balance of guided experiences in which we support the development of our students' thinking skills, as well as investigative experiences in which our students develop skills through trial-and-error experiences.

Did you recognize some of your own students in the vignettes? Did you see some of your own teaching practices in the suggestions throughout the chapter? Consider the following as you reflect on strengthening your students' problem-solving experiences:

Do I routinely provide opportunities for my students to share their solutions and processes with partners, groups, and the whole class?

Do I show my students that I value process (how they did it) rather than simply the correct answer?

Do I pose problems that require perseverance? Do I use thoughtful questions to guide and encourage students as they struggle with problems?

We have come a long way in our understanding of the teaching of problem solving. We want to recognize and celebrate our successes as we continue to refine our teaching.