

Accessible Mathematics

10 Instructional Shifts
That Raise Student Achievement



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*To the thousands of teachers of mathematics
who have welcomed me into their classrooms,
and
with love, to Ann, the best teacher I know.*

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Introduction

It's Instruction That Matters Most

Fortunately, we all agree that our goal should be to ensure that all students know, like, and are able to apply mathematics as a direct result of their school experiences. Unfortunately, we also agree that as a country, we are currently falling far short of this goal. It's a tough situation for which many solutions have been suggested. However, of all the programs, initiatives, and proposals for addressing this situation, I'm increasingly convinced that the answer is one word: *instruction*. That is, the web of plans, actions, and decisions that constitute what teachers actually do behind closed classroom doors makes all the difference.

There is no question that a coherent and rational curriculum guides and organizes an effective mathematics program. Life is a lot easier when the assessments we use, or are subjected to, are closely aligned with this curriculum and hold us accountable for important mathematics. It certainly helps to have sufficient access to print and non-print materials that support implementation of the program. And it's a relief when we have supportive and gutsy leadership and parental support. But none of these program components has anything like the degree of impact on student achievement as the *quality of instruction*. For many reasons, no component of the K–12 mathematics program seems to get as little attention as instruction—the day in and day out of complex interactions between teachers and students that determine who learns how much mathematics.

As I noted in the conclusion of *Sensible Mathematics: A Guide for School Leaders*,

Never stray from a *dogged focus on classroom instruction*. Just as Bill Clinton won in 1992 in part by making “It's the economy, stupid!” a campaign mantra, for educators and educational policy makers at all levels, the appropriate mantra must be “It's the classroom, stupid!” When all is said and done, it's not the buses, nor the buildings, nor the budgets that determine how much and how well students learn, it's the daily interaction between teachers and students, and among students,

in the classrooms of every school that determine how much and how well students will learn. . . . If one seeks to improve the quality of education and the quality of student achievement, enhancing, empowering, energizing, and engaging teaching and teachers has always been and will continue to be the method of choice.

This book begins where *Sensible Mathematics* ended. It focuses on mathematics instruction and is designed to give practical meaning to the challenges of “enhancing, empowering, energizing, and engaging” our teaching of mathematics.

I’ve been blessed by the opportunities that have been presented to me. For the past 30 years, as consultant, as evaluator, and as researcher, I’ve observed over 2000 mathematics classes from kindergarten to calculus. I’ve marveled at a fourth-grade teacher who artfully used the explanations of her students to summarize (and assess) the key understanding that emerged from a rotation through four fraction activity stations. I’ve been in awe of middle school teachers whose planning and classroom management routines kept students thoroughly engaged in mathematical thinking for 47 uninterrupted minutes. And I’ve been wowed by the diversity of approaches to solving a problem that is cultivated in classes that tangibly value multiple ways of thinking.

But I’ve also seen hundreds of missed opportunities and less than stellar lessons. I once watched a teacher direct the students to copy 15 word problems out of their textbook—a task that consumed all 42 minutes of class time. I’ve seen lessons where it was patently obvious that the teacher had not worked out the well-chosen problems the night before and had no idea what opportunities and stumbling blocks lay in the path to getting the right answer. And I’ve observed many lessons where so much time was essentially wasted going over the correct answers to homework problems that not enough time remained to address the day’s new content effectively.

In other words, I’ve been given a rich repository of “data” from which to extract and synthesize what appears to work and what appears to make a difference. We’ve all heard that “the most important variable in determining the quality of education is the teacher.” Of course that’s true. But the next (and far more important) message is that it is instruction—what teachers actually do to present mathematical ideas and to structure learning—that makes all the difference.

The perspective I take is that increasing calls for greater accountability and the pressures of *No Child Left Behind*, to say nothing of our

own personal professional expectations, require us to do something that, as a society, we've never before accomplished: taking this body of knowledge and skills called mathematics and presenting it in ways that result in all or nearly all students being successful. We know that this goal cannot be met by continuing to do what we have always done. We know that traditional and typical approaches rarely work for more than half of our students. We know we can't sprinkle in the same mathematics more gently and with bigger smiles, or pound it in harder and say it louder, and expect things to be different. We know that moving faster and mandating more and more mathematics earlier and earlier only exacerbate the problem for many. But most of all, we know that the only way we'll achieve our goal of broader and deeper mathematical literacy is by changing some of what we expect students to learn and a lot of how we go about teaching it. In other words, the goal of more math for more students is inextricably linked to teaching the math in distinctly different ways from how we were taught.

The good news is that we've got clear answers for how to do this. There are ways of shifting instructional practice that enhance our productivity and our effectiveness. This book focuses on exactly those shifts and strategies that make a difference and guides the provision of exactly the high-quality mathematics instruction that can and does work for far more students.

We've Got Most of the Answers

1

Walk into a kindergarten or first-grade classroom. You're likely to see kids actively engaged, often in play, learning by doing, and sponging up knowledge. They learn about shapes by seeing them all around their classroom. They name and refer to these shapes naturally and in context—not just in Chapter 7. They use materials to join and separate sets in ways that concretize numbers and operations. It's no surprise that, mathematically speaking, kindergarten and first-grade students tend to be a whole lot more successful than students in any other grades.

Look at typical reading instruction. In a large proportion of reading lessons, literal comprehension is nearly always extended to inferential and evaluative comprehension via a systematic progression of questions. As we'll explore in Chapter 3, it's not a surprise that for many students, reading is a far more positive, and more successful, experience than mathematics. Then wander into the gifted classroom and observe the questioning, the focus on reasoning, the frequent use of "Why?" and the attention paid to alternative approaches. It doesn't take much imagination to recognize the chasm between standard operating procedure in most gifted classes and the worksheet-driven, procedure-oriented drudgery that typically passes for mathematics

instruction in the remedial class down the hall. In fact, stroll down the corridor of nearly any school, and you have an immediate sense of which classrooms you would hope your own children would be in. These are the active classes, the questioning classes, the thinking and reasoning classes where students are mentally, and even physically, engaged in the tasks at hand. You can feel the minds-on behaviors and sense the learning going on. And when we look deeper at what explains this engagement and this enhanced learning, whether in first grade or in a reading lesson or a gifted class, we can identify a clear pattern of instructional practices and behaviors that account for this success.

In other words, we know that there are places throughout the educational system where things tend to work consistently better than they do for mathematics. If this is the case, it becomes our job to identify and extract the specific strategies that are used in these successful places and experiences and to ensure that they are employed broadly and regularly to raise achievement, to improve instruction, and to make a difference in mathematics instruction for kids.

None of this is news. When instruction is traditional, rule-oriented teaching by telling and is devoid of number sense, we shouldn't be surprised when students announce that the 5% tax on a \$12 shirt is \$60. Alan Schoenfeld describes relating to students that a shepherd was guarding his flock of 18 sheep when all of a sudden 4 wolves came over the mountain. He then asks them, "How old is the shepherd?" This tends to evoke such appropriate answers as "Who knows?" and "That's silly!" from the majority of first graders and the depressing answer "22" from the majority of sixth graders. We do not need any more compelling evidence that something is seriously amiss. It is clear that for many students, thinking, reasoning, and sense-making are systematically exorcized from teaching because the primary focus in too many classrooms is a mindless emphasis on the one right way to get the one right answer to yet another decontextualized exercise.

Indeed, there are several ways to teach mathematics. We can demonstrate, tell, and let our students practice, or we can engage and focus on understanding and application. Nowhere is the traditional approach more succinctly presented than in *Everybody Counts*, the 1989 epic "Report to the Nation on the Future of Mathematics Education" from the National Research Council, where we are reminded that

evidence from many sources shows that the least effective mode for mathematics learning is the one that prevails in most of America's classrooms: lecturing and listening. Despite daily homework, for most students and most teachers mathematics continues to be primarily a passive activity: teachers prescribe; students transcribe. Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for high-order thinking, and for versatile problem-solving. (57)

Nowhere are the differences in how we might teach mathematics clearer than in the findings of the TIMSS video study. In the original work, Jim Stigler and his colleagues videotaped one hundred randomly chosen eighth-grade classes in the United States, Germany, and Japan. The analysis of these videos revealed two very different cultural scripts for what typically takes place in a mathematics classroom. Not surprisingly, in the United States and Germany, the focus was on skill acquisition and the typical class proceeded as follows:

- The teacher instructs students in a concept or skill.
- The teacher solves example problems with the class.
- The students practice on their own while the teacher assists individual students.

Interestingly, in Japan, one finds a very different cultural script—not necessarily a better script, but certainly different. In this case, the focus was on understanding and the typical class unfolded this way:

- The teacher poses a complex, thought-provoking problem.
- Students struggle with the problem.
- Various students present ideas or solutions to the class.
- The class discusses the various solution methods.
- The teacher summarizes the class' conclusions.
- The students practice similar problems.

If the traditional American mathematics classroom script is working for only about half of our students, one answer is to vary our approach. We are quick to acknowledge that our students come to us with incredibly diverse backgrounds, interests, and learning styles, but

so often they are met with one monolithic approach best characterized as lecture and practice. It seems sensible that one way to address the diversity of our students is with greater diversity in our instructional practices. This diversity of practice implies variation in our approaches, alternative approaches to solving problems and thinking through the mathematics, and a richness of questioning that stimulates these alternatives and this thinking.

The sad reality is that our critics are not all wrong. For too long and for too many students the typical outcomes of school mathematics are mountains of math anxiety, tons of mathematical illiteracy, mediocre test scores, gobs of wasteful remediation, and a slew of entirely warranted criticism. This is not a pretty picture and is very hard to defensively dismiss. But the good news is that we know more than enough to avoid these outcomes. The better news is that we have sensible and accessible solutions. And the best news is that overcoming the obstacles to implementing these solutions does not require more money or expensive interventions, or numerous days of professional development, or more courses in mathematics content. The primary obstacle is a lack of direction and a lack of will to make a few powerful instructional shifts in how we plan, implement, and assess daily mathematics instruction. In short, the answers lie in narrowing the chasm between what we know and what we do!

What exactly are these shifts? I've identified ten that capture much of what I have observed and that appear to work.

1. Incorporate ongoing cumulative review into every day's lesson.
2. Adapt what we know works in our reading programs and apply it to mathematics instruction.
3. Use multiple representations of mathematical entities.
4. Create language-rich classroom routines.
5. Take every available opportunity to support the development of number sense.
6. Build from graphs, charts, and tables.

7. Tie the math to such questions as How big? How much? How far? to increase the natural use of measurement throughout the curriculum.
8. Minimize what is no longer important, and teach what is important when it is appropriate to do so.
9. Embed the mathematics in realistic problems and real-world contexts.
10. Make “Why?” “How do you know?” “Can you explain?” classroom mantras.

Shifts 1, 2, and 10 focus broadly on review, discourse, and stimulating a depth of understanding. Shifts 3, 4, and 5 address the use of representations, communication, and number sense—three critical domains for accessing and demonstrating mathematical understanding. Shifts 6, 7, and 9 ground the mathematics in the world in which we live and build understanding from data and measurement that apply key mathematical ideas. Finally, shift 8 reminds us that we can’t do it all. Taken together, these ten strategies or shifts in how we plan, implement, and assess our mathematics instruction represent a road map for significantly enhancing the quality of our teaching and its impact on student achievement.