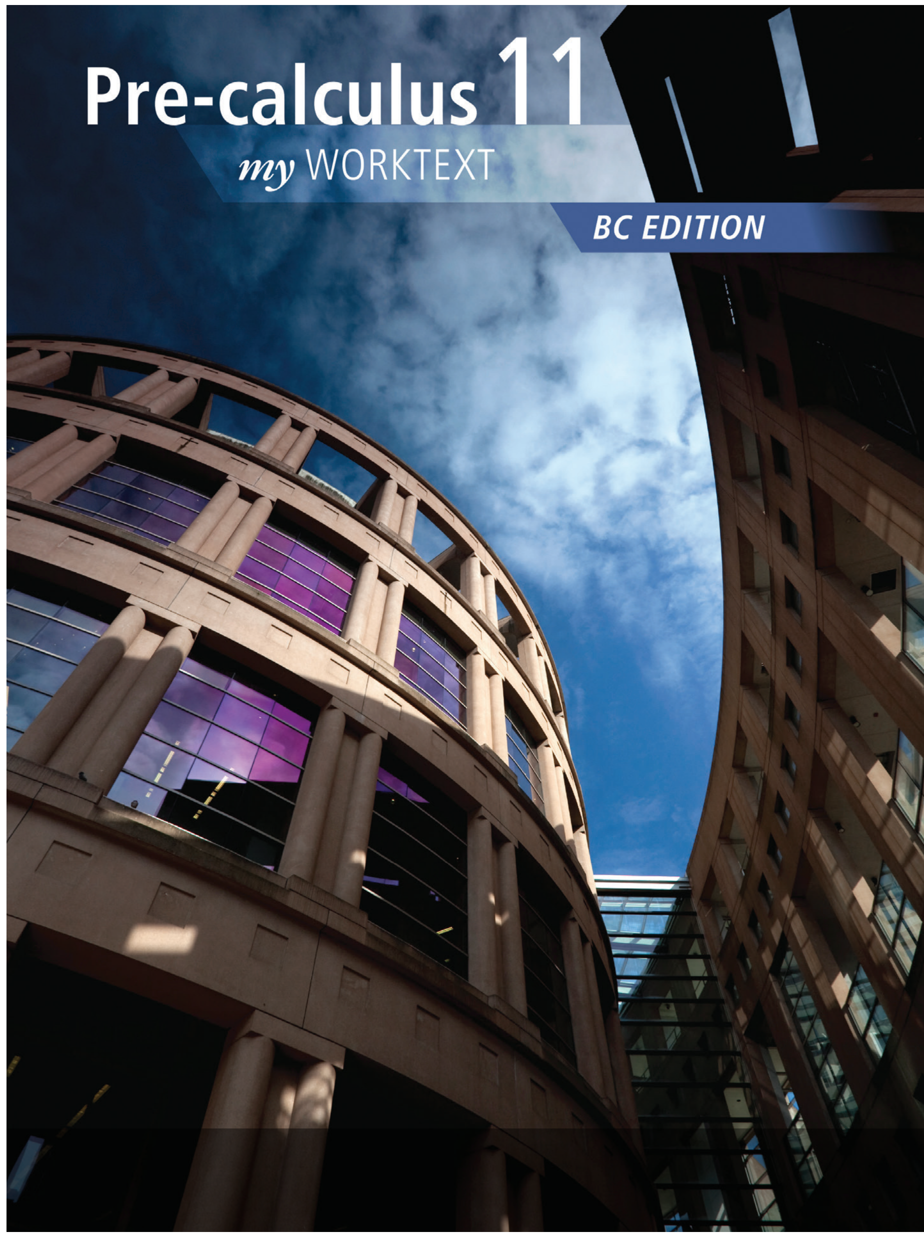
Pre-calculus 11

New BC Edition Sampler

Pearson



Great Value!

BC Pre-Calculus 11 and 12 worktexts are perfect as a **standalone resource** for the new curriculum or as a **supplement** to the resources you already have.

Sampler includes:

- Student Worktext: New lesson 3.1, pp. 2–15
- Student Worktext: Existing lesson with extra practice: lesson 3.2, pp. 16–31
- Teacher Website: Example of Desmos graphing: p. 46
- Teacher Website: Example of bigger grids: p. 43

Pre-calculus 11

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3.1 Factoring Trinomials of the Form $ax^2 + bx + c$

FOCUS **Write a trinomial of the form $ax^2 + bx + c$ as the product of two binomials, or a constant and two binomials.**

Get Started RM, CR1

Factor each trinomial. Explain your work.

Trinomial	Factored Form	Trinomial	Factored Form
$2x^2 + 8x + 2$		$-3x^2 + 9x - 15$	
$x^2 + 7x + 10$		$x^2 - 2x - 8$	
$x^2 - 9$		$x^2 - 49$	

Construct Understanding RM, CR1

Complete the table to determine each binomial product.
What patterns do you notice in the products in columns 3 and 4?
The first row has been completed for you.

Binomial Product	Product: as $ax^2 + b_1x + b_2x + c$	Product: $a \times c$	Product: $b_1 \times b_2$
$(2x + 1)(x + 2)$	$2x^2 + 4x + 1x + 2$	$2 \times 2 = 4$	$4 \times 1 = 4$
$(2x - 3)(x + 4)$			
$(3x - 1)(x - 5)$			
$(3x + 4)(3x - 4)$			

Recall that to factor a trinomial of the form $x^2 + bx + c$, identify two integers that have a sum b and a product c. If these two integers can be determined, then the trinomial is factorable using integers, and the integers are part of the factors.

From the results of *Construct Understanding*, a trinomial of the form $ax^2 + bx + c$ is factorable if ac is equal to the product of the coefficients of the x-terms when the trinomial is written as $ax^2 + b_1x + b_2x + c$.

Example 1 Identifying whether a Trinomial Is Factorable Using Integers

Determine if each trinomial is factorable using integers.

a) $2x^2 + 11x + 5$ **b)** $3x^2 - 5x - 4$ **c)** $4x^2 - 12x + 5$

SOLUTION

a) For $2x^2 + 11x + 5$, $a = 2$, $b = 11$, and $c = 5$
The product $ac = 2 \times 5$, or 10
The coefficient of x, which is 11, can be written as:
$1 + 10$, $2 + 9$, $3 + 8$, $4 + 7$, and $5 + 6$
Identify whether any pair of these integers has product 10.
The integers in the sum, $1 + 10$, have the product $1 \times 10 = 10$,
so the trinomial $2x^2 + 11x + 5$ is factorable using integers.

b) For $3x^2 - 5x - 4$, $a = 3$, $b = -5$, and $c = -4$
The product $ac = (3)(-4)$, or -12
The coefficient of x, which is -5, can be written as:
$1 + (-6)$, $2 + (-7)$, $3 + (-8)$, $4 + (-9)$, and so on
None of these sums has integers whose product is -12,
so the trinomial $3x^2 - 5x - 4$ is not factorable using integers.

RM, CR1

Check Your Understanding

1. Determine if each trinomial is factorable using integers.

a) $6x^2 + x - 12$

b) $3x^2 - 8x + 4$

c) $2x^2 - 3x - 7$

c) For $4x^2 - 12x + 5$, $a = 4$, $b = -12$, and $c = 5$
The product $ac = (4)(5)$, or 20
The coefficient of x, which is -12, can be written as:
$(-1) + (-11)$, $(-2) + (-10)$, $(-3) + (-9)$, $(-4) + (-8)$,
$(-5) + (-7)$, $(-6) + (-6)$
The integers in the sum, $(-2) + (-10)$, have the product $(-2)(-10) = 20$, so the trinomial $4x^2 - 12x + 5$ is factorable using integers.

You will learn two strategies to factor a trinomial of the form $ax^2 + bx + c$. Both strategies require determining if the trinomial is factorable using integers:

- Multiply a and c.
- Identify two integers with a product equal to ac and a sum equal to b.
- If those two integers can be determined, then the trinomial is factorable.

The first strategy is called *systematic trial.*

RM, CR1

Check Your Understanding

2. Factor the trinomial $5x^2 + 14x - 3$.

Example 2 Using Systematic Trial to Factor a Trinomial

Factor the trinomial $2x^2 - 7x + 6$.

SOLUTION

The terms of $2x^2 - 7x + 6$ have no common factor.
For $2x^2 - 7x + 6$, $a = 2$, $b = -7$, and $c = 6$
Test to identify if the trinomial is factorable.
$ac = 2 \times 6$, or 12
The coefficient of x, which is -7, can be written as $(-3) + (-4)$.
And, $(-3)(-4) = 12$
So, the trinomial is factorable.

To factor the trinomial, identify pairs of possible binomial factors.
Check to see which pair produces the middle term: $-7x$
The first terms in the binomial factors have the product $2x^2$, so they are $2x$ and x.
The second terms in the binomial factors have a product equal to the constant term, 6.
Since their product is positive, and the middle term $-7x$ has a negative coefficient, both second terms in the binomial factors are negative.
Possible second terms are: -6, -1; -3, -2
Test the different combinations of first terms and second terms.

Check Your Understanding

Answers

1. **a)** yes **b)** yes **c)** no

2. $(5x - 1)(x + 3)$

Multiply terms as shown, then add the products.

Try $(2x - 1)(x - 6)$.

$2x$ ↘↗ -1
x ↗↘ -6

$(2x)(-6) + (x)(-1) = -13x$
$\neq -7x$

Try $(2x - 2)(x - 3)$.

$2x$ ↘↗ -2
x ↗↘ -3

$(2x)(-3) + (x)(-2) = -8x$
$\neq -7x$

Try $(2x - 3)(x - 2)$.

$2x$ ↘↗ -3
x ↗↘ -2

$(2x)(-2) + (x)(-3) = -7x$

Try $(2x - 6)(x - 1)$.

$2x$ ↘↗ -6
x ↗↘ -1

$(2x)(-1) + (x)(-6) = -8x$
$\neq -7x$

The product, $(2x - 3)(x - 2)$, is a trinomial with a middle term $-7x$.
Therefore, in factored form, $2x^2 - 7x + 6 = (2x - 3)(x - 2)$

For *Example 2*, here is an alternative strategy to test possible factors. It illustrates that, to identify the binomial factors, it is only necessary to identify the value of the x-term in the trinomial.

Try $(2x - 1)(x - 6)$.

$-1x$
$-12x$

$(-1x) + (-12x) = -13x$
$\neq -7x$

Try $(2x - 2)(x - 3)$.

$-2x$
$-6x$

$(-2x) + (-6x) = -8x$
$\neq -7x$

Try $(2x - 3)(x - 2)$.

$-3x$
$-4x$

$(-3x) + (-4x) = -7x$

Try $(2x - 6)(x - 1)$.

$-6x$
$-2x$

$(-6x) + (-2x) = -8x$
$\neq -7x$

The factoring of a trinomial can be checked by multiplying the binomial factors to ensure that their product is equal to the given trinomial.

The second strategy to factor a trinomial of the form $ax^2 + bx + c$ is to use *decomposition*; that is, split, or *decompose*, the middle term bx into two terms b_1x and b_2x, where $b_1 \times b_2 = a \times c$.

Then, factor $ax^2 + b_1x + b_2x + c$ by first removing the greatest common factor of each of $ax^2 + b_1x$ and $b_2x + c$.

RM, CR1

Check Your Understanding

3. Factor the trinomial $4x^2 + 16x + 15$.

Example 3 Using Decomposition to Factor a Trinomial

Factor the trinomial $-6x^2 - 20x + 16$.

SOLUTION

The terms of the trinomial $-6x^2 - 20x + 16$ have a common factor, -2.
So, $-6x^2 - 20x + 16 = -2(3x^2 + 10x - 8)$
For $3x^2 + 10x - 8$, $a = 3$, $b = 10$, $c = -8$

Test to identify if the trinomial $3x^2 + 10x - 8$ is factorable.
$ac = (3)(-8)$, or -24
The coefficient of x, which is 10, can be written as $(-2) + 12$.
And, $(-2)(12) = -24$
So, the trinomial is factorable.

To factor $3x^2 + 10x - 8$, use decomposition.

$3x^2 + 10x - 8$	Decompose the middle term, $10x$.
$= 3x^2 + 12x - 2x - 8$	Remove a common factor from each pair of terms.
$= 3x(x + 4) - 2(x + 4)$	Remove the common binomial factor $(x + 4)$.
$= (3x - 2)(x + 4)$	

So, $3x^2 + 10x - 8 = (3x - 2)(x + 4)$
And, $-6x^2 - 20x + 16 = -2(3x - 2)(x + 4)$

In *Example 3*, the factoring of the trinomial can be checked by multiplying the common factor and binomial factors, to ensure that their product is equal to the given trinomial.

Check Your Understanding

Answer:

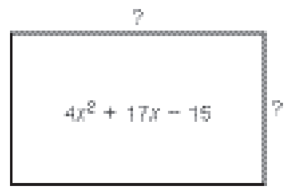
3. $(2x + 5)(2x + 3)$

Example 4 Using Factoring to Solve a Problem

The area of a rectangle is represented by the trinomial $4x^2 + 17x - 15$.

a) Factor the trinomial to determine the possible dimensions of the rectangle.

b) Use the factored form of the trinomial. Determine possible dimension of the rectangle when $x = 6$ cm.



SOLUTION

a) For $4x^2 + 17x - 15$, $a = 4$, $b = 17$, $c = -15$

$ac = (4)(-15)$, or -60

The coefficient of x, which is 17, can be written as $(-3) + 20$.

And, $(-3)(20) = -60$

Use decomposition to factor $4x^2 + 17x - 15$.

$4x^2 + 17x - 15$	Decompose the middle term, $17x$.
$= 4x^2 + 20x - 3x - 15$	Remove a common factor from each pair of terms.
$= 4x(x + 5) - 3(x + 5)$	Remove the common binomial factor $(x + 5)$.
$= (x + 5)(4x - 3)$	

So, $4x^2 + 17x - 15 = (x + 5)(4x - 3)$

The dimensions of the rectangle are $x + 5$ and $4x - 3$.

b) When $x = 6$:

$x + 5 = 6 + 5$, or 11

$4x - 3 = 4(6) - 3$, or 21

The possible dimensions of the rectangle are 21 cm by 11 cm

RM, US, CR1

Check Your Understanding

4. The area of a rectangle is represented by the trinomial $3x^2 - 19x - 14$.

a) Factor the trinomial to determine the possible dimensions of the rectangle.

b) Use the factored form of the trinomial. Determine possible dimension of the rectangle when $x = 9$ cm.

Check Your Understanding

Answer:

4. 29 cm by 2 cm

THINK FURTHER

CR1

In *Example 4,* are there other possible dimensions for the rectangle? Explain.

Discuss the Ideas

CR1

1. How can you determine whether a trinomial of the form $ax^2 + bx + c$ can be written as the product of two binomial factors, using integers?

2. A trinomial of the form $ax^2 + bx + c$ is factorable. How many factors might it have? Explain.

3. After a trinomial has been factored, why should the product of the factors be determined?

Exercises

A

RM, CR1

4. Expand.

a) $(5x + 1)(x - 2)$ **b)** $(6x - 5)(x - 4)$ **c)** $(2x + 1)(2x + 3)$

d) $(3x + 7)(4x - 5)$ **e)** $3(4x + 5)(x + 4)$ **f)** $-4(2x + 3)(2x - 5)$

RM, CR1

5. Determine if each trinomial is factorable. Check for common factors first.

a) $2x^2 + 11x + 9$ **b)** $-4x^2 - 9x + 6$

c) $5x^2 + 3x - 8$ **d)** $6x^2 - 22x + 12$

e) $9x^2 - 5x - 4$ **f)** $-12x^2 + 15x - 9$

g) $5x^2 - 7x + 2$ **h)** $14e^2 - 13e + 3$

RM, CR1

6. Factor each trinomial, if possible.

a) $5x^2 + 16x + 3$

b) $3x^2 - 10x - 8$

c) $4x^2 - 12x + 5$

d) $5x^2 - 7x - 8$

e) $8x^2 + 2x - 15$

f) $9x^2 + 6x - 8$

g) $2m^2 - 11m + 12$

h) $10u^2 - 29u + 10$

B

RM, CR1

7. Fully factor each trinomial, if possible.

a) $-3x^2 - 4x - 1$

b) $4x^2 + 22x + 10$

c) $6x^2 + 39x - 21$

d) $-6x^2 + 10x + 8$

e) $-8x^2 - 12x - 4$

f) $15x^2 - 70x - 25$

RM, CR1

8. The trinomial $3x^2 + 4x - 4$ represents an integer.

a) Factor the trinomial to get expressions for the factors of the integer.

b) Use the factored form of the trinomial. Determine the value of the integer and one possible pair of factors when $x = -1$.

RM, CR1

9. The area of a rectangular patio, in square metres, is represented by the trinomial $15x^2 + 14x - 8$.

a) Determine the factors that represent the dimensions of the patio.

b) Use the factored form of the trinomial. Determine possible dimensions of the patio and the area of the patio when $x = 4$.

c) Use the factored form of the polynomial. Determine the dimensions of the patio and the area of the patio when $x = 3.25$.

RM, CR1

10. Determine the integer value(s) of k so that each trinomial is factorable.

a) $2x^2 + kx + 6$

b) $4x^2 + kx - 5$

C

RM, CR1

11. Determine the integer value(s) of k so that each trinomial is factorable.

a) $3x^2 + 10x + k$

b) $4x^2 - 7x + k$

RM, CR1

12. An integer is the product of a constant and two integers whose values can vary. The integer is represented by the trinomial $-20x^2 + 164x - 32$.

a) Given that the constant is -4, determine the algebraic expressions that represent the two integers.

b) Determine the values of the integers and the trinomial when $x = -2$.

Multiple-Choice Questions

RM

1. Which expression is the factored form of the trinomial $-10x^2 + 22x + 24$?

A. $-2(5x + 3)(x - 4)$ **B.** $-2(5x + 6)(x - 2)$

C. $-2(5x + 4)(x - 3)$ **D.** $-2(5x + 2)(x - 6)$

2. Which trinomial cannot be factored?

A. $4x^2 + 2x - 20$ **B.** $8x^2 - 3x - 6$

C. $6x^2 + 19x + 8$ **D.** $9x^2 - 6x - 35$

3. What is a value of k so that the trinomial $8x^2 + kx - 9$ is factorable?

A. -20 **B.** 15 **C.** 7 **D.** -14

4. For which value of k is the trinomial $9x^2 - kx + 1$ **not** factorable?

A. 6 **B.** -9 **C.** -6 **D.** -10

Study Note

CR1, CR2

You have learned two strategies to factor a trinomial. Give an example of when you might use each strategy, and show how to use the strategy.

ANSWERS

4. a) $5x^2 - 9x - 2$ **b)** $6x^2 - 29x + 20$ **c)** $4x^2 + 8x + 3$ **d)** $12x^2 + 13x - 35$
e) $12x^2 + 63x + 60$ **f)** $-16x^2 + 16x + 60$ **5. a)** factorable **b)** not factorable
c) factorable **d)** factorable **e)** factorable **f)** not factorable **g)** factorable **h)** factorable
6. a) $(5x + 1)(x + 3)$ **b)** $(3x + 2)(x - 4)$ **c)** $(2x - 1)(2x - 5)$ **d)** not factorable
e) $(4x - 5)(2x + 3)$ **f)** $(3x - 2)(3x + 4)$ **g)** $(2m - 3)(m - 4)$ **h)** $(5u - 2)(2u - 5)$
7. a) $-(3x + 1)(x + 1)$ **b)** $2(2x + 1)(x + 5)$ **c)** $3(2x - 1)(x + 7)$ **d)** $-2(3x^2 - 5x - 4)$
e) $-4(2x + 1)(x + 1)$ **f)** $5(3x + 1)(x - 5)$
8. a) $(3x - 2)$ and $(x + 2)$ **b)** factors: -5 and 1; integer: -5
9. a) $(5x - 2)$ by $(3x + 4)$ **b)** dimensions: 18 m by 16 m; area: 288 m^2
c) dimensions: 14.25 m by 13.75 m; area: 195.9375 m^2
10. a) $-13, -8, -7, 7, 8, 13$ **b)** $-19, -8, -1, 1, 8, 19$ **11. a)** $3, 7, -13$ **b)** $-2, -11, 3$
12. a) factors: $(5x - 1)$ and $(x - 8)$ **b)** factors: -11 and -10; trinomial: -440

Multiple Choice

1. C **2.** B **3.** D **4.** B

3.2 Factoring Polynomial Expressions

FOCUS **Factor polynomial expressions that contain functions.**

Get Started US

Factor this trinomial: $x^2 - 4x - 21$

Construct Understanding CR1

Work with a partner.

Factor each polynomial, then identify a perfect square trinomial and a difference of squares. Justify your answer.

$3a^2 - 6a - 24$	$9b^2 + 12b + 4$
$3c^2 + 7c + 4$	$2g^2 - g - 3$
$2x^2 - 2x - 12$	$m^2 - 16n^2$

If a trinomial does not have a common factor, then its factors, if they exist, will be two binomials.

Example 1 Determining whether a Given Binomial Is a Factor of a Given Trinomial

US, CR1

Is $x + 3$ a factor of each trinomial? Justify the answer.

a) $4x^2 + 12x + 9$ **b)** $2x^2 + x - 15$

SOLUTION

Use logical reasoning.
If $x + 3$ is a factor, then each trinomial can be written as:
$(x + 3)(ax + b)$

a) $4x^2 + 12x + 9 = (x + 3)(ax + b)$ Expand.
$4x^2 + 12x + 9 = ax^2 + (3a + b)x + 3b$
↑ ↑

The x^2-terms on both sides must be equal. Compare coefficients:
$a = 4$

The constant terms on both sides must be equal.
$9 = 3b$
So, $b = 3$

The trinomial would be: $(x + 3)(4x + 3)$
Expand to check the x-term.

$$(x + 3)(4x + 3) = 4x^2 + 3x + 12x + 9$$
$$= 4x^2 + 15x + 9$$

Since this trinomial is not equal to the given trinomial, $x + 3$ is not a factor of the given trinomial.

b) $2x^2 + x - 15 = (x + 3)(ax + b)$ Expand.
$= ax^2 + (3a + b)x + 3b$

Compare coefficients. For the x^2-terms: $a = 2$
For the constant terms: $-15 = 3b$
So, $b = -5$

The trinomial would be: $(x + 3)(2x - 5)$
Expand to check the x-term.

$$(x + 3)(2x - 5) = 2x^2 - 5x + 6x - 15$$
$$= 2x^2 + x - 15$$

Since this trinomial is equal to the given trinomial, $x + 3$ is a factor of the given trinomial.

Check Your Understanding

1. Is $d - 4$ a factor of each trinomial? Justify the answer.
a) $2d^2 + 6d - 56$
b) $3d^2 + 13d + 4$

Check Your Understanding

Answers:

1. a) yes **b)** no

THINK FURTHER

RM

In *Example 1a*, what other strategies could you use to check whether the binomial $x + 3$ is a factor?

US

Check Your Understanding

2. Factor each trinomial.

a) $x^2 - 1.5x + 0.5$

b) $x^2 - \frac{17}{3}x - 2$

Example 2 Factoring Trinomials with Rational Coefficients

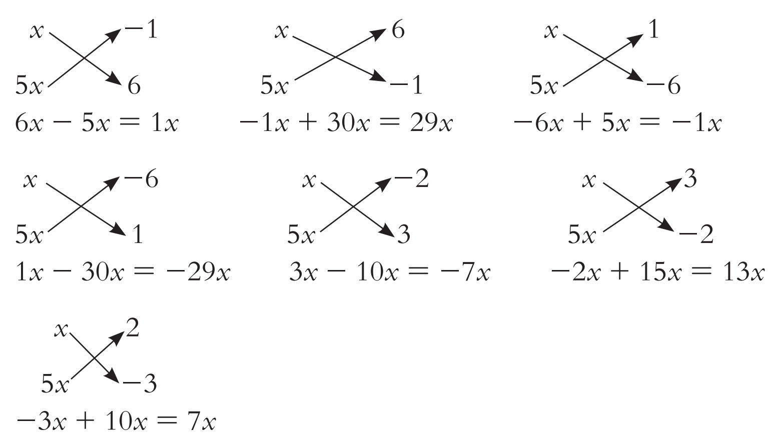
Factor each trinomial.

a) $x^2 + 1.4x - 1.2$ **b)** $3x^2 - \frac{29}{2}x + 14$

SOLUTION

a) Remove 0.1 as a common factor to get integer coefficients.
To divide by 0.1, multiply by 10.
$x^2 + 1.4x - 1.2 = 0.1(10x^2 + 14x - 12)$
Remove the common factor, 2, from the trinomial.
So, $x^2 + 1.4x - 1.2 = 0.1[2(5x^2 + 7x - 6)]$
$= 0.2(5x^2 + 7x - 6)$

Use logical reasoning to factor $5x^2 + 7x - 6$.
The factors of $5x^2$ are: x and $5x$
The factors of -6 are: -1 and 6, or 1 and -6; or -2 and 3, or 2 and -3.
Combine each pair of factors of $5x^2$ with each pair of factors of -6 until you find a sum of $7x$.



So, $5x^2 + 7x - 6 = (x + 2)(5x - 3)$
And, $x^2 + 1.4x - 1.2 = 0.2(x + 2)(5x - 3)$

Check Your Understanding

Answers:

2. a) $0.5(x - 1)(2x - 1)$

b) $\frac{1}{3}(3x + 1)(x - 6)$

b) Remove $\frac{1}{2}$ as a common factor to get integer coefficients.

To divide by $\frac{1}{2}$, multiply by 2.

$3x^2 - \frac{29}{2}x + 14 = \frac{1}{2}(6x^2 - 29x + 28)$

Use decomposition to factor $6x^2 - 29x + 28$.
The product of the coefficient of x^2 and the constant term is:
$6(28) = 168$
Write $-29x$ as the sum of two terms whose coefficients have a product of 168.

Factors of 168	Sum of Factors
−1, −168	−1 − 168 = −169
−2, −84	−2 − 84 = −86
−3, −56	−3 − 56 = −59
−4, −42	−4 − 42 = −46
−6, −28	−6 − 28 = −34
−8, −21	−8 − 21 = −29

Since the *x*-coefficient is negative, list only negative factors of 168. Stop when the sum of the factors is −29.

The two coefficients are −8 and −21, so write the trinomial $6x^2 - 29x + 28$ as $6x^2 - 8x - 21x + 28$.

Remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$6x^2 - 8x - 21x + 28$
$= 2x(3x - 4) - 7(3x - 4)$
$= (3x - 4)(2x - 7)$

Each product has a common binomial factor.

So, $6x^2 - 29x + 28 = (3x - 4)(2x - 7)$

And, $3x^2 - \frac{29}{2}x + 14 = \frac{1}{2}(3x - 4)(2x - 7)$

THINK FURTHER RM

How can you check that you have factored a trinomial correctly?

Some polynomial expressions contain functions of a variable; for example, an expression such as $(x + 3)^2 - 6(x + 3) - 16$ contains $f(x) = x + 3$.

The expression $(x + 3)^2 - 6(x + 3) - 16$ has the form $[f(x)]^2 - 6f(x) - 16$.

US

Example 3 Factoring Using a Trinomial Pattern

Check Your Understanding

3. Factor each polynomial expression.

a) $x^2 + 5x - 24$

b) $2(x - 6)^2 + 10(x - 6) - 48$

c) $3(2x + 5)^2 + 10(2x + 5) - 8$

Factor each polynomial expression.

a) $x^2 - 6x - 16$

b) $(x + 3)^2 - 6(x + 3) - 16$

c) $6(3x - 4)^2 - 21(3x - 4) + 15$

SOLUTION

a) Use logical reasoning. Find two numbers whose product is −16 and whose sum is −6: 2 and −8.
So, $x^2 - 6x - 16 = (x + 2)(x - 8)$

b) The expression has the form $[f(x)]^2 - 6f(x) - 16$, where $f(x) = x + 3$.
To factor, replace $x + 3$ with a variable such as z.

$$(x + 3)^2 - 6(x + 3) - 16$$
$$\uparrow \qquad\qquad \uparrow$$
$$z^2 \quad - \quad 6z \quad - 16$$

Factor the trinomial $z^2 - 6z - 16$.
From part a, $x^2 - 6x - 16 = (x + 2)(x - 8)$
So, $z^2 - 6z - 16 = (z + 2)(z - 8)$
Substitute $z = x + 3$ in the expressions above.

$(x + 3)^2 - 6(x + 3) - 16$
$= [(x + 3) + 2][(x + 3) - 8]$ Simplify.
So, $(x + 3)^2 - 6(x + 3) - 16 = (x + 5)(x - 5)$

c) Remove the common factor, 3.
$6(3x - 4)^2 - 21(3x - 4) + 15$
$= 3[2(3x - 4)^2 - 7(3x - 4) + 5]$
To factor $2(3x - 4)^2 - 7(3x - 4) + 5$, substitute $3x - 4 = z$ to get: $2z^2 - 7z + 5$

$2z^2 - 7z + 5 = (z - 1)(2z - 5)$
Substitute: $z = 3x - 4$
$2(3x - 4)^2 - 7(3x - 4) + 5$
$= [(3x - 4) - 1][2(3x - 4) - 5]$
$= (3x - 5)(6x - 8 - 5)$
$= (3x - 5)(6x - 13)$
And, $6(3x - 4)^2 - 21(3x - 4) + 15 = 3(3x - 5)(6x - 13)$

Check Your Understanding

Answers:

3. a) $(x + 8)(x - 3)$

b) $2(x + 2)(x - 9)$

c) $(2x + 9)(6x + 13)$

THINK FURTHER

CR1

What other strategy could you use to factor the polynomial expression in *Example 3c*? Which strategy is more efficient?

Example 4 Factoring Using the Difference of Squares Pattern

Factor each polynomial expression.

a) $4x^2 - 25y^2$ **b)** $(2x - 1)^2 - (y + 4)^2$

c) $32(x + 2)^2 - 18(2y - 3)^2$

SOLUTION

a) This is a difference of squares: $4x^2 - 25y^2 = (2x)^2 - (5y)^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$

b) The polynomial expression has the form $[f(x)]^2 - [f(y)]^2$, where $f(x) = 2x - 1$ and $f(y) = y + 4$.

To factor, replace $2x - 1$ with a; replace $y + 4$ with b.

$$\underset{\substack{\uparrow\\ a^2}}{(2x-1)^2} \;\underset{-}{-}\; \underset{\substack{\uparrow\\ b^2}}{(y+4)^2}$$

Factor: $a^2 - b^2 = (a + b)(a - b)$

Substitute $a = 2x - 1$ and $b = y + 4$ in the expressions above.

$$\begin{aligned}&(2x - 1)^2 - (y + 4)^2\\ &= [(2x - 1) + (y + 4)][(2x - 1) - (y + 4)]\\ &= [2x - 1 + y + 4][2x - 1 - y - 4]\\ &= [2x + y + 3][2x - y - 5]\end{aligned}$$

Simplify inside the square brackets.

So, $(2x - 1)^2 - (y + 4)^2 = (2x + y + 3)(2x - y - 5)$

c) Remove the common factor, 2.

$32(x + 2)^2 - 18(2y - 3)^2 = 2[16(x + 2)^2 - 9(2y - 3)^2]$

And $16(x + 2)^2 - 9(2y - 3)^2$ can be written as:

$$\begin{aligned}&4^2(x + 2)^2 - 3^2(2y - 3)^2\\ &= [4(x + 2)]^2 - [3(2y - 3)]^2\\ &= [4(x + 2) + 3(2y - 3)][4(x + 2) - 3(2y - 3)]\\ &= [4x + 8 + 6y - 9][4x + 8 - 6y + 9]\\ &= [4x + 6y - 1][4x - 6y + 17]\end{aligned}$$

Visualize the substitution used in part b.

So, $16(x + 2)^2 - 9(2y - 3)^2 = (4x + 6y - 1)(4x - 6y + 17)$

And, $32(x + 2)^2 - 18(2y - 3)^2 = 2(4x + 6y - 1)(4x - 6y + 17)$

Check Your Understanding

4. Factor.

a) $16a^2 - 9b^2$

b) $(3x + 4)^2 - (2y - 1)^2$

c) $27(2x - 3)^2 - 75(y - 4)^2$

Check Your Understanding

Answers:

4. a) $(4a + 3b)(4a - 3b)$

b) $(3x + 2y + 3)(3x - 2y + 5)$

c) $3(6x + 5y - 29)(6x - 5y + 11)$

Discuss the Ideas

CR1

1. How can you determine if a polynomial has been factored fully?

2. When a polynomial expression contains a function of a variable, what is an advantage of substituting a variable for the function before you factor the expression?

Exercises

A

US **3.** Factor.

a) $x^2 - 100$

b) $49x^2 - 1$

c) $121 - 81t^2$

d) $4 - 49n^2$

e) $64x^2 - 25y^2$

f) $36p^2 - 169q^2$

US **4.** Factor.

a) $x^2 + 7x + 10$

b) $x^2 - 11x + 30$

c) $m^2 + 9m - 36$

d) $n^2 - 21n + 108$

e) $q^2 - 4q - 77$

f) $r^2 + 2r - 48$

g) $m^2 + 4m - 45$ h) $y^2 - 2y - 63$ i) $m^2 - 19m - 120$

US **5.** Factor.

a) $5x^2 + 9x + 4$ b) $4x^2 - 7x + 3$ c) $7y^2 - 12y - 4$

d) $6x^2 + x - 2$ e) $2x^2 - 9x + 4$ f) $8n^2 + 14n - 15$

US **6.** Factor.

a) $9x^2 + 6x + 1$ b) $25x^2 + 20x + 4$ c) $16n^2 - 24n + 9$

d) $36x^2 - 60x + 25$ e) $4y^2 - 20y + 25$ f) $10p^2 - 9p + 2$

B

US **7.** Determine whether $x + 5$ is a factor of each polynomial.

a) $2x^2 - 2x - 40$ b) $3x^2 + 13x - 10$ c) $5x^2 - 27x + 10$

US **8.** Determine whether $2x + 1$ is a factor of each polynomial.

a) $6x^2 - 13x - 8$ **b)** $2x^2 - 8$ **c)** $10x^2 - 7x - 6$

US **9.** Factor.

a) $2x^2 - 50y^2$ **b)** $0.1x^2 - 0.001$ **c)** $0.5t^2 - 0.245$

d) $20x^2 - 125y^2$ **e)** $\frac{1}{100}x^2 - \frac{1}{25}y^2$ **f)** $\frac{1}{64}p^2 - \frac{1}{196}q^2$

US **10.** Factor.

a) $2x^2 + 16x + 24$ **b)** $3x^2 - 9x - 30$ **c)** $n^2 + 1.75n - 0.5$

d) $x^2 + \frac{5}{2}x - 6$ **e)** $x^2 + 2.5x - 1.5$ **f)** $\frac{1}{2}x^2 + \frac{11}{3}x + 4$

g) $6a^2 + 26a - 20$ **h)** $h^2 + 2.5h + 1$ **i)** $4 + 7d - \frac{15}{2}d^2$

US **11.** Factor each polynomial.

a) $\frac{x^2}{9} - \frac{4}{25}$

b) $6 + 5x - x^2$

c) $-x^2 + \frac{121}{64}$

d) $7 - \frac{5}{3}x - 2x^2$

US **12.** Factor each polynomial expression.

a) **i)** $9x^2 - 4y^2$

ii) $9(x - 3)^2 - 4(2y + 1)^2$

b) **i)** $50x^2 - 162y^2$

ii) $50(2x - 5)^2 - 162(3y - 2)^2$

c) **i)** $25m^2 - \frac{1}{4}n^2$

ii) $25(2m + 3)^2 - \frac{1}{4}(3n - 5)^2$

US **13.** Factor each polynomial expression.

a) $16(2x - 7)^2 - 25(y + 2)^2$

b) $121(x + 3)^2 - 36(2y - 5)^2$

CR1 **14.** Use two different strategies to factor this polynomial expression: $(2x + 3)^2 - (x - 5)^2$. Which strategy is more efficient?

US **15.** Factor each polynomial expression.

a) i) $3x^2 + 19x + 16$ **ii)** $3(2x - 1)^2 + 19(2x - 1) + 16$

b) i) $12x^2 + 17x - 5$ **ii)** $12(4x - 1)^2 + 17(4x - 1) - 5$

US **16.** Factor each polynomial expression.

a) $(7x - 5)^2 - 8(7x - 5) + 15$

b) $9(2x + 1)^2 - 42(2x + 1) + 49$

c) $(x^2 - x)^2 - 8(x^2 - x) + 12$

d) $(4x^2 + 4x + 3)^2 - 8(4x^2 + 4x + 3) + 12$

C

US **17.** Is $x + 1$ a factor of each polynomial expression?

a) $(x + 1)^2 + 4(x + 1) - 32$

b) $2(2x + 3)^2 + 5(2x + 3) - 7$

US **18.** Factor each polynomial.

a) $x^2 + x + \frac{1}{4}$

b) $\frac{1}{2}x^2 + \frac{9}{8}x + \frac{1}{4}$

c) $x^2 + \frac{3}{2}x + \frac{1}{2}$

d) $-\frac{4}{9}p^2 + \frac{1}{4}q^2$

e) $\frac{x^2}{2} - \frac{x}{4} - \frac{3}{4}$

f) $\frac{4}{3}x^2 + \frac{1}{3}x - \frac{1}{2}$

g) $\frac{4}{3}x^2 - \frac{1}{12}y^2$

h) $\frac{3}{2}m^2 + m - \frac{4}{3}$

CR1 **19.** Factor each polynomial. Explain your strategy.

a) $x^2 - 6x + 9 - y^2$

b) $4x^2 + 4x + 1 - 9y^2$

US **20.** Consider the polynomial $3x^2 + nx - 4$. Determine a value for n so that $3x - 2$ is a factor of the polynomial.

Multiple-Choice Questions

RM

1. Which polynomial is a perfect square trinomial?

A. $4x^2 - 25$ **B.** $x^2 - 10x + 9$

C. $9x^2 + 20x + 4$ **D.** $36x^2 - 60x + 25$

2. Which polynomial expression has $x - 2$ as a factor?

A. $2(x - 1)^2 + 8(x - 1) - 10$ **B.** $(x - 2)^2 + 8(x - 2) + 15$

C. $(x^2 - x)^2 + 14(x^2 - x) + 24$ **D.** $6(x + 1)^2 + 13(x + 1) - 5$

Study Note

CR2

Explain how the strategies for factoring a trinomial can be used to factor an expression such as $a(f(x))^2 + b(f(x)) + c$.

ANSWERS

3. a) $(x - 10)(x + 10)$ **b)** $(7x - 1)(7x + 1)$ **c)** $(11 + 9t)(11 - 9t)$ **d)** $(2 - 7n)(2 + 7n)$ **e)** $(8x - 5y)(8x + 5y)$ **f)** $(6p + 13q)(6p - 13q)$ **4. a)** $(x + 5)(x + 2)$ **b)** $(x - 6)(x - 5)$ **c)** $(m - 3)(m + 12)$ **d)** $(n - 9)(n - 12)$ **e)** $(q + 7)(q - 11)$ **f)** $(r + 8)(r - 6)$ **g)** $(m + 9)(m - 5)$ **h)** $(y - 9)(y + 7)$ **i)** $(m - 24)(m + 5)$ **5. a)** $(5x + 4)(x + 1)$ **b)** $(4x - 3)(x - 1)$ **c)** $(7y + 2)(y - 2)$ **d)** $(3x + 2)(2x - 1)$ **e)** $(2x - 1)(x - 4)$ **f)** $(2n + 5)(4n - 3)$ **6. a)** $(3x + 1)^2$ **b)** $(5x + 2)^2$ **c)** $(4n - 3)^2$ **d)** $(6x - 5)^2$ **e)** $(2y - 5)^2$ **f)** $(5p - 2)(2p - 1)$ **7. a)** no **b)** yes **c)** no **8. a)** yes **b)** no **c)** yes **9. a)** $2(x - 5y)(x + 5y)$ **b)** $0.001(10x - 1)(10x + 1)$ **c)** $0.5(t + 0.7)(t - 0.7)$ **d)** $5(2x - 5y)(2x + 5y)$ **e)** $\frac{1}{100}(x - 2y)(x + 2y)$ **f)** $\frac{1}{4}\left(\frac{1}{4}p - \frac{1}{7}q\right)\left(\frac{1}{4}p + \frac{1}{7}q\right)$ **10. a)** $2(x + 6)(x + 2)$ **b)** $3(x - 5)(x + 2)$ **c)** $0.25(4n - 1)(n + 2)$ **d)** $\frac{1}{2}(2x - 3)(x + 4)$ **e)** $0.5(2x - 1)(x + 3)$ **f)** $\frac{1}{6}(3x + 4)(x + 6)$ **g)** $2(3a - 2)(a + 5)$ **h)** $0.5(2h + 1)(h + 2)$ **i)** $\frac{1}{2}(4 - 3d)(2 + 5d)$ **11. a)** $\left(\frac{x}{3} - \frac{2}{5}\right)\left(\frac{x}{3} + \frac{2}{5}\right)$ **b)** $(1 + x)(6 - x)$ **c)** $\left(\frac{11}{8} - x\right)\left(\frac{11}{8} + x\right)$ **d)** $\frac{1}{3}(7 + 3x)(3 - 2x)$
12. a) i) $(3x - 2y)(3x + 2y)$ **ii)** $(3x - 4y - 11)(3x + 4y - 7)$ **b) i)** $2(5x - 9y)(5x + 9y)$ **ii)** $2(10x - 27y - 7)(10x + 27y - 43)$ **c) i)** $\frac{1}{4}(10m - n)(10m + n)$ **ii)** $\frac{1}{4}[20m - 3n + 35][20m + 3n + 25]$
13. a) $(8x - 5y - 38)(8x + 5y - 18)$ **b)** $(11x - 12y + 63)(11x + 12y + 3)$ **14.** $(x + 8)(3x - 2)$
15. a) i) $(3x + 16)(x + 1)$ **ii)** $2x(6x + 13)$ **b) i)** $(4x - 1)(3x + 5)$ **ii)** $2(16x - 5)(6x + 1)$
16. a) $(7x - 10)(7x - 8)$ **b)** $4(3x - 2)^2$ **c)** $(x - 3)(x + 2)(x - 2)(x + 1)$ **d)** $(2x + 3)(2x - 1)(2x + 1)^2$
17. a) no **b)** yes **18. a)** $\frac{1}{4}(2x + 1)^2$ **b)** $\frac{1}{8}(4x + 1)(x + 2)$ **c)** $\frac{1}{2}(2x + 1)(x + 1)$ **d)** $-\frac{1}{36}(4p - 3q)(4p + 3q)$ **e)** $\frac{1}{4}(2x - 3)(x + 1)$ **f)** $\frac{1}{6}(4x + 3)(2x - 1)$ **g)** $\frac{1}{12}(4x - y)(4x + y)$ **h)** $\frac{1}{6}(3m - 2)(3m + 4)$
19. a) $(x - y - 3)(x + y - 3)$ **b)** $(2x - 3y + 1)(2x + 3y + 1)$ **20.** 4

Multiple Choice

1. D **2.** A

CHECKPOINT 1

Self-Assess

Can you . . .	Try *Checkpoint* question	For review, see
factor a polynomial expression that requires identifying common factors?	2	Page 6 in Lesson 3.1 (Example 3)
determine whether a binomial is a factor of a given polynomial expression?	8	Page 17 in Lesson 3.2 (Example 1)
factor a polynomial expression of the form $ax^2 + bx + c$, where $a \neq 0$?	3	Page 4 in Lesson 3.1 (Example 2)
factor a polynomial expression of the form $a^2x^2 - b^2y^2$, where $a \neq 0$ and $b \neq 0$?	6a	Page 21 in Lesson 3.2 (Example 4a)
factor a polynomial expression of the form $a(f(x))^2 + b(f(x)) + c$, where $a \neq 0$?	7b	Page 20 in Lesson 3.2 (Example 3b, c)
factor a polynomial expression of the form $a^2(f(x))^2 - b^2(g(y))^2$, where $a \neq 0$ and $b \neq 0$?	7a	Page 21 in Lesson 3.2 (Example 4b, c)

Assess Your Understanding

3.1

1. **Multiple Choice** Which expression is the factored form of $-2x^2 - 6x + 36$?

A. $-2(x + 3)(x - 6)$ **B.** $-2(x - 3)(x + 6)$

C. $-2(x - 9)(x + 2)$ **D.** $-2(x + 9)(x - 2)$

2. Remove the greatest common factor from each trinomial.

a) $2x^2 - 14x + 16$ **b)** $-y^2 - 8y + 12$

c) $6n^2 - 15n - 27$ **d)** $-32 + 48x - 36x^2$

e) $30y^2 - 45y + 75$ **f)** $-45x^2 + 27x + 36$

3. Factor each trinomial, if possible.

a) $x^2 + 16x + 15$ **b)** $y^2 + 5y - 36$

c) $n^2 + 7n + 8$ **d)** $2x^2 - 16x + 14$

e) $12y^2 + 36y + 15$ **f)** $-8n^2 + 16n - 20$

4. The area of a rectangle, in square units, is represented by the polynomial $3x^2 + 28x - 20$. Factor the polynomial to determine the dimensions of the rectangle.

3.2

RM **5. Multiple Choice** Which expression is the factored form of $3x^2 + 11x - 4$?

A. $(3x + 4)(x - 1)$ **B.** $(3x + 1)(x - 4)$

C. $(3x - 1)(x + 4)$ **D.** $(3x - 4)(x + 1)$

US **6.** Factor.

a) $36x^2 - 49y^2$ **b)** $0.5x^2 - 3.5x + 5$

c) $10x^2 + 29x - 21$ **d)** $\frac{1}{5}x^2 - \frac{1}{180}y^2$

US **7.** Factor.

a) $(7x + 4)^2 - (3y - 2)^2$ **b)** $3(2x - 1)^2 + 14(2x - 1) + 8$

US **8.** Determine whether $2x - 5$ is a factor of each polynomial.

a) $10x^2 + 23x - 5$ **b)** $6x^2 - 17x + 5$

ANSWERS

1. B **2. a)** $2(x^2 - 7x + 8)$ **b)** no common factors **c)** $3(2n^2 - 5n - 9)$ **d)** $-4(8 - 12x + 9x^2)$ **e)** $15(2y^2 - 3y + 5)$ **f)** $-9(5x^2 - 3x - 4)$ **3. a)** $(x + 1)(x + 15)$ **b)** $(y - 4)(y + 9)$ **c)** does not factor **d)** $2(x - 7)(x - 1)$ **e)** $3(2y + 1)(2y + 5)$ **f)** $-4(2n^2 - 4n + 5)$ **4.** $(3x - 2)$ units and $(x + 10)$ units **5.** C **6. a)** $(6x - 7y)(6x + 7y)$ **b)** $0.5(x - 2)(x - 5)$ **c)** $(5x - 3)(2x + 7)$ **d)** $\frac{1}{180}(6x - y)(6x + y)$ **7. a)** $(7x - 3y + 6)(7x + 3y + 2)$ **b)** $(6x - 1)(2x + 3)$ **8. a)** no **b)** yes

Extra Material

Graphs for *Check Your Understanding* answers

THINK FURTHER

RM, CR1

In *Example 1a*, why were the values of x chosen so that $x + 1$ is a perfect square?

To make it easier to graph the radical function

In *Example 1*, what is the domain of each radical function?

Example 1a: for $y = \sqrt{x + 1}$, the domain is: $x \geq -1$
Example 1b: for $y = \sqrt{2x + 7}$, the domain is: $x \geq -3.5$
Example 1c: for $y = \sqrt{4 - x}$, the domain is: $x \leq 4$

When the root of a radical equation is not an integer, graphing technology can be used to solve the equation.

RM, US, CR1

Check Your Understanding

2. Use graphing technology to solve: $3 + 4x = \sqrt{2 - 3x}$
Give the solution to the nearest tenth.

Graph the related functions:
$y = 3 + 4x$ and $y = \sqrt{2 - 3x}$
The root of $3 + 4x = \sqrt{2 - 3x}$ is: $x \doteq -0.3$

Example 2 — Solving a Radical Equation Using Graphing Technology

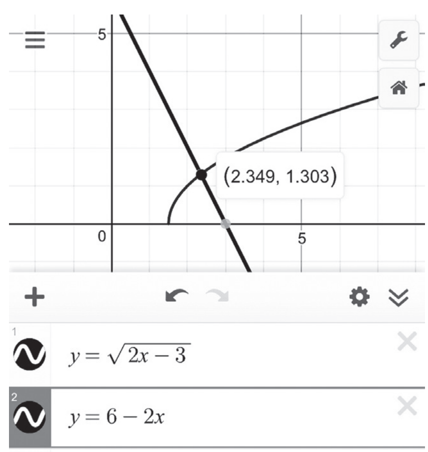
Use graphing technology to solve: $\sqrt{2x - 3} = 6 - 2x$
Give the solution to the nearest tenth.

SOLUTION

$\sqrt{2x - 3} = 6 - 2x$

Graph each related function:
$y = \sqrt{2x - 3}$ and $y = 6 - 2x$

The root of the equation $\sqrt{2x - 3} = 6 - 2x$ is the x-coordinate of the point where the graphs of $y = \sqrt{2x - 3}$ and $y = 6 - 2x$ intersect.

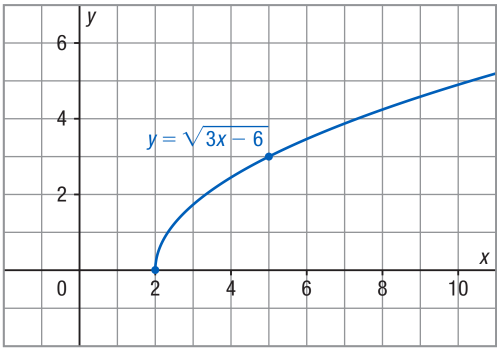


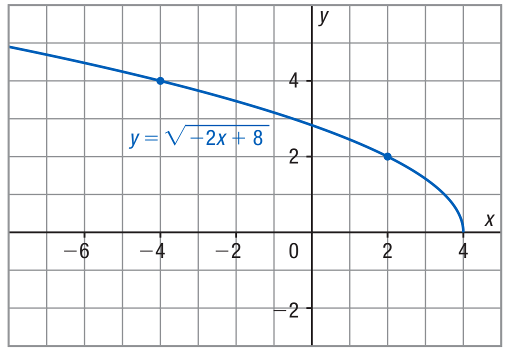
The graphs intersect at the point with coordinates: (2.349, 1.303)
So, the root of $\sqrt{2x - 3} = 6 - 2x$ is: $x \doteq 2.3$

TECHNOLOGY NOTE

If students do not have internet access, they could use a graphing calculator.

Check Your Understanding
Answer:
2. $x \doteq -0.3$



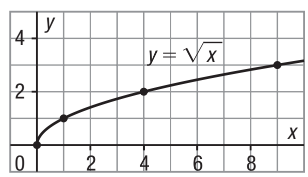


Compare the equations:
$y = \sqrt{3x - 6}$
$0 = \sqrt{3x - 6}$
The solution of $\sqrt{3x - 6} = 0$ is the x-intercept of the graph of $y = \sqrt{3x - 6}$; that is, the value of x when $y = 0$.
The solution is $x = 2$.

Compare the equations:
$y = \sqrt{-2x + 8}$
$0 = \sqrt{-2x + 8}$
The solution of $\sqrt{-2x + 8} = 0$ is the x-intercept of the graph of $y = \sqrt{-2x + 8}$; that is, the value of x when $y = 0$.
The solution is $x = 4$.

A **radical function** is a function that contains only a radical expression. Functions such as $y = \sqrt{3x - 6}$ and $y = \sqrt{-2x + 8}$ are **radical functions**.

Here is the graph of the radical function $y = \sqrt{x}$.



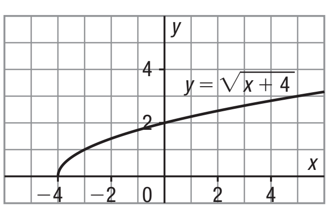
The square root of a number x is only defined for non-negative values of x, so the domain of $y = \sqrt{x}$ is $x \geq 0$, and the range is $y \geq 0$.

Since $x \geq 0$ and $y \geq 0$, the graph of $y = \sqrt{x}$ starts at the origin and extends in only one direction.

A **radical equation** is an equation with at least one radical whose radicand contains a variable.

A solution to a radical equation is a **root** of the equation.

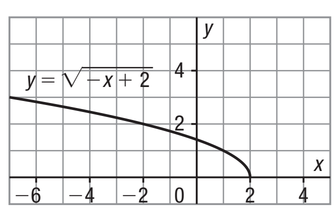
Here is the graph of $y = \sqrt{x + 4}$.



Since $x + 4 \geq 0$, then $x \geq -4$
So, the domain of $y = \sqrt{x + 4}$ is $x \geq -4$;
and the range is $y \geq 0$.

From the graph, the root of the radical equation $\sqrt{x + 4} = 0$ is the value of x when $y = 0$; that is, the x-intercept. The root is $x = -4$.

Here is the graph of $y = \sqrt{-x + 2}$.



Since $-x + 2 \geq 0$, then $x \leq 2$
So, the domain of $y = \sqrt{-x + 2}$ is $x \leq 2$; and the range is $y \geq 0$.

From the graph, the root of the radical equation $\sqrt{-x + 2} = 0$ is the value of x when y = 0; that is the x-intercept.
The root is $x = 2$.

TEACHER NOTE

In *Example 1*, students could use graphing technology to generate each table of values.

TEACHER NOTE

For *Example 1b*, students may think they can collect like terms, and want to write $\sqrt{2x + 7} + x = 4$ as $\sqrt{2x} + x = 4 - \sqrt{7}$. Remind students that the square root of the sum of terms is not equal to the sum of the square roots of the terms.

TEACHER NOTE

DI: Common Difficulties

In *Example 1*, for students who have difficulty understanding why the solution of a radical equation is determined by finding the coordinates of the point of intersection of two graphs, review solving a system of linear equations graphically.