NUMBER TALKS

FRACTIONS, DECIMALS, AND PERCENTAGES

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With Video Streaming

SHERRY PARRISH AND ANN DOMINICK

Foreword by Steve Leinwand

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Foreword

Number Talks is a powerful, accessible, and informative journey through mathematical understanding and the pedagogical strategies that support its development. Broadly and consistently implementing the ideas and techniques presented by the authors can change lives—ours as teachers of mathematics and those of our students—as we work to build a deep and lasting sense of fractions, decimals, and percentages in our students.

Done right, there is something so engaging, so mathematically rich and yes, even so magical, about a number talk. Pose a straightforward exercise or simple word problem, give students time to solve it mentally, and then open the floodgates of thinking and reasoning with little more than asking, "How did you get your answer?" "Who did it differently?" and "How are our approaches similar and different?"

As authors Sherry Parrish and Ann Dominick remind us in Chapter 1, we all share a common goal when it comes to mathematics: We want our students to "become mathematically proficient, reason mathematically, and compute with accuracy, flexibility, and efficiency." It's one thing to state such a mom-and-apple-pie goal. It's quite another to propose and describe practical and powerful strategies for meeting this goal. However, that is exactly why this resource is so helpful. *Number Talks* models how the techniques of conducting mathematical number talks and number strings enable students to grapple with number relationships, analyze their justifications and explanations, and communicate and solidify these critical understandings.

Parrish's first book, *Number Talks: Whole Number Computation,* popularized the very concept and approach of number talks with a primary focus on whole numbers in grades K–5. This second resource in the series tackles the much more intricate realm of fractions, decimals, and percentages. It extends what students have hopefully experienced previously to these far more complex and often seriously confusing topics that build on whole number fluency and understanding to rational numbers.

Just consider how many fourth and fifth graders are taught, confused, and even mathematically damaged by the approach of remembering and regurgitating that the "one correct way" to add $\frac{1}{2}$ and $\frac{7}{8}$ is to mindlessly get a common denominator of 8, convert the $\frac{1}{2}$ to $\frac{4}{8}$, and then add the numerators and "keep the denominator"—because "that's the rule"—to get the correct answer of $\frac{11}{8}$. And based on years of test scores, this makes frighteningly little sense to hundreds of millions of students.

Now envision, as described on pages 23–24, this same class where, through the process of number talks, we see a decidedly different mindset about teaching mathematics in practice.

- One student argues that $\frac{7}{8}$ is the same as $\frac{1}{2}+\frac{3}{8}$ so $\frac{1}{2}+\frac{7}{8}$ is the same as $\frac{1}{2}+\frac{1}{2}+\frac{3}{8}$ or $1+\frac{3}{8}$ or $1\frac{3}{8}$.
- Another student proposes that you can rename $\frac{1}{2}$ as $\frac{4}{8}$ and that means that there is a total of $\frac{11}{8}$.
- A third student announces that, thinking of her ruler, she can decompose $\frac{1}{2}$ into $\frac{3}{8}$ and $\frac{1}{8}$, and since $\frac{1}{8}+\frac{7}{8}$ is 1, the sum must be $1\frac{3}{8}$.

When each of these approaches is recorded and discussed in the manner so clearly modeled throughout this resource, we move from the limits of "remember and regurgitate" to the impact of a classroom dominated by "pause, reason, explain, and connect."

What has always impressed me about both the simplicity and the power of number talks done well is how they help to shift our teaching mindsets from "our telling" to "their thinking." What has dazzled me in dozens of classrooms is how effortlessly a number talk surfaces alternative approaches and multiple representations that, although raised by only a few students, inform the learning of the entire class. What has amazed me is how seamlessly a number talk raises big ideas like equivalents, place value, representations, the meaning of operations and properties of numbers—all essential and unifying ideas for all mathematics learning. And what has astounded me is how frequently a number talk surfaces common misconceptions and common errors that can be resolved well before they grow into major impediments.

We often think that the mathematical practices of reasoning and problem solving require rich and complex problems. But then we discover that a sixth-grade classroom discussion launched by a number talk about $17\frac{1}{4} - 9\frac{5}{8}$ can thoroughly engage students in constructing viable arguments and critiquing the reasoning of others—thereby operationalizing one of the key Common Core Standards for Mathematical Practice in a much more efficient and accessible manner.

As the reader sees over and over again in the diverse examples that Parrish and Dominick present and describe in great detail, a number talk is the diametric opposite of teaching students the one right way to get the one right answer. Instead, an effective number talk celebrates different approaches and values different ways of thinking. As such, number talks place sense making exactly where it belongs—in the forefront of every mathematics lesson.

Helping students develop deep and lasting number sense around fractions, decimals, and percentages is *not* easy. Eliciting, justifying, and connecting alternative approaches—including some that are at first alien to us—is *not easy.* Initially implementing successful number talks with our students is *not* easy. But by taking some of the same small risks as teachers that we expect our students to take, together we can blend the critical goals of deeper number sense and computational fluency with the techniques of number talks to create far more effective learning experiences and much greater success for our students.

I sincerely hope that every reader of this book learns as much from it as I have and, starting today, is as motivated to transfer these ideas into daily classroom practice.

—Steve Leinwand, American Institutes for Research

CHAPTER 1

What Is a Number Talk?

Number Talks \'nəm-bər\ \'tōks\

1. A five- to fifteen-minute classroom conversation around purposefully crafted problems that are solved mentally.
2. The best part of a teacher's day.

This chapter builds an understanding of number talks by first addressing their importance (Why number talks?), then thoughtfully looking at four foundational principles and concluding with a look at how number talks elicit the Standards for Mathematical Practice. Through video clips, classroom dialogue, and numerous mathematical examples, the chapter emphasizes the importance of creating a learning community, selecting purposeful problems, and purposefully recording (writing) students' thinking during a number talk. All of this prepares readers for a focus on the last foundational principle—know when to ask and when to tell—and why it's crucial for educators to shift from the traditional role of "teacher as teller" to that of "teacher as facilitator, questioner, listener, and learner."

OVERVIEW

$16 \times \frac{2}{8}$: Why Number Talks?
Classroom Clip 1.1

To view this video clip, scan the QR code or access via http://hein.pub/MathOLR

Consider the following questions as you watch this number talk from a fifth-grade classroom:

1. What do you see happening in this number talk?
2. What do you notice about what the students are doing and saying?
3. What do you notice about what the teacher is doing and saying?
4. What surprises you?
5. What is familiar to you?
6. In what ways does this number talk differ from a traditional lesson teaching students how to multiply a whole number times a fraction?
7. What questions do you have after watching this number talk?

Why Number Talks?

The work-related challenges that most people face in today's world demand the best of creative thinking and problem-solving skills. To meet this call as teachers, we must develop our own conceptual understanding of mathematics. Specifically, teachers must be able to

- reason about quantitative information;
- utilize number sense;
- discern whether procedures make sense;
- identify which procedures are applicable to specific situations;

- check for reasonableness of solutions and answers; and
- communicate mathematically to others.

We must expect this not only of ourselves but also of our students. To summarize the previous statements, today's mathematics curricula and instruction must focus on preparing students to compute accurately, efficiently, and flexibly and to be mathematically proficient.

So what does it mean to reason and compute with accuracy, efficiency, and flexibility? *Accuracy* implies the ability to produce a correct answer; *efficiency* refers to the ability to choose an appropriate, expedient strategy for a specific problem; and *flexibility* refers to the ability to use number relationships with ease. Let's take a look at an example.

Sophia, a sixth grader, needs to compare $\frac{24}{50}$ and $\frac{33}{64}$. She chooses to use $\frac{1}{2}$ to prove that $\frac{33}{64}$ is greater than $\frac{24}{50}$. She explains that $\frac{25}{50}$ is exactly $\frac{1}{2}$, so $\frac{24}{50}$ would be a little less than $\frac{1}{2}$. She applies this same reasoning to note that $\frac{32}{64}$ is exactly $\frac{1}{2}$, so $\frac{33}{64}$ is a little more than $\frac{1}{2}$, which means it is greater than $\frac{24}{50}$.

Typically, students learn to use either the cross-multiplication algorithm or the common-denominator procedure to compare fractions, but as Sophia's example shows, using $\frac{1}{2}$ as a benchmark makes solving this problem much more efficient.

Number Talk Tip

Students need to be able to compute accurately, efficiently, and flexibly. *Accuracy* implies the ability to produce a correct answer; *efficiency* refers to the ability to choose an appropriate, expedient strategy for a specific problem; and *flexibility* refers to the ability to use number relationships with ease.

To help students become mathematically proficient, reason mathematically, and compute with accuracy, flexibility, and efficiency, teachers must provide opportunities for students to grapple with constructing number relationships, test these relationships, and analyze and discuss their reasoning. As educators, we want students to have strong number sense: "an awareness and understanding about what numbers are, their relationships, their magnitude, [and] the relative effect of operating on numbers, including the use of mental mathematics and estimation" (Fennell and Landis 1994, 187).

With this in mind, the use of classroom number talks is a powerful opportunity teachers can provide students. During a number talk, teachers ask students to mentally solve problems. These problems help students focus on number relationships, encourage and elicit students' individual strategies, and help students construct important mathematical ideas. The classroom conversations and discussions that ensue around these purposefully crafted problems are at the core of number

Number Talk Tip

Part of a successful number talk is encouraging students to solve problems mentally in various ways.

talks. As students share and defend their solutions and strategies, they have opportunities to collectively reason about numbers while building their mathematical understanding.

Number talks can be especially effective at helping students develop fractional reasoning. In particular, fraction number talks foster efficient, flexible, and accurate computation strategies that build on foundational ideas of rational numbers (e.g., part-to-whole relationships, equivalence, and partitioning). Consider the following "Inside the Classroom" excerpt.

Inside the Classroom

A Number Talk for $\frac{1}{2} + \frac{3}{4}$

Recently, we had the opportunity to visit Ms. Harris's fifth-grade classroom, where number talks are an integral component of mathematics instruction. Ms. Harris asked her fifth graders to gather on the rug without any paper or pencils. She then wrote $\frac{1}{2} + \frac{3}{4}$ where everyone could see it.

Ms. Harris: Show a thumb to your chest when you have a solution. *[Several students place their thumbs up to indicate they have one way to solve the problem.]*

If you find a second way to solve this problem, show you have another way by placing two fingers on your chest. *[Ms. Harris waits until a majority of the students show they have at least one way to think about the problem. Then she begins to request answers.]*

Who has an answer to share?

Leslie: $1\frac{1}{4}$.

Miguel: $\frac{5}{4}$.

Ms. Harris: Does anyone have a different answer?

Kevin: $\frac{4}{6}$.

Ms. Harris: Anyone else? *[Ms. Harris records all of the different answers, whether or not they are correct.]* Who would like to defend or prove one of the answers?

Susan: I want to defend $1\frac{1}{4}$. I thought about $\frac{3}{4}$ as $\frac{1}{2} + \frac{1}{4}$, so I put the two halves together to get one whole. Then I added in the other $\frac{1}{4}$ to get $1\frac{1}{4}$. *[As students explain their answers, Ms. Harris records each student's strategy where the whole class can see it. Ms. Harris records Susan's thinking as follows.]*

$$\frac{1}{2} + \frac{3}{4}$$

$$= \frac{1}{2} + \left(\frac{1}{2} + \frac{1}{4}\right)$$

$$= \left(\frac{1}{2} + \frac{1}{2}\right) + \frac{1}{4}$$

$$= 1 + \frac{1}{4}$$

$$= 1\frac{1}{4}$$

[Ms. Harris intentionally uses parentheses to highlight the associative property in Susan's strategy.]

Ms. Harris: So when Susan grouped addends together, she was using what mathematicians call the *associative property.*

Charlie: I got the same answer, but I did it differently. I changed the $\frac{1}{2}$ to $\frac{1}{4} + \frac{1}{4}$, so I added $\frac{1}{4} + \frac{3}{4}$ to get one whole, and then had $\frac{1}{4}$ left. I put that with the whole to get $1\frac{1}{4}$.

(continued)

Learn More...

Chapter 6, Chapter 8, and Chapter 10, discuss the importance of properties in developing computation strategies.

Blakely: I followed what Charlie did, but I got $\frac{5}{4}$ as my answer because $\frac{1}{4}+\frac{1}{4}+\frac{3}{4}=\frac{5}{4}$.

[Ms. Harris rewrites the original problem each time another student shares. She records Susan's, then Charlie's, and finally Blakely's solutions. All solutions and strategies for a given problem are recorded and left up so they are available for comparison.]

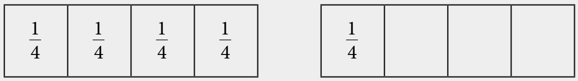
Susan	Charlie	Blakely
$\frac{1}{2}+\frac{3}{4}$	$\frac{1}{2}+\frac{3}{4}$	$\frac{1}{2}+\frac{3}{4}$
$=\frac{1}{2}+\left(\frac{1}{2}+\frac{1}{4}\right)$	$=\left(\frac{1}{4}+\frac{1}{4}\right)+\frac{3}{4}$	$=\left(\frac{1}{4}+\frac{1}{4}\right)+\frac{3}{4}$
$=\left(\frac{1}{2}+\frac{1}{2}\right)+\frac{1}{4}$	$=\frac{1}{4}+\left(\frac{1}{4}+\frac{3}{4}\right)$	$=\frac{1}{4}+\frac{1}{4}+\frac{3}{4}$
$=1+\frac{1}{4}$	$=1\frac{1}{4}$	$=\frac{5}{4}$
$=1\frac{1}{4}$		

Ms. Harris: So how could we get two answers, $1\frac{1}{4}$ and $\frac{5}{4}$? Could they both be correct?

Sara: I think $\frac{5}{4}$ and $1\frac{1}{4}$ are the same, because you could break $\frac{5}{4}$ apart into $\frac{4}{4}$ and $\frac{1}{4}$. Since $\frac{4}{4}$ is the same as one whole, it could be $1\frac{1}{4}$ or $\frac{5}{4}$. They mean the same thing.

Ms. Harris: Would you please turn and talk to someone near you about Sara's reasoning? Do you agree or disagree, and why? *[After about thirty seconds, Ms. Harris pulls the students back together to share their ideas with the whole group.]*

Miguel: Charlie and I agree with Sara. We pictured a rectangle divided into fourths and four fourths made a whole rectangle. If an extra fourth is added, that would be a whole and a fourth, or five fourths. Either way, it's the same. *[Ms. Harris captures Miguel's thinking by drawing an area model.]*



Kevin: I want to change my answer. When I first saw the problem, I added the numbers across the top and across the bottom. That's how I got $\frac{4}{6}$, but now I've changed my mind. I don't think there are sixths in this problem. Now I'm thinking about fourths, and I agree with $1\frac{1}{4}$ and $\frac{5}{4}$.

Understanding Number Talks: Four Foundational Principles

Let's take a few minutes to explore how Ms. Harris's classroom number talk contains four foundational principles that are inherent in every number talk. Keep in mind that while each of these principles is discussed separately on the following pages, they are interdependent and tightly interwoven.

Four Foundational Principles of Number Talks

1. Establish a safe learning community.
2. Use purposeful problems.
3. Record with purpose.
4. Know when to ask and when to tell.

Establish a Safe Learning Community

It's essential for teachers to build a cohesive classroom community and a safe, risk-free learning environment for effective number talks. The culture of the classroom should be based on mutual respect, and it should be driven by a common quest for learning and understanding. Students should be comfortable offering responses for discussion, questioning themselves and their peers, and investigating new strategies. Although it takes time to establish a community of learners built on mutual respect, if you consistently model and set this expectation from the beginning, students will respond.

The following are some key actions teachers can take to build a learning community prior to orchestrating a number talk.

- Be curious about what students think. Knowing that students have something important to contribute is an important first step.
- Ask open-ended questions like, "How did you know? Will it always work?"
- Wait for students to respond. Waiting even 15 seconds after you pose a problem or ask a question can make a big difference in the responses you get.
- Listen carefully to students' responses.
- Expect students to listen to and respect each other's ideas.

Number talks can be a powerful vehicle for promoting a safe learning environment. Let's highlight this by walking through how to facilitate a number talk.

As shown and discussed thus far, number talks start when teachers display a problem for the class and give students time to solve the problem mentally without paper or pencil. Students hold one fist to their chest as they think about the problem. Then, without moving their fist, they quietly put their thumb up to indicate that they have a solution. When teachers see that most students have found an answer—that is, when multiple students have raised a thumb—they encourage students to find additional efficient strategies while others continue thinking. Learners who find additional solutions raise a finger for each additional response. This finger-signal approach ensures that there is ample wait time for all

Number Talk Tip

Using finger signals in a number talk allows wait time for students to think, and it allows teachers to challenge and stretch the thinking of those who already have an answer.

students to think, and it simultaneously allows teachers to challenge and stretch the thinking of those students who already have an answer.

Once most of the students indicate they have a solution and strategy, teachers call for and record all answers—correct and incorrect—for everyone to see. It's important to do this without comment or change of facial expression. By acknowledging all answers, teachers communicate acceptance and a willingness to consider each student's reasoning.

Number Talk Tip

In a number talk, the teacher records all answers—correct and incorrect—for everyone to see. It's important to do this without comment or change of facial expression.

Next, students have the opportunity to share their strategies and justifications with their peers. There are several benefits of sharing and discussing strategies, summarized here.

Benefits of Sharing and Discussing Strategies

Students have the opportunity to

1. clarify their thinking;
2. consider and test other strategies to see if they are mathematically logical;
3. investigate and apply mathematical relationships;
4. build a repertoire of efficient strategies; and
5. make decisions about choosing efficient strategies for specific problems.

Encouraging social interaction, or the exchange of ideas, is an essential way that teachers build an effective learning community. In a number talk, when students exchange ideas they are explaining their thinking and deepening their own mathematical understanding. In turn, listening to others' ideas, considering how these ideas connect or conflict with their own ideas, and then adjusting their own thinking also gives students access to new ways to solve problems.

Do incorrect answers arise during a number talk? Absolutely. However, teachers may ask—and students may ask each other—to defend or justify their answers to prove their mathematical reasoning. When students participate in number talks, they have a sense of shared authority in determining whether an answer is accurate and students know they are expected to think carefully about the solutions and strategies presented.