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Cumulative Review Chapters 1–8
Sequences and Series

BUILDING ON
■ graphing linear functions
■ properties of linear functions
■ expressing powers using exponents
■ solving equations

BIG IDEAS
■ An arithmetic sequence is related to a linear function and is created by repeatedly adding a constant to an initial number. An arithmetic series is the sum of the terms of an arithmetic sequence.
■ A geometric sequence is created by repeatedly multiplying an initial number by a constant. A geometric series is the sum of the terms of a geometric sequence.
■ Any finite series has a sum, but an infinite geometric series may or may not have a sum.

LEADING TO
■ applying the properties of geometric sequences and series to functions that illustrate growth and decay

NEW VOCABULARY

<table>
<thead>
<tr>
<th>Arithmetic sequence</th>
<th>Common ratio</th>
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<tr>
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1.1 Arithmetic Sequences

FOCUS Relate linear functions and arithmetic sequences, then solve problems related to arithmetic sequences.

Get Started

When the numbers on these plates are arranged in order, the differences between each number and the previous number are the same.

What are the missing numbers?

Construct Understanding

Saket took guitar lessons.
The first lesson cost $75 and included the guitar rental for the period of the lessons.
The total cost for 10 lessons was $300.
Suppose the lessons continued.
What would be the total cost of 15 lessons?

Find the cost of one extra lesson.
Cost of 9 lessons/total cost of 10 lessons/total cost of 1st lesson
$300/$75/
$225
So, the cost of 1 lesson is:
The cost of 15 lessons is: 1 lesson @ $75/14 lessons @ $25
So, 15 lessons cost: $75/14($25)/$425
In an arithmetic sequence, the difference between consecutive terms is constant. This constant value is called the common difference.

This is an arithmetic sequence:
4, 7, 10, 13, 16, 19, . . .
The first term of this sequence is: \( t_1 = 4 \)
The second term is: \( t_2 = 7 \)

Let \( d \) represent the common difference. For the sequence above:

\[
\begin{align*}
    d &= t_2 - t_1 &= 7 - 4 &= 3 \\
    &= t_3 - t_2 &= 10 - 7 &= 3 \\
    &= t_4 - t_3 &= 13 - 10 &= 3 \\
    &= t_5 - t_4 &= 16 - 13 &= 3 \\
    &= t_6 - t_5 &= 19 - 16 &= 3 \\
    &\vdots & & \vdots \\
    &= t_{n} - t_{n-1} & & \vdots
\end{align*}
\]

The dots indicate that the sequence continues forever; it is an infinite arithmetic sequence.
To graph this arithmetic sequence, plot the term value, \( t_n \), against the term number, \( n \).

![Graph of an Arithmetic Sequence]

The graph represents a linear function because the points lie on a straight line. A line through the points on the graph has slope 3, which is the common difference of the sequence.

In an arithmetic sequence, the common difference can be any real number.

Here are some other examples of arithmetic sequences.

- This is an increasing arithmetic sequence because \( d \) is positive and the terms are increasing:
  \[ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots; \text{ with } d = \frac{1}{4} \]

- This is a decreasing arithmetic sequence because \( d \) is negative and the terms are decreasing:
  \[ 5, -1, -7, -13, -19, \ldots; \text{ with } d = -6 \]
Example 1  Writing an Arithmetic Sequence

Write the first 5 terms of:

a) an increasing arithmetic sequence
b) a decreasing arithmetic sequence

**SOLUTION**

**a)** Choose any number as the first term; for example, \( t_1 = -7 \).
The sequence is to increase, so choose a positive common difference; for example, \( d = 2 \). Keep adding the common difference until there are 5 terms.

\[
\begin{align*}
&\ldots -1, 1, \ldots \\
&t_1, \quad t_2 = t_1 + d, \quad t_3 = t_2 + d, \quad t_4 = t_3 + d, \quad t_5 = t_4 + d \\
&-7, \quad -5, \quad -3, \quad -1, \quad 1, \ldots 
\end{align*}
\]

The arithmetic sequence is: \(-7, -5, -3, -1, 1, \ldots\)

**b)** Choose the first term; for example, \( t_1 = 5 \).
The sequence is to decrease, so choose a negative common difference; for example, \( d = -3 \).

\[
\begin{align*}
&\ldots +7, +4, +1, \ldots \\
&t_1, \quad t_2 = t_1 + d, \quad t_3 = t_2 + d, \quad t_4 = t_3 + d, \quad t_5 = t_4 + d \\
&5, \quad 2, \quad -1, \quad -4, \quad -7, \ldots 
\end{align*}
\]

The arithmetic sequence is: \(5, 2, -1, -4, -7, \ldots\)

Consider this arithmetic sequence: \(3, 7, 11, 15, 19, 23, \ldots\)

To determine an expression for the general term, \( t_n \), use the pattern in the terms. The common difference is 4. The first term is 3.

\[
\begin{align*}
&t_1 = 3 = 3 + 4(0) \\
&t_2 = 7 = 3 + 4(1) \\
&t_3 = 11 = 3 + 4(2) \\
&t_4 = 15 = 3 + 4(3) \\
&\vdots \\
&t_n = 3 + 4(n - 1)
\end{align*}
\]

For each term, the second factor in the product is 1 less than the term number.

The second factor in the product is 1 less than \( n \), or \( n - 1 \).

**Check Your Understanding**

1. Write the first 6 terms of:
   a) an increasing arithmetic sequence
   b) a decreasing arithmetic sequence

**SOLUTION**

**a)** Sample response: Choose \( t_1 = 20 \) and \( d = 2 \).
The sequence is:

\[
20, 20, 20, 20, 20, 20, \ldots
\]

Simplify. An arithmetic sequence is:

\(-20, -18, -16, -14, -12, -10, \ldots\)

**b)** Sample response: Choose \( t_1 = 100 \) and \( d = -3 \).
The sequence is:

\[
100, 97, 94, 91, 88, 85, \ldots
\]

Simplify. An arithmetic sequence is:

\[
5, 2, -1, -4, -7, \ldots
\]
The General Term of an Arithmetic Sequence

An arithmetic sequence with first term, \( t_1 \), and common difference, \( d \), is:

\[ t_n = t_1 + d(n - 1) \]

Example 2
Calculating Terms in a Given Arithmetic Sequence

For this arithmetic sequence: \(-3, 2, 7, 12, \ldots\)

a) Determine \( t_{20} \)

b) Which term in the sequence has the value 212?

SOLUTION

\(-3, 2, 7, 12, \ldots\)

a) Calculate the common difference: \( 2 - (-3) = 5 \)

Use: \( t_n = t_1 + d(n - 1) \)

Substitute: \( n = 20, t_1 = -3, d = 5 \)

\[ t_{20} = -3 + 5(20 - 1) \]

Use the order of operations.

\[ t_{20} = -3 + 5(19) \]

\[ t_{20} = 92 \]

b) Use: \( t_n = t_1 + d(n - 1) \)

Substitute: \( t_n = 212, t_1 = -3, d = 5 \)

\[ 212 = -3 + 5(n - 1) \]

Solve for \( n \).

\[ 212 = -3 + 5n - 5 \]

\[ 220 = 5n \]

\[ \frac{220}{5} = n \]

\[ n = 44 \]

The term with value 212 is \( t_{44} \).

THINK FURTHER

In Example 2, how could you show that 246 is not a term of the sequence?
Example 3  Calculating a Term in an Arithmetic Sequence, Given Two Terms

Two terms in an arithmetic sequence are \( t_3 = 4 \) and \( t_8 = 34 \). What is \( t_i \)?

**SOLUTION**

\( t_3 = 4 \) and \( t_8 = 34 \)

Sketch a diagram. Let the common difference be \( d \).

\[
\begin{align*}
&+d \quad +d \quad +d \quad +d \\
&\text{\( t_1 \)} \quad \text{\( t_2 \)} \quad \text{\( 4 \)} \quad \text{\( t_4 \)} \quad \text{\( t_5 \)} \quad \text{\( t_6 \)} \quad \text{\( t_7 \)} \quad \text{\( t_8 \)} \quad \text{\( 34 \)}
\end{align*}
\]

From the diagram,

\[
t_8 = t_3 + 5d
\]

Substitute: \( t_8 = 34 \), \( t_3 = 4 \)

\[
34 = 4 + 5d
\]

Solve for \( d \).

\[
d = 6
\]

Then, \( t_1 = t_3 - 2d \)

Substitute: \( t_3 = 4 \), \( d = 6 \)

\[
t_1 = 4 - 2(6)
\]

\[
t_1 = 4 - 12
\]

\[
t_1 = -8
\]

---

Example 4  Using an Arithmetic Sequence to Model and Solve a Problem

Some comets are called periodic comets because they appear regularly in our solar system. The comet Kojima appears about every 7 years and was last seen in the year 2007. Halley’s comet appears about every 76 years and was last seen in 1986. Determine whether both comets should appear in 3043.

**SOLUTION**

The years in which each comet appears form an arithmetic sequence. The arithmetic sequence for Kojima has \( t_1 = 2007 \) and \( d = 7 \).

To determine whether Kojima should appear in 3043, determine whether 3043 is a term of its sequence.

\[
t_n = t_1 + d(n - 1)
\]

Substitute: \( t_n = 3043 \), \( t_1 = 2007 \), \( d = 7 \)

\[
3043 = 2007 + 7(n - 1)
\]

Solve for \( n \).

\[
3043 = 2000 + 7n
\]

\[
1043 = 7n
\]

\[
149 = n
\]
Since the year 3043 is the 149th term in the sequence, Kojima should appear in 3043.

The arithmetic sequence for Halley's comet has \( t_1 = 1986 \) and \( d = 76 \). To determine whether Halley's comet should appear in 3043, determine whether 3043 is a term of its sequence.

\[
t_n = t_1 + d(n - 1)
\]
Substitute: \( t_n = 3043, t_1 = 1986, d = 76 \)
\[
3043 = 1986 + 76(n - 1)
\]
Solve for \( n \).

\[
3043 = 1986 + 76n
\]
\[
1133 = 76n
\]
\[
n = 14.9078...\]

Since \( n \) is not a natural number, the year 3043 is not a term in the arithmetic sequence for Halley's comet; so the comet will not appear in that year.

---

**Discuss the Ideas**

1. How can you tell whether a sequence is an arithmetic sequence? What do you need to know to be certain?

2. The definition of an arithmetic sequence relates any term after the first term to the preceding term. Why is it useful to have a rule for determining any term?

3. Suppose you know a term of an arithmetic sequence. What information do you need to determine any other term?
Exercises

A

4. Circle each sequence that could be arithmetic. Determine its common difference, \(d\).
   a) 6, 10, 14, 18, . . .  
   b) 9, 7, 5, 3, . . .
   c) \(-11, -4, 3, 10, . . .\)  
   d) \(2, -4, 8, -16, . . .\)

5. Each sequence is arithmetic. Determine each common difference, \(d\), then list the next 3 terms.
   a) 12, 15, 18, . . .  
   b) 25, 21, 17, . . .

6. Determine the indicated term of each arithmetic sequence.
   a) 6, 11, 16, . . .; \(t_7\)  
   b) 2, \(\frac{1}{2}, 1, . . .; t_{15}\)

7. Write the first 4 terms of each arithmetic sequence, given the first term and the common difference.
   a) \(t_1 = -3, d = 4\)  
   b) \(t_1 = -0.5, d = -1.5\)
8. When you know the first term and the common difference of an arithmetic sequence, how can you tell if it is increasing or decreasing? Use examples to explain.

9. a) Create your own arithmetic sequence. Write the first 7 terms. Explain your method.

b) Use technology or grid paper to graph the sequence in part a. Plot the Term value on the vertical axis and the Term number on the horizontal axis. Print the graph or sketch it on this grid.

i) How do you know that you have graphed a linear function?

ii) What does the slope of the line through the points represent? Explain why.
10. Two terms of an arithmetic sequence are given. Determine the indicated terms.
   
a) \( t_4 = 24, \ t_{10} = 66; \) determine \( t_1 \)

b) \( t_3 = 81, \ t_{12} = 27; \) determine \( t_{23} \)

11. Create an arithmetic sequence for each description below. For each sequence, write the first 6 terms and a rule for \( t_n \).
   
a) an increasing sequence  
   b) a decreasing sequence  
   c) every term is negative  
   d) every term is an even number

12. Claire wrote the first 3 terms of an arithmetic sequence: 3, 6, 9, . . .
    When she asked Alex to extend the sequence to the first 10 terms, he wrote:
    3, 6, 9, 3, 6, 9, 3, 6, 9, 3, . . .
    
a) Is Alex correct? Explain.

b) What fact did Alex ignore when he extended the sequence?

c) What is the correct sequence?
13. Determine whether 100 is a term of an arithmetic sequence with \( t_3 = 250 \) and \( t_6 = 245.5 \).

14. The Chinese zodiac associates years with animals. Ling was born in 1994, the Year of the Dog.
   a) The Year of the Dog repeats every 12 years. List the first three years that Ling will celebrate her birthday in the Year of the Dog.
   b) Why do the years in part a form an arithmetic sequence?
   c) In 2099, Nunavut will celebrate its 100th birthday. Will that year also be the Year of the Dog? Explain.

15. In this arithmetic sequence: 3, 8, 13, 18, \ldots; which term has the value 123?

16. For two different arithmetic sequences, \( t_5 = -1 \). What are two possible sequences? Explain your reasoning.
17. A sequence is created by adding each term of an arithmetic sequence to the preceding term.
   a) Show that the new sequence is arithmetic.

   b) How are the common differences of the two sequences related?

18. In this arithmetic sequence, $k$ is a natural number: $k, \frac{2k}{3}, \frac{k}{3}, 0, \ldots$
   a) Determine $t_n$.

   b) Write an expression for $t_n$.

   c) Suppose $t_{20} = -16$; determine the value of $k$. 
Multiple-Choice Questions

1. How many of these sequences have a common difference of $-4$?
   $-19, -15, -11, -7, -3, \ldots$
   $19, 15, 11, 7, 3, \ldots$
   $3, 7, 11, 15, 19, \ldots$
   $-3, 7, -11, 15, -19, \ldots$
   A. 0  B. 1  C. 2  D. 3

2. Which number below is a term of this arithmetic sequence?
   $97, 91, 85, 79, 73, \ldots$
   A. $-74$  B. $-75$  C. $-76$  D. $-77$

3. The first 6 terms of an arithmetic sequence are plotted on a grid. The coordinates of two points on the graph are $(3, 11)$ and $(6, 23)$. What is an expression for the general term of the sequence?
   A. $6n - 3$  B. $3n + 11$  C. $4n - 1$  D. $1 + 4n$

Study Note

How are arithmetic sequences and linear functions related?

ANSWERS

4. a) 4  b) $-2$  c) 7  5. a) 3; 21, 24, 27  b) $-4$; 13, 9, 5  6. a) 36  b) $-15$
   7. a) $-3, 1, 5, 9$  b) $-0.5, -2, -3.5, -5$  10. a) 3  b) $-39$
   12. a) no  c) 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, $\ldots$  13. 100 is a term.
   14. a) 2006, 2018, 2030  c) no  15. $r_3$  18. a) $-\frac{2k}{3}$  b) $\frac{4k}{3} - \frac{kn}{3}$  c) 3

Multiple Choice
Chapter 1: Sequences and Series

1.2 Arithmetic Series

**FOCUS** Derive a rule to determine the sum of $n$ terms of an arithmetic series, then solve related problems.

**Get Started**
Suppose this sequence continues.
What is the value of the 8th term?

What is an expression for the $n$th term?

Is 50 a term in this sequence? How do you know?

**Construct Understanding**
Talise displayed 90 photos of the Regina Dragon Boat Festival in 5 rows. The difference between the numbers of photos in consecutive rows was constant.
How many different sequences are possible? Justify your answer.
A series is a sum of the terms in a sequence.

An arithmetic series is the sum of the terms in an arithmetic sequence.

For example, an arithmetic sequence is: 5, 8, 11, 14, . . .
The related arithmetic series is: 5 + 8 + 11 + 14 + . . .
The term, $S_n$, is used to represent the sum of the first $n$ terms of a series.
The $n$th term of an arithmetic series is the $n$th term of the related arithmetic sequence.

For the arithmetic series above:

\[
\begin{align*}
S_1 &= t_1 \\
S_2 &= t_1 + t_2 \\
S_3 &= t_1 + t_2 + t_3 \\
S_4 &= t_1 + t_2 + t_3 + t_4 \\
S_1 &= 5 \\
S_2 &= 5 + 8 \\
S_3 &= 5 + 8 + 11 \\
S_4 &= 5 + 8 + 11 + 14 \\
S_2 &= 13 \\
S_3 &= 24 \\
S_4 &= 38
\end{align*}
\]

These are called partial sums.

If there are only a few terms, $S_n$ can be determined using mental math.
To develop a rule to determine $S_n$, use algebra.
Write the sum on one line, reverse the order of the terms on the next line, then add vertically. Write the sum as a product.
The Sum of \( n \) Terms of an Arithmetic Series

For an arithmetic series with 1st term, \( t_1 \), common difference, \( d \), and \( n \)th term, \( t_n \), the sum of the first \( n \) terms, \( S_n \), is:

\[
S_n = \frac{n(t_1 + t_n)}{2} \quad \text{or} \quad S_n = \frac{n[2t_1 + d(n - 1)]}{2}
\]

Example 1  Determining the Sum, Given the Series

Determine the sum of the first 6 terms of this arithmetic series:

\(-75 \; -69 \; -63 \; -57 \; -51 \; -45 \; \ldots\)

**SOLUTION**

\( t_1 \) is \(-75 \) and \( t_6 \) is \(-45 \).

\[
S_6 = \frac{n(t_1 + t_n)}{2} \quad \text{or} \quad S_6 = \frac{n[2t_1 + d(n - 1)]}{2}
\]

Substitute: \( n = 6, \; t_1 = -75, \; t_n = -45 \)

\[
S_6 = \frac{6(-75 - 45)}{2} \quad \text{or} \quad S_6 = \frac{6[2(-75) + d(6 - 1)]}{2}
\]

\[
S_6 = -360
\]

The sum of the first 6 terms is \(-360\).
**Example 2** Determining the Sum, Given the First Term and Common Difference

An arithmetic series has \( t_1 = 5.5 \) and \( d = -2.5 \); determine \( S_{40} \).

**SOLUTION**

Use: \( S_n = \frac{n[2t_1 + d(n - 1)]}{2} \)

Substitute: \( n = 40, t_1 = 5.5, d = -2.5 \)

\[
S_{40} = \frac{40[2(5.5) - 2.5(40 - 1)]}{2} = -1730
\]

**Check Your Understanding**

2. An arithmetic series has \( t_1 = 3 \) and \( d = -4 \); determine \( S_{15} \).

---

**Example 3** Determining the First Few Terms Given the Sum, Common Difference, and One Term

An arithmetic series has \( S_{20} = 143 \frac{1}{3}, \) \( d = \frac{1}{3} \), and \( t_{20} = 10 \frac{1}{3} \); determine the first 3 terms of the series.

**SOLUTION**

\( S_{20} \) and \( t_{20} \) are known, so use this rule to determine \( t_i \):

\[
S_n = \frac{n(t_1 + t_n)}{2}
\]

Substitute: \( n = 20, S_{20} = 143 \frac{1}{3}, t_{20} = 10 \frac{1}{3} \)

\[
143 \frac{1}{3} = \frac{2(20t_1 + 10 \frac{1}{3})}{2}
\]

Simplify.

\[
143 \frac{1}{3} = 10t_1 + 10 \frac{1}{3}
\]

\[
143 \frac{1}{3} = 10t_1 + 103 \frac{1}{3}
\]

Solve for \( t_i \).

\[
40 = 10t_1
\]

\[
4 = t_1
\]

The first term is 4 and the common difference is \( \frac{1}{3} \).

So, the first 3 terms of the series are written as the partial sum:

\[
4 + 4 \frac{1}{3} + 4 \frac{2}{3}
\]

**THINK FURTHER**

In Example 3, which partial sums are natural numbers? Why?

**Check Your Understanding**

Answers:

2. \(-1125\)

3. \(1 + 1.75 + 2.5\)
Example 4  Using an Arithmetic Series to Model and Solve a Problem

Students created a trapezoid from the cans they had collected for the food bank. There were 10 rows in the trapezoid. The bottom row had 100 cans. Each consecutive row had 5 fewer cans than the previous row. How many cans were in the trapezoid?

**SOLUTION**

The numbers of cans in the rows form an arithmetic sequence with first 3 terms 100, 95, 90, \ldots

The total number of cans is the sum of the first 10 terms of the arithmetic series:

\[ 100 + 95 + 90 + \ldots \]

Use: Substitute: \( n, t_1 = 100, d = -5 \)

\[
S_{10} = \frac{n[2t_1 + d(n - 1)]}{2}
\]

\[
S_{10} = \frac{10[2(100) - 5(10 - 1)]}{2}
\]

\[
S_{10} = 775
\]

There were 775 cans in the trapezoid.

**Check Your Understanding**

4. The bottom row in a trapezoid had 49 cans. Each consecutive row had 4 fewer cans than the previous row. There were 11 rows in the trapezoid. How many cans were in the trapezoid?

**Answer:**

4. 319 cans

1. How are an arithmetic series and an arithmetic sequence related?

2. Suppose you know the 1st and \( n \)th terms of an arithmetic series. What other information do you need to determine the value of \( n \)?
Exercises

A

3. Use each arithmetic sequence to write the first 4 terms of an arithmetic series.
   a) $2, 4, 6, 8, \ldots$
   b) $-2, 3, 8, 13, \ldots$
   c) $4, 0, -4, -8, \ldots$
   d) $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \ldots$

4. Determine the sum of the given terms of each arithmetic series.
   a) $12 + 10 + 8 + 6 + 4$
   b) $-2 - 4 - 6 - 8 - 10$

5. Determine the sum of the first 20 terms of each arithmetic series.
   a) $3 + 7 + 11 + 15 + \ldots$
   b) $-21 - 15.5 - 10 - 4.5 - \ldots$

B

6. For each arithmetic series, determine the indicated value.
   a) $-4 - 11 - 18 - 25 - \ldots$; determine $S_{20}$

   b) $1 + 3.5 + 6 + 8.5 + \ldots$; determine $S_{42}$
7. Use the given data about each arithmetic series to determine the indicated value.
   a) $S_{20} = -850$ and $t_{20} = -90$; determine $t_1$

   b) $S_{15} = 322.5$ and $t_1 = 4$; determine $d$

   c) $S_n = -126$, $t_1 = -1$, and $t_n = -20$; determine $n$

   d) $t_1 = 1.5$ and $t_{20} = 58.5$; determine $S_{15}$

8. Two hundred seventy-six students went to a powwow. The first bus had 24 students. The numbers of students on the buses formed an arithmetic sequence. What additional information do you need to determine the number of buses? Explain your reasoning.
9. Ryan’s grandparents loaned him the money to purchase a BMX bike. He agreed to repay $25 at the end of the first month, $30 at the end of the second month, $35 at the end of the third month, and so on. Ryan repaid the loan in 12 months. How much did the bike cost? How do you know your answer is correct?

10. Determine the sum of the indicated terms of each arithmetic series.
   a) $31 + 35 + 39 + \ldots + 107$
   b) $-13 - 10 - 7 - \ldots + 62$

11. a) Explain how this series would be arithmetic.
    \[ 1 + 3 + \ldots \]

   b) What information do you need to be certain that this is an arithmetic series?

12. An arithmetic series has $S_{10} = 100$, $t_1 = 1$, and $d = 2$. How can you use this information to determine $S_{11}$ without using a rule for the sum of an arithmetic series? What is $S_{11}$?
13. The side lengths of a quadrilateral form an arithmetic sequence. The perimeter is 74 cm. The longest side is 29 cm. What are the other side lengths?

14. Derive a rule for the sum of the first \( n \) natural numbers:
\[ 1 + 2 + 3 + \ldots + n \]

15. The sum of the first 5 terms of an arithmetic series is 170. The sum of the first 6 terms is 225. The common difference is 7. Determine the first 4 terms of the series.

16. The sum of the first \( n \) terms of an arithmetic series is: \( S_n = 3n^2 - 8n \)
Determine the first 4 terms of the series.
17. Each number from 1 to 60 is written on one of 60 index cards. The cards are arranged in rows with equal lengths, and no cards are left over. The sum of the numbers in each row is 305. How many rows are there?

18. Determine the arithmetic series that has these partial sums: \( S_1 = 26 \), \( S_5 = 40 \), and \( S_6 = 57 \)

Multiple-Choice Questions

1. Which of these series is arithmetic?
   - A. \( 2.5 + 5 + 7.5 + 11 + \ldots \)
   - B. \( -2.5 - 5 - 7.5 - 11 - \ldots \)
   - C. \( 3.5 + 6 + 8.5 + 11 + \ldots \)
   - D. \( 3.5 - 6 - 8.5 - 11 + \ldots \)

2. For which series below is 115 the sum of 10 terms?
   - A. \( 34 + 29 + 25 + 20 + 16 + \ldots \)
   - B. \( -11 - 6 - 1 + 4 + 9 + \ldots \)
   - C. \( 11 + 6 + 1 - 4 - 9 - \ldots \)
   - D. \( 34 - 29 - 25 - 20 - 16 - \ldots \)
3. How many of these expressions could be used to determine the sum to \( n \) terms of an arithmetic series?

\[
\begin{align*}
\frac{n[2t_1 + d(n + 1)]}{2} & \quad \frac{n[2t_1 - d(n + 1)]}{2} & \quad \frac{n(t_1 + t_n)}{2} & \quad \frac{n(t_1 - t_n)}{2}
\end{align*}
\]

A. 4 \quad B. 3 \quad C. 2 \quad D. 1

Study Note

There are two forms of the rule to determine the sum of \( n \) terms of an arithmetic series. When would you use each form of the rule?

ANSWERS

3. a) \(2 + 4 + 6 + 8\) \quad b) \(−2 + 3 + 8 + 13\) \quad c) \(4 + 0 - 4 - 8\) 

\(\frac{1}{2} + \frac{1}{4} + 0 - \frac{1}{4}\) \quad 4. a) 40 \quad b) −30 \quad 5. a) 820 \quad b) 625 \quad 6. a) −2758 

b) 2194.5 \quad 7. a) 5 \quad b) 2.5 \quad c) 12 \quad d) 337.5 \quad 9. \(5630\) \quad 10. a) 1380 \quad b) 637 

12. 121 \quad 13. 8 cm, 15 cm, and 22 cm \quad 14. \(S_n = \frac{n(n + 1)}{2}\) \quad 15. 20 + 27 + 34 + 41 

16. \(−5 + 1 + 7 + 13\) \quad 17. 6 \quad 18. \(2 + 5 + 8 + 11 + 14 + 17 + \ldots\)

Multiple Choice

1. C \quad 2. B \quad 3. D
# CHECKPOINT 1

## Self-Assess

<table>
<thead>
<tr>
<th>Can you . . .</th>
<th>To check, try question . . .</th>
<th>For review, see . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>write an example of an arithmetic sequence and explain how you know it is arithmetic?</td>
<td>2</td>
<td>Page 4 in Lesson 1.1 (Example 1)</td>
</tr>
<tr>
<td>explain the meaning of the symbols ( n, t_1, t_n ), and ( d )?</td>
<td></td>
<td>Page 3 in Lesson 1.1</td>
</tr>
<tr>
<td>identify the assumptions made when defining an arithmetic sequence or series?</td>
<td></td>
<td>Page 7 in Lesson 1.1</td>
</tr>
<tr>
<td>use a rule to determine the ( n )th term in an arithmetic sequence?</td>
<td>3</td>
<td>Page 4 in Lesson 1.1</td>
</tr>
<tr>
<td>describe the relationship between an arithmetic sequence and a linear function?</td>
<td></td>
<td>Page 3 in Lesson 1.1</td>
</tr>
<tr>
<td>use a rule to determine ( t_n ) and ( n ) in an arithmetic sequence given the values of ( t_1 ) and ( d )?</td>
<td>3b, 4b</td>
<td>Page 5 in Lesson 1.1 (Example 2)</td>
</tr>
<tr>
<td>use a rule to determine ( t_n ) and ( d ) in an arithmetic sequence given the values of ( t_n ) and ( n )?</td>
<td>4a</td>
<td>Page 6 in Lesson 1.1 (Example 3)</td>
</tr>
<tr>
<td>solve problems involving arithmetic sequences?</td>
<td>5</td>
<td>Pages 6-7 in Lesson 1.1 (Example 4)</td>
</tr>
<tr>
<td>use a rule to determine the sum ( S_n ) of an arithmetic sequence?</td>
<td>7b, 8</td>
<td>Page 14 in Lesson 1.2</td>
</tr>
<tr>
<td>use a rule to determine ( S_n ) in an arithmetic series given the values of ( n, t_n ), and ( t_f )?</td>
<td></td>
<td>Page 14 in Lesson 1.2 (Example 1)</td>
</tr>
<tr>
<td>use a rule to determine ( S_n ) in an arithmetic series given the values of ( n, t_n ), and ( d )?</td>
<td></td>
<td>Page 15 in Lesson 1.2 (Example 2)</td>
</tr>
<tr>
<td>use a rule to determine ( t_n ) in an arithmetic series given the values of ( S_n, t_n ), and ( d )?</td>
<td>9a</td>
<td>Page 15 in Lesson 1.2 (Example 3)</td>
</tr>
<tr>
<td>solve problems involving arithmetic series?</td>
<td></td>
<td>Page 16 in Lesson 1.2 (Example 4)</td>
</tr>
</tbody>
</table>
Assess Your Understanding

1. **Multiple Choice** Which sequence has \( d = -8 \) and \( t_{10} = -45 \)?
   - A. 27, 19, 11, 3, . . .
   - B. -8, -12, -15, -20, . . .
   - C. -5, -13, -21, -29, . . .
   - D. -27, -19, -11, -3, . . .

2. Write the first 4 terms of an arithmetic sequence with its 5th term equal to -4.

3. This sequence is arithmetic: -8, -11, -14, . . .
   a) Write a rule for the \( n \)th term.
   b) Use your rule to determine the 17th term.
4. Use the given data about each arithmetic sequence to determine the indicated values.
   a) \( t_4 = -5 \) and \( t_7 = -20 \); determine \( d \) and \( t_1 \)

b) \( t_1 = 3 \), \( d = 4 \), and \( t_{59} = 59 \); determine \( n \)

5. The steam clock in the Gastown district of Vancouver, B.C., displays the time on four faces and announces the quarter hours with a whistle chime that plays the tune *Westminster Quarters*. This sequence represents the number of tunes played from 1 to 3 days: 96, 192, 288, \ldots. Determine the number of tunes played in one year.

1.2

6. **Multiple Choice** For which series would you use \( S_n = \frac{n(t_1 + t_n)}{2} \) to determine its sum?
   
   A. \( 3 + 5 + 7 + 10 + \ldots + 29 \)
   
   B. \( 3 - 1 - 5 - 9 - \ldots - 93 \)
   
   C. \( -3 - 5 - 8 - 10 - \ldots - 29 \)
   
   D. \( 3 - 1 - 5 - 9 - \ldots + 93 \)
7. a) Create the first 5 terms of an arithmetic series with a common difference of $-3$.

b) Determine $S_n$ for your series.

8. Determine the sum of this arithmetic series:

$$-2 + 3 + 8 + 13 + \ldots + 158$$

9. Use the given data about each arithmetic series to determine the indicated value.

a) $S_{17} = 106.25$ and $t_{17} = 8.25$; determine $t_1$

b) $S_{15} = 337.5$ and $t_1 = -2$; determine $d$

**ANSWERS**

1. A  3. a) $t_n = -5 - 3n$  b) $-56$

4. a) $-5; 10$  b) 15  5. 35 040  6. B

8. 2574  9. a) 4.25  b) 3.5
FOCUS  Investigate the graphs of geometric sequences and geometric series.

Get Started

Here are 4 geometric sequences:

A. 1, 2, 4, 8, 16, ...  
B. 1, −2, 4, −8, 16, ...  
C. 1, 1/2, 1/4, 1/8, 1/16, ...  
D. 1, 1/2, 1/4, 1/8, 1/16, ...

Compare the sequences. How are they alike? How are they different?

Construct Understanding

Use a graphing calculator or graphing software to investigate graphs of geometric sequences and geometric series that have the same first term but different common ratios.

A. Choose a positive first term. Choose a common ratio, \( r \), in each of the intervals in the table below. For each common ratio, create the first 5 terms of a geometric sequence.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Common ratio, ( r )</th>
<th>Geometric sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r &gt; 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 0 &lt; r &lt; 1 )</td>
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<td></td>
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<tr>
<td>( -1 &lt; r &lt; 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r &lt; -1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
B. For each sequence

- Graph the term numbers on the horizontal axis and the term values on the vertical axis. Sketch and label each graph on a grid below, or print each graph.

- What happens to the term values as more points are plotted?

For $r < 1$, the term values decrease and approach 0.

For $0 < r < 1$, the term values alternate between positive and negative, and approach 0.

For $r > 1$, the term values increase in numerical value.
C. Use the four geometric sequences in Part A to create four corresponding geometric series.

For each series
• Complete the table below by calculating these partial sums: $S_1$, $S_2$, $S_3$, $S_4$, $S_5$

<table>
<thead>
<tr>
<th>Interval</th>
<th>Common ratio, $r$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
</tr>
</thead>
<tbody>
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</table>

• Graph the numbers of terms in the partial sums on the horizontal axis and the partial sums on the vertical axis. Sketch and label each graph on a grid below, or print each graph.
• What happens to the partial sums as more points are plotted?

D. Without graphing

• Describe the graph of this geometric sequence: 3, 2, 4, 8, 16, ...

• Describe the graph of the partial sums of this geometric series:
  \[3 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \ldots\]

Verify your descriptions by graphing. Sketch and label each graph on a grid below, or print each graph.
Assess Your Understanding

1. Create the first 5 terms of a geometric sequence for each description of a graph.
   a) The term values approach 0 as more points are plotted.

   b) The term values increase as more points are plotted.

   c) The term values alternate between positive and negative as more points are plotted.

2. Create a geometric series for each description of a graph.
   a) The partial sums approach a constant value as more points are plotted.

   b) The partial sums increase as more points are plotted.
Chapter 1: Sequences and Series

**Concept Summary**

<table>
<thead>
<tr>
<th>Big Ideas</th>
<th>Applying the Big Ideas</th>
</tr>
</thead>
</table>
| • An arithmetic sequence is related to a linear function and is created by repeatedly adding a constant to an initial number. An arithmetic series is the sum of the terms of an arithmetic sequence. | This means that:  
• The common difference of an arithmetic sequence is equal to the slope of the line through the points of its related linear function.  
• Rules can be derived to determine the nth term of an arithmetic sequence and the sum of the first n terms of an arithmetic series. |
| • A geometric sequence is created by repeatedly multiplying an initial number by a constant. A geometric series is the sum of the terms of a geometric sequence. | • The common ratio of a geometric sequence can be determined by dividing any term after the first term by the preceding term.  
• Rules can be derived to determine the nth term of a geometric sequence and the sum of the first n terms of a geometric series. |
| • It may or may not be possible to determine the sum of an infinite geometric series. | • The common ratio determines whether an infinite series has a finite sum. |

**Chapter Study Notes**

• What information do you need to know about an arithmetic sequence and a geometric sequence to determine \( t_n \)?

• What information do you need to know about an arithmetic series and a geometric series to determine the sum \( S_n \)?
# Skills Summary

<table>
<thead>
<tr>
<th>Skill</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
</table>
| **A.** Determine the general term, \( t_n \), for an arithmetic sequence. <br>**(1.1, 1.2)**<br>**Question 2** | A rule is:
\[ t_n = t_1 + d(n - 1) \]
where \( t_1 \) is the first term, \( d \) is the common difference, and \( n \) is the number of terms. | For this arithmetic sequence:
\(-9, -3, 3, 9, \ldots\)
the 20th term is:
\[ t_{20} = -9 + 6(20 - 1) \]
\[ t_{20} = -9 + 6(19) \]
\[ t_{20} = 105 \] |
| **B.** Determine the sum to \( n \) terms, \( S_n \), for an arithmetic series. <br>**(1.2)**<br>**Question 4** | When \( n \) is the number of terms, \( t_1 \) is the first term, \( t_n \) is the \( n \)th term, and \( d \) is the common difference
One rule is:
\[ S_n = \frac{n(t_1 + t_n)}{2} \]
Another rule is:
\[ S_n = \frac{n[2t_1 + d(n - 1)]}{2} \] | For this arithmetic series:
\(5 + 7 + 9 + 11 + 13 + 15 + 17\);
the sum of the first 7 terms is:
\[ S_7 = \frac{7(5 + 17)}{2} \]
\[ S_7 = \frac{7(22)}{2} \]
\[ S_7 = 77 \] |
| **C.** Determine the general term, \( t_n \), for a geometric sequence. <br>**(1.3, 1.4)**<br>**Question 8** | A rule is:
\[ t_n = t_1 r^{n-1} \]
where \( t_1 \) is the first term, \( r \) is the common ratio, and \( n \) is the number of terms. | For this geometric sequence:
\(1, -0.25, 0.0625, \ldots\)
the 6th term is:
\[ t_6 = (-0.25)^{6-1} \]
\[ t_6 = (-0.25)^5 \]
\[ t_6 = -0.000\ 976\ 5 \ldots \] |
| **D.** Determine the sum to \( n \) terms, \( S_n \), of a geometric series. <br>**(1.4)**<br>**Question 10** | A rule is:
\[ S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1 \]
where \( t_1 \) is the first term, \( r \) is the common ratio, and \( n \) is the number of terms. | For this geometric series:
\(4, 2, 1, \ldots\)
the sum of the first 10 terms is:
\[ S_{10} = \frac{4(1 - 0.5^{10})}{1 - 0.5} \]
\[ S_{10} = 7.9921 \ldots \text{or approximately 8} \] |
| **E.** Determine the sum, \( S_\infty \), of a convergent infinite geometric series. <br>**(1.6)**<br>**Question 14** | When \( r \) is between \(-1 \) and 1, use this rule:
\[ S_\infty = \frac{t_1}{1 - r} \]
where \( t_1 \) is the first term and \( r \) is the common ratio. | For this geometric series:
\(100 + 50 + 25 + \ldots\)
the sum is:
\[ S_\infty = \frac{100}{1 - (-0.5)} \]
\[ S_\infty = 66.\overline{6} \] |
1. a) Why does the rule \( t_n = 5 + 2(n - 1) \) create an arithmetic sequence for all natural-number values of \( n \)?

b) Why does the rule create odd numbers greater than 5?

c) i) Use technology or grid paper to graph the sequence in part a. Plot the Term value on the vertical axis and the Term number on the horizontal axis. Print the graph or sketch it on a grid.

ii) How does the graph show the relationship between an arithmetic sequence and a linear function?

2. During the 2003 fire season, the Okanagan Mountain Park fire was the most significant wildfire event in B.C. history. By September 7, the fire had reached about 24 900 ha and was burning at a rate of about 150 ha/h.

a) Suppose the fire continued to burn at the same rate. Create terms of a sequence to represent the area of the fire for each of the next 6 h. Why is the sequence arithmetic?
b) Write a rule for the general term of the sequence in part a. Use the rule to predict the area of the fire after 24 h. What assumptions did you make?

3. Use the given data about each arithmetic sequence to determine the indicated value.
   a) 5, −1, −7, −13, …; determine $t_{20}$
   
   b) $t_4 = −5$ and $t_8 = −17$; determine $t_4$
   
   c) $t_1 = 5$, $d = 0.5$, and $t_n = 14.5$; determine $n$
   
   d) $t_1 = −1$ and $t_{10} = \frac{1}{2}$; determine $d$
1.2

4. Use the given data about each arithmetic series to determine the indicated value.
   
   a) $5 + 3\frac{1}{2} + 2 + \frac{1}{2} - 1 - \ldots$; determine $S_{21}$

   b) $S_{12} = 78$ and $t_1 = -21$; determine $t_{12}$

   c) $S_{25} = -1025$ and $t_{25} = -77$; determine $t_1$

   d) $S_n = 123.5$, $t_1 = -25$, and $t_n = 38$; determine $n$
5. An employee of a national movie chain drew names from a barrel to award movie passes. The first name drawn won 5 movie passes. The second name drawn won 10 movie passes. This pattern continued, with each new name drawn receiving 5 more movie passes than the preceding name. On the last draw, 200 movie passes were won. What is the total number of movie passes won? Show your work.

6. Explain the meaning of this newspaper headline.

**I-Pod Sales Grew Geometrically from 2001 to 2006**

7. a) Why does the rule \( t_n = -3(-2)^{n-1} \) create a geometric sequence for all natural-number values of \( n \)?

b) Why do the terms alternate between negative and positive?
8. a) Use the given data about each geometric sequence to determine the indicated value.
   i) \( t_1 = -3 \) and \( r = -5 \); write a rule for \( t_n \), then determine \( t_7 \)
   
   ii) \( t_1 = 36 \) and \( r = \frac{2}{3} \); determine the term with value \( \frac{576}{81} \)

   iii) \( t_1 = 1.25 \) and \( t_5 = 20 \); determine the term with value 2560

b) In part a, is each sequence divergent or convergent? How do you know?
9. A soapstone carving was appraised at $2500. The value of the carving is estimated to increase by 12% each year. What will be the approximate value of the carving after 15 years?

10. Determine each sum. Give the answer to 3 decimal places when necessary.
   a) \(12 + 36 + 108 + 324 + 972 + 2916 + 8748\)
   
   b) the sum of the first 5 terms of the geometric series with \(t_1 = \frac{1}{2}\) and \(r = \frac{3}{4}\)
   
   c) \(-700 + 350 - 175 + \ldots + 5.46875\)
11. Determine the terms in this partial sum of a geometric series:
\[ S_5 = 14707, r = -7 \]

12. Use a graphing calculator or graphing software.
   a) Choose one sequence from question 8. Graph the first 5 terms.
      Explain how the graph shows whether the sequence converges or diverges.

   b) Choose one series from question 10. Graph the first 5 partial sums.
      Explain how the graph shows whether the series converges or diverges.

13. Explain how you can use the common ratio of a geometric series to identify whether the series is convergent or divergent.
14. Identify each infinite geometric series that converges. Determine the sum of any series that converges.
   a) $2 - 3 + 4.5 - 6.75 + \ldots$  
   b) $\frac{1}{2} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \ldots$

15. Danielle has $700 in a bank account. Each week she spends 10% of the amount remaining in the account.
   a) Determine how much Danielle spends in the first 3 weeks. Write the sum as a series. Is the series geometric? Explain.
   b) Determine how much Danielle spends in 12 weeks.
   c) Suppose Danielle could continue this pattern of spending indefinitely. How much money would she spend? Justify your answer.
16. Use the given data about each geometric series to determine the indicated value.
   a) \( t_1 = 3, S_\infty = 6 \); determine \( r \)
   b) \( r = \frac{1}{6} \) and \( S_\infty = \frac{6}{5} \); determine \( t_i \)

17. A small steel ball bearing is moving vertically between two electromagnets whose relative strength varies each second. The ball bearing moves 10 cm up in the 1st second, then 5 cm down in the 2nd second, then 2.5 cm up in 3rd second, and so on. This pattern continues.
   a) Assume the distance the ball bearing moves up is positive; the distance it moves down is negative.
      i) Write a series to represent the distance travelled in 5 s.
      ii) Calculate the sum of the series. What does this sum represent?
   b) Suppose this process continues indefinitely. What is the sum of the series

ANSWERS
3. a) \(-109\)  b) 4  c) 20  d) \(\frac{1}{6}\)  4. a) \(-210\)  b) 34  c) \(-5\)  d) 19  5. 4100
8. a) i) \(-46\,875\)  ii) \(t_5\)  iii) \(t_{12}\)  b) i) divergent  ii) convergent  iii) divergent
9. \$13 684  10. a) 13 116  b) approximately 1.525  c) \(-464.843\,75\)
11. \(7 - 49 + 343 - 2401 + 16\,807\)  14. a) diverges  b) converges; 1
15. a) \$700(0.1) + \$700(0.9)(0.1) + \$700(0.9)^2(0.1); geometric  b) \$502.30
   c) \$700  16. a) 0.5  b) 1  17. a) i) \(-5 - 2.5 - 1.25 + 0.625\)  ii) 6.875 cm
   b) 6.6 cm
1. **Multiple Choice** What is the sum of the first 30 terms of this arithmetic series? \(-5 - 2 + 1 + 4 + \ldots\)
   
   A. 1152  
   B. 1155  
   C. 1158  
   D. 1161

2. **Multiple Choice** What is the sum of the first 10 terms of this geometric series? \(-12 800 + 6400 - 3200 + 1600 - \ldots\)
   
   A. 8525  
   B. \(-8525\)  
   C. \(-8537.5\)  
   D. 8537.5

3. a) Which sequence below appears to be arithmetic? Justify your answer.
   i) 4, \(-10, 16, -22, 28, \ldots\)  
   ii) 4, \(-10, -24, -38, -52, \ldots\)

   b) Assume that the sequence you identified in part a is arithmetic. Determine:
      i) a rule for \(t_n\)  
      ii) \(t_{17}\)

   iii) the term that has value \(-332\)
4. For a geometric sequence, \( t_4 = -1000 \) and \( t_7 = 1 \); determine:
   a) \( t_1 \)  
   b) the term with value 0.0001

5. a) For the infinite geometric series below, identify which series converges and which series diverges. Justify your answer.
   i) \( 100 - 150 + 225 - 337.5 + \ldots \)
   ii) \( 10 + 5 + 2.5 + 1.25 + \ldots \)

   b) For which series in part a can you determine its sum? Explain why, then determine this sum.
6. This sequence represents the approximate lengths in centimetres of a spring that is stretched by loading it with from one to four 5-kg masses: 50, 54, 58, 62, ... Suppose the pattern in the sequence continues. What will the length of the spring be when it is loaded with ten 5-kg masses? Explain how you found out.

7. As part of his exercise routine, Earl uses a program designed to help him eventually do 100 consecutive push-ups. He started with 17 push-ups in week 1 and planned to increase the number of push-ups by 2 each week.
   a) In which week does Earl expect to reach his goal?
   b) What is the total number of push-ups he will have done when he reaches his goal? Explain how you know.
c) What assumptions did you make in your calculations in parts a and b?

8. Madalyn sent an email message to 4 people asking them to vote online for her favourite singer. The next hour, each of these people forwarded her message to 4 other people. This pattern continued each hour.

a) How many people were sent Madalyn’s message in the 12th hour?

b) What is the total number of messages sent and forwarded in 12 h?

ANSWERS
1. B  2. B  3. a) i) not arithmetic  ii) arithmetic  b) i) \( t_n = 4 - 14(n - 1) \)
   ii) -220  iii) \( t_{25} \)
4. a) 1 000 000  b) \( t_{10} \)  5. a) i) diverges  ii) converges  b) week 20
6. 86 cm  7. a) 43  b) 2536
8. a) 16 772 216  b) 22 369 620