RETHINKING FRACTIONS

8 CORE CONCEPTS TO SUPPORT ASSESSMENT AND LEARNING

CATHERINE D. BRUCE • TARA FLYNN • SHELLEY YEARLEY

CATHERINE D. BRUCE
TARA FLYNN
SHELLEY YEARLEY

Rethinking Fractions

8 Core Concepts to Support Assessment and Learning

Pearson

Pearson Canada Inc., 26 Prince Andrew Place, North York, Ontario M3C 2H4

ISBN 978-0-13-756821-5

ScoutAutomatedPrintCode

Printed and bound in the United States of America

Marketing Manager: Patti Henderson
Managing Editor: Joanne Close
Developmental Editor: Sarah Mawson
Project Managers, Editorial: Lisa Dimson, Cathy Fraccaro
Copy Editor: Cathy Fraccaro
Proofreader: Kathleen ffolliott
Project Manager, Production: Haley Muñoz
Manager, Project Management K–12: Alison Dale
Cover Design: Alex Li
Cover Image: Shutterstock
Interior Design: Alex Li
Composition: Straive
Vice-President, Product Development: Mark Cobham

Table of Contents

Acknowledgements

Rethinking Fractions is grounded in classroom-based research with teachers and students over many years, across multiple Canadian school districts, and in consultation with the national and international research community. We are grateful for the unfailing support from our collaborators, our funding partners, and the broader mathematics education sphere. Tackling the teaching and learning of fractions head on has been equal parts exciting and daunting. It was only through our partnerships that this was made possible.

The authors would like to acknowledge, with our deep gratitude, the many educators who participated in collaborative action research to explore fractions teaching and learning, some of you over several years. Your curiosity, openness, and dedication to students have been a constant source of inspiration. Thank you for putting your trust in us and in the process:

Hastings and Prince Edward District School Board (HPEDSB): Chris Aikman, Claudia Allanson, Lisa Allery, Jeannie Curtis, Danica Donald, David Fry, Chris Lee, Maurice McFadden, Tracy Vanslyke

Kawartha Pine Ridge District School Board (KPRDSB): Devin Arnold, Stephanie Cavanagh, Lisa Collison, Gorica Davis, Tasha Deline, Marie Elliott, Karen Fortin, Ashley Gall, Jennifer Gemmiti, Heather Hedges, Brandi Hollinger, Suzanne Hunt, Ben Imeson, Deb Knox, Kerry Lee, Janice MacKenzie, Gerry McBride, Laurie Moher, Matthew Parsons, Amy Simmons, Heather Taylor, Kim Usher, Melanie Whitehead, Jennifer Willshaw, Kerry Zinkiewich

Ottawa-Carleton District School Board (OCDSB): Dana Daley, Kelly Dykes, Suhana Kadoura, Julie LeFleur, Lisa Potter, Karen Prytalak, Hilary Saari

Simcoe County District School Board (SCDSB): Anna-Marie Boulding, Lisa Carl, Judy Cathcart, Scott Dowling, Chris Hogg, Carol Imhoff, Cathy Lethbridge, Kim Limoges, Kit Luce, Michael McAlpine, Stephanie McCuaig, Cheryl McGinnis, Kristen Muscat-Fennell, Susan Nicholson, Jan Olson, Jennifer Patterson, Jasmine Robertson, Jay Speijer, Trish Steele, Heidi Zupancic

Trillium Lakelands District School Board (TLDSB): Marianne Auger, Jennifer Bellevue, Dustin Belton, Andrea Borsyiu, Brenda Browers, Kevin Buchanan, Sherry Bush, Jane Crockford, Alexis David, Marcia Delgado, Chris Dowell, Sharon Fennell, Daniel Fockler, Martin Gage, Anne Grant, Erin Jenkins, Jim Kearse, Michelle Kernohan, Amy Klose, Carolyn Lewis, Sam Little, Matt Lowell, Patti Lyons, June McCallum, Melissa McEachran, Steve Messer, Jennifer Mills, Jennifer Paton, Greg Pitts, Ed Poropot, Jennifer Richter, Maggie Rodgers, April Sanderson, Delane Sawyer, Jennifer Simpson, Mark Stamp, Melissa Stephens, Jennifer Thompson, Nancy Wilkes, Deb Wines, Cheyenne Woods, Boyd Young

Thank you to all the additional educators and students who participated in field testing at different stages of the research from the following school boards: District School Board of Niagara, Durham District School Board, Hamilton–Wentworth District School Board, Kawartha Pine Ridge District School Board, and York Catholic District School Board.

For your additional input and support, our deepest thanks go to

- Lisa Collison (KPRDSB) and April Sanderson (TLDSB) whose explorations in Kindergarten inspired us and helped to inform our understanding of the very beginnings of fractions understanding;
- our long-term partners and friends at the Ontario Association of Mathematics Educators and the Ontario Mathematics Coordinators Association; and
- the Ontario Ministry of Education staff who supported this research at various stages: Shirley Dalrymple, Sandy DiLena, Myrna Ingalls, Trish Steele, and Jennipher Torney.

A special shout out goes to participants in various fractions learning sessions, including Math CAMPPP alumni, whose conversations helped us deepen our learning through the years.

This research builds on solid foundations laid by other researchers on whose shoulders we stand, including Janette Bobis (University of Sydney), Yukari Okamoto (UC Santa Barbara), John Ross (University of Toronto), and Tad Watanabe (Kennesaw State University).

This research would not have been possible without financial support from the Ontario Ministry of Education, the Canadian Social Sciences and Humanities Research Council, the Knowledge Network for Applied Education Research, and Trent University.

We acknowledge a very special thank you to the members of the Trent Math Education Research Collaborative over the years, including research assistants Sarah Bennett, Diana Chang, Mackenzie Condon, Nansi Harris, Rich McPherson, and Ralph Shaw.

We were fortunate to have two reviewers provide valuable guidance and insight during the development of this resource. We are grateful to Jen Carter, K–12 Numeracy Coordinator SD#22 and Jonathan So, Peel District School Board Math Resource Teacher K–12.

This resource would not have come to pass if it were not for the persistence of Joanne Close—a special thank you goes to you for your belief in the project and all your work to help make it happen. Thank you also to Sarah Mawson, Lisa Dimson, and Cathy Fraccaro for bringing your laser focus to these pages. It has been humbling to have such talented individuals supporting our vision and helping to bring it to life.

Finally, on a personal note, we would like to thank some very special people for their support over the years.

- *Cathy Bruce:* Throughout my math education research career, my two sons, Devon and Patrick, have been asked many times to "just try something out." Patrick brings visual–spatial and mental math approaches to his thinking. He spent many a car ride solving and depicting trial balloon problems. Devon brings a love for math matrices and logic to his thinking. In some research and writing stages for this book, Devon showed me his understanding of fractions using grids as his object-to-think-with. I would ask him to explain his thinking over and over again, which he did with patience. Thanks to Devon and Patrick, and a special thank you to Paul, for your steady support.
- *Tara Flynn:* Thank you to Noreen and Pat Flynn, Megan Flynn, Eamon and Lily Flynn Knell, Ralph Shaw, and Devlin Flynn for engaging with curiosity in conversations about fractions, for trying out questions and tasks, and letting us use examples of your thinking in our work. My deepest gratitude for your constant loving support, always. Special thanks and extra cookies to Mr. Peanutbutter for literally staying by my side during these last crucial months.
- *Shelley Yearley:* I am grateful to many friends, family, and colleagues who have been willing to think deeply about fractions. The time to travel and write would not have been possible without the support of my husband, Mark Lane. He was also willing to help me better understand contexts in which fractions occur in construction and business. Thank you to Isabella Duguid and Trinity Spry for being so willing to explore fractions tasks with me. A very special thank you to my daughter, Jayla, for agreeing to be an "on-demand" hand model and for putting up with endless fractions exploration at home.

Finally, thank you, reader, for joining us on this journey. To educators everywhere—thank you for the deeply important work you do and the efforts you take to show up for your students in the classroom every day. Our hearts are there with you.

CHAPTER 1

WELCOME TO *RETHINKING FRACTIONS*

We are delighted to bring you this resource, which is the culmination of over a decade of collaboration, research, curriculum development, and writing on fractions. Through the years, our focus has been on examining fractions teaching on both local and international levels, on deconstructing the core concepts of fractions understanding, and on finding ways to make the teaching and learning of fractions effective and engaging. Since the project began in 2011, our research team of Cathy Bruce, Tara Flynn, and Shelley Yearley has been fortunate to collaborate with educators and students in classrooms across Ontario. Through a process called collaborative action research, we were able to engage deeply in thinking about fractions with 86 educators, and to try things out in classrooms involving 2292 students. We were also fortunate to engage with many other educators who participated in professional learning about fractions. Dozens of educators field tested research artefacts and shared their data with us.

As a result of this collaboration, we were able to build strong evidence and confidence in "what works." Through in-depth analysis of both quantitative and qualitative data gathered through the research process, we have identified eight core concepts that underpin essential fractions learning and form the foundation for thinking about and working with fractions. These eight core concepts, and the related assessment and learning materials in this resource, have been collaboratively developed, field tested, and refined in a variety of real-world contexts. Our experience and data show that when students engage with these core concepts and tasks, they develop the understanding and skills to capably and confidently work with fractions.

Our hope is that this resource will help students to develop a deep understanding of

- unit fractions (the base of all fractions); and
- the eight core concepts (the keys to unlocking fractions understanding).

For educators, we hope that this resource will help you to assess students' understanding and possible misconceptions via the use of targeted, field-tested questions and recommended next steps that

- reveal student understanding; and
- respond to student needs (with precise and differentiated tasks and instruction).

The empowerment of students is an important underlying goal of our research program. We hope this resource will empower your students by helping to scaffold their understanding of this critical area of mathematics.

Overview

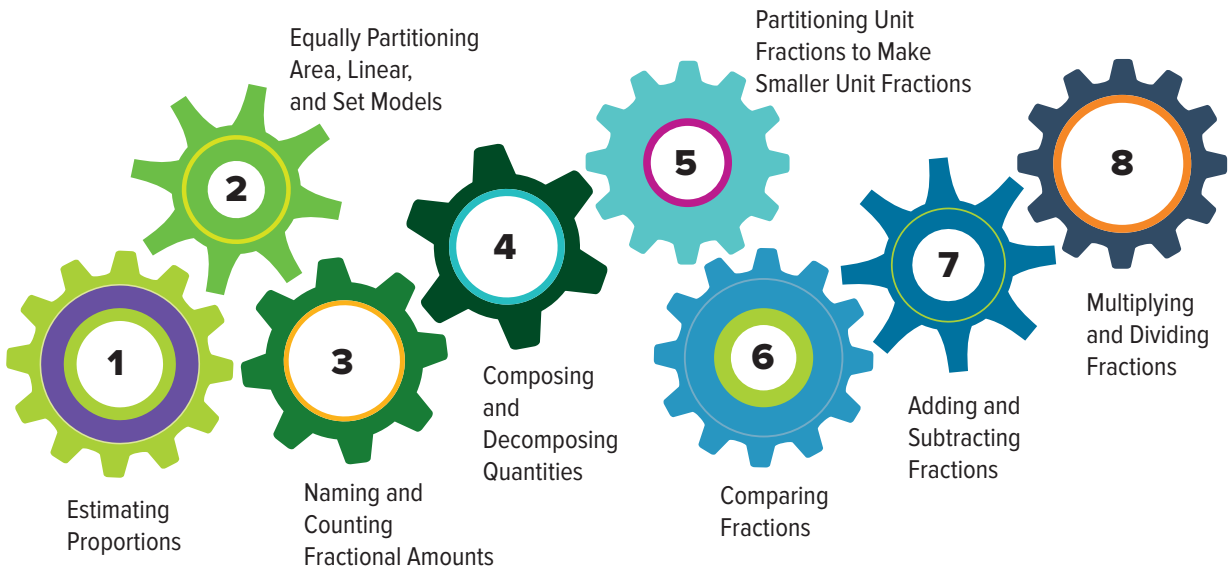
This overview outlines what you will find in *Rethinking Fractions*, both in the book and on the website, and provides guidance for how to use the resource to support instruction.

The 8 Core Concepts

See Chapter 4 for more information about the trajectory of learning and each of the eight core concepts.

Eight core concepts form a trajectory of learning that moves generally from foundational to more complex ideas. The trajectory is designed to help students form strong schemas and mental representations of fractions, allowing them to work with increasingly difficult fractions ideas with understanding.

Figure 1.1 The core concepts and trajectory were developed through our research with educators from Kindergarten to Grade 12 classrooms. Across all eight core concepts, there is a focus on the unit fraction as the foundation to understanding and working with fractions.



Core Concepts 1 to 4 lay a strong foundation for understanding fractions, while Core Concepts 5 to 8 allow students to apply this understanding to more complex thinking about fractions. Although the core concepts move in a direction of increasing complexity, they do not act as a linear prescription for learning. Rather, relationships across the core concepts allow us, as educators, to identify students' needs and support their learning by moving among concepts. For example, a student who is struggling with addition and subtraction of fractions (Core Concept 7) may be supported by opportunities to name and count fractional amounts (Core Concept 3) or to compose and decompose fractions (Core Concept 4). The assessment questions that are unpacked in detail in this resource will help to identify student areas of strength as well as areas of need.

Why Gears?

We have used gears in this resource as a metaphor for illustrating the manner in which the eight core concepts interconnect and, together, propel understanding of fractions forward. When gears are at work, one gear rotates clockwise, which causes the next gear to rotate counter-clockwise. The first "gear" or core concept puts our thinking in motion, then causes the next gear to move in a responsive and complementary direction toward new and more complex ideas. One gear gives us traction to move to the next powerful idea. We were inspired by the work of Seymour Papert, the mathematician and computer science pioneer who designed the *Logo* program to teach children computer language and programming at the Massachusetts Institute of Technology (MIT) in the 1970s and 1980s. He used gears as objects-to-think-with all his life—and connected gears to ratios, multiplication, algebra, and most other areas of mathematics and computer science in which he worked, including coding. We wanted to find a way to display the core concepts visually without implying a hierarchy or a strictly linear progression. For us, the visual of the gears reminds us of the movement and dynamic interplay of ideas across the core concepts—they work together to support a deep and flourishing understanding of fractions.

What's in the Book?

The book consists of two sections.

Section A

Chapters 2 through 4 give you background information on why fractions are important and the long-term value of teaching fractions with an emphasis on unit fractions; the types of reasoning that support fractions understanding; and an overview of each of the eight core fractions concepts that are developed in the book.

Section B

There are eight core concepts in this resource. Each concept is explored in its own chapter that includes the following components:

- A detailed explanation of the **core concept**
- Two or more quick-to-use **assessment questions** to gain a better sense of what students understand about the concept. Sample student responses provide insights into how students might respond to a question and what their response may tell you about their understanding. Based on student responses, **next steps** outline suggestions for follow-up and point to a task (or tasks) to extend student understanding.
- Two or more classroom **tasks** that focus on deepening student understanding of the core concept. These tasks include
  - an overview;
  - a description of the mathematics in the task;
  - a clearly defined task sequence;
  - questions to pose during the task;
  - precise instruction notes;

- suggestions of what to look for as students engage with the task; and
- notes about what we observed during field testing.

What's on the Website?

The website is accessed via a unique code that is printed on the inside front cover of your book. It contains both necessary and supplementary material to the book, and includes

- line masters with the individual assessment questions from the book as well as parallel questions that can be used for variety;
- line masters needed to complete the tasks;
- full assessments, with support to help you choose the right assessment according to your goals and the age/stage of your students;
- tools to analyze student results from the assessment questions;
- a glossary of selected mathematical terms used in this book (these terms are usually indicated with boldface type the first time they are used); and
- curriculum maps that identify how the core concepts are aligned with mathematics curricula across Canada as well as with NCTM standards.

Using This Resource

There are two main ways you might use this resource.

1. You may want to start by using the assessment questions to gather diagnostic data in order to gain formative insights about your students' knowledge, skills, and understanding related to fractions. This data will help you to decide where to focus your energies—which concept(s) may require only a light touch, and which concepts may require a deeper focus to guide your instruction. The assessment questions, student samples, analysis, and next steps are designed to help give you deeper insight into your students' understanding and support your learning and comfort with fractions.
2. You may decide to jump directly into the tasks that are aligned with each of the eight core concepts. In this case, we recommend starting with the tasks in Core Concept 1 (*Estimating Proportions*) and working through the eight core concepts in order, since they form an intentional sequence or progression. If taking this approach, the assessment questions may still be used to gather formative data (for example, as practice questions or exit tickets) or even summative data, if that fits with your assessment goals. Although the assessment questions tend to be written through a diagnostic lens, the unpacking of the questions will support you in gaining insights into student understanding in whatever way you choose to use them.

How This Resource Works through the Grades

This resource spans Grades 3 through 9, with some reach below and above these grades. You will notice that there aren't grade-specific recommendations for any of the assessment questions or tasks. During the research process that led to the writing of this resource, our collaborative research teams of course considered grade-level and curricular expectations, as this is the immediate context of every classroom teacher. However, our primary lens was on the mechanisms by which fractions understanding is built. We set aside our own assumptions and biases about how students come to learn fractions (which are often informed by curricula) to focus on the complex web of concepts that underpin fractions and how the understanding of these concepts grows. We wanted to deconstruct the building blocks—or, using what for us is a more apt metaphor, take apart the gears—of these concepts to understand how they connect and how they might be sequenced in instruction to move student thinking forward. What resulted is the eight core concepts—a developmental continuum focused on the concepts that underpin fractions understanding.

Throughout the research, educators found that the questions and tasks worked across multiple grades. They were intentionally designed to be open enough to be accessed by students from a wide range of ages and stages and to provide challenge for students who are ready to show more complex or nuanced understanding. Additionally, we note that students across a range of grades may not have had opportunities to develop all of these concepts, and so may have gaps that have led to a shaky understanding of fractions.

The core concepts build on one another. A sequential approach starting with Core Concept 1 (no matter your grade or level) is good for all students, and essential for some. For later grades and older ages, you might think that it would be better to start later in the series of core concepts. Through our classroom research, we learned that it is important for all students to start with Core Concept 1. Some earlier concepts may be familiar to older students, and they may not need much time with these (e.g., estimating proportions). You may be able to move through those core concepts at an accelerated pace with older students to establish a foundation before moving to more complex ideas. In our experience, the investment was "time-positive—meaning that although starting with Core Concept 1 took a small investment of time up front, it paid off later as students tackled more complex ideas with confidence. This applied not only to later fractions concepts but also to areas beyond fractions—algebra, in particular. (Each core concept chapter includes a "Connecting to Other Mathematics" section that describes one or two math topics that link to the core concept being considered. Additional related topics are described on the *Rethinking Fractions* website.) Many tasks provide options for variations and extensions to further tailor the learning to your students' needs.

Digging Deeper into the Components of *Rethinking Fractions*

This resource centres on three main components:

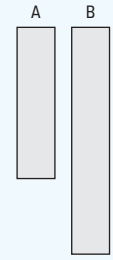
- a trajectory of eight core concepts of fractions that serves as the framework for the resource;
- assessment questions for each core concept to help you assess students' understanding; and
- tasks aligned to the assessment questions that consolidate and extend students' understanding.

Assessment Questions

The assessment questions are simple paper-and-pencil questions that can be answered in approximately 10 minutes. A line master for each assessment question is provided on the website. There are two copies of a question per page so you can print, copy, and cut the page in half to distribute to students. If using electronic displays and discussion format, you can project an image of the question from the line master file. Alternatively, the assessment questions are also available in the form of multi-question assessments on the website.

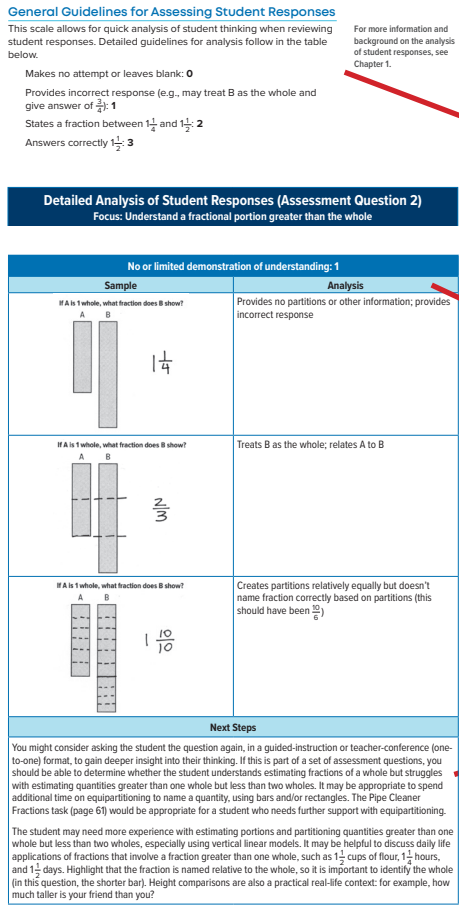
Figure 1.2 Here is a sample assessment question from Core Concept 1.

ASSESSMENT QUESTION 2: Estimating the Value of a Quantity Greater than a Whole

Assessment Question	**If A is 1 whole, what fraction does B show?** 
What Is Being Assessed	Students' understanding of a fractional portion greater than the whole
Implementation	To administer the individual question, use Line Master 1-2 (estimated time: under 10 minutes). To administer as part of a larger assessment, see the options available on the website (estimated time: varies). *Note: See Chapter 1 for additional information on possible strategies for implementing assessment questions.*

Support for Analyzing Student Responses

Along with each assessment question, we unpack the mathematics underlying the question (What Mathematics Lives in This Question?). We also provide explanations of what we typically saw in student responses during the research (What to Look For in Student Responses). In addition, you will find the following supports to help you unpack student thinking in response to each question:



General Guidelines for Assessing Student Responses

This scale allows for quick analysis of student thinking when reviewing student responses. Detailed guidelines for analysis follow in the table below.

For more information and background on the analysis of student responses, see Chapter 1.

Makes no attempt or leaves blank: **0**

Provides incorrect response (e.g., may treat B as the whole and give answer of $\frac{3}{4}$): **1**

States a fraction between $1\frac{1}{4}$ and $1\frac{1}{2}$: **2**

Answers correctly $1\frac{1}{2}$: **3**

Detailed Analysis of Student Responses (Assessment Question 2)
Focus: Understand a fractional portion greater than the whole

No or limited demonstration of understanding: 1

Sample	Analysis
If A is 1 whole, what fraction does B show? A B $1\frac{1}{4}$	Provides no partitions or other information; provides incorrect response
If A is 1 whole, what fraction does B show? A B $\frac{2}{3}$	Treats B as the whole; relates A to B
If A is 1 whole, what fraction does B show? A B $1\frac{10}{10}$	Creates partitions relatively equally but doesn't name fraction correctly based on partitions (this should have been $\frac{10}{6}$)

Next Steps

You might consider asking the student the question again, in a guided-instruction or teacher-conference (one-to-one) format, to gain deeper insight into their thinking. If this is part of a set of assessment questions, you should be able to determine whether the student understands estimating fractions of a whole but struggles with estimating quantities greater than one whole but less than two wholes. It may be appropriate to spend additional time on equipartitioning to name a quantity, using bars and/or rectangles. The Pipe Cleaner Fractions task (page 61) would be appropriate for a student who needs further support with equipartitioning.

The student may need more experience with estimating portions and partitioning quantities greater than one whole but less than two wholes, especially using vertical linear models. It may be helpful to discuss daily life applications of fractions that involve a fraction greater than one whole, such as $1\frac{1}{2}$ cups of flour, $1\frac{1}{4}$ hours, and $1\frac{1}{2}$ days. Highlight that the fraction is named relative to the whole, so it is important to identify the whole (in this question, the shorter bar). Height comparisons are also a practical real-life context: for example, how much taller is your friend than you?

For ease of reference while marking, we provide **general guidelines for assessing student responses**. Our scale for assessing student responses is based on analysis of more than 45 000 individual assessment responses and was refined over many years—often with extensive discussion during moderated marking of assessments with educators who participated in the research. Responses are categorized on a scale of 0 to 3.

For each question, we provide a **detailed analysis**, including samples of student responses. In the creation of this resource, we recreated all student samples to reduce visual distractions of poor handwriting, extraneous information (e.g., patterns or colours), and messy (but accurate) representations. Sometimes, in order to reduce the overall number of samples, aspects from multiple samples were combined into one recreated sample.

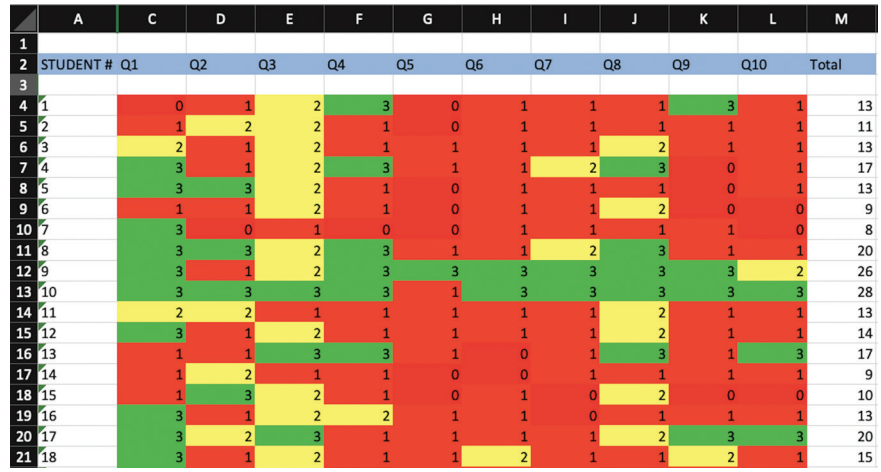
For each type of response, we provide concrete **next steps**, including recommendations for which task(s) to try next.

Acknowledging the Tensions of Scoring Student Responses

The guidelines for scoring student responses uses a scale from 0 to 3, with 0 indicated for "no response," 1 indicated for "no or limited understanding," 2 indicated for "some understanding," and 3 indicated for "clear understanding." We understand that it can be uncomfortable to assign such a simple scale to written representations of student thinking. We share this discomfort. It is especially challenging when evaluating the work of your own students, whom you know so well. You *know* from conversations and other modalities of showing their thinking that they know more than they are putting on the page. They may have been hungry or distracted or just having an "off" day. We encourage you to focus on what they have shown on the page, without making inferences about what they know based on other information. This is exceedingly difficult but can glean very interesting data, particularly if you are using a pre–post strategy, because it will allow you to see the change in their expression of what they understand.

The scale was designed to be simple and quick to score, so that you can quickly get to a stage of using the assessment data to inform instruction. Also note that assigning a numeric scale to the range of responses allows you to work with a heat map, if you choose, which is a powerful way of making your class data visible and guiding where to go with individuals and with the group.

Figure 1.3 A formatted Excel spreadsheet that will allow you to use the class results of an assessment to generate a heat map is available on the website. The heat map is a visual display of the data from student responses that helps you to see the areas of strength and need.



	A	C	D	E	F	G	H	I	J	K	L	M
1												
2	STUDENT #	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
3												
4	1	0	1	2	3	0	1	1	1	3	1	13
5	2	1	2	2	1	0	1	1	1	1	1	11
6	3	2	1	2	1	1	1	1	2	1	1	13
7	4	3	1	2	3	1	1	2	3	0	1	17
8	5	3	3	2	1	0	1	1	1	0	1	13
9	6	1	1	2	1	0	1	1	2	0	0	9
10	7	3	0	1	0	0	1	1	1	1	0	8
11	8	3	3	2	3	1	1	2	3	1	1	20
12	9	3	1	2	3	3	3	3	3	3	2	26
13	10	3	3	3	3	1	3	3	3	3	3	28
14	11	2	2	1	1	1	1	1	2	1	1	13
15	12	3	1	2	1	1	1	1	2	1	1	14
16	13	1	1	3	3	1	0	1	3	1	3	17
17	14	1	2	1	1	0	0	1	1	1	1	9
18	15	1	3	2	1	0	1	0	2	0	0	10
19	16	3	1	2	2	1	1	0	1	1	1	13
20	17	3	2	3	1	1	1	1	2	3	3	20
21	18	3	1	2	1	1	2	1	1	2	1	15

You may not necessarily agree at first glance with the parameters we've established for scoring, but we ask you to trust the guidelines as much as you are able when marking the assessment questions and, as much as possible, to follow the parameters outlined.

Debate is healthy and welcomed. If you are able to use this resource as part of a collaborative inquiry process and have an opportunity to do moderated marking of the assessments, you may find yourselves in lively conversation over scoring while examining individual student responses. We found that it was during these discussions that some of our deepest learning about the mathematics itself and student learning took place.

Suggestions for Using the Assessment Questions

There are two main ways to use the assessment questions.

Administer One Assessment Question at a Time

You can have students complete one assessment question at a time rather than administering a full assessment with multiple questions. This may be of interest to you if you are mainly focused on using the resource to support your own professional learning and want to take more time to explore one core concept at a time. This could also be a strategy for using the resource to support a collaborative inquiry or action research project, or for a book study. This approach might also be of interest if you want to punctuate your fractions instruction over the course of the school year to introduce or revisit concepts over time. Information from a student's answer can be used in the following ways:

- Diagnostic (assessment *for* and *of* learning): Gain insights into a student's thinking in relation to a specific fractions concept.
- Formative (assessment *as* and *for* learning): Follow up with the whole class or in a teacher-conference (one-to-one) format to explore student thinking further.
- Summative (assessment *of* learning): Use results of the question as part of a consolidation strategy that provides insights into the

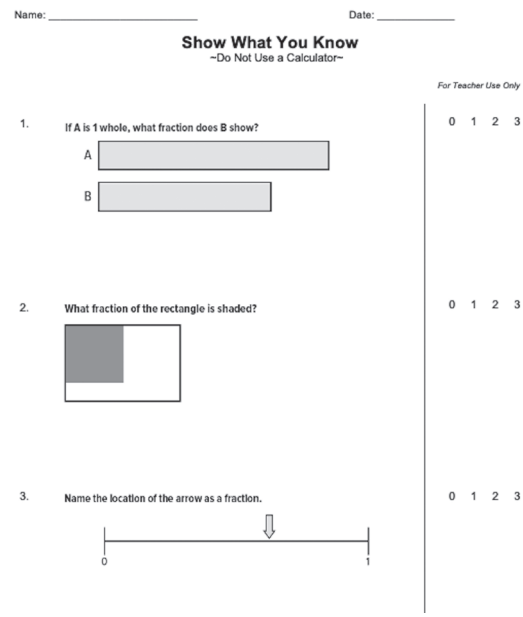
You may notice that the analysis of student samples and next steps are given primarily through a diagnostic lens—leaning toward detailed recommendations for what students may need and where to go next based on what their responses indicate about their understanding. However, even if you are using the information from assessment questions for a purpose other than diagnostic assessment, this same information is still highly relevant and should help to inform your unpacking of student responses.

level of learning the student has developed after engaging in learning tasks and class discussions related to the core concept.

Administer a Full Assessment

You can administer a full assessment with multiple questions designed to gain a snapshot of student understanding across several core concepts. Ready-made sample assessments are available on the website. There you can choose from a number of assessments depending on the age and stage of your students. (Guidance for choosing which assessment to use can also be found on the website.) It is also possible for you to generate your own assessments using the assessment questions and/or the parallel questions, which are provided on the website in reproducible form.

Figure 1.4 Full assessments are available on the website. They are formatted to allow you to select a score for each response from a scale of 0 to 3. Scoring guides are provided for each question.


Name: _____ Date: _____

Show What You Know

~Do Not Use a Calculator~

For Teacher Use Only

1. If A is 1 whole, what fraction does B show? 0 1 2 3

A

B

2. What fraction of the rectangle is shaded? 0 1 2 3

3. Name the location of the arrow as a fraction. 0 1 2 3

0 1

Information from the student responses can be used in the following ways:

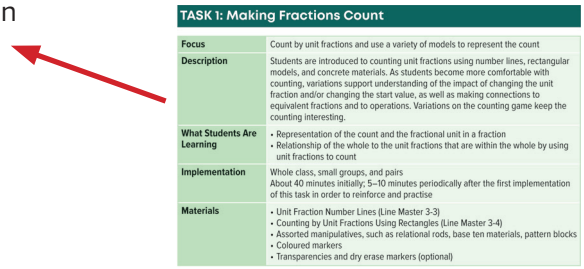
- Diagnostic (assessment *for* and *of* learning): Prior to teaching, assess what students already understand and in what areas they need additional attention to support learning. Data from student responses to the assessment questions should inform instruction (what to focus on, in particular, and with which students).
- Formative (assessment *as* and *for* learning): Use an exploratory strategy where students work in homogeneous pairs to answer each question together. Follow up with a whole-group or pairs discussion that helps to determine what needs further exploration. Assessments can also be administered at the midpoint of instruction to determine what key ideas need more attention based on student responses.

- Summative (assessment *of* learning): Use results of the questions as part of a consolidation strategy that provides insights into the level of learning the student has developed after engaging in learning tasks and whole-group discussions related to the cluster of core concepts in focus.
- Pre- and post-assessment: This approach usually has the goal of looking at change over time and is based on specific instruction and learning opportunities for students. Parallel questions are available on the website so that students have slightly different questions for each assessment.

Classroom Tasks

Each core concept chapter includes two or more field-tested classroom tasks that can be used to deepen student understanding.

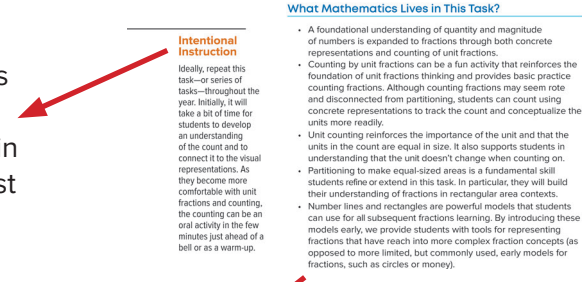
An **overview chart** describes the key information you'll want to know before trying a task, including a description of what students will be learning and doing as well as the timing and materials needed to implement the task in your classroom. Depending on the age and prior experiences of your students, some tasks may require less time than indicated.



TASK 1: Making Fractions Count

Focus	Count by unit fractions and use a variety of models to represent the count
Description	Students are introduced to counting unit fractions using number lines, rectangular models, and concrete materials. As students become more comfortable with counting, variations support understanding of the impact of changing the unit fraction and/or changing the start value, as well as making connections to equivalent fractions and to operations. Variations on the counting game keep the counting interesting.
What Students Are Learning	• Representation of the count and the fractional unit in a fraction • Relationship of the whole to the unit fractions that are within the whole by using unit fractions to count
Implementation	Whole class, small groups, and pairs About 40 minutes initially, 5–10 minutes periodically after the first implementation of this task in order to reinforce and practise
Materials	• Unit Fraction Number Lines (Line Master 3-3) • Counting by Unit Fractions Using Rectangles (Line Master 3-4) • Assorted manipulatives, such as relational rods, base ten materials, pattern blocks • Coloured markers • Transparencies and dry erase markers (optional)

Intentional Instruction margin notes provide suggestions for successfully implementing tasks with your class or describe the rationale behind implementation strategies. Other margin notes in the tasks describe possible variations or suggest questions you might pose at different points during a task.



Intentional Instruction

Ideally, repeat this task—or series of tasks—throughout the year. Initially, it will take a bit of time for students to develop an understanding of the count and to connect it to the visual representations. As they become more comfortable with unit fractions and counting, the counting can be an oral activity in the few minutes just ahead of a bell or as a warm-up.

What Mathematics Lives in This Task?

- A foundational understanding of quantity and magnitude of numbers is expanded to fractions through both concrete representations and counting of unit fractions.
- Counting by unit fractions can be a fun activity that reinforces the foundation of unit fractions thinking and provides basic practice counting fractions. Although counting fractions may seem rote and disconnected from partitioning, students can count using concrete representations to track the count and conceptualize the units more readily.
- Unit counting reinforces the importance of the unit and that the units in the count are equal in size. It also supports students in understanding that the unit doesn't change when counting on.
- Partitioning to make equal-sized areas is a fundamental skill students refine or extend in this task. In particular, they will build their understanding of fractions in rectangular area contexts.
- Number lines and rectangles are powerful models that students can use for all subsequent fractions learning. By introducing these models early, we provide students with tools for representing fractions that have reach into more complex fraction concepts (as opposed to more limited, but commonly used, early models for fractions, such as circles or money).

Every task is unpacked within a **What Mathematics Lives in This Task?** section, which describes the mathematics underlying the task.

The tasks also include the following features:

- a **Task Sequence**, which outlines steps you can follow to implement the task with your class, which often includes photos or diagrams;
- a **Consolidation** section that offers suggestions for class or small-group discussions to share and summarize the understandings students have developed during the task;
- a section called **What to Look For in Student Responses**; and
- a feature called **In our research** that describes insights from the field.

Some tasks also include **Extensions** or **Variations** that can be used to extend or adapt a task.