

PEARSON

# Pre-calculus 11

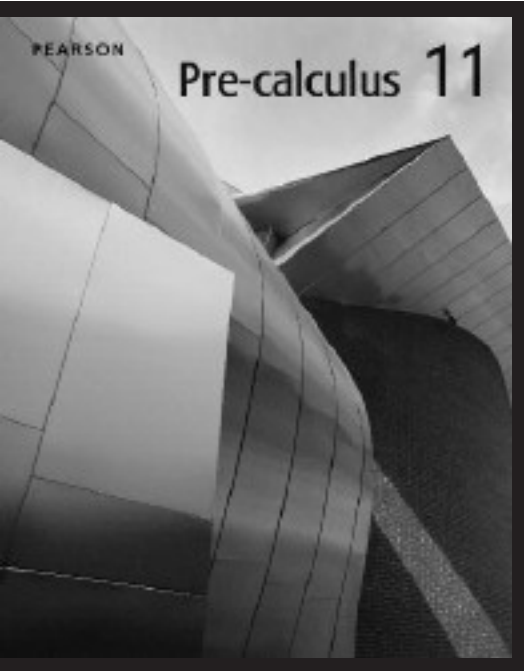
*my*WORKTEXT

**Sample  
Chapter 1**



PEARSON

# Pre-calculus 11



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PEARSON



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Pearson Canada Inc.  
26 Prince Andrew Place  
Don Mills, ON M3C 2T8  
Customer Service: 1-800-361-6128

ISBN-13 978-0-321-62449-9

**Publisher:** Mike Czukar

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**Design and Art Direction:** David Cheung

**Composition:** MPS Limited, a Macmillan Company

**Vice-President, Publishing:** Mark Cobham

1 2 3 4 5 – WC – 15 14 13 12 11

Printed and bound in Canada

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Pearson would like to thank its advisers and reviewers who helped shape the vision for *Pre-Calculus 11* through discussions and reviews of prototype materials and manuscript.

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# CONTENTS

<b>1 Sequences and Series</b>	<b>1</b>	<b>3 Solving Quadratic Equations</b>	<b>169</b>
1.1 Arithmetic Sequences . . . . .	2	3.1 Factoring Polynomial Expressions . . . . .	170
1.2 Arithmetic Series . . . . .	14	3.2 Solving Quadratic Equations by Factoring . .	184
Checkpoint 1 . . . . .	25	Checkpoint . . . . .	197
1.3 Geometric Sequences . . . . .	29	3.3 Using Square Roots to Solve Quadratic Equations . . . . .	200
1.4 Geometric Series . . . . .	43	3.4 Developing and Applying the Quadratic Formula . . . . .	212
Checkpoint 2 . . . . .	55	3.5 Interpreting the Discriminant . . . . .	228
1.5 Math Lab: Graphing Geometric Sequences and Series . . . . .	58	Study Guide . . . . .	240
1.6 Infinite Geometric Series . . . . .	63	Review . . . . .	242
Study Guide . . . . .	74	Practice Test . . . . .	246
Review . . . . .	76		
Practice Test . . . . .	80		
		<b>4 Analyzing Quadratic Functions</b>	<b>249</b>
<b>2 Absolute Value and Radicals</b>	<b>83</b>	4.1 Properties of a Quadratic Function . . . . .	250
2.1 Absolute Value of a Real Number . . . . .	84	4.2 Math Lab: Solving a Quadratic Equation Graphically . . . . .	263
2.2 Simplifying Radical Expressions . . . . .	95	4.3 Math Lab: Transforming the Graph of $y = x^2$ . . . . .	268
Checkpoint 1 . . . . .	107	Checkpoint 1 . . . . .	272
2.3 Adding and Subtracting Radical Expressions . . . . .	110	4.4 Analyzing Quadratic Functions of the Form $y = a(x - p)^2 + q$ . . . . .	276
2.4 Multiplying and Dividing Radical Expressions . . . . .	120	4.5 Equivalent Forms of the Equation of a Quadratic Function . . . . .	291
Checkpoint 2 . . . . .	135	4.6 Analyzing Quadratic Functions of the Form $y = ax^2 + bx + c$ . . . . .	301
2.5 Solving Radical Equations . . . . .	139	Checkpoint 2 . . . . .	313
Study Guide . . . . .	154	4.7 Modelling and Solving Problems with Quadratic Functions . . . . .	318
Review . . . . .	156	Study Guide . . . . .	328
Practice Test . . . . .	162	Review . . . . .	330
Cumulative Review Chapters 1–2 . . . . .	165	Practice Test . . . . .	336

## **5 Graphing Inequalities and Systems of Equations 339**

- 5.1 Solving Quadratic Inequalities in One Variable . . . . . 340
- 5.2 Graphing Linear Inequalities in Two Variables . . . . . 354
- 5.3 Graphing Quadratic Inequalities in Two Variables . . . . . 369
  - Checkpoint . . . . . 382
- 5.4 Math Lab: Solving Systems of Equations Graphically . . . . . 386
- 5.5 Solving Systems of Equations Algebraically. . 391
  - Study Guide . . . . . 406
  - Review. . . . . 408
  - Practice Test . . . . . 415
  - Cumulative Review Chapters 1–5 . . . . . 419

## **6 Trigonometry 425**

- 6.1 Angles in Standard Position in Quadrant 1. . 426
- 6.2 Angles in Standard Position in All Quadrants . . . . . 440
  - Checkpoint . . . . . 460
- 6.3 Math Lab: Constructing Triangles . . . . . 465
- 6.4 The Sine Law . . . . . 470
- 6.5 The Cosine Law . . . . . 491
  - Study Guide . . . . . 508
  - Review. . . . . 510
  - Practice Test . . . . . 516

## **7 Rational Expressions and Equations 521**

- 7.1 Equivalent Rational Expressions. . . . . 522
- 7.2 Multiplying and Dividing Rational Expressions . . . . . 533
  - Checkpoint 1 . . . . . 545
- 7.3 Adding and Subtracting Rational Expressions with Monomial Denominators. . . . . 549
- 7.4 Adding and Subtracting Rational Expressions with Binomial and Trinomial Denominators . . . . . 561
  - Checkpoint 2 . . . . . 575
- 7.5 Solving Rational Equations. . . . . 578
- 7.6 Applications of Rational Equations . . . . . 592
  - Study Guide . . . . . 604
  - Review. . . . . 606
  - Practice Test . . . . . 610

## **8 Absolute Value and Reciprocal Functions 613**

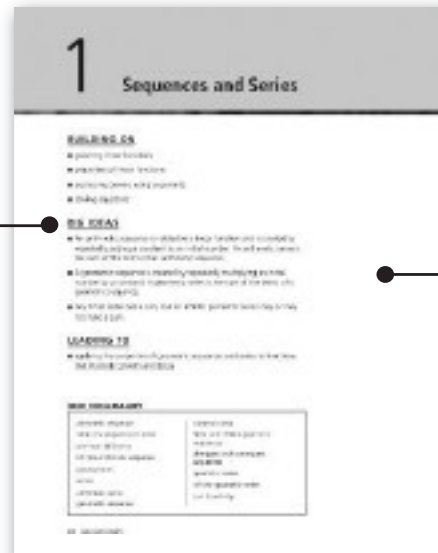
- 8.1 Absolute Value Functions . . . . . 614
- 8.2 Solving Absolute Value Equations . . . . . 631
  - Checkpoint . . . . . 648
- 8.3 Graphing Reciprocals of Linear Functions . . 652
- 8.4 Math Lab: Using Technology to Graph Reciprocals of Quadratic Functions. . . . . 667
- 8.5 Graphing Reciprocals of Quadratic Functions . . . . . 674
  - Study Guide . . . . . 690
  - Review. . . . . 692
  - Practice Test . . . . . 698
  - Cumulative Review Chapters 1–8. . . . . 701



# BUILD GOOD STUDY HABITS NOW.

## 1. Know What You Need to Learn


Use a **highlighter** to reinforce the **Big Ideas** of the chapter.

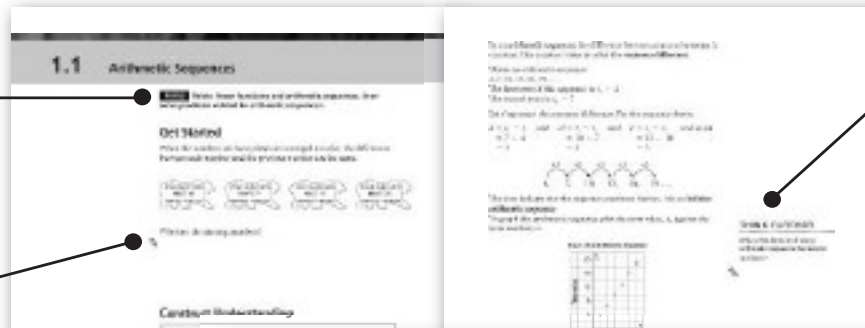


Use the **margin** throughout the worktext to record important ideas from class.

## 2. Build Your Understanding

Use a **highlighter** to reinforce the lesson **Focus**.

Use your worktext like a notebook. Write **solutions** on the page wherever you see the .

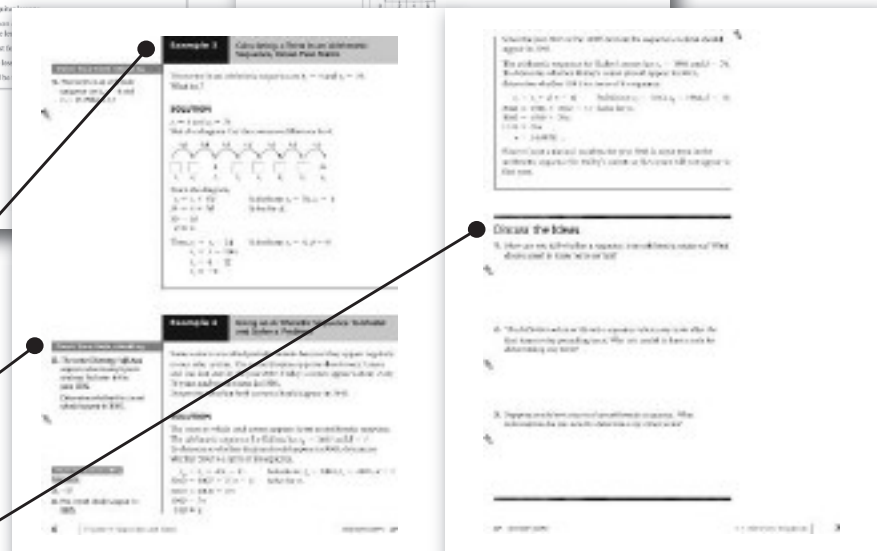


Use **Think Further** prompts to extend your thinking and apply your learning.

Refer to the **Examples** for strategies on solving problems.

Apply these strategies to **Check Your Understanding**.

Discuss the **Ideas** of each lesson and record your thoughts.





# PRACTICAL TIPS FOR USING myWORKTEXT

## 3. Practise What You've Learned

Try a range of the exercise questions.

Use Multiple-Choice questions to help you prepare for exams.

Record Study Notes to summarize your learning.

Use the margins to jot down notes about your thinking as the homework is taken up.

## 4. Develop Your Study Skills

Checkpoints occur at key intervals in the chapter to let you check your understanding so far.

The Study Guide summarizes important Concepts and key Skills from the chapter. Use this as a model to make your own study guides.

# PRACTICAL TIPS FOR USING myWORKTEXT

## 5. Prepare for Tests and Exams

Use **Review** pages for additional practice.



Try the **Practice Test** as a sample before you take a class test.

## Get the most out of your worktext

### Make it your own.

Highlight key words or important ideas. Work out solutions in the space provided. Write down your questions and ideas.

### Keep it for later.

The worktext becomes a record of your learning. It can be helpful later, when you take Grade 12 Pre-calculus and university math courses.



Try **MathXL**, an online homework and tutoring system.  
[www.pearsoncanada.ca/mathxl](http://www.pearsoncanada.ca/mathxl)

# 1

# Sequences and Series

## **BUILDING ON**

- graphing linear functions
- properties of linear functions
- expressing powers using exponents
- solving equations

## **BIG IDEAS**

- An arithmetic sequence is related to a linear function and is created by repeatedly adding a constant to an initial number. An arithmetic series is the sum of the terms of an arithmetic sequence.
- A geometric sequence is created by repeatedly multiplying an initial number by a constant. A geometric series is the sum of the terms of a geometric sequence.
- Any finite series has a sum, but an infinite geometric series may or may not have a sum.

## **LEADING TO**

- applying the properties of geometric sequences and series to functions that illustrate growth and decay

## **NEW VOCABULARY**

arithmetic sequence

term of a sequence or series

common difference

infinite arithmetic sequence

general term

series

arithmetic series

geometric sequence

common ratio

finite and infinite geometric sequences

divergent and convergent sequences

geometric series

infinite geometric series

sum to infinity

# 1.1 Arithmetic Sequences

**FOCUS** Relate linear functions and arithmetic sequences, then solve problems related to arithmetic sequences.

## Get Started

When the numbers on these plates are arranged in order, the differences between each number and the previous number are the same.



What are the missing numbers?



## Construct Understanding

Saket took guitar lessons.

The first lesson cost \$75 and included the guitar rental for the period of the lessons.

The total cost for 10 lessons was \$300.

Suppose the lessons continued.

What would be the total cost of 15 lessons?



In an **arithmetic sequence**, the difference between consecutive **terms** is constant. This constant value is called the **common difference**.

This is an arithmetic sequence:

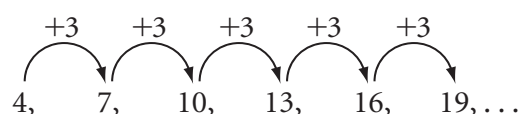
4, 7, 10, 13, 16, 19, ...

The first term of this sequence is:  $t_1 = 4$

The second term is:  $t_2 = 7$

Let  $d$  represent the common difference. For the sequence above:

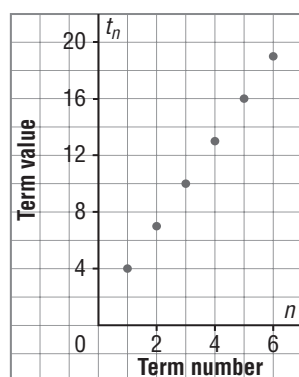
$$\begin{array}{llll} d = t_2 - t_1 & \text{and} & d = t_3 - t_2 & \text{and} & d = t_4 - t_3 & \text{and so on} \\ = 7 - 4 & & = 10 - 7 & & = 13 - 10 & \\ = 3 & & = 3 & & = 3 & \end{array}$$



The dots indicate that the sequence continues forever; it is an **infinite arithmetic sequence**.

To graph this arithmetic sequence, plot the term value,  $t_n$ , against the term number,  $n$ .

Graph of an Arithmetic Sequence



The graph represents a linear function because the points lie on a straight line. A line through the points on the graph has slope 3, which is the common difference of the sequence.

In an arithmetic sequence, the common difference can be any real number.

Here are some other examples of arithmetic sequences.

- This is an *increasing* arithmetic sequence because  $d$  is positive and the terms are increasing:  
 $\frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, \dots$ ; with  $d = \frac{1}{4}$
- This is a *decreasing* arithmetic sequence because  $d$  is negative and the terms are decreasing:  
 $5, -1, -7, -13, -19, \dots$ ; with  $d = -6$

## THINK FURTHER

Why is the domain of every arithmetic sequence the natural numbers?



## THINK FURTHER

What sequence is created when the common difference is 0?



## Example 1

## Writing an Arithmetic Sequence

### Check Your Understanding

1. Write the first 6 terms of:
  - a) an increasing arithmetic sequence
  - b) a decreasing arithmetic sequence

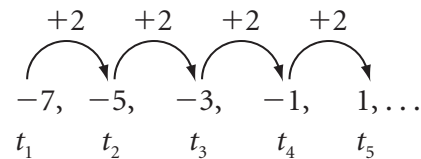


Write the first 5 terms of:

- a) an increasing arithmetic sequence
- b) a decreasing arithmetic sequence

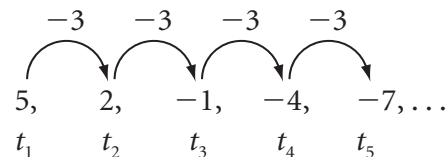
### SOLUTION

- a) Choose any number as the first term; for example,  $t_1 = -7$ .  
The sequence is to increase, so choose a positive common difference; for example,  $d = 2$ . Keep adding the common difference until there are 5 terms.



The arithmetic sequence is:  $-7, -5, -3, -1, 1, \dots$

- b) Choose the first term; for example,  $t_1 = 5$ .  
The sequence is to decrease, so choose a negative common difference; for example,  $d = -3$ .



The arithmetic sequence is:  $5, 2, -1, -4, -7, \dots$

Consider this arithmetic sequence:  $3, 7, 11, 15, 19, 23, \dots$

To determine an expression for the **general term**,  $t_n$ , use the pattern in the terms. The common difference is 4. The first term is 3.

$t_1$	$3 = 3 + 4(0)$
$t_2$	$7 = 3 + 4(1)$
$t_3$	$11 = 3 + 4(2)$
$t_4$	$15 = 3 + 4(3)$
$\vdots$	
$t_n$	$3 + 4(n - 1)$

For each term, the second factor in the product is 1 less than the term number.

The second factor in the product is 1 less than  $n$ , or  $n - 1$ .

### Check Your Understanding

#### Answers:

1. a)  $-20, -18, -16, -14, -12, -10, \dots$
- b)  $100, 97, 94, 91, 88, 85, \dots$

Write:

$$\begin{array}{ccccccc}
 t_n & = & 3 & + & 4(n - 1) \\
 \uparrow & & \uparrow & & \uparrow \\
 \text{general} & & \text{first} & & \text{common} \\
 \text{term} & & \text{term} & & \text{difference}
 \end{array}$$

### The General Term of an Arithmetic Sequence

An arithmetic sequence with first term,  $t_1$ , and common difference,  $d$ , is:  $t_1, t_1 + d, t_1 + 2d, t_1 + 3d, \dots$

The general term of this sequence is:  $t_n = t_1 + d(n - 1)$

#### Example 2

#### Calculating Terms in a Given Arithmetic Sequence

For this arithmetic sequence:  $-3, 2, 7, 12, \dots$

- a) Determine  $t_{20}$ .
- b) Which term in the sequence has the value 212?

#### SOLUTION

$-3, 2, 7, 12, \dots$

- a) Calculate the common difference:  $2 - (-3) = 5$

Use:  $t_n = t_1 + d(n - 1)$       Substitute:  $n = 20, t_1 = -3, d = 5$

$$t_{20} = -3 + 5(20 - 1) \quad \text{Use the order of operations.}$$

$$t_{20} = -3 + 5(19)$$

$$t_{20} = 92$$

- b) Use:  $t_n = t_1 + d(n - 1)$       Substitute:  $t_n = 212, t_1 = -3, d = 5$

$$212 = -3 + 5(n - 1) \quad \text{Solve for } n.$$

$$212 = -3 + 5n - 5$$

$$220 = 5n$$

$$\frac{220}{5} = n$$

$$n = 44$$

The term with value 212 is  $t_{44}$ .

#### Check Your Understanding

- 2. For this arithmetic sequence:  
 $3, 10, 17, 24, \dots$ 
  - a) Determine  $t_{15}$ .
  - b) Which term in the sequence has the value 220?

#### THINK FURTHER

In Example 2, how could you show that 246 is *not* a term of the sequence?

#### Check Your Understanding

#### Answers:

- 2. a) 101
- b)  $t_{32}$



**Example 3****Calculating a Term in an Arithmetic Sequence, Given Two Terms****Check Your Understanding**

3. Two terms in an arithmetic sequence are  $t_4 = -4$  and  $t_7 = 23$ . What is  $t_1$ ?

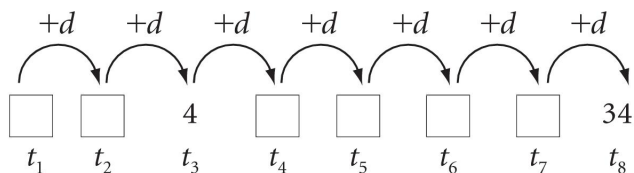


Two terms in an arithmetic sequence are  $t_3 = 4$  and  $t_8 = 34$ . What is  $t_1$ ?

**SOLUTION**

$$t_3 = 4 \text{ and } t_8 = 34$$

Sketch a diagram. Let the common difference be  $d$ .



From the diagram,

$$t_8 = t_3 + 5d$$

$$\text{Substitute: } t_8 = 34, t_3 = 4$$

$$34 = 4 + 5d$$

Solve for  $d$ .

$$30 = 5d$$

$$d = 6$$

$$\text{Then, } t_1 = t_3 - 2d$$

$$\text{Substitute: } t_3 = 4, d = 6$$

$$t_1 = 4 - 2(6)$$

$$t_1 = 4 - 12$$

$$t_1 = -8$$

**Example 4****Using an Arithmetic Sequence to Model and Solve a Problem****Check Your Understanding**

4. The comet Denning-Fujikawa appears about every 9 years and was last seen in the year 2005.

Determine whether the comet should appear in 3085.



Some comets are called periodic comets because they appear regularly in our solar system. The comet Kojima appears about every 7 years and was last seen in the year 2007. Halley's comet appears about every 76 years and was last seen in 1986.

Determine whether both comets should appear in 3043.

**SOLUTION**

The years in which each comet appears form an arithmetic sequence.

The arithmetic sequence for Kojima has  $t_1 = 2007$  and  $d = 7$ .

To determine whether Kojima should appear in 3043, determine whether 3043 is a term of its sequence.

$$t_n = t_1 + d(n - 1)$$

$$\text{Substitute: } t_n = 3043, t_1 = 2007, d = 7$$

$$3043 = 2007 + 7(n - 1) \quad \text{Solve for } n.$$

$$3043 = 2000 + 7n$$


$$1043 = 7n$$

$$149 = n$$

**Check Your Understanding****Answers:**

3. -31

4. The comet should appear in 3085.

Since the year 3043 is the 149th term in the sequence, Kojima should appear in 3043. 

The arithmetic sequence for Halley's comet has  $t_1 = 1986$  and  $d = 76$ . To determine whether Halley's comet should appear in 3043, determine whether 3043 is a term of its sequence.

$$\begin{aligned}t_n &= t_1 + d(n - 1) && \text{Substitute: } t_n = 3043, t_1 = 1986, d = 76 \\3043 &= 1986 + 76(n - 1) && \text{Solve for } n. \\3043 &= 1910 + 76n \\1133 &= 76n \\n &= 14.9078 \dots\end{aligned}$$

Since  $n$  is not a natural number, the year 3043 is not a term in the arithmetic sequence for Halley's comet; so the comet will not appear in that year.

---

## Discuss the Ideas

1. How can you tell whether a sequence is an arithmetic sequence? What do you need to know to be certain?



2. The definition of an arithmetic sequence relates any term after the first term to the preceding term. Why is it useful to have a rule for determining any term?



3. Suppose you know a term of an arithmetic sequence. What information do you need to determine any other term?



## Exercises

### A

4. Circle each sequence that could be arithmetic. Determine its common difference,  $d$ .
- a) 6, 10, 14, 18, ...                      b) 9, 7, 5, 3, ...
- c) -11, -4, 3, 10, ...                      d) 2, -4, 8, -16, ...
5. Each sequence is arithmetic. Determine each common difference,  $d$ , then list the next 3 terms.
- a) 12, 15, 18, ...                      b) 25, 21, 17, ...
6. Determine the indicated term of each arithmetic sequence.
- a) 6, 11, 16, ...;  $t_7$                       b)  $2, 1\frac{1}{2}, 1, \dots; t_{35}$
7. Write the first 4 terms of each arithmetic sequence, given the first term and the common difference.
- a)  $t_1 = -3, d = 4$                       b)  $t_1 = -0.5, d = -1.5$

### B

8. When you know the first term and the common difference of an arithmetic sequence, how can you tell if it is increasing or decreasing? Use examples to explain.

9. a) Create your own arithmetic sequence. Write the first 7 terms.  
Explain your method.
- b) Use technology or grid paper to graph the sequence in part a.  
Plot the *Term value* on the vertical axis and the *Term number* on the horizontal axis. Print the graph or sketch it on this grid.
- i) How do you know that you have graphed a linear function?
- ii) What does the slope of the line through the points represent?  
Explain why.
10. Two terms of an arithmetic sequence are given. Determine the indicated terms.
- a)  $t_4 = 24, t_{10} = 66$ ;  
determine  $t_1$
- b)  $t_3 = 81, t_{12} = 27$ ;  
determine  $t_{23}$
11. Create an arithmetic sequence for each description below. For each sequence, write the first 6 terms and a rule for  $t_n$ .
- a) an increasing sequence      b) a decreasing sequence

c) every term is negative

d) every term is an even number

**12.** Claire wrote the first 3 terms of an arithmetic sequence: 3, 6, 9, ...

When she asked Alex to extend the sequence to the first 10 terms, he wrote:

3, 6, 9, 3, 6, 9, 3, 6, 9, 3, ...

a) Is Alex correct? Explain.

b) What fact did Alex ignore when he extended the sequence?

c) What is the correct sequence?

**13.** Determine whether 100 is a term of an arithmetic sequence with

$t_3 = 250$  and  $t_6 = 245.5$ .

- 14.** The Chinese zodiac associates years with animals. Ling was born in 1994, the Year of the Dog.
- a) The Year of the Dog repeats every 12 years. List the first three years that Ling will celebrate her birthday in the Year of the Dog.
  - b) Why do the years in part a form an arithmetic sequence?
  - c) In 2099, Nunavut will celebrate its 100th birthday. Will that year also be the Year of the Dog? Explain.
- 15.** In this arithmetic sequence: 3, 8, 13, 18, . . . ; which term has the value 123?
- 16.** For two different arithmetic sequences,  $t_5 = -1$ . What are two possible sequences? Explain your reasoning.

- 17.** A sequence is created by adding each term of an arithmetic sequence to the preceding term.
- Show that the new sequence is arithmetic.
  - How are the common differences of the two sequences related?
- 18.** In this arithmetic sequence,  $k$  is a natural number:  $k, \frac{2k}{3}, \frac{k}{3}, 0, \dots$
- Determine  $t_6$ .
  - Write an expression for  $t_n$ .
  - Suppose  $t_{20} = -16$ ; determine the value of  $k$ .



## Multiple-Choice Questions

---

1. How many of these sequences have a common difference of  $-4$ ?  
 $-19, -15, -11, -7, -3, \dots$   
 $19, 15, 11, 7, 3, \dots$   
 $3, 7, 11, 15, 19, \dots$   
 $-3, 7, -11, 15, -19, \dots$   
A. 0            B. 1            C. 2            D. 3
2. Which number below is a term of this arithmetic sequence?  
 $97, 91, 85, 79, 73, \dots$   
A.  $-74$             B.  $-75$             C.  $-76$             D.  $-77$
3. The first 6 terms of an arithmetic sequence are plotted on a grid.  
The coordinates of two points on the graph are  $(3, 11)$  and  $(6, 23)$ .  
What is an expression for the general term of the sequence?  
A.  $6n - 3$             B.  $3n + 11$             C.  $4n - 1$             D.  $1 + 4n$

---

## Study Note

How are arithmetic sequences and linear functions related?



## ANSWERS

4. a) 4   b)  $-2$    c) 7   5. a) 3; 21, 24, 27   b)  $-4$ ; 13, 9, 5   6. a) 36   b)  $-15$   
7. a)  $-3, 1, 5, 9$    b)  $-0.5, -2, -3.5, -5$    10. a) 3   b)  $-39$   
12. a) no   c) 3, 6, 9, 12, 15, 18, 21, 24, 27, 30,  $\dots$    13. 100 is a term.  
14. a) 2006, 2018, 2030   c) no   15.  $t_{25}$    18. a)  $-\frac{2k}{3}$    b)  $\frac{4k}{3} - \frac{kn}{3}$    c) 3

## Multiple Choice

1. B   2. D   3. C

# 1.2 Arithmetic Series

**FOCUS** Derive a rule to determine the sum of  $n$  terms of an arithmetic series, then solve related problems.

## Get Started

Suppose this sequence continues.  
What is the value of the 8th term?



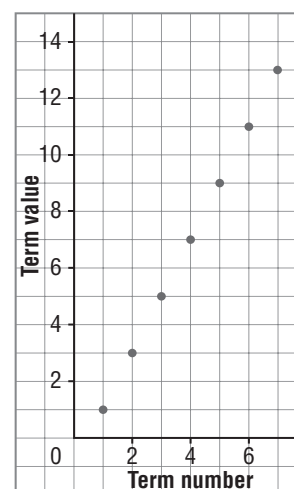
What is an expression for the  $n$ th term?



Is 50 a term in this sequence? How do you know?



Graph of an Arithmetic Sequence



## Construct Understanding

Talise displayed 90 photos of the Regina Dragon Boat Festival in 5 rows. The difference between the numbers of photos in consecutive rows was constant.

How many different sequences are possible? Justify your answer.





A **series** is a sum of the terms in a sequence.

An **arithmetic series** is the sum of the terms in an arithmetic sequence.

For example, an arithmetic sequence is: 5, 8, 11, 14, . . .

The related arithmetic series is:  $5 + 8 + 11 + 14 + \dots$

The term,  $S_n$ , is used to represent the sum of the first  $n$  terms of a series.

The  $n$ th term of an arithmetic series is the  $n$ th term of the related arithmetic sequence.

For the arithmetic series above:

$$\begin{array}{llll} S_1 = t_1 & S_2 = t_1 + t_2 & S_3 = t_1 + t_2 + t_3 & S_4 = t_1 + t_2 + t_3 + t_4 \\ S_1 = 5 & S_2 = 5 + 8 & S_3 = 5 + 8 + 11 & S_4 = 5 + 8 + 11 + 14 \\ & S_2 = 13 & S_3 = 24 & S_4 = 38 \end{array}$$

These are called *partial sums*.

If there are only a few terms,  $S_n$  can be determined using mental math.

To develop a rule to determine  $S_n$ , use algebra.

Write the sum on one line, reverse the order of the terms on the next line, then add vertically. Write the sum as a product.

## THINK FURTHER

Why can the  $(n - 1)$ th term be written as  $t_n - d$ ?



$$\begin{aligned} S_n &= t_1 + (t_1 + d) + (t_1 + 2d) + \dots + (t_n - d) + t_n \\ + S_n &= t_n + (t_n - d) + (t_n - 2d) + \dots + (t_1 + d) + t_1 \\ 2S_n &= (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + \dots + (t_1 + t_n) + (t_1 + t_n) \\ 2S_n &= n(t_1 + t_n) \quad \text{Solve for } S_n. \\ S_n &= \frac{n(t_1 + t_n)}{2} \end{aligned}$$

The rule above is used when  $t_1$  and  $t_n$  are known. Substitute for  $t_n$  to write  $S_n$  in a different way, so it can be used when  $t_1$  and the common difference,  $d$ , are known.

$$\begin{aligned} S_n &= \frac{n(t_1 + t_n)}{2} && \text{Substitute: } t_n = t_1 + d(n - 1) \\ S_n &= \frac{n[t_1 + t_1 + d(n - 1)]}{2} && \text{Combine like terms.} \\ S_n &= \frac{n[2t_1 + d(n - 1)]}{2} \end{aligned}$$

### The Sum of $n$ Terms of an Arithmetic Series

For an arithmetic series with 1st term,  $t_1$ , common difference,  $d$ , and  $n$ th term,  $t_n$ , the sum of the first  $n$  terms,  $S_n$ , is:

$$S_n = \frac{n(t_1 + t_n)}{2} \quad \text{or} \quad S_n = \frac{n[2t_1 + d(n - 1)]}{2}$$

### Check Your Understanding

- Determine the sum of the first 6 terms of this arithmetic series:  $25 + 14 + 3 - 8 - 19 - 30 - \dots$



### Example 1

### Determining the Sum, Given the Series

Determine the sum of the first 6 terms of this arithmetic series:

$$-75 - 69 - 63 - 57 - 51 - 45 - \dots$$

### SOLUTION

$$-75 - 69 - 63 - 57 - 51 - 45 - \dots$$

$t_1$  is  $-75$  and  $t_6$  is  $-45$ .

$$\text{Use: } S_n = \frac{n(t_1 + t_n)}{2} \quad \text{Substitute: } n = 6, t_1 = -75, t_n = -45$$

$$S_6 = \frac{6(-75 - 45)}{2}$$

$$S_6 = -360$$

The sum of the first 6 terms is  $-360$ .

### Check Your Understanding

**Answer:**

- $-15$

**Example 2****Determining the Sum, Given the First Term and Common Difference**

An arithmetic series has  $t_1 = 5.5$  and  $d = -2.5$ ; determine  $S_{40}$ .

**SOLUTION**

Use:  $S_n = \frac{n[2t_1 + d(n-1)]}{2}$       Substitute:  $n = 40, t_1 = 5.5, d = -2.5$

$$S_{40} = \frac{40[2(5.5) - 2.5(40-1)]}{2}$$

$$S_{40} = -1730$$

**Check Your Understanding**

2. An arithmetic series has  $t_1 = 3$  and  $d = -4$ ; determine  $S_{25}$ .

**Example 3****Determining the First Few Terms Given the Sum, Common Difference, and One Term**

An arithmetic series has  $S_{20} = 143\frac{1}{3}$ ,  $d = \frac{1}{3}$ , and  $t_{20} = 10\frac{1}{3}$ ; determine the first 3 terms of the series.

**SOLUTION**

$S_{20}$  and  $t_{20}$  are known, so use this rule to determine  $t_1$ :

$$S_n = \frac{n(t_1 + t_n)}{2} \quad \text{Substitute: } n = 20, S_{20} = 143\frac{1}{3}, t_{20} = 10\frac{1}{3}$$

$$143\frac{1}{3} = \frac{20(t_1 + 10\frac{1}{3})}{2} \quad \text{Simplify.}$$

$$143\frac{1}{3} = 10(t_1 + 10\frac{1}{3})$$

$$143\frac{1}{3} = 10t_1 + 103\frac{1}{3} \quad \text{Solve for } t_1.$$

$$40 = 10t_1$$

$$4 = t_1$$

The first term is 4 and the common difference is  $\frac{1}{3}$ .

So, the first 3 terms of the series are written as the partial sum:

$$4 + 4\frac{1}{3} + 4\frac{2}{3}$$

**Check Your Understanding**

3. An arithmetic series has  $S_{15} = 93.75$ ,  $d = 0.75$ , and  $t_{15} = 11.5$ ; determine the first 3 terms of the series.

**THINK FURTHER**

In *Example 3*, which partial sums are natural numbers? Why?

**Check Your Understanding****Answers:**

2.  $-1125$   
3.  $1 + 1.75 + 2.5$

**Example 4****Using an Arithmetic Series to Model and Solve a Problem****Check Your Understanding**

4. The bottom row in a trapezoid had 49 cans. Each consecutive row had 4 fewer cans than the previous row. There were 11 rows in the trapezoid. How many cans were in the trapezoid?



Students created a trapezoid from the cans they had collected for the food bank. There were 10 rows in the trapezoid. The bottom row had 100 cans. Each consecutive row had 5 fewer cans than the previous row. How many cans were in the trapezoid?

**SOLUTION**

The numbers of cans in the rows form an arithmetic sequence with first 3 terms 100, 95, 90, ...

The total number of cans is the sum of the first 10 terms of the arithmetic series:

$$100 + 95 + 90 + \dots$$

$$\text{Use: } S_n = \frac{n[2t_1 + d(n - 1)]}{2} \quad \text{Substitute: } n = 10, t_1 = 100, d = -5$$

$$S_{10} = \frac{10[2(100) - 5(10 - 1)]}{2}$$

$$S_{10} = 775$$

There were 775 cans in the trapezoid.

---

**Discuss the Ideas**

1. How are an arithmetic series and an arithmetic sequence related?



2. Suppose you know the 1st and  $n$ th terms of an arithmetic series. What other information do you need to determine the value of  $n$ ?

**Check Your Understanding****Answer:**

4. 319 cans

## Exercises

### A

3. Use each arithmetic sequence to write the first 4 terms of an arithmetic series.

a)  $2, 4, 6, 8, \dots$

b)  $-2, 3, 8, 13, \dots$

c)  $4, 0, -4, -8, \dots$

d)  $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \dots$

4. Determine the sum of the given terms of each arithmetic series.

a)  $12 + 10 + 8 + 6 + 4$

b)  $-2 - 4 - 6 - 8 - 10$

5. Determine the sum of the first 20 terms of each arithmetic series.

a)  $3 + 7 + 11 + 15 + \dots$

b)  $-21 - 15.5 - 10 - 4.5 - \dots$

### B

6. For each arithmetic series, determine the indicated value.

a)  $-4 - 11 - 18 - 25 - \dots$ ; determine  $S_{28}$

b)  $1 + 3.5 + 6 + 8.5 + \dots$ ; determine  $S_{42}$



**7.** Use the given data about each arithmetic series to determine the indicated value.

- a)  $S_{20} = -850$  and  $t_{20} = -90$ ; determine  $t_1$       b)  $S_{15} = 322.5$  and  $t_1 = 4$ ; determine  $d$

- c)  $S_n = -126$ ,  $t_1 = -1$ , and  $t_n = -20$ ; determine  $n$       d)  $t_1 = 1.5$  and  $t_{20} = 58.5$ ; determine  $S_{15}$

**8.** Two hundred seventy-six students went to a powwow. The first bus had 24 students. The numbers of students on the buses formed an arithmetic sequence. What additional information do you need to determine the number of buses? Explain your reasoning.

- 9.** Ryan's grandparents loaned him the money to purchase a BMX bike. He agreed to repay \$25 at the end of the first month, \$30 at the end of the second month, \$35 at the end of the third month, and so on. Ryan repaid the loan in 12 months. How much did the bike cost? How do you know your answer is correct?

- 10.** Determine the sum of the indicated terms of each arithmetic series.

a)  $31 + 35 + 39 + \dots + 107$       b)  $-13 - 10 - 7 - \dots + 62$

- 11. a)** Explain how this series could be arithmetic.

$$1 + 3 + \dots$$

- b) What information do you need to be certain that this is an arithmetic series?

**12.** An arithmetic series has  $S_{10} = 100$ ,  $t_1 = 1$ , and  $d = 2$ . How can you use this information to determine  $S_{11}$  without using a rule for the sum of an arithmetic series? What is  $S_{11}$ ?

**13.** The side lengths of a quadrilateral form an arithmetic sequence. The perimeter is 74 cm. The longest side is 29 cm. What are the other side lengths?

**14.** Derive a rule for the sum of the first  $n$  natural numbers:  
 $1 + 2 + 3 + \dots + n$

- 15.** The sum of the first 5 terms of an arithmetic series is 170. The sum of the first 6 terms is 225. The common difference is 7. Determine the first 4 terms of the series.

**C**

- 16.** The sum of the first  $n$  terms of an arithmetic series is:  $S_n = 3n^2 - 8n$   
Determine the first 4 terms of the series.
- 17.** Each number from 1 to 60 is written on one of 60 index cards. The cards are arranged in rows with equal lengths, and no cards are left over. The sum of the numbers in each row is 305. How many rows are there?

- 18.** Determine the arithmetic series that has these partial sums:  $S_4 = 26$ ,  $S_5 = 40$ , and  $S_6 = 57$

### Multiple-Choice Questions

- 1.** Which of these series is arithmetic?

A.  $2.5 + 5 + 7.5 + 11 + \dots$       B.  $-2.5 - 5 - 7.5 - 11 - \dots$   
 C.  $3.5 + 6 + 8.5 + 11 + \dots$       D.  $3.5 - 6 - 8.5 - 11 + \dots$

- 2.** For which series below is 115 the sum of 10 terms?

A.  $34 + 29 + 25 + 20 + 16 + \dots$   
 B.  $-11 - 6 - 1 + 4 + 9 + \dots$   
 C.  $11 + 6 + 1 - 4 - 9 - \dots$   
 D.  $34 - 29 - 25 - 20 - 16 - \dots$

- 3.** How many of these expressions could be used to determine the sum to  $n$  terms of an arithmetic series?

$\frac{n[2t_1 + d(n + 1)]}{2}$        $\frac{n[2t_1 - d(n + 1)]}{2}$        $\frac{n(t_1 + t_n)}{2}$        $\frac{n(t_1 - t_n)}{2}$   
 A. 4      B. 3      C. 2      D. 1

### Study Note

There are two forms of the rule to determine the sum of  $n$  terms of an arithmetic series. When would you use each form of the rule?



### ANSWERS

3. a)  $2 + 4 + 6 + 8$     b)  $-2 + 3 + 8 + 13$     c)  $4 + 0 - 4 - 8$   
 d)  $\frac{1}{2} + \frac{1}{4} + 0 - \frac{1}{4}$     4. a) 40    b) -30    5. a) 820    b) 625    6. a) -2758  
 b) 2194.5    7. a) 5    b) 2.5    c) 12    d) 337.5    9. \$630    10. a) 1380    b) 637  
 12. 121    13. 8 cm, 15 cm, and 22 cm    14.  $S_n = \frac{n(n + 1)}{2}$     15.  $20 + 27 + 34 + 41$   
 16.  $-5 + 1 + 7 + 13$     17. 6    18.  $2 + 5 + 8 + 11 + 14 + 17 + \dots$

### Multiple Choice

1. C    2. B    3. D



# CHECKPOINT 1

## Self-Assess

Can you . . .	To check, try question . . .	For review, see . . .
write an example of an arithmetic sequence and explain how you know it is arithmetic?	2	Page 4 in Lesson 1.1 (Example 1)
explain the meaning of the symbols $n$ , $t_1$ , $t_n$ , and $d$ ?		Page 3 in Lesson 1.1
identify the assumptions made when defining an arithmetic sequence or series?		Page 7 in Lesson 1.1
use a rule to determine the $n$ th term in an arithmetic sequence?	3	Page 4 in Lesson 1.1
describe the relationship between an arithmetic sequence and a linear function?		Page 3 in Lesson 1.1
use a rule to determine $t_n$ and $n$ in an arithmetic sequence given the values of $t_1$ and $d$ ?	3b, 4b	Page 5 in Lesson 1.1 (Example 2)
use a rule to determine $t_1$ and $d$ in an arithmetic sequence given the values of $t_n$ and $n$ ?	4a	Page 6 in Lesson 1.1 (Example 3)
solve problems involving arithmetic sequences?	5	Pages 6-7 in Lesson 1.1 (Example 4)
use a rule to determine the sum $S_n$ of an arithmetic series?	7b, 8	Page 16 in Lesson 1.2
use a rule to determine $S_n$ in an arithmetic series given the values of $n$ , $t_1$ , and $t_n$ ?		Page 16 in Lesson 1.2 (Example 1)
use a rule to determine $S_n$ in an arithmetic series given the values of $n$ , $t_1$ , and $d$ ?		Page 17 in Lesson 1.2 (Example 2)
use a rule to determine $t_1$ in an arithmetic series given the values of $S_n$ , $t_n$ , and $d$ ?	9a	Page 17 in Lesson 1.2 (Example 3)
solve problems involving arithmetic series?		Page 18 in Lesson 1.2 (Example 4)

## Assess Your Understanding

### 1.1

1. **Multiple Choice** Which sequence has  $d = -8$  and  $t_{10} = -45$ ?  
A. 27, 19, 11, 3, ...      B. -8, -12, -15, -20, ...  
C. -5, -13, -21, -29, ...      D. -27, -19, -11, -3, ...
  
2. Write the first 4 terms of an arithmetic sequence with its 5th term equal to -4.
  
  
  
  
  
  
  
  
  
  
3. This sequence is arithmetic: -8, -11, -14, ...
  - a) Write a rule for the  $n$ th term.
  
  
  
  
  
  
  
  - b) Use your rule to determine the 17th term.
  
  
  
  
  
  
  
4. Use the given data about each arithmetic sequence to determine the indicated values.
  - a)  $t_4 = -5$  and  $t_7 = -20$ ; determine  $d$  and  $t_1$



b)  $t_1 = 3$ ,  $d = 4$ , and  $t_n = 59$ ; determine  $n$

5. The steam clock in the Gastown district of Vancouver, B.C., displays the time on four faces and announces the quarter hours with a whistle chime that plays the tune *Westminster Quarters*. This sequence represents the number of tunes played from 1 to 3 days: 96, 192, 288, ... Determine the number of tunes played in one year.

## 1.2

6. **Multiple Choice** For which series would you use  $S_n = \frac{n(t_1 + t_n)}{2}$  to determine its sum?
- A.  $3 + 5 + 7 + 10 + \dots + 29$
  - B.  $3 - 1 - 5 - 9 - \dots - 93$
  - C.  $-3 - 5 - 8 - 10 - \dots - 29$
  - D.  $3 - 1 - 5 - 9 - \dots + 93$
7. a) Create the first 5 terms of an arithmetic series with a common difference of  $-3$ .

b) Determine  $S_{26}$  for your series.

**8.** Determine the sum of this arithmetic series:

$$-2 + 3 + 8 + 13 + \dots + 158$$

**9.** Use the given data about each arithmetic series to determine the indicated value.

a)  $S_{17} = 106.25$  and  $t_{17} = 8.25$ ; determine  $t_1$

b)  $S_{15} = 337.5$  and  $t_1 = -2$ ; determine  $d$

### **ANSWERS**

1. A    3. a)  $t_n = -5 - 3n$     b)  $-56$     4. a)  $-5; 10$     b)  $15$     5. 35 040    6. B  
8. 2574    9. a) 4.25    b) 3.5

# 1.3 Geometric Sequences

**FOCUS** Solve problems involving geometric sequences.

## Get Started

For each sequence below, what are the next 2 terms? What is the rule?

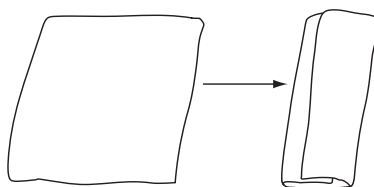
- $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$
- $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots$
- $1, -3, 9, -27, 81, \dots$



## Construct Understanding

A French pastry called *mille feuille* or “thousand layers” is made from dough rolled into a square, buttered, and then folded into thirds to make three layers. This process is repeated many times. Each step of folding and rolling is called a turn.

How many turns are required to get more than 1000 layers?



A **geometric sequence** is formed by multiplying each term after the 1st term by a constant, to determine the next term.

For example, 4, 4(3), 4(3)<sup>2</sup>, 4(3)<sup>3</sup>, . . . , is the geometric sequence:  
4, 12, 36, 108, . . .

The first term,  $t_1$ , is 4 and the constant is 3.

The constant is the **common ratio**,  $r$ , of any term after the first, to the preceding term.

The common ratio is any non-zero real number.

To determine the common ratio, divide any term by the preceding term.

For the geometric sequence above:

$$r = \frac{12}{4} \quad \text{and} \quad r = \frac{36}{12} \quad \text{and} \quad r = \frac{108}{36}$$

$$r = 3 \quad \quad \quad r = 3 \quad \quad \quad r = 3$$

The sequence 4, 12, 36, 108, . . . , is an **infinite geometric sequence** because it continues forever.

The sequence 4, 12, 36, 108 is a **finite geometric sequence** because the sequence is limited to a fixed number of terms.

Here are some other examples of geometric sequences.

- This is an *increasing* geometric sequence because the terms are increasing: 2, 10, 50, 250, 1250, . . .  
The sequence is **divergent** because the terms do not approach a constant value.
- This is a geometric sequence that neither increases, nor decreases because consecutive terms have numerically greater values and different signs: 1, -2, 4, -8, 16, . . .  
The sequence is *divergent* because the terms do not approach a constant value.
- This is a *decreasing* geometric sequence because the terms are decreasing:  
 $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots$

The sequence is **convergent** because the terms approach a constant value of 0.

## THINK FURTHER

Why must  $r$  be non-zero?



**Example 1****Determining a Term of a Given Geometric Sequence**

- a) Determine the 12th term of this geometric sequence:  
512, -256, 128, -64, ...
- b) Identify the sequence as convergent or divergent.

**SOLUTION**

- a) 512, -256, 128, -64, ...

The common ratio is:

$$r = \frac{-256}{512}$$

$$r = -\frac{1}{2}$$

Multiply the first term, 512, by consecutive powers of  $-\frac{1}{2}$ .

$$t_1 = 512 \quad t_2 = 512\left(-\frac{1}{2}\right) \quad t_3 = 512\left(-\frac{1}{2}\right)^2 \quad t_4 = 512\left(-\frac{1}{2}\right)^3$$

The exponent of each power of the common ratio is 1 less than the term number.

So, the 12th term is:

$$t_{12} = 512\left(-\frac{1}{2}\right)^{12-1}$$

$$t_{12} = 512\left(-\frac{1}{2}\right)^{11}$$

$$t_{12} = -\frac{1}{4}$$

The 12th term is  $-\frac{1}{4}$ .

- b) Since consecutive terms alternate between positive and negative values, and approach a constant value of 0, the sequence is convergent.

**Check Your Understanding**

1. a) Determine the 10th term of this geometric sequence:  
2, -6, 18, -54, ...
- b) Identify the sequence as convergent or divergent.

**THINK FURTHER**

In *Example 1*, how can you predict whether a term is positive or negative?

**Check Your Understanding****Answers:**

1. a) -39 366  
b) divergent

## Example 2

## Creating a Geometric Sequence

### Check Your Understanding

2. Create a geometric sequence whose 6th term is 27.



Create a geometric sequence whose 5th term is 48.

### SOLUTION

Work backward.

$$t_5 = 48$$

Choose a common ratio that is a factor of 48, such as 2.

Repeatedly divide 48 by 2.

$$\begin{array}{cccccc} t_5 = 48 & t_4 = \frac{48}{2} & t_3 = \frac{24}{2} & t_2 = \frac{12}{2} & t_1 = \frac{6}{2} \\ & t_4 = 24 & t_3 = 12 & t_2 = 6 & t_1 = 3 \end{array}$$

A possible geometric sequence is: 3, 6, 12, 24, 48, ...

## THINK FURTHER

In *Example 2*, why does it make sense to choose a value for  $r$  that is a factor of 48? Could you choose any value for  $r$ ?



A geometric sequence with first term,  $t_1$ , and common ratio,  $r$ , can be written as:

$$\begin{array}{cccccc} t_1, & t_1r, & t_1r^2, & t_1r^3, & t_1r^4, & \dots, & t_1r^{n-1} \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & & \uparrow \\ t_1 & t_2 & t_3 & t_4 & t_5 & & t_n \end{array}$$

The exponent of each power of the common ratio is 1 less than the term number.

## The General Term of a Geometric Sequence

For a geometric sequence with first term,  $t_1$ , and common ratio,  $r$ , the general term,  $t_n$ , is:

$$t_n = t_1r^{n-1}$$

Recall that the product of two negative numbers is positive. So, a square number may be the product of two equal negative numbers or two equal positive numbers. For example,

$$\text{when } r^2 = 4$$

$$\text{then } r = \pm\sqrt{4}$$

$$\text{and } r = 2 \text{ or } r = -2$$

### Check Your Understanding

**Answer:**

2.  $\frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots$

To determine the common ratio of a geometric sequence, you may need to solve an equation of this form:

$$\begin{aligned} r^4 &= 81 \\ \text{then } r^2 &= 9 \\ \text{and } r &= 3 \text{ or } r = -3 \end{aligned}$$

## THINK FURTHER

Why is  $r^2 \neq -9$ ?



### Example 3

### Determining Terms and the Number of Terms in a Finite Geometric Sequence

In a finite geometric sequence,  $t_1 = 5$  and  $t_5 = 1280$

- Determine  $t_2$  and  $t_6$ .
- The last term of the sequence is 20 480. How many terms are in the sequence?

### SOLUTION

- Determine the common ratio.

Use:  $t_n = t_1 r^{n-1}$       Substitute:  $n = 5, t_5 = 1280, t_1 = 5$

$$1280 = 5r^{5-1} \quad \text{Simplify.}$$

$$1280 = 5r^4 \quad \text{Divide each side by 5.}$$

$$256 = r^4 \quad \text{Take the fourth root of each side.}$$

$$\sqrt[4]{256} = r$$

$$r = -4 \text{ or } r = 4$$

There are 2 possible values for  $r$ .

When  $r = -4$ , then

$$t_2 \text{ is } 5(-4) = -20$$

When  $r = 4$ , then

$$t_2 \text{ is } 5(4) = 20$$

To determine  $t_6$ , use:  $t_n = t_1 r^{n-1}$

Substitute:  $n = 6, t_1 = 5, r = -4$       Substitute:  $n = 6, t_1 = 5, r = 4$

$$t_6 = 5(-4)^{6-1}$$

$$t_6 = 5(4)^{6-1}$$

$$t_6 = 5(-4)^5$$

$$t_6 = 5(4)^5$$

$$t_6 = -5120$$

$$t_6 = 5120$$

So,  $t_2$  is  $-20$  or  $20$ , and  $t_6$  is  $-5120$  or  $5120$ .

- Since the last term is positive, use the positive value of  $r$ .

$t_n = t_1 r^{n-1}$       Substitute:  $t_n = 20\,480, t_1 = 5, r = 4$

$$20480 = 5(4)^{n-1} \quad \text{Divide each side by 5.}$$

$$4096 = 4^{n-1}$$

Use guess and test to determine which power of 4 is equal to 4096.

$$\text{Guess: } 4^4 = 256$$

This is too low.

$$\text{Guess: } 4^6 = 4096$$

This is correct.

$$\text{So, } 4^6 = 4^{n-1}$$

Equate exponents.

$$6 = n - 1$$

$$n = 7$$

There are 7 terms in the sequence.

### Check Your Understanding

- In a finite geometric sequence,  $t_1 = 7$  and  $t_5 = 567$ 
  - Determine  $t_2$  and  $t_6$ .
  - The last term is 45 927. How many terms are in the sequence?



### Check Your Understanding

#### Answers:

- $-21$  or  $21, -1701$  or  $1701$
  - 9 terms

**Example 4****Using a Geometric Sequence to Solve a Problem****Check Your Understanding**

4. Statistics Canada estimates the population growth of Canadian cities, provinces, and territories. The population of Nunavut is expected to grow annually by 0.8%. In 2009, its population was about 30 000. Estimate the population in each year to the nearest thousand.

- a) 2013  
b) 2049; Nunavut's 50th birthday

The population of Airdrie, Alberta, experienced an average annual growth rate of about 9% from 2001 to 2006. The population in 2006 was 28 927. Estimate the population in each year to the nearest thousand.

- a) 2011  
b) 2030, the 125th anniversary of Alberta becoming part of Canada  
What assumption did you make? Is this assumption reasonable?

**SOLUTION**

- a) A growth rate of 9% means that each year the population increases by 9%, or 0.09.

The population in 2006 was 28 927.

So, the population in 2007 was:

$$28\,927 + 9\% \text{ of } 28\,927$$

$$= 28\,927 + 0.09(28\,927) \quad \text{Remove } 28\,927 \text{ as a common factor.}$$

$$= 28\,927(1 + 0.09)$$

$$= 28\,927(1.09)$$

Increasing a quantity by 9% is the same as multiplying it by 1.09.

So, to determine a population with a growth rate of 9%, multiply the current population by 1.09.

The annual populations form a geometric sequence with 1st term 28 927 and common ratio 1.09.

The population in 2006 is the 1st term. So, the population in 2011 is the 6th term:  $28\,927(1.09)^5 = 44\,507.7751\dots$

The population in 2011 is approximately 45 000.

- b) To predict the population in 2030, determine  $n$ , the number of years from 2006 to 2030:

$$n = 2030 - 2006$$

$$n = 24$$

$$\text{The population in 2030 is: } 28\,927(1.09)^{24} = 228\,843.903\dots$$

The population in 2030 will be approximately 229 000.

We assume that the population increase of 9% annually continues. This assumption may be false because the rate of growth may change in future years. This assumption is reasonable for a short time span, but not for a longer time span, such as 100 years.

**Check Your Understanding****Answers:**

4. a) approximately 31 000  
b) approximately 41 000



---

## Discuss the Ideas

1. How do you determine whether a given sequence is geometric? What assumptions do you make?



2. Which geometric sequences are created when  $r = 1$ ?  $r = -1$ ?



---

## Exercises

### A

3. Which sequences could be geometric? If a sequence is geometric, state its common ratio.

a) 1, 2, 4, 8, 16, ...

b) 4, 9, 16, 25, 36, ...

c) -3, 2, 7, 12, 17, ...

d) 6, 0.6, 0.06, 0.006, ...

e) 10, 100, 1000, 10 000

f) 2, 4, 6, 8, 10, ...

4. State the common ratio, then write the next 3 terms of each geometric sequence.

a)  $-1, -3, -9, \dots$

b)  $48, 24, 12, \dots$

c)  $4, -2, 1, \dots$

d)  $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \dots$

5. For each geometric sequence, determine the indicated value.

a)  $3, 6, 12, \dots$ ; determine  $t_7$

b)  $18, 9, 4.5, \dots$ ; determine  $t_6$

c)  $23, -46, 92, \dots$ ; determine  $t_{10}$

d)  $2, \frac{1}{2}, \frac{1}{8}, \dots$ ; determine  $t_5$

## B

6. Write the first 4 terms of each geometric sequence, given the 1st term and the common ratio. Identify the sequence as decreasing, increasing, or neither. Justify your answers.

a)  $t_1 = -3; r = 4$

b)  $t_1 = 5; r = 2$

c)  $t_1 = -0.5; r = -3$

d)  $t_1 = \frac{1}{2}; r = \frac{2}{3}$

**7.** Write the first 5 terms of each geometric sequence.

a) the 6th term is 64

b) the 1st term is  $\frac{3}{4}$

c) every term is negative

d) every term is an even number

**8.** Use the given data about each finite geometric sequence to determine the indicated values.

**a)** Given  $t_1 = -1$  and  $r = -2$

**i)** Determine  $t_9$ .

**ii)** The last term is  $-4096$ . How many terms are in the sequence?

**b)** Given  $t_1 = 0.002$  and  $t_4 = 2$

**i)** Determine  $t_7$ .

**ii)** Determine which term has the value  $20\,000$ .

**9. a)** Insert three numbers between 8 and 128, so the five numbers form an arithmetic sequence. Explain what you did.

**b)** Insert three numbers between 8 and 128, so the five numbers form a geometric sequence. Explain what you did.

**c)** What was similar about your strategies in parts a and b? What was different?

**10.** Suppose a person is given 1¢ on the first day of April; 3¢ on the second day; 9¢ on the third day, and so on. This pattern continues throughout April.

**a)** About how much money will the person be given on the last day of April?

**b)** Why might it be difficult to determine the exact amount using a calculator?

**11.** In a geometric sequence,  $t_3 = 9$  and  $t_6 = 1.125$ ; determine  $t_7$  and  $t_9$ .

**12.** An arithmetic sequence and a geometric sequence have the same first term. The common difference and common ratio are equal and greater than 1. Which sequence increases more rapidly as more terms are included? Use examples to explain.

**13.** A ream of paper is about 2 in. thick. Imagine a ream of paper that is continually cut in half and the two halves stacked one on top of the other. How many cuts have to be made before the stack of paper is taller than 318 ft., the height of Le Chateau York in Winnipeg, Manitoba?

- 14.** Between the Canadian censuses in 2001 and 2006, the number of people who could converse in Cree had increased by 7%. In 2006, 87 285 people could converse in Cree. Assume the 5-year increase continues to be 7%. Estimate to the nearest hundred how many people will be able to converse in Cree in 2031.

**C**

- 15.** A farmer in Saskatchewan wants to estimate the value of a new combine after several years of use. A new combine worth \$370 000 depreciates in value by about 10% each year.
- a) Estimate the value of the combine at the end of each of the first 5 years. Write the values as a sequence.
- b) What type of sequence did you write in part a? Explain your reasoning.
- c) Predict the value of the combine at the end of 10 years.
- 16.** a) Show that squaring each term in a geometric sequence produces the same type of sequence. What is the common ratio?

- b) Show that raising each term in a geometric sequence to the  $m$ th power of each term produces the same type of sequence. What is the common ratio?

### Multiple-Choice Questions

- Which expression below represents the  $n$ th term of this sequence?  
9, -6, 4,  $-\frac{8}{3}$ , ...  
A.  $9\left(\frac{2}{3}\right)^{n-1}$     B.  $9\left(-\frac{2}{3}\right)^{n-1}$     C.  $\frac{2}{3}(9^{n-1})$     D.  $-\frac{2}{3}(9^{n-1})$
- Which geometric sequence does not have a common ratio of  $-0.5$ ?  
A. -5, 2.5, -1.25, 0.625, ...    B. 6, -3, 1.5, -0.75, ...  
C.  $\frac{1}{200}, -\frac{1}{100}, \frac{1}{50}, -\frac{1}{25}, \dots$     D.  $-\frac{1}{3}, \frac{1}{6}, -\frac{1}{12}, \frac{1}{24}, \dots$
- The value of a car in each of its first 3 years is: \$24 000, \$20 400, \$17 340. These amounts are the first 3 terms of a sequence. Which statement describes this sequence?  
A. arithmetic with common difference \$3600  
B. geometric with common ratio approximately 1.18  
C. geometric with common ratio 0.85  
D. neither arithmetic nor geometric

### Study Note

Can a sequence be both arithmetic and geometric? Explain.



### ANSWERS

3. a) 2    d) 0.1    e) 10    4. a) 3; -27, -81, -243    b) 0.5; 6, 3, 1.5  
c) -0.5; -0.5, 0.25, -0.125    d)  $\frac{1}{3}, \frac{1}{54}, \frac{1}{162}, \frac{1}{486}$     5. a) 192    b) 0.5625    c) -11 776  
d)  $\frac{1}{128}$     6. a) -3, -12, -48, -192; decreasing    b) 5, 10, 20, 40; increasing  
c) -0.5, 1.5, -4.5, 13.5; neither    d)  $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}$ ; decreasing  
8. a) i) -256    ii) 13    b) i) 2000    ii)  $t_8$     9. a) 8, 38, 68, 98, 128, ...  
b) 8, -16, 32, -64, 128, ... or 8, 16, 32, 64, 128, ...    10. a) approximately  $\$6.863 \times 10^{11}$   
11. 0.5625; 0.140 625    12. geometric    13. 11 cuts    14. approximately 122 400 people  
15. a) \$333 000, \$299 700, \$269 730, \$242 757, \$218 481    b) geometric    c) \$129 011

### Multiple Choice

1. B    2. C    3. C



# 1.4 Geometric Series

**FOCUS** Derive a rule to determine the sum of  $n$  terms of a geometric series, then solve related problems.

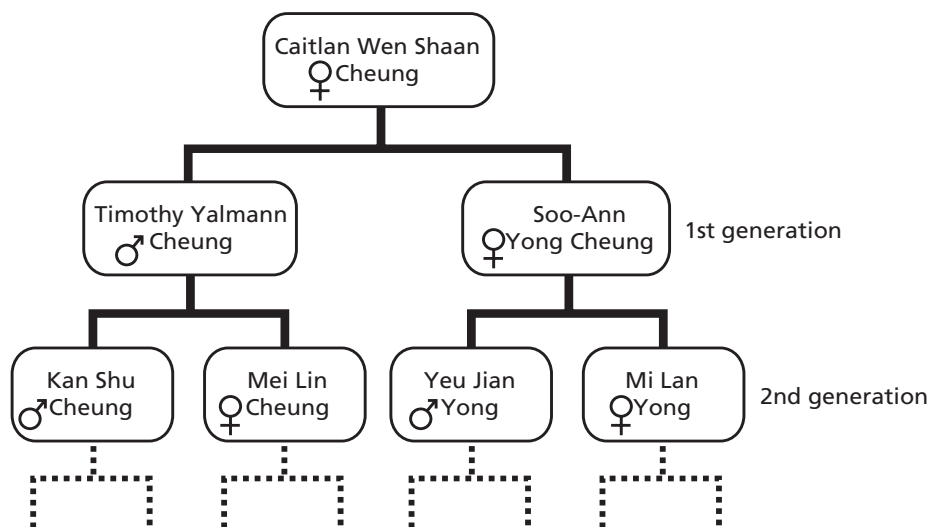
## Get Started

Two geometric sequences have the same first term but the common ratios are opposite integers. Which corresponding terms are equal? Which corresponding terms are different? Use an example to explain.



## Construct Understanding

Caitlan traced her direct ancestors, beginning with her 2 parents, 4 grandparents, 8 great-grandparents, and so on.



Determine the total number of Caitlan's direct ancestors in 20 generations.





## THINK FURTHER

How could you determine  $S_n$  when  $r = 1$ ?



A **geometric series** is the sum of the terms of a geometric sequence.

For example, a geometric sequence is: 6, 12, 24, 48, ...

The related geometric series is:  $6 + 12 + 24 + 48 + \dots$

$S_n$  represents the partial sum of  $n$  terms of the series; for example,

$$\begin{array}{lll} S_1 = t_1 & S_2 = t_1 + t_2 & S_3 = t_1 + t_2 + t_3 \\ S_1 = 6 & S_2 = 6 + 12 & S_3 = 6 + 12 + 24 \\ & S_2 = 18 & S_3 = 42 \end{array}$$

The  $n$ th term of a geometric series is the  $n$ th term of the related geometric sequence.

To derive a rule for determining the sum of the first  $n$  terms in any geometric series:  $t_1 + t_1r + t_1r^2 + t_1r^3 + t_1r^4 + \dots + t_1r^{n-1}$

write the sum, multiply each side by the common ratio,  $r$ , write the new sum, then subtract vertically.

$$\begin{array}{l} S_n = t_1 + t_1r + t_1r^2 + t_1r^3 + \dots + t_1r^{n-2} + t_1r^{n-1} \\ rS_n = \quad \quad t_1r + t_1r^2 + t_1r^3 + t_1r^4 + \dots + t_1r^{n-1} + t_1r^n \\ \hline S_n - rS_n = t_1 - t_1r^n \quad \quad \text{Factor each side.} \\ S_n(1 - r) = t_1(1 - r^n) \quad \quad \text{Divide each side by } (1 - r) \text{ to solve for } S_n. \\ S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1 \end{array}$$

## THINK FURTHER

Why is 1 not a permissible value for  $r$ ?



### The Sum of $n$ Terms of a Geometric Series

For the geometric series  $t_1 + t_1r + t_1r^2 + \dots + t_1r^{n-1}$ ,

the sum of  $n$  terms,  $S_n$ , is:  $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

## THINK FURTHER

Why can the rule also be written as  $S_n = \frac{t_1(r^n - 1)}{r - 1}$ ,  $r \neq 1$ ?

When does it make sense to use this form of the rule?



### Example 1

#### Determining the Sum of Given Terms of a Geometric Series

Determine the sum of the first 15 terms of this geometric series:

$$40 - 20 + 10 - 5 + 2.5 - \dots$$

Write the sum to the nearest hundredth.

#### SOLUTION

$$40 - 20 + 10 - 5 + 2.5 - \dots$$

$$t_1 = 40 \text{ and } r \text{ is } \frac{-20}{40} = -\frac{1}{2}$$

$$\text{Use: } S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$$

$$\text{Substitute: } n = 15, t_1 = 40, r = -\frac{1}{2}$$

$$S_{15} = \frac{40\left(1 - \left(-\frac{1}{2}\right)^{15}\right)}{1 - \left(-\frac{1}{2}\right)}$$

Use a calculator.

$$S_{15} = 26.6674\dots$$

The sum of the first 15 terms is approximately 26.67.

#### Check Your Understanding

- Determine the sum of the first 12 terms of this geometric series:

$$3 + 12 + 48 + 192 + \dots$$



### Example 2

#### Determining Terms of a Geometric Series

The sum of the first 10 terms of a geometric series is  $-29\,524$ .

The common ratio is  $-3$ . Determine the 1st term.

#### SOLUTION

Suppose the geometric series has 1st term,  $t_1$ , and common ratio,  $r$ .

$$\text{Use: } S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1 \quad \text{Substitute: } n = 10, S_n = -29\,524, r = -3$$

$$-29\,524 = \frac{t_1(1 - (-3)^{10})}{1 - (-3)}$$

#### Check Your Understanding

- The sum of the first 14 terms of a geometric series is 16 383. The common ratio is  $-2$ . Determine the 1st term.



#### Check Your Understanding

#### Answers:

- 16 777 215
- $-3$

$$\begin{aligned}
 -29\,524 &= \frac{t_1(1 - 59\,049)}{4} && \text{Simplify.} \\
 -29\,524 &= -14\,762t_1 && \text{Solve for } t_1. \\
 \frac{-29\,524}{-14\,762} &= t_1 \\
 2 &= t_1
 \end{aligned}$$

The 1st term is 2.

## THINK FURTHER

In *Example 2*, how can you check the answer?

### Example 3

### Determining the Sum of a Geometric Series

#### Check Your Understanding

3. Calculate the sum of this geometric series:  
 $-3 - 15 - 75 - \dots - 46\,875$

Calculate the sum of this geometric series:

$$6 + 12 + 24 + 48 + \dots + 12\,288$$

#### SOLUTION

$$6 + 12 + 24 + 48 + \dots + 12\,288$$

$$t_1 = 6 \text{ and } r \text{ is } \frac{12}{6} = 2$$

Determine  $n$ , the number of terms in the series.

$$\text{Use: } t_n = t_1 r^{n-1}$$

$$12\,288 = 6(2)^{n-1}$$

$$2048 = 2^{n-1}$$

$$2^{11} = 2^{n-1}$$

$$11 = n - 1$$

$$n = 12$$

$$\text{Substitute: } t_n = 12\,288, t_1 = 6, r = 2$$

Simplify. Divide each side by 6.

Use guess and test to write 2048 as a power of 2.

Equate exponents.

There are 12 terms in the series.

$$\text{Use: } S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$$

$$\text{Substitute: } n = 12, t_1 = 6, r = 2$$

$$S_{12} = \frac{6(1 - 2^{12})}{1 - 2}$$

$$S_{12} = 24\,570$$

The sum is 24 570.

#### Check Your Understanding

**Answer:**

3.  $-58\,593$

**Example 4****Using a Geometric Series to Model and Solve a Problem**

A person takes tablets to cure an ear infection. Each tablet contains 200 mg of an antibiotic. About 12% of the mass of the antibiotic remains in the body when the next tablet is taken. Determine the mass of antibiotic in the body after each number of tablets has been taken.

- a)** 3 tablets      **b)** 12 tablets

**SOLUTION**

- a)** Determine the mass of the antibiotic in the body for 1 to 3 tablets.

Number of tablets	Mass of antibiotic (mg)
1	200
2	$200 + 200(0.12) = 224$
3	$200 + 200(0.12) + 200(0.12)^2 = 226.88$

The problem can be modelled by a geometric series, which has 3 terms because 3 tablets were taken:

$$200 + 200(0.12) + 200(0.12)^2$$

The sum is 226.88. So, after taking the 3rd tablet, the total mass of antibiotic in the person's body is 226.88 mg or just under 227 mg.

- b)** Determine the sum of a geometric series whose terms are the masses of the antibiotic in the body after 12 tablets.

The series is:

$$200 + 200(0.12) + 200(0.12)^2 + 200(0.12)^3 + \dots + 200(0.12)^{11}$$

Use:  $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$       Substitute:  $n = 12, t_1 = 200, r = 0.12$

$$S_{12} = \frac{200(1 - 0.12^{12})}{1 - 0.12}$$

$$S_{12} = 227.2727\dots$$

The mass of antibiotic in the body after 12 tablets is approximately 227.27 mg, or just over 227 mg.

**Check Your Understanding**

- 4.** A person takes tablets to cure a chest infection. Each tablet contains 500 mg of an antibiotic. About 15% of the mass of the antibiotic remains in the body when the next tablet is taken. Determine the mass of antibiotic in the body after each number of tablets:

- a)** 3 tablets      **b)** 10 tablets

**THINK FURTHER**

In *Example 4*, compare the masses of antibiotic remaining in the body for parts a and b. What do you notice about the masses?

**Check Your Understanding****Answers:**

- 4.** **a)** 586.25 mg, or about 586 mg  
**b)** approximately 588.24 mg, or about 588 mg

---

## Discuss the Ideas

1. Why do the terms in some geometric series alternate between positive and negative numbers, but the terms in an arithmetic series never alternate?



2. How can you identify when a problem may be modelled by an arithmetic series or modelled by a geometric series?



---

## Exercises

### A

3. Write a geometric series for each geometric sequence.

a)  $1, 4, 16, 64, 256, \dots$

b)  $20, -10, 5, -2.5, 1.25, \dots$

4. Which series appear to be geometric? If the series is geometric, determine  $S_5$ .

a)  $2 + 4 + 8 + 16 + 32 + \dots$       b)  $2 - 4 + 8 - 16 + 32 - \dots$

c)  $1 + 4 + 9 + 16 + 25 + \dots$       d)  $-3 + 9 - 27 + 81 - 243 + \dots$

5. Use the given data about each geometric series to determine the indicated value. Give the answers to 3 decimal places where necessary.
- a)  $t_1 = 1, r = 0.3$ ; determine  $S_8$       b)  $t_1 = \frac{3}{4}, r = \frac{1}{2}$ ; determine  $S_4$

**B**

6. Determine  $S_6$  for each geometric series.

a)  $2 + 10 + 50 + \dots$

b)  $80 - 40 + 20 - \dots$

7. Determine  $S_{10}$  for each geometric series. Give the answers to 3 decimal places.

a)  $0.1 + 0.01 + 0.001 + 0.0001 + \dots$       b)  $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$

8. a) Explain why this series appears to be geometric:

$1 + 5 + 25 + 125 + \dots$

- b) What information do you need to be certain that this is a geometric series?
- c) What assumptions do you make when you identify or extend a geometric series?
9. For each geometric series, determine how many terms it has then calculate its sum.
- a)  $1 - 2 + 4 - 8 + \dots - 512$

b)  $-6561 + 2187 - 729 + 243 - \dots - 1$



- 10.** Identify the terms in each partial sum of a geometric series.
- a)  $S_5 = 62, r = 2$                       b)  $S_8 = 1111.1111; r = 0.1$

- 11.** On Monday, Ian had 3 friends visit his personal profile on a social networking website. On Tuesday, each of these 3 friends had 3 different friends visit Ian's profile. On Wednesday, each of the 9 friends on Tuesday had 3 different friends visit Ian's profile.
- a) Write the total number of friends who visited Ian's profile as a geometric series. What is the first term? What is the common ratio?
- b) Suppose this pattern continued for 1 week. What is the total number of people who visited Ian's profile? How do you know your answer is correct?

- 12.** Each stroke of a vacuum pump extracts 5% of the air in a  $50\text{-m}^3$  tank. How much air had been removed after the 50th stroke? Give the answer to the nearest tenth of a cubic metre.

- 13.** The sum of the first 10 terms of a geometric series is 1705. The common ratio is  $-2$ . Determine  $S_{11}$ . Explain your reasoning.

**C**

- 14.** Binary notation is used to represent numbers on a computer. For example, the number 1111 in base two represents  $1(2)^3 + 1(2)^2 + 1(2)^1 + 1$ , or 15 in base ten.
- a) Why is the sum above an example of a geometric series?

- b) Which number in base ten is represented by  
11 111 111 111 111 111 111 in base two? Explain your reasoning.

- 15.** Show how you can use geometric series to determine this sum:

$$1 + 2 + 3 + 4 + 8 + 9 + 16 + 27 + 32 + 64 + 81 + 128 + 243 + 256 + 512$$

- 16.** Determine the common ratio of a geometric series that has these  
partial sums:  $S_3 = -\frac{49}{8}$ ,  $S_4 = -\frac{105}{16}$ ,  $S_5 = -\frac{217}{32}$

## Multiple-Choice Questions

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1. For which geometric series is  $-1023$  the sum to 10 terms?

- A.  $1 - 2 + 4 - 8 + \dots$       B.  $1 + 2 + 4 + 8 + \dots$   
C.  $-1 + 2 - 4 + 8 - \dots$       D.  $-1 - 2 - 4 - 8 - \dots$

2. The sum of the first  $n$  terms of a geometric series is:  $S_n = 4^n - 1$   
For this series:

- I. The common ratio is 4.  
II. The first 3 terms are 3, 12, and 48.  
III.  $S_{2n} = 2^{4n} - 1$

- A. Statements I, II, and III are correct.  
B. Statements I and II are correct.  
C. Statements II and III are correct.  
D. Statements I and III are correct.

---

## Study Note

The rule for the sum of the first  $n$  terms of a geometric series has the restriction  $r \neq 1$ . Identify the geometric series with first term  $a$  and  $r = 1$ , then determine the sum of  $n$  terms.



## ANSWERS

3. a)  $1 + 4 + 16 + 64 + 256 + \dots$     b)  $20 - 10 + 5 - 2.5 + 1.25 - \dots$   
4. a) 62    b) 22    d)  $-183$     5. a) approximately 1.428    b) 1.406 25    6. a) 7812  
b) 52.5    7. a) approximately 0.111    b) approximately 0.750    9. a) 10 terms;  $-341$   
b) 9 terms;  $-4921$     10. a) 2, 4, 8, 16, 32    b) 1000, 100, 10, 1, 0.1, 0.01, 0.001, 0.0001  
11. a)  $3 + 9 + 27; 3; 3$     b) 3279 people    12.  $46.2 \text{ m}^3$     13.  $-3415$     14. b) 1 048 575  
15. 1386    16.  $\frac{1}{2}$

## Multiple Choice

1. D      2. A



## CHECKPOINT 2

### Self-Assess

Can you . . .	To check, try question . . .	For review, see . . .
write a geometric sequence and explain how you know it is geometric?		Page 32 in Lesson 1.3 (Example 2)
identify the assumptions you make when you create a geometric sequence or series?		Page 30 in Lesson 1.3
use a rule to determine the $n$ th term in a geometric sequence?	1, 2	Page 31 in Lesson 1.3 (Example 1)
use a rule to determine any of $n$ , $t_1$ , $t_n$ , or $r$ in a geometric sequence?	3	Page 33 in Lesson 1.3 (Example 3)
model and solve problems involving geometric sequences?		Page 34 in Lesson 1.3 (Example 4)
use a rule to determine the sum $S_n$ of a geometric series?	5	Page 45 in Lesson 1.4 (Example 1)
use a rule to determine $t_1$ in a geometric series, given the values of $r$ , $n$ , or $S_n$ ?		Pages 45–46 in Lesson 1.4 (Example 2)
use a rule to determine $n$ and $S_n$ in a geometric series, given the values of $t_1$ and $r$ ?	5	Page 46 in Lesson 1.4 (Example 3)
model and solve problems involving geometric series?	6	Page 47 in Lesson 1.4 (Example 4)

### Assess Your Understanding

#### 1.3

- Multiple Choice** For this geometric sequence:  $-5000, 500, -50, \dots$ ; which number below is the value of  $t_9$ ?  
 A. 0.0005      B.  $-0.0005$       C. 0.000 05      D.  $-0.000\ 05$
- This sequence is geometric:  $2, -6, 18, -54, \dots$ 
  - Write a rule to determine the  $n$ th term.
  - Use your rule to determine the 10th term.

3. Use the given data about each geometric sequence to determine the indicated value.

a)  $t_4 = -5$  and  $t_7 = 135$ ; determine  $t_1$

b)  $t_1 = -1$  and  $t_4 = -19\,683$ ; determine  $r$

#### 1.4

4. **Multiple Choice** The sum of the first 5 terms of a geometric series is  $\frac{121}{81}$ . The common ratio is  $\frac{1}{3}$ . What is the 2nd term?

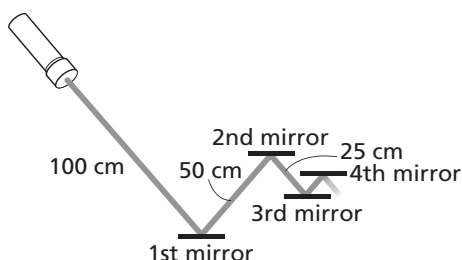
A.  $\frac{1}{3}$       B. 1      C.  $\frac{121}{3}$       D.  $\frac{1}{9}$

5. Use the given data about each geometric series to determine the indicated value.

a)  $t_1 = -4, r = 3$ ;  
determine  $S_5$

b)  $3125 + 625 + 125 + \dots + \frac{1}{25}$ ;  
determine  $n$

6. The diagram shows a path of light reflected by mirrors. After the first path, the length of each path is one-half the preceding length.



- a) What is the length of the path from the 4th mirror to the 5th mirror?

- b) To the nearest hundredth of a centimetre, what is the total length of the path from the 1st mirror to the 10th mirror?

### ANSWERS

1. D    2. a)  $t_n = 2(-3)^{n-1}$     b) -39 366    3. a)  $\frac{5}{27}$     b) 27    4. A  
5. a) -484    b) 8    6. a) 6.25 cm    b) 199.80 cm

## 1.5

## Graphing Geometric Sequences and Series

**FOCUS** Investigate the graphs of geometric sequences and geometric series.

## Get Started

Here are 4 geometric sequences:

**A.**  $1, 2, 4, 8, 16, \dots$       **B.**  $1, -2, 4, -8, 16, \dots$

**C.**  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$       **D.**  $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \dots$

Compare the sequences. How are they alike? How are they different?



## Construct Understanding

Use a graphing calculator or graphing software to investigate graphs of geometric sequences and geometric series that have the same first term but different common ratios.

- A.** Choose a positive first term. Choose a common ratio,  $r$ , in each of the intervals in the table below. For each common ratio, create the first 5 terms of a geometric sequence.

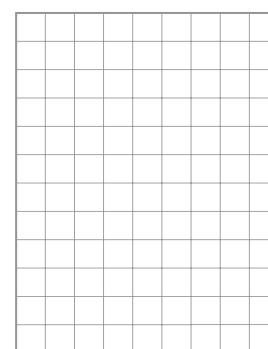
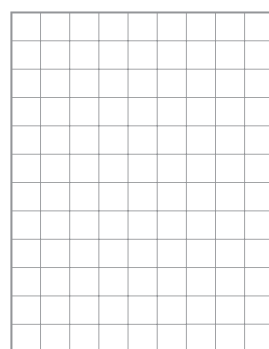
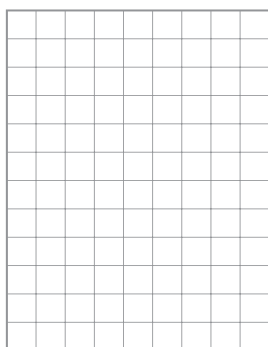
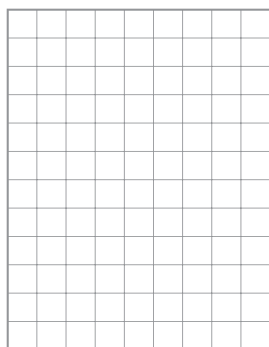


Interval	Common ratio, $r$	Geometric sequence
$r > 1$		
$0 < r < 1$		
$-1 < r < 0$		
$r < -1$		



**B.** For each sequence

- Graph the term numbers on the horizontal axis and the term values on the vertical axis. Sketch and label each graph on a grid below, or print each graph.



- What happens to the term values as more points are plotted?



C. Use the four geometric sequences in Part A to create four corresponding geometric series.

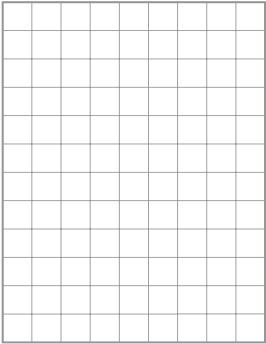
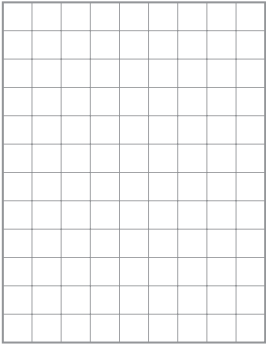
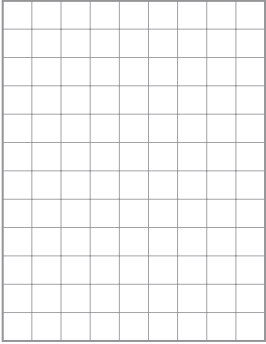
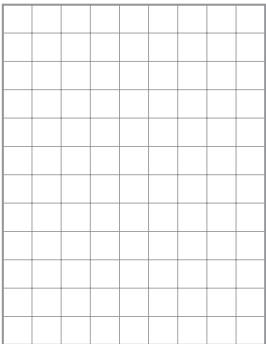
For each series

- Complete the table below by calculating these partial sums:  
 $S_1, S_2, S_3, S_4, S_5$



Interval	Common ratio, $r$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$
$r > 1$						
$0 < r < 1$						
$-1 < r < 0$						
$r < -1$						

- Graph the numbers of terms in the partial sums on the horizontal axis and the partial sums on the vertical axis. Sketch and label each graph on a grid below, or print each graph.



- What happens to the partial sums as more points are plotted?



**D.** Without graphing

- Describe the graph of this geometric sequence:  $3, 2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$

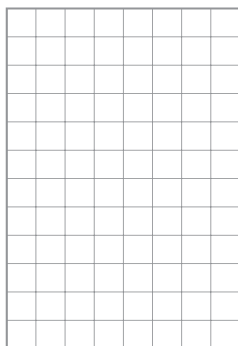
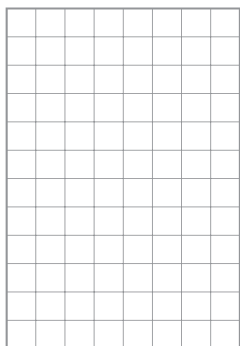


- Describe the graph of the partial sums of this geometric series:

$$3 + 2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \dots$$



Verify your descriptions by graphing. Sketch and label each graph on a grid below, or print each graph.



## Assess Your Understanding

1. Create the first 5 terms of a geometric sequence for each description of a graph.

a) The term values approach 0 as more points are plotted.



b) The term values increase as more points are plotted.



c) The term values alternate between positive and negative as more points are plotted.



2. Create a geometric series for each description of a graph.

a) The partial sums approach a constant value as more points are plotted.



b) The partial sums increase as more points are plotted.



# 1.6 Infinite Geometric Series

**FOCUS** Determine the sum of an infinite geometric series.

## Get Started

Write  $0.\overline{6}$  as a series.

What type of series is it? How do you know?



## Construct Understanding

Draw a square.

Divide it into 4 equal squares, then shade 1 smaller square.

Divide one smaller square into 4 equal squares, then shade 1 square.

Continue dividing smaller squares into 4 equal squares and shading 1 square, for as long as you can.

Suppose you could continue this process indefinitely.

Estimate the total area of the shaded squares. Explain your reasoning.



Square, $n$	Area of square	$S_n$
1		
2		
3		
4		
5		

An **infinite geometric series** has an infinite number of terms.  
 For an infinite geometric series, if the sequence of partial sums converges to a constant value as the number of terms increases, then the geometric series is convergent and the constant value is the finite sum of the series. This sum is called the **sum to infinity** and is denoted by  $S_{\infty}$ .

### Example 1

### Estimating the Sum of an Infinite Geometric Series

#### Check Your Understanding

1. Determine whether each infinite geometric series has a finite sum. Estimate each finite sum.

a)  $\frac{1}{3} + \frac{1}{12} + \frac{1}{48} + \frac{1}{192} + \dots$

b)  $-4 - 8 - 16 - 32 - \dots$

c)  $\frac{1}{10} - \frac{1}{100} + \frac{1}{1000} - \frac{1}{10\,000} + \dots$

Determine whether each infinite geometric series has a finite sum. Estimate each finite sum.

a)  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

b)  $0.5 + 1 + 2 + 4 + \dots$

c)  $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$

#### SOLUTION

For each geometric series, calculate some partial sums.

a)  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

$$S_1 = \frac{1}{2}$$

$$S_2 = S_1 + \frac{1}{4}$$

$$S_3 = S_2 + \frac{1}{8}$$

$$S_4 = S_3 + \frac{1}{16}$$

$$S_2 = \frac{1}{2} + \frac{1}{4}$$

$$S_3 = \frac{3}{4} + \frac{1}{8}$$

$$S_4 = \frac{7}{8} + \frac{1}{16}$$

$$S_2 = \frac{3}{4}$$

$$S_3 = \frac{7}{8}$$

$$S_4 = \frac{15}{16}$$

The next term in the series is  $\frac{1}{32}$ .

$$S_5 = S_4 + \frac{1}{32}$$

$$S_5 = \frac{15}{16} + \frac{1}{32}$$

$$S_5 = \frac{31}{32}$$

These partial sums appear to get closer to 1.

An estimate of the finite sum is 1.

b)  $0.5 + 1 + 2 + 4 + \dots$

$$S_1 = 0.5$$

$$S_2 = S_1 + 1$$

$$S_3 = S_2 + 2$$

$$S_4 = S_3 + 4$$

$$S_2 = 0.5 + 1$$

$$S_3 = 1.5 + 2$$

$$S_4 = 3.5 + 4$$

$$S_2 = 1.5$$

$$S_3 = 3.5$$

$$S_4 = 7.5$$

As the number of terms increases, the partial sums increase, so the series does not have a finite sum.

#### Check Your Understanding

#### Answers:

1. a) approximately 0.44  
 b) does not have a finite sum  
 c) approximately 0.09

c)  $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$

$S_1 = \frac{1}{2}$	$S_2 = S_1 - \frac{1}{4}$	$S_3 = S_2 + \frac{1}{8}$	$S_4 = S_3 - \frac{1}{16}$
$S_1 = 0.5$	$S_2 = \frac{1}{2} - \frac{1}{4}$	$S_3 = \frac{1}{4} + \frac{1}{8}$	$S_4 = \frac{3}{8} - \frac{1}{16}$
	$S_2 = \frac{1}{4}$	$S_3 = \frac{3}{8}$	$S_4 = \frac{5}{16}$
	$S_2 = 0.25$	$S_3 = 0.375$	$S_4 = 0.3125$

The next term is  $\frac{1}{32}$ .

The next term is  $-\frac{1}{64}$ .

$S_5 = S_4 + \frac{1}{32}$	$S_6 = S_5 - \frac{1}{64}$
$S_5 = \frac{5}{16} + \frac{1}{32}$	$S_6 = \frac{11}{32} - \frac{1}{64}$
$S_5 = \frac{11}{32}$	$S_6 = \frac{21}{64}$
$S_5 = 0.34375$	$S_6 = 0.328125$

The partial sums alternately increase and decrease, but appear to get close to 0.3.

An estimate of the finite sum is 0.3.

## THINK FURTHER

In *Example 1*, what other strategy could you use to estimate each finite sum?

In *Example 1*, why is each partial sum written as a decimal?

In *Example 1*, each series has the same first term but different common ratios. So, from *Example 1* and Lesson 1.5, it appears that the value of  $r$  determines whether an infinite geometric series converges or diverges.

Consider the rule for the sum of  $n$  terms.

For a geometric series,

$$S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$$

When  $-1 < r < 1$ ,  $r^n$  approaches 0 as  $n$  increases indefinitely.

So,  $S_n$  approaches  $\frac{t_1(1 - 0)}{1 - r}$

$$\text{and, } S_\infty = \frac{t_1}{1 - r}$$

## THINK FURTHER

Why does this rule not apply when  $r \leq -1$  or  $r \geq 1$ ?



### The Sum of an Infinite Geometric Series

For an infinite geometric series with first term,  $t_1$ , and common ratio,  $-1 < r < 1$ , the sum of the series,  $S_\infty$ , is:

$$S_\infty = \frac{t_1}{1 - r}$$

#### Check Your Understanding

2. Determine whether each infinite geometric series converges or diverges. If it converges, determine its sum.

a)  $32 + 8 + 2 + 0.5 + \dots$

b)  $100 - 10 + 1 - 0.1 + \dots$



#### Example 2

#### Determining the Sum of an Infinite Geometric Series

Determine whether each infinite geometric series converges or diverges. If it converges, determine its sum.

a)  $27 - 9 + 3 - 1 + \dots$       b)  $4 - 8 + 16 - 32 + \dots$

#### SOLUTION

a)  $27 - 9 + 3 - 1 + \dots$

$t_1 = 27$  and  $r$  is:  $\frac{-9}{27} = -\frac{1}{3}$

The common ratio is between  $-1$  and  $1$ , so the series converges.

Use the rule for the sum of an infinite geometric series:

$$S_\infty = \frac{t_1}{1 - r} \quad \text{Substitute: } t_1 = 27, r = -\frac{1}{3}$$

$$S_\infty = \frac{27}{1 - \left(-\frac{1}{3}\right)}$$

$$S_\infty = \frac{27}{1 + \frac{1}{3}}$$

$$S_\infty = \frac{27}{\frac{4}{3}}$$

$$S_\infty = 27\left(\frac{3}{4}\right)$$

$$S_\infty = 20.25$$

The sum of the infinite geometric series is 20.25.

b)  $4 - 8 + 16 - 32 + \dots$

$t_1 = 4$  and  $r$  is:  $\frac{-8}{4} = -2$

The common ratio is not between  $-1$  and  $1$ , so the series diverges.

The infinite geometric series does not have a finite sum.

#### Check Your Understanding

#### Answers:

2. a) converges; the sum is  $42.\overline{6}$

b) converges; the sum is  $90.\overline{90}$



## THINK FURTHER

In Example 2, how could you check that the sum in part a is reasonable?



**Example 3****Using an Infinite Geometric Series to Solve a Problem**

Determine a fraction that is equal to  $0.4\overline{9}$ .

**SOLUTION**

The repeating decimal  $0.4\overline{9}$  can be expressed as:

$$0.4 + 0.09 + 0.009 + 0.0009 + \dots$$

The repeating digits form an infinite geometric series.

The series converges because  $-1 < r < 1$ . Use the rule for  $S_\infty$ .

$$S_\infty = \frac{t_1}{1 - r} \quad \text{Substitute: } t_1 = 0.09, \text{ or } \frac{9}{100}; r = 0.1, \text{ or } \frac{1}{10}$$

$$S_\infty = \frac{\frac{9}{100}}{1 - \frac{1}{10}}$$

$$S_\infty = \frac{\frac{9}{100}}{\frac{9}{10}}, \text{ or } \frac{1}{10}$$

Add  $\frac{1}{10}$  to 0.4, or  $\frac{4}{10}$ , the non-repeating part of the decimal:

$$\frac{4}{10} + \frac{1}{10} = \frac{5}{10}, \text{ or } \frac{1}{2}$$

$$\text{So, } 0.4\overline{9} = \frac{1}{2}$$

**Check Your Understanding**

3. Determine a fraction that is equal to  $0.1\overline{6}$ .

**Discuss the Ideas**

1. How do you determine whether an infinite geometric series diverges or converges?



2. Why is an infinite geometric series with  $r = 1$  or  $r = -1$  a divergent series?

**Check Your Understanding****Answer:**

3.  $\frac{1}{6}$

## Exercises

### A

3. Determine whether each infinite geometric series has a finite sum.

How do you know?

a)  $2 + 3 + 4.5 + 6.75 + \dots$

b)  $-0.5 - 0.05 - 0.005 - 0.0005 - \dots$

c)  $\frac{1}{2} - \frac{3}{8} + \frac{9}{32} - \frac{27}{128} + \dots$

d)  $0.1 + 0.2 + 0.4 + 0.8 + \dots$

4. Write the first 4 terms of each infinite geometric series.

a)  $t_1 = -4, r = 0.3$

b)  $t_1 = 1, r = -0.25$

c)  $t_1 = 4, r = \frac{1}{5}$

d)  $t_1 = -\frac{3}{2}, r = -\frac{3}{8}$

5. Each infinite geometric series converges. Determine each sum.

a)  $8 + 2 + 0.5 + 0.125 + \dots$       b)  $-1 - \frac{3}{4} - \frac{9}{16} - \frac{27}{64} - \dots$

c)  $10 - \frac{20}{3} + \frac{40}{9} - \frac{80}{27} + \dots$       d)  $-2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \dots$

**B**

6. What do you know about the common ratio of an infinite geometric series whose sum is finite?

7. Use the given data about each infinite geometric series to determine the indicated value.

a)  $t_1 = 21, S_\infty = 63$ ; determine  $r$       b)  $r = -\frac{3}{4}, S_\infty = \frac{24}{7}$ ; determine  $t_1$

- 8.** Use an infinite geometric series to express each repeating decimal as a fraction.

a)  $0.\overline{497}$

b)  $1.\overline{143}$

- 9.** The hour hand on a clock is pointing to 12. The hand is rotated clockwise  $180^\circ$ , then another  $60^\circ$ , then another  $20^\circ$ , and so on. This pattern continues.

- a) To which number would the hour hand converge if this rotation continued indefinitely? Explain what you did.

- b) What assumptions did you make?

- 10.** Brad has a balance of \$500 in a bank account. Each month he spends 40% of the balance remaining in the account.
- a) Express the total amount Brad spends in the first 4 months as a series. Is the series geometric? Explain.
- b) Determine the approximate amount Brad spends in 10 months. Explain what you did.
- c) Suppose Brad could continue this pattern of spending indefinitely. Would he eventually empty his bank account? Explain.

**C**

- 11.** Write the product of  $0.\overline{a}$  and  $0.\overline{b}$  as a fraction, where  $a$  and  $b$  represent 1-digit natural numbers. Explain your strategy.
- 12.** Create 2 different infinite geometric series with a sum of 4. Explain what you did.

13. Determine the sum of this infinite series:

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{32}} + \frac{1}{\sqrt{128}} + \frac{1}{\sqrt{512}} + \dots$$

### Multiple-Choice Questions

1. What is the sum of this infinite geometric series?

$$10 - \frac{20}{3} + \frac{40}{9} - \frac{80}{27} + \dots$$

- A. 30      B. 4      C. 6      D. 20

2. Which infinite geometric series has the sum  $-8.\bar{3}$ ?

A.  $-5 - 2 - 0.8 - 0.32 - \dots$       B.  $-5 + 2 - 0.8 + 0.32 - \dots$

C.  $5 - 2 + 0.8 - 0.32 + \dots$       D.  $5 + 2 + 0.8 + 0.32 + \dots$

3. How many of these geometric series have finite sums?

$1 + 0.5 + 0.125 + 0.0625 + \dots$        $3 - 9 + 27 - 81 + \dots$

$1 + \frac{4}{3} + \frac{16}{9} + \frac{64}{27} + \dots$        $-12 - 6 - 3 - 1.5 - \dots$

- A. 1 series      B. 2 series      C. 3 series      D. 4 series

### Study Note

What is a rule for determining the sum of an infinite geometric series?

When is it appropriate to apply this rule? When is it not appropriate?



### ANSWERS

3. a) not finite   b) finite   c) finite   d) not finite   4. a)  $-4 - 1.2 - 0.36 - 0.108$

b)  $1 - 0.25 + 0.0625 - 0.015625$    c)  $4 + \frac{4}{5} + \frac{4}{25} + \frac{4}{125}$

d)  $-\frac{3}{2} + \frac{9}{16} - \frac{27}{128} + \frac{81}{1024}$    5. a)  $10.\bar{6}$    b)  $-4$    c) 6   d)  $-1.5$    7. a)  $\frac{2}{3}$    b) 6

8. a)  $\frac{493}{990}$    b)  $\frac{1142}{999}$    9. a) 9   10. a)  $\$500(0.4) + \$500(0.6)(0.4) + \$500(0.6)^2(0.4) +$

$\$500(0.6)^3(0.4)$ ; geometric   b) \$496.98   c) yes   11.  $\frac{ab}{81}$    13.  $\sqrt{2}$

### Multiple Choice

1. C   2. A   3. B

# STUDY GUIDE

## Concept Summary

Big Ideas	Applying the Big Ideas
<ul style="list-style-type: none"><li>• An arithmetic sequence is related to a linear function and is created by repeatedly adding a constant to an initial number. An arithmetic series is the sum of the terms of an arithmetic sequence.</li></ul>	<p>This means that:</p> <ul style="list-style-type: none"><li>• The common difference of an arithmetic sequence is equal to the slope of the line through the points of its related linear function.</li><li>• Rules can be derived to determine the <math>n</math>th term of an arithmetic sequence and the sum of the first <math>n</math> terms of an arithmetic series.</li></ul>
<ul style="list-style-type: none"><li>• A geometric sequence is created by repeatedly multiplying an initial number by a constant. A geometric series is the sum of the terms of a geometric sequence.</li></ul>	<ul style="list-style-type: none"><li>• The common ratio of a geometric sequence can be determined by dividing any term after the first term by the preceding term.</li><li>• Rules can be derived to determine the <math>n</math>th term of a geometric sequence and the sum of the first <math>n</math> terms of a geometric series.</li></ul>
<ul style="list-style-type: none"><li>• Any finite series has a sum, but an infinite geometric series may or may not have a sum.</li></ul>	<ul style="list-style-type: none"><li>• The common ratio determines whether an infinite series has a finite sum.</li></ul>

## Chapter Study Notes

- What information do you need to know about an arithmetic sequence and a geometric sequence to determine  $t_n$ ?



- What information do you need to know about an arithmetic series and a geometric series to determine the sum  $S_n$ ?





## Skills Summary

Skill	Description	Example
Determine the general term, $t_n$ , for an arithmetic sequence. (1.1, 1.2)	A rule is: $t_n = t_1 + d(n - 1)$ where $t_1$ is the first term, $d$ is the common difference, and $n$ is the number of terms.	For this arithmetic sequence: $-9, -3, 3, 9, \dots$ the 20th term is: $t_{20} = -9 + 6(20 - 1)$ $t_{20} = -9 + 6(19)$ $t_{20} = 105$
Determine the sum of $n$ terms, $S_n$ , for an arithmetic series. (1.2)	When $n$ is the number of terms, $t_1$ is the first term, $t_n$ is the $n$ th term, and $d$ is the common difference One rule is: $S_n = \frac{n(t_1 + t_n)}{2}$ Another rule is: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$	For this arithmetic series: $5 + 7 + 9 + 11 + 13 + 15 + 17$ ; the sum of the first 7 terms is: $S_7 = \frac{7(5 + 17)}{2}$ $S_7 = \frac{7(22)}{2}$ $S_7 = 77$
Determine the general term, $t_n$ , for a geometric sequence. (1.3, 1.4)	A rule is: $t_n = t_1 r^{n-1}$ where $t_1$ is the first term, $r$ is the common ratio, and $n$ is the number of terms.	For this geometric sequence: $1, -0.25, 0.0625, \dots$ the 6th term is: $t_6 = (-0.25)^{6-1}$ $t_6 = (-0.25)^5$ $t_6 = -0.000\ 976\ 5 \dots$
Determine the sum of $n$ terms, $S_n$ , of a geometric series. (1.4)	A rule is: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$ where $t_1$ is the first term, $r$ is the common ratio, and $n$ is the number of terms.	For this geometric series: $4, 2, 1, \dots$ the sum of the first 10 terms is: $S_{10} = \frac{4(1 - 0.5^{10})}{1 - 0.5}$ $S_{10} = 7.9921 \dots$ , or approximately 8
Determine the sum, $S_\infty$ , of a convergent infinite geometric series. (1.6)	When $r$ is between $-1$ and $1$ , use this rule: $S_\infty = \frac{t_1}{1 - r}$ where $t_1$ is the first term and $r$ is the common ratio.	For this geometric series: $100 - 50 + 25 - \dots$ the sum is: $S_\infty = \frac{100}{1 - (-0.5)}$ $S_\infty = 66.\overline{6}$

# REVIEW

## 1.1

1. During the 2003 fire season, the Okanagan Mountain Park fire was the most significant wildfire event in B.C. history. By September 7, the fire had reached about 24 900 ha and was burning at a rate of about 150 ha/h.
  - a) Suppose the fire continued to burn at the same rate. Create terms of a sequence to represent the area of the fire for each of the next 6 h. Why is the sequence arithmetic?
  - b) Write a rule for the general term of the sequence in part a. Use the rule to predict the area of the fire after 24 h. What assumptions did you make?

## 1.2

2. Use the given data about each arithmetic series to determine the indicated value.
  - a)  $5 + 3\frac{1}{2} + 2 + \frac{1}{2} - 1 - \dots$ ; determine  $S_{21}$
  - b)  $S_{12} = 78$  and  $t_1 = -21$ ; determine  $t_{12}$

### 1.3

3. Explain the meaning of this newspaper headline.

**I-Pod Sales Grew Geometrically from 2001 to 2006**

4. A soapstone carving was appraised at \$2500. The value of the carving is estimated to increase by 12% each year. What will be the approximate value of the carving after 15 years?

### 1.4

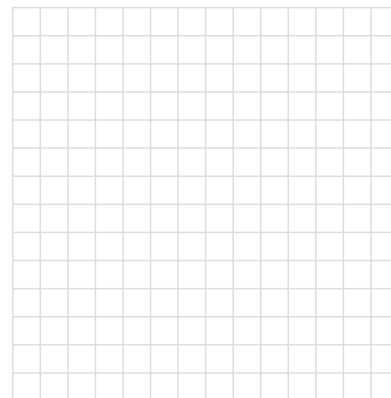
5. Determine the sum of the series below. Give the answer to 3 decimal places.

$$-700 + 350 - 175 + \dots + 5.468\ 75$$

## 1.5

6. Use a graphing calculator or graphing software.

Use the series from question 5. Graph the first 5 partial sums. Explain how the graph shows whether the series converges or diverges.



## 1.6

7. Explain how you can use the common ratio of a geometric series to identify whether the series is convergent or divergent.

8. Identify each infinite geometric series that converges. Determine the sum of any series that converges.

a)  $2 - 3 + 4.5 - 6.75 + \dots$       b)  $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots$

9. A small steel ball bearing is moving vertically between two electromagnets whose relative strength varies each second. The ball bearing moves 10 cm up in the 1st second, then 5 cm down in the 2nd second, then 2.5 cm up in 3rd second, and so on. This pattern continues.

a) Assume the distance the ball bearing moves up is positive; the distance it moves down is negative.

i) Write a series to represent the distance travelled in 5 s.

ii) Calculate the sum of the series. What does this sum represent?

b) Suppose this process continues indefinitely. What is the sum of the series?

### **ANSWERS**

1. a) 25 050, 25 200, 25 350, 25 500, 25 650, 25 800, ... b) approximately 28 500 ha  
 2. a) -210 b) 34 4. \$13 684 5. -464.843 75 8. a) diverges b) converges; 1  
 9. a) i)  $10 - 5 + 2.5 - 1.25 + 0.625$  ii) 6.875 cm b)  $6.\overline{6}$  cm

# PRACTICE TEST

- 1. Multiple Choice** What is the sum of the first 30 terms of this arithmetic series?  $-5 - 2 + 1 + 4 + \dots$

A. 1152      B. 1155      C. 1158      D. 1161

- 2. Multiple Choice** What is the sum of the first 10 terms of this geometric series?  $-12\,800 + 6400 - 3200 + 1600 - \dots$

A. 8525      B.  $-8525$       C.  $-8537.5$       D. 8537.5

- 3. a)** Which sequence below appears to be arithmetic? Justify your answer.

i)  $4, -10, 16, -22, 28, \dots$     ii)  $4, -10, -24, -38, -52, \dots$

- b)** Assume that the sequence you identified in part a is arithmetic. Determine:

i) a rule for  $t_n$       ii)  $t_{17}$

iii) the term that has value  $-332$

**4.** For a geometric sequence,  $t_4 = -1000$  and  $t_7 = 1$ ; determine:

**a)**  $t_1$

**b)** the term with value 0.0001

**5. a)** For the infinite geometric series below, identify which series converges and which series diverges. Justify your answer.

**i)**  $100 - 150 + 225 - 337.5 + \dots$

**ii)**  $10 + 5 + 2.5 + 1.25 + \dots$

**b)** For which series in part a can you determine its sum? Explain why, then determine this sum.

6. This sequence represents the approximate lengths in centimetres of a spring that is stretched by loading it with from one to four 5-kg masses: 50, 54, 58, 62, . . .

Suppose the pattern in the sequence continues. What will the length of the spring be when it is loaded with ten 5-kg masses? Explain how you found out.

7. As part of his exercise routine, Earl uses a program designed to help him eventually do 100 consecutive push-ups. He started with 17 push-ups in week 1 and planned to increase the number of push-ups by 2 each week.

a) In which week does Earl expect to reach his goal?

b) What is the total number of push-ups he will have done when he reaches his goal? Explain how you know.

## **ANSWERS**

1. B   2. B   3. a) i) not arithmetic   ii) arithmetic   b) i)  $t_n = 4 - 14(n - 1)$   
ii) -220   iii)  $t_{25}$    4. a) 1 000 000   b)  $t_{11}$    5. a) i) diverges   ii) converges   b) 20  
6. 86 cm   7. a) week 43   b) 2537





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