First Steps in Mathematics

Measurement

Resource Book 1

Understand Units and Direct Measure

Improving the mathematics outcomes of students

PEARSON

Department of Education and Training

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First Steps in Mathematics: Measurement Resource Book 1
Understand Units and Direct Measure

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Diagnostic Map: Measurement

Emergent Phase

During the Emergent Phase

Students initially attend to overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small) rather than to describe comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgements of size.

As a result, they begin to understand and use the everyday language of attributes and comparison used within their home and school environment, differentiating between attributes that are obviously perceptually different.

By the end of the Emergent phase, students typically:

- distinguish tallness, heaviness, fatness, and how much things hold
- start to distinguish different forms of length and to use common contextual length distinctions; e.g., distinguish wide from tall
- use different bipolar pairs to describe things; e.g., thin—fat, heavy—light, tall—short
- describe two or three obvious measurement attributes of the same thing; e.g., tall, thin, and heavy
- describe something as having more or less of an attribute than something else, e.g., as being taller than or as being fatter than.

Matching and Comparing Phase

Most students will enter the Matching and Comparing phase between 5 and 7 years of age.

As students move from the Emergent phase, to the Matching and Comparing phase, they:

- may not "conserve" measures; e.g., thinking that moving a rod changes its length, pouring changes "how much," cutting up paper makes more surface area
- may visually compare the size of two things, but make no effort to match; e.g., saying which stick is longer without lining up the bases or which sheet of paper is bigger without superimposing
- compare time spans but may not take into account different starting times; e.g., deciding that the TV program that finished latest was on longest
- use bipolar pairs but may have difficulty with some comparative terms; e.g., lift to decide which is heavier but say both are heavy because both hands go down
- may distinguish two attributes (such as tallness and mass) but not understand that the two attributes may lead to different orders of size for a collection, expecting the order for tallness and the order for mass to be the same
- while describing different attributes of the same thing (tall, thin, and heavy) may be confused by a request to compare two things by different attributes, particularly if the comparisons lead to different orders
- often do not think to use counting to say how big or how much bigger; e.g., they may "weigh" something by putting it into one side of a balance and smaller objects into the other side but not count the objects

During the Matching and Comparing Phase

Students match in a conscious way in order to decide which is bigger by familiar readily perceived and distinguished attributes such as length, mass, capacity, and time. They also repeat copies of objects, amounts, and actions to decide how many fit (balance or match) a provided object or event.

As a result, they learn to directly compare things to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as "How long (tall, wide or heavy, much time, much does it hold)?"or when explicitly asked to measure something.

By the end of the Matching and Comparing phase, students typically:

- attempt to focus on a particular attribute to compare two objects or events; e.g., how much the jar holds
- know that several objects or events may be in different orders when compared by different attributes
- line up the base of two sticks when comparing their lengths and fit regions on top of each other to compare area
- use the everyday notion of "how many fit" and count how many repeats of an object fit into or match another; e.g., How many pens fit along the table? How many potato prints cover the sheet? How many blocks fit in the box?
- count units and call it "measuring;" e.g., *I measured and found the jar holds a bit more than 7 scoops.*
- use "between" to describe measurements of uni-dimensional quantities (length, mass, capacity, time); e.g., *It weights between 7 and 8 marbles.*
- refer informally to part-units when measuring uni-dimensional quantities; e.g., *Our room is 6 and a bit metres long.*

Quantifying Phase

Most students will enter the Quantifying phase between 7 and 9 years of age.

As students move from the Matching and Comparing phase to the Quantifying phase, they:

- while knowing that ordering objects by different attributes may lead to different orders, may still be influenced by the more dominant perceptual features; e.g., they may still think the tallest container holds the most
- may count "units" in order to compare two things but be fairly casual in their repetition of units, not noticing gaps or overlaps; e.g., placing the first "unit" away from the end when measuring length, not worrying about spills when measuring how much a container holds, not stopping their claps immediately the music stops
- do not necessarily expect the same "answer" each time when deciding how many fit
- many not think to use unit information to answer questions such as: Which cup holds more? Will the table slide through the door?
- may not see the significance of using a common unit to compare two things and, when using different units, let the resulting number override their perceptual judgement
- while many will have learned to use the centimetre marks on a conventional rule to "measure" lengths, they often do not see the connection between the process and the repetition of units

During the Quantifying Phase

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit" and so come to an understanding that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared without directly matching them.

As a result, they trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

By the end of the Quantifying phase, students typically:

- attempt to ensure uniformity of representations of the unit; e.g., check that the cup is always full, the pencil does not change length, the balls are the same size
- use the representations of their unit carefully to make as close a match as possible, avoiding gaps and overlaps; e.g., choose a flexible tape to measure the perimeter of a curved shape
- know why they need to choose the same size objects to use as units when comparing two quantities
- see repeating one representation of the unit over and over as equivalent to filling or matching with multiple copies of it
- connect the repetition of a 'unit' with the numbers on a whole-number calibrated scale
- make things to a specified length in uniform units (including centimetres and metres)
- use provided measurements to make a decision about comparative size; e.g., use the fact that a friend's frog weighs 7 marbles to decide whether their own frog is heavier or lighter
- count units as a strategy to solve comparison problems such as: Whose frog is heavier? Put the jars in order from the one that holds the most to the one that holds the least.
- are prepared to say which is longer (heavier) based on information about the number of units matching each object
- think of different things having the same "size"; e.g., use grid paper to draw different shapes with the same perimeter
- add measurements that they can readily think of in terms of repetitions of units; e.g., find the perimeter of a shape by measuring the sides and adding

What is the Diagnostic Map for Measurement?

How students currently think about measurement attributes and units will influence how they respond to the activities provided for them, and hence what they are able to learn from them. As students' thinking about measurement develops, it goes through a series of characteristic phases. Recognizing these common patterns of thinking should help you to interpret students' responses to activities, to understand why they seem to be able to do some things and not others, and also why some students may be having difficulty in achieving certain outcomes while others are not. It should also help you to provide the challenges students need to move their thinking forward, refine their half-formed ideas, overcome any misconceptions they might have to and hence achieve the outcomes.

See over

Diagnostic Map: Measurement cont.

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- add measurements that they can readily think of in terms of repetitions of units; e.g., find the perimeter of a shape by measuring the sides and adding

Measuring Phase

Most students will enter the Measuring phase between 9 and 11 years of age.

As students move from the Quantifying phase to the Measuring phase, they:

- while trying to make as close a match as possible to the thing to be measured, may find the desire to match closely overriding the need for consistency of unit; e.g., they may resort to "filling" a region with a variety of different objects in order to cover it as closely as possible
- may not understand that the significance of having no gaps and overlaps is that the "true" measurement is independent of the placement of the units
- may still think of the unit as an object and of measuring as "fitting" in the social sense of the word (How many people fit in the elevator? How many beans in the jar?) and so have difficulty with the idea of combining part-units as is often needed in order to find the area of a region
- many confuse the unit (a quantity) with the instrument (or object) used to represent it; e.g., they may think a square metre has to be a square with sides of 1 m, may count cubes for area and not think of the face of each as the unit
- may interpret whole numbered marks on a calibrated scale as units but may not interpret the meaning of unlabelled graduations

During the Measuring Phase

Students come to understand the unit as an amount (rather than an object or a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and trust the measurement as a property or description of the object being measured that does not change as a result of the choice or placement of units.

By the end of the Measuring phase, students typically:

- expect the same number of copies of the representation of their unit to match the object being measured regardless of how they arrange or place the copies
- understand that the smaller the unit the greater the number; e.g., are able to say which is the longer of a 1-km walk and a 1400-m walk.
- compose "part-units" into wholes, understanding, for example, that a narrow garden bed may have an area of 5 or 6 m^2 even though no whole "metre squares" fit into the bed
- can themselves partition a rectangle into appropriate squares and use the array structure to work out how many squares are in the rectangle
- interpret the unnumbered graduations on a familiar whole-number scale
- understand the relationship between "part-units" and the common metric prefixes; e.g., know that a unit can be broken into one hundred parts and each part will be a centi-unit
- work with provided measurement information alone; e.g., order measurements of capacity provided in different standard units, make things that meet measurement specifications

Relating Phase

Most students will enter the Relating phase between 11 and 13 years of age

As students move from the Measuring phase to the Relating phase, they:

- while partitioning a rectangle into appropriate squares and using the array structure to find its area, may not connect this with multiplying the lengths of the sides of a rectangle to find its area
- while understanding the inverse relationship between the unit and the number of units needed, may still be distracted by the numbers in measurements and ignore the units; e.g., say that 350 g is more than 2 kg
- while converting between known standard units, may treat related metric measures just as they would any other unit, not seeing the significance of the decimal structure built into all metric measures

During the Relating Phase

Students come to trust measurement information even when it is about things they cannot see or handle and to understand measurement relationships, both those between attributes and those between units.

As a result, they work with measurement information itself and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

By the end of the Relating phase, students typically:

- understand that known relationships between attributes can be used to find measurements that cannot be found directly; e.g., understand that we can use length measurements to work out area
- know that for figures of the same shape (that is, similar) the greater the length measures the greater the area measures, but this is not so if the figures are different shapes
- understand why the area of a rectangle and the volume of a rectangular prism can be found by multiplying its length dimensions and can use this for fractional side lengths
- think of the part-units themselves as units; e.g., a particular unit can be divided into one hundred parts and each part is then a centi-unit
- subdivide units to make measurements more accurate
- choose units that are sufficiently small (that is, accurate) to make the needed comparisons
- use their understanding of the multiplicative structure built into the metric system to move flexibly between related standard units; e.g., they interpret the 0.2 kg mark on a scale as 200 g
- notice and reject unrealistic estimates and measurements, including of objects or events they have not actually seen or experienced
- use relationships between measurements to find measures indirectly; e.g., knowing that 1 mL = 1 cm^3 they can find the volume of an irregular solid in cubic centimetres by finding how many millilitres of water it displaces using a capacity cylinder

CONTENTS

INTRODUCTION

The *First Steps in Mathematics* resource books and professional learning program are designed to help teachers plan, implement, and evaluate the mathematics program they provide for students. The series describes the key mathematical ideas students need to understand in order to achieve the principal learning goals of mathematics curricula across Canada and around the world.

Unlike many resources that present mathematical concepts that have been logically ordered and prioritized by mathematicians or educators, First Steps in Mathematics follows a sequence derived from the mathematical development of real children. Each resource book is based on five years of research by a team of teachers from the Western Australia Department of Education and Training, and tertiary consultants led by Professor Sue Willis at Murdoch University.

The *First Steps in Mathematics* project team conducted an extensive review of international research literature, which revealed gaps in the field of knowledge about students' learning in mathematics. Many of these findings are detailed in the Background Notes that supplement the Key Understandings described in the *First Steps in Mathematics* resource books for Measurement.

Using tasks designed to replicate those in the research literature, team members interviewed hundreds of elementary school children in diverse locations. Analysis of the data obtained from these interviews identified characteristic phases in the development of students' thinking about mathematical concepts.

The Diagnostic Maps—which appear in the resource books for Number, Measurement, Geometry and Space, and Data Management and Probability—describe these phases of development, exposing specific markers where students often lose, or never develop, the connection between mathematics and meaning. Thus, *First Steps in Mathematics* helps teachers systematically observe not only what mathematics individual children do, but how the children do the mathematics, and how to advance the children's learning.

It has never been more important to teach mathematics well. Globalization and the increasing use of technology have created changing demands for the application of mathematics in all aspects of our lives. Teaching mathematics well to all students requires a high level of understanding of teaching and learning in mathematics and of mathematics itself. The *First Steps in Mathematics* series and professional learning program help teachers provide meaningful learning experiences and enhance their capacity to decide how best to help all students achieve the learning goals of mathematics.

Chapter 1

An Overview of *First Steps in Mathematics*

First Steps in Mathematics is a professional learning program and series of teacher resource books that are organized around mathematics curricula for Number, Measurement, Geometry and Space, and Data Management and Probability.

The aim of *First Steps in Mathematics* is to improve students' learning of mathematics.

First Steps in Mathematics examines mathematics within a developmental framework to deepen teachers' understanding of teaching and learning mathematics. The developmental framework outlines the characteristic phases of thinking that students move through as they learn key mathematical concepts. As teachers internalize this framework, they make more intuitive and informed decisions around instruction and assessment to advance student learning.

First Steps in Mathematics helps teachers to:

- build or extend their own knowledge of the mathematics underpinning the curriculum
- understand how students learn mathematics so they can make sound professional decisions
- plan learning experiences that are likely to develop the mathematics outcomes for all students
- recognize opportunities for incidental teaching during conversations and routines that occur in the classroom

This chapter details the beliefs about effective teaching and learning that *First Steps in Mathematics* is based on and shows how the elements of the teacher resource books facilitate planning and instruction.

Beliefs about Teaching and Learning

Focus Improves by Explicitly Clarifying Outcomes for Mathematics

Learning is improved if the whole-school community has a shared understanding of the mathematics curriculum goals, and an implementation plan and commitment to achieving them. A common understanding of these long-term aims helps individuals and groups of teachers decide how best to support and nurture students' learning, and how to tell when this has happened.

All Students Can Learn Mathematics to the Best of Their Ability

A commitment to common goals signals a belief that **all** students can be successful learners of mathematics. A situation where less is expected of and achieved by certain groups of students is not acceptable. School systems, schools and teachers are all responsible for ensuring that **each** student has access to the learning conditions he or she requires to achieve the curricular goals to the best of his or her ability.

Learning Mathematics Is an Active and Productive Process

Learning is not simply about the transfer of knowledge from one person to another. Rather, students need to construct their own mathematical knowledge in their own way and at a pace that enables them to make sense of the mathematical situations and ideas they encounter. A developmental learning approach is based on this notion of learning. It recognizes that not all students learn in the same way, through the same processes, or at the same rate.

Common Curricular Goals Do Not Imply Common Instruction

The explicit statement of the curricular goals expected for all students helps teachers to make decisions about the classroom program. However, the list of content and process goals for mathematics is not a curriculum. If all students are to succeed to the best of their ability on commonly agreed concepts, different curriculum implementations will not only be possible, but also be necessary. Teachers must decide what type of instructional activities are needed for their students to achieve the learning goals.

> *A curriculum that enables all students to learn must allow for different starting points and pathways to learning so that students are not left out or behind.*
>
> ***—Darling-Hammond, National standards and assessments, p. 480***

Professional Decision-Making Is Central in Teaching

It is the responsibility of teachers to provide all students with the conditions necessary for them to achieve the curricular goals of mathematics. This responsibility requires teachers to make many professional decisions simultaneously, such as what to teach, to whom, and how, and making these professional decisions requires a synthesis of knowledge, experience, and evidence.

> *Professionalism has one essential feature; …(it) requires the exercise of complex, high level professional judgments…(which) involve various mixes of specialised knowledge; high level cognitive skills; sensitive and sophisticated personal skills; broad and relevant background and tacit knowledge.*
>
> **—Preston, *Teacher professionalism*, p. 2, 20**

The personal nature of each student's learning journey means that the decisions teachers make are often "non-routine," and the reasoning processes involved can be complex. These processes cannot be reduced to a set of instructions about what to do in any given situation. Teachers must have the freedom and encouragement to adapt existing curricula flexibly to best meet their students' needs and to move them forward. The improvement of students' learning is most likely to take place when teachers have good information about tasks, response range, and intervention techniques on which to base their professional decisions.

"Risk" Relates to Future Mathematics Learning

Risk cannot always be linked directly to students' current achievement. Rather, it refers to the likelihood that their future mathematical progress is "at risk."

Some students who can answer questions correctly might not have the depth of understanding needed for ongoing progress. Others might have misconceptions that could also put their future learning "at risk." A number of students may make errors that are common when they try to make sense of new mathematical ideas and, therefore, show progress. For example, a student who tries to count tiles using an array may count the corners twice, which is incorrect. However, the use of the array signals progress because the student is using his or her knowledge of the repeating nature of the area unit.

Students who are learning slowly, or whose previous experiences are atypical, might nevertheless progress steadily if their stage of learning is accommodated with appropriate, but challenging, learning experiences.

Successful Mathematics Learning Is Robust Learning

Robust learning, which focuses on students developing mathematics concepts fully and deeply, is essential if learning is to be sustained over the long term.

A focus on short-term performance or procedural knowledge at the expense of robust knowledge places students "at risk" of not continuing to progress throughout the years of schooling.

Learning Mathematics: Implications for the Classroom

Learning mathematics is an active and productive process on the part of the learner. The following section illustrates how this approach influences the ways in which mathematics is taught in the classroom.

Learning Is Built on Existing Knowledge

Learners' interpretations of mathematical experiences depend on what they already know and understand. For example, many young students may distinguish two attributes (such as tallness and heaviness) but not understand that the two attributes may lead to different orders of size for a collection, expecting the order for height and the order for mass to be the same. Other students may compare time spans but may not take into account different starting times; e.g., deciding that the television program that finished latest was on longest.

In each case, students' existing knowledge should be recognized and used as the basis for further learning. Their learning should be developed to include the complementary knowledge with the new knowledge being linked to and building on students' existing ideas.

Learning Requires That Existing Ideas Be Challenged

Learning requires that students extend or alter what they know as a result of their knowledge being challenged or stretched in some way. For example, a challenge may occur when a student predicts that the tallest container will hold the most water, then measures and finds that it does not.

Another challenge may occur when a student believes that when size increases, mass increases. Or, it may happen when the student finds that peers think about a problem in a different way. The student must find some way of dealing with the challenge or conflict provided by the new information in order to learn.

Learning Occurs when the Learner Makes Sense of the New Ideas

Teaching is important—but learning is done *by* the learner rather than *to* the learner. This means the learner acts on and makes sense of new information. Students almost always try to do this. However, in trying to make sense of their mathematical experiences, some students will draw conclusions that are not quite what their teachers expect.

Also, when students face mathematical situations that are not meaningful, or are well beyond their current experience and reach, they often conclude that the mathematics does not make sense or that they are incapable of making sense of it. This may encourage students to resort to learning strategies based on the rote imitation of procedures. The result is likely to be short-term rather than effective long-term learning. Teachers have to provide learning experiences that are meaningful and challenging, but within the reach of their students.

Learning Involves Taking Risks and Making Errors

In order to learn, students have to be willing to try a new or different way of doing things, and stretch a bit further than they think they can. At times, mistakes can be a sign of progress. For example, students may count "units" in order to compare two things but be fairly casual in their repetition of units, not noticing gaps or overlaps; e.g., placing the first "unit" away from the end when measuring length, not worrying about spills when measuring how much a container holds, or not stopping their claps immediately when the music stops. Such errors can be positive signs that students are matching and comparing as they move to understand the more precise meaning of quantity or "how much."

Errors can provide a useful source of feedback, challenging students to adjust their conceptions before trying again. Errors may also suggest that learners are prepared to work on new or difficult problems where increased error is likely. Or, learners may try improved ways of doing things that mean giving up old and safe, but limited, strategies. For example, a student who can successfully find the number of paper tiles to tile a rectangular room by laying them all out in order takes a risk when trying to multiply the lengths of the sides of the rectangle, since multiplying may result in increased mistakes in the short term.

Learners Get Better with Practice

Students should get adequate opportunities to practise mathematics, but this involves much more than the rote or routine repetition of facts and procedures. For example, the very language of "square unit" convinces many students that units of area have to be square shapes. They will need considerable experience in cutting and rearranging shapes to convince themselves that rearranging the cut shape does not change the area and that a "square unit" is an amount of area not a shape or thing.

Likewise, if students are to develop a rich understanding of measurement they will need spaced and varied opportunities to notice and reject unrealistic estimates and measurements, including things they have not actually seen or experienced. Repetitious procedures of routine questions are unlikely to provide this rich understanding. In fact, they are more likely to interfere with it.

Learning Is Helped by Clarity of Purpose for Students as well as Teachers

Learning is likely to be enhanced if students understand what kind of learning activity they should be engaged in at any particular time. This means helping students to distinguish between tasks that provide practice of an already learned procedure and tasks that are intended to develop understanding of mathematical concepts and processes. In the former case, little that is new is involved, and tasks are repetitive, so they become habitual and almost unthinking. Students should expect to be able to start almost immediately and, if they cannot, realize that they may need to know more and seek help.

With tasks that are intended to develop understanding, non-routine tasks and new ideas may be involved. Students should not expect to know what to do or to be able to get started immediately.

Students may spend a considerable amount of time on a single task, and they will often be expected to work out for themselves what to do. They should recognize that, for such activities, persistence, thoughtfulness, struggle, and reflection are expected.

Teaching Mathematics

Teachers assume considerable responsibility for creating the best possible conditions for learning. The kind of learning tasks and environment teachers provide depend on their own view of how learning is best supported. The perspective that learning is an active and productive process has two significant implications for teaching.

First, teachers cannot predict or control exactly what and when students learn. They need to plan curricula that provide students with a wider and more complex range of information and experiences than they would be expected to understand fully at any given time. For example, teachers often provide opportunities for their students to explore the number system using calculators. Similar experiences can assist students in making parallel connections to the multiplicative structure of the metric system. Before formally introducing a concept, a teacher can present situations that challenge students to use their prior understanding of number to experiment with ideas about measurement, such as, "Is 0.2 km equivalent to 2 m, 20 m, 200 m, or 2000 m?" In this way teachers can stimulate their students' curiosity about connections within mathematics, helping students develop notions about the structure of the metric system at many different levels preceding the prescribed teaching of these connections.

This represents a significant change in curriculum planning. It is a movement away from an approach that only exposes students to content and ideas that they should be able to understand or do at a particular point in time.

Second, for students to become effective learners of mathematics, they must be engaged fully and actively. Students will want, and be able, to take on the challenge, persistent effort, and risks involved. Equal opportunities to learn mathematics means teachers will:

- provide an environment for learning that is equally supportive of all students
- offer each student appropriate mathematical challenges
- foster in all students processes that enhance learning and contribute to successful achievment of goals

Understanding the Elements of *First Steps in Mathematics*

The elements of *First Steps in Mathematics* embody the foregoing beliefs about teaching and learning and work together to address three main questions:

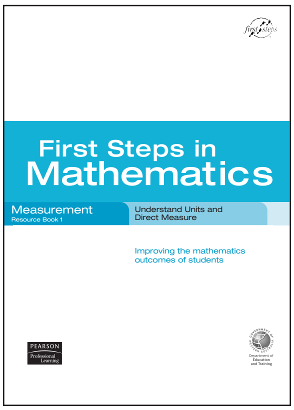
- What are students expected to learn?
- How does this learning develop?
- How do teachers advance this learning?

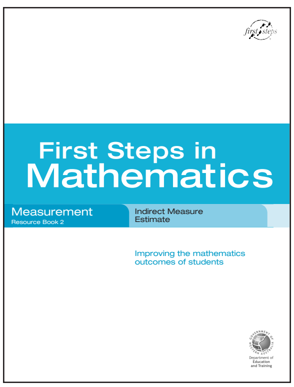
Learning Outcomes for the Measurement Strand

The Measurement strand focuses on the basic principles of measurement: the range of measures in common use and the skills needed for everyday purposes. As a result of their learning, students will develop confidence and proficiency in using direct and indirect measurement and estimating skills to describe, compare, evaluate, plan, and construct.

To achieve these outcomes, students require an understanding of the nature of the different physical attributes that can be measured and the way units are used to quantify amounts of such attributes to needed levels of accuracy. It also requires the ability and understandings needed to make informed judgements about measurements for a range of purposes and to calculate measurement indirectly using measurement relationships. Learning experiences should be provided that will enable students to understand units, directly and indirectly measure, and estimate measurements.

As a result of their learning experiences, students at all levels should be able to achieve the following outcomes.





Understand Units

Decide what needs to be measured by selecting what attributes to measure and what units to use.

Direct Measure

Carry out measurements of length, capacity, volume, mass, area, time, and angle to needed levels of accuracy.

Indirect Measure

Select, interpret, and combine measurements, measurement relationships and formulas to determine other measures indirectly.

Estimate

Make sensible direct and indirect estimates of quantities and be alert to the reasonableness of measurements and results.

Integrating the Outcomes

The outcomes suggested above for Measurement are each dealt with in a separate chapter. This is to emphasize the importance of each and the difference between them. For example, students need to learn about what attributes to measure and what units to use (Understand Units) as well as developing the skill to reliably and accurately use units to directly measure each of these attributes (Direct Measure). By paying separate and special attention to each outcome, teachers can make sure that both areas receive sufficient attention and that important ideas about each are drawn out of the learning experiences they provide.

This does not mean, however, that the ideas and skills underpinning each of the outcomes should be taught separately or that they will be learned separately. The outcomes are inextricably linked. Consequently, many of the activities will provide opportunities for students to develop their ideas about more than one of the outcomes. This will help teachers to ensure that the significant mathematical ideas are drawn from the learning activities so that students achieve each of the outcomes for Measurement.

How Does This Learning Develop?

First Steps in Mathematics: Measurement describes characteristic phases in students' thinking about the major mathematical concepts of the Measurement strand. These developmental phases are organized in a Diagnostic Map.

Diagnostic Map

The Diagnostic Map for Measurement details five developmental phases. It helps teachers to:

- understand why students seem to be able to do some things and not others
- realize why some students may be experiencing difficulty while others are not
- indicate the challenges students need to move their thinking forward, to refine their preconceptions, overcome any misconceptions, and so develop deep reflective understanding about concepts
- interpret students' responses to activities

Diagnostic Map: Measurement

Emergent Phase

During the Emergent Phase

Students initially attend to overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small) rather than to describe comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgements of size.

As a result, they begin to understand and use the everyday language of attributes and comparison used within their home and school environment, differentiating between attributes that are obviously perceptually different.

By the end of the Emergent phase, students typically:

- distinguish tallness, heaviness, fatness, and how much things hold

Matching and Comparing Phase

Most students will enter the Matching and Comparing phase between 5 and 7 years of age.

As students move from the Emergent phase, to the Matching and Comparing phase, they:

- may not "conserve" measures; e.g., thinking that moving a rod changes its length, pouring changes "how much," cutting up paper makes more surface area
- may visually compare the size of two things, but make no effort to match; e.g., saying which stick is longer without lining up the bases or which sheet of paper is bigger without superimposing
- compare time spans but may not take into account different starting times; e.g., deciding that the TV program that finished latest was on longest
- use bipolar pairs but may have difficulty with some comparative terms; e.g., lift to decide which is heavier but say both are heavy because both hands go down
- may distinguish two attributes (such as tallness and mass) but not understand that the two attributes may lead to different orders of size for a colle...

Quantifying Phase

Most students will enter the Quantifying phase between 7 and 9 years of age.

As students move from the Matching and Comparing phase to the Quantifying phase, they:

- while knowing that ordering objects by different attributes may lead to different orders, may still be influenced by the more dominant perceptual features; e.g., they may still think the tallest container holds the most
- may count "units" in order to compare two things but be fairly casual in their repetition of units, not noticing gaps or overlaps; e.g., placing the first "unit" away from the end when measuring length, not worrying about spills when measuring how much a container holds, not stopping their claps immediately the music stops
- do not necessarily expect the same "answer" each time when deciding how many fit
- many not think to use unit information to answer questions such as: Which cup holds more? Will the ...

The Diagnostic Map includes key indications and consequences of students' understanding and growth. This information is crucial for teachers making decisions about their students' level of understanding of mathematics. It enhances teachers' decisions about what to teach, to whom, and when to teach it.

Each developmental phase of the Diagnostic Map has three aspects. The first aspect describes students' major preoccupations *during that phase*. At the centre of each phase is the learning focus *during that phase*. This learning results in typical thinking and behaviour patterns *by the end of the phase*. Preconceptions, partial conception, or misconceptions, however, may still exist for students at the end of the phase. This final aspect provides the learning challenges and teaching emphases *as students move to the next phase*.

Diagnostic Tasks

First Steps in Mathematics: Measurement provides a series of short, focused Diagnostic Tasks in the *Course Book*. These tasks have been validated through extensive research with students and help teachers locate individual students on the Diagnostic Map.

How Do Teachers Advance This Learning?

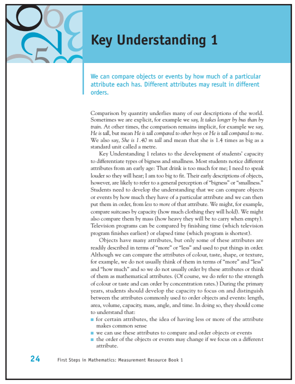
To advance student learning, teachers identify the big mathematical ideas or key understandings of the outcomes or curricular goals. Teachers plan learning activities to develop these key understandings. As learning activities provide students with opportunities and support to develop new insights, students begin to move to the next developmental phase of mathematical thinking.

Key Understandings

The Key Understandings are the cornerstone of *First Steps in Mathematics*. The Key Understandings:

- describe the mathematical ideas, or concepts, which students need to know in order to achieve curricular goals
- explain how these mathematical ideas form the underpinnings of the mathematics curriculum statements
- suggest what experiences teachers should plan for students so that they move forward in a developmentally appropriate way
- provide a basis for the recognition and assessment of what students already know and still need to know in order to progress along the developmental continuum and deepen their knowledge
- indicate the emphasis of the curriculum at particular stages
- provide content and pedagogic advice to assist with planning the curriculum at the classroom and whole-school levels

The number of Key Understandings for each mathematics curricular goal varies according to the number of "big mathematical ideas" students need to achieve the goal.


Key Understanding 1

We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.

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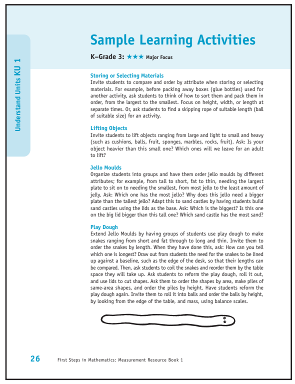
Sample Learning Activities

For each Key Understanding, there are Sample Learning Activities that teachers can use to develop the mathematical ideas of the Key Understanding. The activities are organized into three broad groups:

- activities suitable for students in Kindergarten to Grade 3
- activities for students in Grades 3 to 5
- activities for students in Grades 5 to 8

If students in Grades 5 to 8 have not had enough prior experience, then teachers may need to select and adapt activities from earlier groups.


Sample Learning Activities

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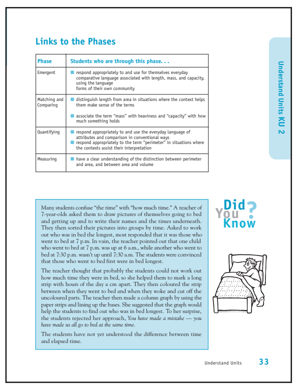
Case Studies

The Case Studies illustrate some of the ways in which students have responded to Sample Learning Activities. The emphasis is on how teachers can focus students' attention on the mathematics during the learning activities.

"Did You Know?" Sections

For some of the Key Understandings, there are "Did You Know?" sections. These sections highlight common understandings and misunderstandings that students have. Some "Did You Know?" sections also suggest diagnostic activities that teachers may wish to try with their students.


Links to the Phases

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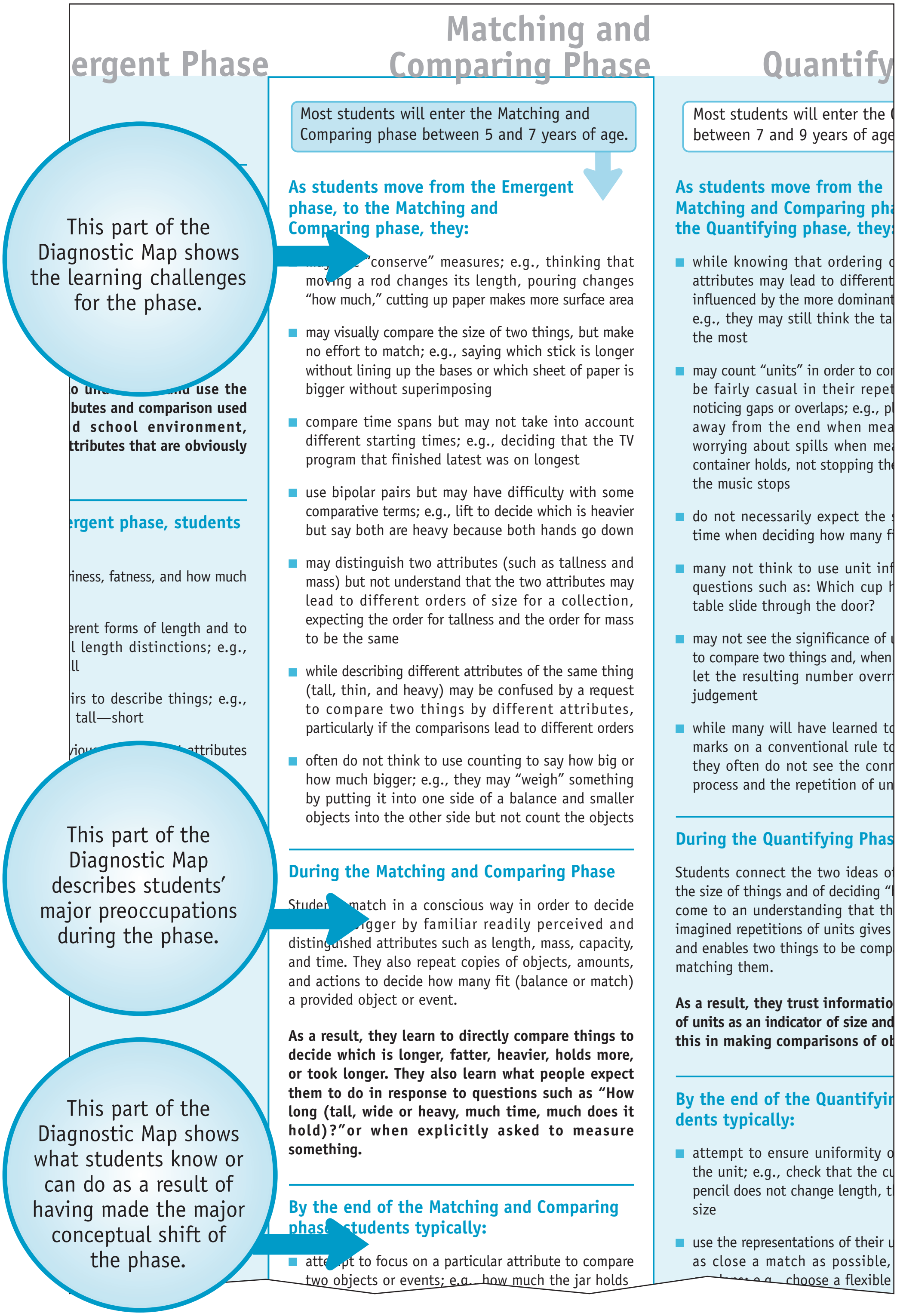
How to Read the Diagnostic Map

The Diagnostic Map for Measurement has five phases: Emergent, Matching and Comparing, Quantifying, Measuring, and Relating. The diagram on this page shows the second phase, the Matching and Comparing phase.



The text in the "During the phase" section describes students' major preoccupations, or focuses, during that phase of thinking about Measurement.

The "By the end" section of each phase provides examples of what students typically think and are able to do as a result of having worked through the phase.

The achievements described in the "By the end" section should be read in conjunction with the "As students move" section. Together, these two sections illustrate that although students might have developed a range of important understandings as they passed through the phase, they might also have developed some unconventional or unhelpful ideas. Both of these sections of the Diagnostic Map are intended as a useful guide only. Teachers will recognize more examples of similar thinking in the classroom.

How Do Students Progress Through the Phases?

Students who have passed through one phase of the Diagnostic Map are entering the next phase. They bring behaviours and understandings from one phase to the next. For example, the text in the "As students move from the Emergent Phase" section describes the behaviours students bring to the Matching and Comparing phase. This section includes the preconceptions, partial conceptions, and misconceptions that students may have developed along the way. These provide the learning challenges for the next phase.

Linking the Diagnostic Maps and Learning Goals

Students are unlikely to achieve full conceptual understanding unless they have moved through certain phases of the Diagnostic Map. However, passing through the phase does not guarantee that the concept has been mastered. Students might have the conceptual development necessary for deepening their understanding, but without access to a classroom program that enables them to learn the necessary foundation concepts described in a particular phase, they will be unable to do so.

The developmental phases help teachers interpret students' responses in terms of pre- and partial conceptions. If, for example, a student believes that large objects weigh more (have a greater mass) than small objects, then the phases can help explain what the problem might be. In this case, a student might not "conserve" mass, and no amount of telling the students that bigger things are not always heavier will help. The student needs multiple experiences matching and comparing objects of different density, first holding the objects, then placing the objects on balance scales. In this manner students will begin to change their incorrect perceptual understanding of bigger always being heavier—which was generally correct, so they trusted it was always correct—to the more reliable act of testing before predicting.

How Will Teachers Use the Diagnostic Map?

The Diagnostic Map is intended to assist teachers as they plan for mathematics teaching and learning. The descriptions of the phases help teachers make informed decisions about students' understandings of the mathematical concepts. The map will help teachers understand why students can do some things and not others, and why some students may be having difficulty achieving certain outcomes.

Initially, teachers may use the Diagnostic Map to extend their own knowledge about how students typically learn mathematics. Knowing about the major conceptual shifts in each phase and their links to mathematical learning goals will help teachers decide which Key Understandings should be the major focus for classroom planning.

Familiarity with the behaviours described in the phases will enhance the informed decisions teachers make about what they observe students doing and saying during lessons. The information obtained over time about the major preoccupations of students informs ongoing planning. As teachers begin to understand the typical behaviours of each phase, this planning process will become more efficient.

Planning with *First Steps in Mathematics*

Using Professional Decision-Making to Plan

The *First Steps in Mathematics* resource books and professional development support the belief that teachers are in the best position to make informed decisions about how to help their students achieve conceptual understanding in mathematics. Teachers will base these decisions on knowledge, experience, and evidence.

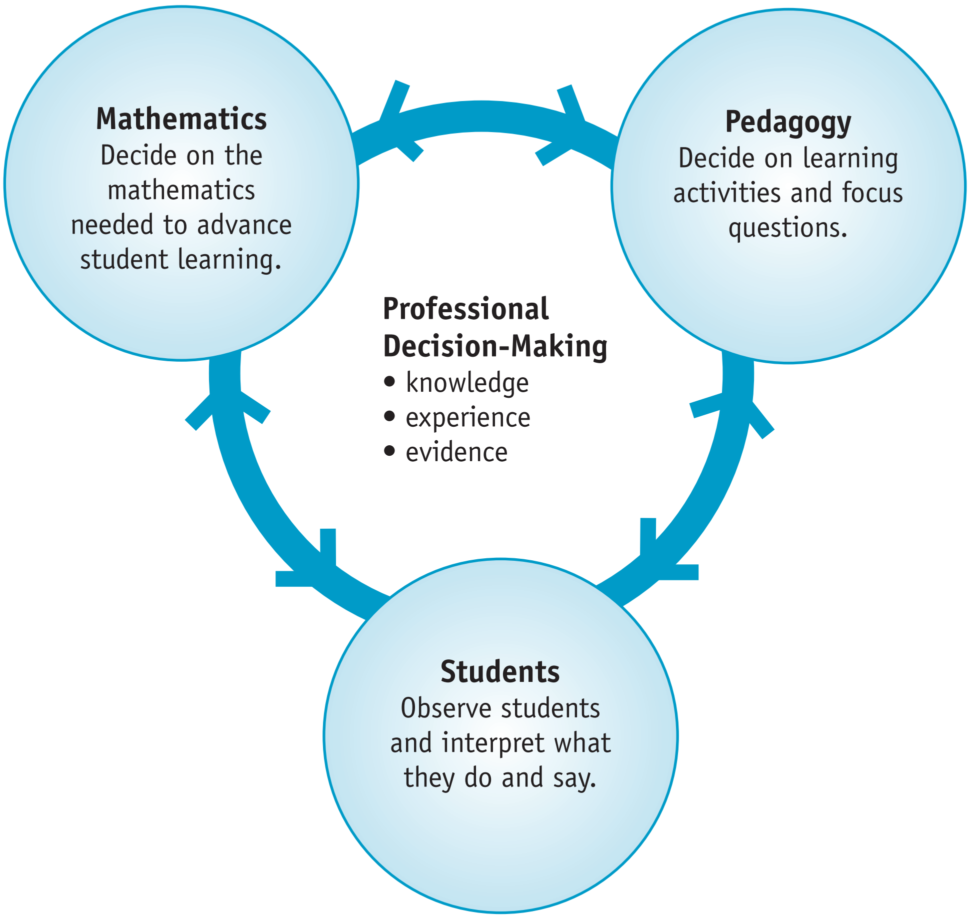
The process of using professional decision-making to plan classroom experiences for students is fluid, dependent on the situation and context, and varies from teacher to teacher. The selection of learning activities and appropriate focus questions will be driven by each teacher's knowledge of his or her students and their learning needs, the mathematics, and mathematics-related pedagogy. The *First Steps in Mathematics* resource books and professional development focus on developing this pedagogical content knowledge.

The diagram on the next page illustrates how these components combine to inform professional decision-making. There is no correct place to start or finish, or process to go through. Circumstances and experience will determine both the starting point and which component takes precedence at any given time.

Different teachers working with different students may make different decisions about what to teach, to whom, when, and how.

The process is about selecting activities that enable all students to learn the mathematics described in curriculum focus statements. More often than not, teachers' choice of activities and focus questions will be driven by their knowledge of their students and the mathematics. At other times, teachers might select an activity to help them assess students' existing knowledge or because of the specific mathematics in the task. Whatever the starting point, the *First Steps in Mathematics* resource books and professional development will help teachers to ensure that their mathematics pedagogy is well informed.

The examples on the opposite page show some of the different ways teachers can begin planning using *First Steps in Mathematics*.



Focusing on the Mathematics

Teachers may choose to focus on the mathematics, deciding on the mathematics they think they need to move students on.

What mathematics do my students need to know?

Mathematics
Decide on the mathematics needed to advance student learning.

What sections of ***First Steps in Mathematics*** do I look at?

- Key Understandings and Key Understandings descriptions

Understanding What Students Already Know

Teachers may choose to start by finding out what mathematics their students already know.

What do my students know about these mathematics concepts?

Students
Observe students and interpret what they do and say.

What sections of ***First Steps in Mathematics*** do I look at?

- Key Understandings and Key Understandings descriptions
- "Did You Know?" sections
- Diagnostic Map
- Diagnostic Tasks

Developing Students' Knowledge

Teachers may begin by planning and implementing some activities to develop their knowledge of students' learning.

What activities will help my students develop these ideas? How will I draw out the mathematical ideas from the learning activity?

Pedagogy
Decide on learning activities and focus questions.

What sections of ***First Steps in Mathematics*** do I look at?

- Sample Learning Activities
- Case Studies
- Key Understandings and Key Understandings descriptions

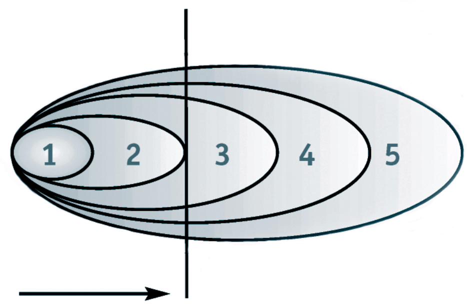
Planning

The mathematics curriculum goals and developmental phases described in the Diagnostic Map help teachers to know where students have come from and where they are heading. This has implications for planning. While day-to-day planning may focus on the mathematics of particular Key Understandings, teachers must keep in mind the learning required for progression through the developmental phases.

If a student has reached the end of the Matching and Comparing phase, then the majority of experiences the teacher provides will relate to reaching the end of the Quantifying phase. However, some activities will also be needed that, although unnecessary for reaching the Quantifying phase, will lay important groundwork for reaching the Measuring phase and even the Relating phase.

For example, students do not typically understand that they can partition a rectangle into appropriate squares and use the array structure to work out how many squares are in a rectangle, until approximately Grades 4–6 (ages 9–11). Therefore, understanding the significance of having no gaps and overlaps when "filling" rectangular arrays is not expected for reaching the end of the Quantifying phase, but it is for reaching the end of the Measuring phase. By Grade 6 (age 11), given access to an appropriate program in Measurement, most students should be able to reach the Measuring phase and be able to use an appropriate array structure without gaps to work out how many squares are in a rectangle. If students are to develop these ideas in a timely manner, however, then the ideas cannot be left until after reaching the end of the Quantifying phase.

Developmental Phases



There are a number of reasons for this approach. First, it is expected that a considerable number of students will enter Grade 4 having reached the end of the Measuring phase. Second, if teachers are to wait until this time to start teaching about partitioning a rectangle into appropriate squares, having no gaps and overlaps, and using the array structure to work out the number of squares in a rectangle, then it is unlikely those students would develop all the necessary concepts and skills in a timely fashion. Third, work in Grades 4–6 should not only focus on the Measuring phase, but also provide the groundwork for students to reach the Relating phase in the next year or two, understanding that we can use length measurements to work out area.

Teachers, who plan on the basis of deepening the understanding of the concepts, would think about the expected long-term learning in the early years of schooling. They would provide experiences that lead to the learning goals at the Measuring and Relating phases. This means students may be challenged about the significance of having no gaps or overlaps when finding how many squares are in a rectangle during earlier years of schooling. The students may not yet be ready to fully understand the significance of not leaving spaces or having overlaps when filling a two-dimensional space. It will take several years of learning experiences in a variety of contexts to culminate in a full understanding. (See, for example, Case Study 2, pages 51–53.)

Monitoring Students over Time

By describing progressive conceptual development that spans the elementary-school years, teachers can monitor students' individual long-term mathematical growth as well as their long-term progress against an external standard. This long-term monitoring is one of the reasons why a whole-school approach is essential. For example, Sarah has reached the end of the Quantifying phase for each of the Measurement concepts while another student, Maria, has only just reached the Matching and Comparing phase.

By comparing Maria and Sarah's levels against the standard, their teacher is able to conclude that Sarah is progressing as expected, but Maria is not. This prompts Maria's teacher to investigate Maria's thinking about Measurement and to plan specific support.

However, if two years later, Sarah has not reached the end of the Relating phase while Maria has reached the end of the Measuring phase and is progressing well towards reaching the Relating phase, they would both now be considered "on track" against an external standard. Sarah's achievement is more advanced than Maria's, but in terms of individual mathematical growth, Sarah appears to have stalled. Her progress may now be of greater concern than Maria's.

Reflecting on the Effectiveness of Planned Lessons

The fact that activities were chosen with particular mathematical learning goals in mind does not mean that they will have the desired result. Sometimes, students deal with an activity successfully, but they use different mathematics than teachers anticipated. Different activities related to the mathematics that has not been learned may need to be provided in the future.

On other occasions, what students actually learn may not be what teachers intended them to learn. Students may surprise teachers and cause them to rethink the activity.

In some instances, activities, which teachers think will help students develop particular mathematical ideas, do not generate those ideas. This can occur even when students complete the activity as designed.

The evidence about what students are actually thinking and doing during their learning experiences is the most important source of professional learning and decision-making. At the end of every activity, teachers need to ask themselves: *Have the students learned what was intended for this lesson? If not, why not?* These questions are at the heart of improving teaching and learning. Teachers make constant professional, informed evaluations about whether the implemented curriculum is resulting in the intended learning goals for students. If it is not, then teachers need to change the experiences provided.

Teachers' decisions, when planning and adjusting learning activities as they teach, are supported by a clear understanding of:

- the desired mathematics conceptual goal of the selected activities
- what progress in mathematics looks like
- what to look for as evidence of students' deepening understanding

When planning day-to-day lessons, it is important for teachers to appreciate that many of the same activities will be appropriate for students who are working within a range of developmental phases. Teachers can accommodate the differences in understanding and development among students by:

- asking different questions of individual students and groups of students
- providing extension activities for selected students
- giving particular students opportunities to do different things with the activities

Chapter 2

Understand Units

Decide what needs to be measured by selecting what attributes to measure and what units to use.

Overall Description

Students know that objects or events can be compared and ordered by different attributes, such as length, capacity or mass, depending on the purpose of the measurement. They understand that choosing a different attribute will make a difference to the order. They use appropriate comparative language to describe their comparisons, such as taller/shorter, wider/narrower.

Students understand that we use a unit when we want to quantify "how big" or "how much bigger" and that generally measurements are only as accurate as the unit they choose. When deciding what attribute to measure and how accurate they need to be (that is, what unit to use), students consider both the purpose of the measurement and the closeness of the comparisons to be made. They know, for example, that hand spans are probably good enough units to use to check whether a table will fit in a corner, but if it looks like it might be a tight squeeze, a smaller unit should be chosen.

Students understand that standard units are not more accurate than non-standard units, but that using standard units can help them in record-keeping and communication and are usually necessary when using formulas.

Students use common measuring equipment and graduated scales, such as rulers, clocks, and kitchen scales. They choose equipment or techniques to suit their situation. They express measurements in correct units and use their understanding of the common metric prefixes to move flexibly between units and to judge size.

Understand Units:
Key Understandings Overview

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their grade level.

Key Understanding		Description
KU1	We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.	page 24
KU2	There are special words and phrases that help us to describe and compare quantity.	page 32
KU3	To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.	page 40
KU4	The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.	page 54
KU5	Measurements of continuous quantities are always approximate. Measurements can be made more accurate by choosing smaller units, subdividing units, and using other strategies.	page 62
KU6	Our choice of attributes and unit depends on what we are trying to measure and why.	page 70
KU7	Standard units help us to interpret, communicate, and calculate measurements.	page 82
KU8	The relationships among standard units in the metric system help us to judge size, move between units, and do computations.	page 90

Grade Levels— Degree of Emphasis			Sample Learning Activities
K-3	3-5	5-8	
★ ★ ★	★ ★	★ ★	K-Grade 3, page 26 Grades 3-5, page 28 Grades 5-8, page 30
★ ★	★ ★	★ ★	K-Grade 3, page 34 Grades 3-5, page 36 Grades 5-8, page 38
★ ★ ★	★ ★ ★	★ ★	K-Grade 3, page 42 Grades 3-5, page 44 Grades 5-8, page 46
★ ★	★ ★ ★	★ ★	K-Grade 3, page 56 Grades 3-5, page 58 Grades 5-8, page 60
★	★ ★ ★	★ ★ ★	K-Grade 3, page 64 Grades 3-5, page 66 Grades 5-8, page 68
★	★ ★	★ ★ ★	K-Grade 3, page 72 Grades 3-5, page 74 Grades 5-8, page 76
★	★ ★	★ ★ ★	K-Grade 3, page 84 Grades 3-5, page 86 Grades 5-8, page 88
★	★ ★	★ ★ ★	K-Grade 3, page 92 Grades 3-5, page 94 Grades 5-8, page 96

Key

★ ★ ★ **Major Focus**
The development of this Key Understanding is a major focus of planned activities.

★ ★ **Important Focus**
The development of this Key Understanding is an important focus of planned activities.

★ **Introduction, Consolidation, or Extension**
Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

Key Understanding 1

We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.

Comparison by quantity underlies many of our descriptions of the world. Sometimes we are explicit, for example we say, *It takes longer by bus than by train*. At other times, the comparison remains implicit, for example we say, *He is tall*, but mean *He is tall compared to other boys* or *He is tall compared to me*. We also say, *She is 1.40 m tall* and mean that she is 1.4 times as big as a standard unit called a metre.

Key Understanding 1 relates to the development of students' capacity to differentiate types of bigness and smallness. Most students notice different attributes from an early age: That drink is too much for me; I need to speak louder so they will hear; I am too big to fit. Their early descriptions of objects, however, are likely to refer to a general perception of "bigness" or "smallness." Students need to develop the understanding that we can compare objects or events by how much they have of a particular attribute and we can then put them in order, from *less* to *more* of that attribute. We might, for example, compare suitcases by capacity (how much clothing they will hold). We might also compare them by mass (how heavy they will be to carry when empty). Television programs can be compared by finishing time (which television program finishes earliest) or elapsed time (which program is shortest).

Objects have many attributes, but only some of these attributes are readily described in terms of "more" or "less" and used to put things in order. Although we can compare the attributes of colour, taste, shape, or texture, for example, we do not usually think of them in terms of "more" and "less" and "how much" and so we do not usually order by these attributes or think of them as mathematical attributes. (Of course, we do refer to the strength of colour or taste and can order by concentration rates.) During the primary years, students should develop the capacity to focus on and distinguish between the attributes commonly used to order objects and events: length, area, volume, capacity, mass, angle, and time. In doing so, they should come to understand that:

- for certain attributes, the idea of having less or more of the attribute makes common sense
- we can use these attributes to compare and order objects or events
- the order of the objects or events may change if we focus on a different attribute.

Links to the Phases

Phase	Students who are through this phase. . .
Emergent	■ are aware of length, mass, capacity, and time as attributes of objects and events
Matching and Comparing	■ can use length, mass, capacity, and time to put two or three obviously different objects or events in order ■ attempt to focus on a particular attribute in practical familiar situations ■ understand that comparing by one attribute may produce a different order to comparing by another *For example*: A student may have found that sometimes the tallest jar holds the most and sometimes it does not. The student may conclude that to get the most to drink, we need to focus on the capacity of the jar, not the height.
Quantifying	■ have a more generalized understanding that different attributes may lead to different orders, although students are likely to be tricked by complex situations *For example*: A student may incorrectly assume that if the distance around one rectangle is greater, then that rectangle must have a greater area.
Measuring	■ consistently distinguish the time it is now from how much time some event has taken ■ distinguish perimeter from area and notice that the figure with the greatest area may not have the greatest perimeter and vice versa

Sample Learning Activities

K–Grade 3: ★★★ Major Focus

Storing or Selecting Materials

Invite students to compare and order by attribute when storing or selecting materials. For example, before packing away boxes (glue bottles) used for another activity, ask students to think of how to sort them and pack them in order, from the largest to the smallest. Focus on height, width, or length at separate times. Or, ask students to find a skipping rope of suitable length (ball of suitable size) for an activity.

Lifting Objects

Invite students to lift objects ranging from large and light to small and heavy (such as cushions, balls, fruit, sponges, marbles, rocks, fruit). Ask: Is your object heavier than this small one? Which ones will we leave for an adult to lift?

Jello Moulds

Organize students into groups and have them order jello moulds by different attributes; for example, from tall to short, fat to thin, needing the largest plate to sit on to needing the smallest, from most jello to the least amount of jelly. Ask: Which one has the most jello? Why does this jello need a bigger plate than the tallest jello? Adapt this to sand castles by having students build sand castles using the lids as the base. Ask: Which is the biggest? Is this one on the big lid bigger than this tall one? Which sand castle has the most sand?

Play Dough

Extend Jello Moulds by having groups of students use play dough to make snakes ranging from short and fat through to long and thin. Invite them to order the snakes by length. When they have done this, ask: How can you tell which one is longest? Draw out from students the need for the snakes to be lined up against a baseline, such as the edge of the desk, so that their lengths can be compared. Then, ask students to coil the snakes and reorder them by the table space they will take up. Ask students to reform the play dough, roll it out, and use lids to cut shapes. Ask them to order the shapes by area, make piles of same-area shapes, and order the piles by height. Have students reform the play dough again. Invite them to roll it into balls and order the balls by height, by looking from the edge of the table, and mass, using balance scales.

Length

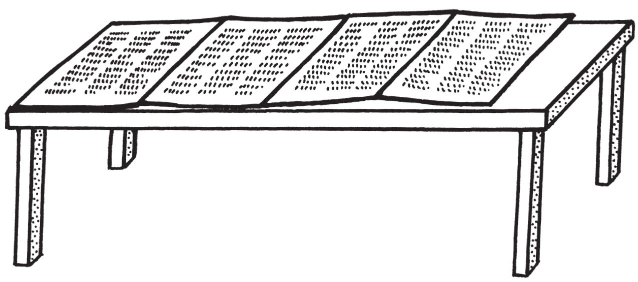
Provide lengths of string (wool, rope, paper tape, straws, chopsticks, craft sticks) for students to choose from and use in play, construction, and collage activities. Ask: How long? Which length will be high enough (long enough) to go around (to go along, to reach the other side, to go far enough down)?

Animal Stories

Read some stories to the class that feature animals of different sizes. Ask students to explain how the animals vary in size in different ways. Ask: Which animal is biggest? Which animal could you lift up? Which animal could fit through a hole in the hedge? Follow the same process with other stories.

Area

Invite students to select and cover regions (choose a rug to cover a toy, spread paste using a knife, squeegee, or brush, help cover a table top with paper to design a model of a town). Ask: Will it cover the surface? Are there any gaps? Did it go over the edge? Encourage students to explain to others what they have to consider to cover a region in the same way.



Lifting Closed Containers

Store construction and collage materials (dried pasta, washers, craft sticks) in identical closed containers. Invite students to lift each container and say what they think each container is holding because of its mass. Open containers for students to talk about the contents and why one container was heavier than another. Ask: Why is the pasta container lighter than the washer container? Add an empty closed container and invite students to lift it. Ask: What do you think is inside this container? Is this container the lightest? As students view and handle containers, ask: How could we order the containers on a shelf from heaviest to lightest? What other ways could the containers be ordered?

Pour to Decide

Ask students to place cups in order of those that hold the most to those that hold the least. Invite them to check by filling the cup they think holds more and pouring the amount into the second cup and so on. Encourage them to talk about why the order was right or wrong (*I thought the tall one was biggest, but that fat one holds more*). Repeat with the same cups over a few days to enable students to compare the capacity of the cups with their predictions.

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Stretching

Have students compare and order materials (string, wool, elastic) by how much each will stretch. Ask: Which attribute (length, area, volume) is being used to order them?

Changing Attributes of a Balloon

Organize students into groups and give each group a selection of different-shaped balloons. Have one student blow up a balloon. Ask: Which attributes of the balloon have changed? (length, area, volume). Invite students to blow up the rest of their balloons and order them by something they can measure. Ask: How would this order change if we used distance around the balloon instead of volume?

Judging the Biggest

Invite students to identify the attributes that could be compared to judge which is the biggest pumpkin (apple, potato). For example, the largest pumpkin (apple, potato) could be the one with the longest circumference or distance around (length), the one that is the heaviest (mass), the one that would give the most to eat (volume or capacity), the one with the largest amount of skin (surface area), the one that is tallest (length). Through discussion, arrive at attributes that are reasonable for the largest pumpkin (apple, potato). Ask students to use the attributes to order them from smallest to largest.

Organizing Drink Containers

Discuss with students how drink containers may be organized in a supermarket. Invite small groups of students to try different arrangements based on attributes such as height, capacity, mass, brand, type of drink, colour. Ask: Which arrangements focus on quantities (how much) and which focus on other things? Why is mass not as important for stacking? (because stability and not wasting space are more important)

Time Taken Getting to School

Have students order the time taken for various activities compared to ordering by other attributes. For example, invite them to work out how long it usually takes to get to school. Ask: Who takes the longest to get to school? How does that student get to school? (by car, by bus, riding a bicycle, walking) How far away does that person live? Does the student who lives closest to the school take the least time to get there? What else do you need to take into account?

Comparing Polygons

Organize students into pairs and give each pair a pattern block hexagon, square, and triangle. Ask students to order them by area. When they have done this, ask them to compare angles by superimposing one shape on top of another to see which angle is larger. Have them order the shapes by the size of the angles. Ask: How else could you order them?

Start and Finish Times

Give groups of students information about the start and finish times of nearby schools and the start and finish times of their recess and lunch breaks. Ask them to order the schools by which start first, which finish first and which have the longest lunch and recess time. Ask: Do the schools that start early finish early? Do the schools that have short lunch and recess time finish early? How did you work out the order of the start times? How did you work out the order of the amount of time taken for lunch and recess? How was working out the order of the start times different to working out how much time was taken for lunch and recess?

Mass, Volume, or Capacity

Present students with a collection of items from the supermarket and ask them to classify them according to whether they use mass, volume or capacity measures. Ask: Why is ice cream and milk sold in litres? How are these two items the same? Why is rice sold by mass, not volume?

Twelve Tiles

Ask students to use 12 tiles to construct as many different rectangles as possible and record each one on grid paper. Have students order the shapes by the distance around each one. Ask: Can you order the shapes by their area? Why? Why not?

Perimeter and Area

Organize students into pairs and give each pair a geoboard and a piece of string. Have them tie a knot in the piece of string at 12 cm and use it to construct a shape with a perimeter of 12 cm. Invite students to make other shapes with the same perimeter. Have them record each shape on grid paper. Ask: How does the size of the area change?

Sample Learning Activities

Grades 5–8: ★★ Important Focus

Comparing Two People

Organize students into pairs and invite them to think of what measurements they could take to compare one another. Ask them to take the measurements and write comparisons of themselves; for example, *I am 4 cm taller than my partner, but my arm span is 3 cm shorter and my mass is 1 kg less*. Invite students to classify their measures into length, mass, volume, and area. Ask: Which attributes were measured most often? Which attributes were not measured? What would you have to do to measure these attributes? Be aware that some students may be sensitive about comparing their measurements.

Ordering Animals

Invite pairs of students to choose ten animals and decide what they could measure to order the animals. Ask: Can you order according to colour (habitat, diet)? Why? Why not? Ask students to specify what measurement they will take for each attribute. Ask: What length are you measuring? Have students order the animals according to each attribute they have measured. Ask: Are the animals in the same order for each attribute? Why? Why not?

Candies

Provide candies that are wrapped individually to groups of students. Say: Whoever thought of making this candy would have had to think of a lot of different things when they were designing it. Ask: What attributes would that person have had to think about? Draw out the range of possibilities related to mass, volume and length, such as the weight of the candy, the dimensions of the candy, the wrapping paper, the print on the wrapping paper, and the various attributes involved in packaging the candies in boxes or bags. Invite students to invent a new kind of candy and write up its specifications with all the measurements they have discussed.

Identikit

Display about 15 objects at the front of the class. Organize students into pairs and ask each pair to choose one of the objects and write a measurement identikit for it using attributes of length, area, mass, and volume or capacity. Return all the objects to the front of the class, collect the identikits and invite each pair to choose a different identikit to solve by matching the descriptions to an object. Encourage them to justify their selections; for example, *I measured the surface area of the cube and found it was 54 cm*^{2}.

Birth Measurements

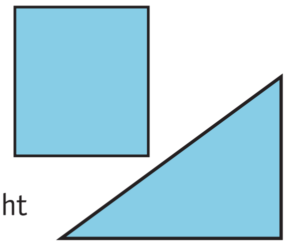
Ask students to bring in their birth measurements. Invite them to take the same measurements now and compare them. Ask: Which attributes have increased the most? Which have increased the least? Why?

Doctor Measurements

Invite students who have been to the doctor recently to describe some of the measurements the doctor took. Discuss the attributes that were measured and what they mean. For example, ask: What are doctors measuring when they measure lung capacity? What capacity are they talking about? What other things can have their capacity measured? What are doctors measuring when they measure blood pressure? What other things involve measuring pressure? What measurements involve time?

Different Orders

Have students explain why the same things may have been ordered in different ways. For example, say: Here is a problem some students were asked to work out. Their school had two pieces of land, one a rectangle and one a triangle. The school wanted to choose the largest one for a garden bed. Joshua thought the rectangular garden was bigger than the triangular bed, but Su-Lin thought the triangle shape was bigger. Ask: Why did the students think different things? What could they use to help them make their decision? How could they tell for sure which piece of land was bigger? Then, say: Another group of students were asked to put four full lunch boxes in order of size. They all came up with different answers. Ask: How could the same objects have been put into different orders? (some could have measured the capacity, some the surface area, some the height) (See Case Study 2, page 51.)



Swimming Pools

Have students draw different shapes that have the same area. Say: A swimming pool company charges the same price to build any swimming pool, no matter what shape, so long as it has a water surface area of 18 m^2. Invite students to design different-shaped pools and demonstrate that the pools all have a surface area of 18 m^2.

Pool Perimeters

Ask students to work out the perimeter of each pool designed in the Swimming Pools activity. Ask: Which pool needs the longest length of tiling around the edge? Which needs the shortest? If you owned the pool company and were paying for the line of tiles around the pool, which shaped pool (with an area of 18m^2) would be the cheapest to build?

Key Understanding 2

There are special words and phrases that help us to describe and compare quantity.

From a very early age, students will have ideas about "more," "less," and "equal amounts" and will have some of the language of comparison. Their language should develop and be refined throughout the primary years so that they are able to use general comparative language (big, small, more, less), different words associated with one attribute (tall, short, wide, narrow, long, length, distance) and comparative language associated with particular attributes (tall, taller, tallest, as tall as).

Students are likely to understand some comparative pairs before others (e.g., multi-dimensional pairs such as big/small before uni-dimensional pairs such as high/low) and "positive" members of pairs before "negative" members (e.g., tall before short, heavy before light). Comparative pairs need explicit attention. Also, the subtleties of comparative terms are to an extent culture specific. For example, some people might determine a "bigger" fish by its length, but other people might use mass to compare the size. Assuming we speak a common language is often to the mathematical disadvantage of students for whom English is a second language. Equally, however, avoiding the use of standard comparative language will disadvantage students in the longer term. All students should experience the use of the language of quantity and comparison in a wide range of contexts and the subtleties and cultural differences in the use of terms such as "wide" and "narrow" should be addressed throughout all of the primary years.

Students also need to learn to use the terms length, area, volume, capacity, and mass appropriately as they engage in activities in direct and indirect measurement. They might, for example, initially talk about "distance around" and "amount of fence needed" for a pasture as well as "how much grass" is inside the fence. In order to develop this Key Understanding, the language of length and area should be modelled by the teacher and situations structured so that the students themselves have plenty of opportunity to try out their use of terms such as length, width, perimeter, and area in sensible contexts.

Links to the Phases

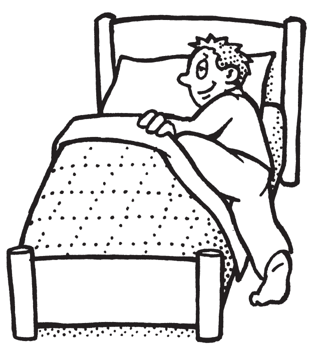
Phase	Students who are through this phase. . .
Emergent	■ respond appropriately to and use for themselves everyday comparative language associated with length, mass, capacity, and time using the language forms of their own community
Matching and Comparing	■ distinguish length from area in situations where the context helps them make sense of the terms ■ associate the term "mass" with heaviness and "capacity" with how much something holds
Quantifying	■ respond appropriately to and use the everyday language of attributes and comparison in conventional ways ■ respond appropriately to the term "perimeter" in situations where the contexts assist their interpretation
Measuring	■ have a clear understanding of the distinction between perimeter and area, area and volume, and time from elapsed time

Did You Know?

Many students confuse "the time" with "how much time." A teacher of 7-year-olds asked them to draw pictures of themselves going to bed and getting up and to write their names and the times underneath. They then sorted their pictures into groups by time. Asked to work out who was in bed the longest, most responded that it was those who went to bed at 7 p.m. In vain, the teacher pointed out that one child who went to bed at 7 p.m. was up at 6 a.m., while another who went to bed at 7:30 p.m. wasn't up until 7:30 a.m. The students were convinced that those who went to bed first were in bed longest.

The teacher thought that probably the students could not work out how much time they were in bed, so she helped them to mark a long strip with hours of the day a cm apart. They then coloured the strip between when they went to bed and when they woke and cut off the uncoloured parts. The teacher then made a column graph by using the paper strips and lining up the bases. She suggested that the graph would help the students to find out who was in bed longest. To her surprise, the students rejected her approach, *You have made a mistake — you have made us all go to bed at the same time.*

The students have not yet understood the difference between time and elapsed time.



Sample Learning Activities

K–Grade 3: ★★ Important Focus

Describing and Comparing Quantity

Give students instructions and ask them questions involving language that describes and compares quantities. For example, ask: Did you use the heaviest or lightest truck today? How do you know your line is longer than your partner's? Or, say: Turn the paper so that it is wider than it is high. Make a narrower gap between the lines.

Animal Stories

Extend *Animal Stories,* page 27, by asking: Which animal is the smallest? Encourage students to name different lengths; for example, tall, thin legs; wide, thick neck; long, thin tail; short, wide snout; narrow ears. Ask: Which animal is wider (lower, shorter, faster, slower, could fit through this space)? Extend to other literature for students to notice and use terms to describe the different attributes of the props and characters.

Full and Empty

Have students use various containers to pour from one to another during play and food preparation. Focus students on "full," "overflowing," "empty," "not very full," "some left over," and "needs more."

Pour to Decide

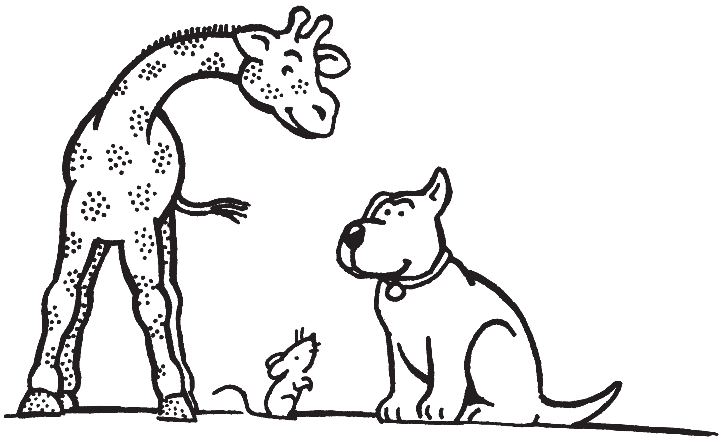
Extend *Pour to Decide,* page 27. As students pour from one container to another to compare capacity, focus them on describing how much a container will hold (holds more, holds less, holds a lot, doesn't hold very much, left over, more than, less than, level, heaping).

Sorting

Invite students to sort objects (shells, leaves, rocks) into groups and then order them by size. Ask: Which part of the shell (leaf, rock) did you look at to judge the size? Have them describe, draw, and label how they grouped and ordered the shells (leaves, rocks).

Short and Tall Posters

Invite students to make posters to show "what is short to a giraffe," and "what is tall to a mouse." Ask: Does anything appear in both posters? How can the dog be both short and tall?



Colloquial Terms

Model the correct word when students use colloquial terms to describe order. For example, when describing by height, they may say, *This one is really tall, this one is a big bit tall, this is a little bit tall, and this one is a bit tall*. Respond by affirming their ordering and say, for example: Yes, that one is the tallest, that one is tall, that one is shorter, and that one is the shortest of all.

What Am I?

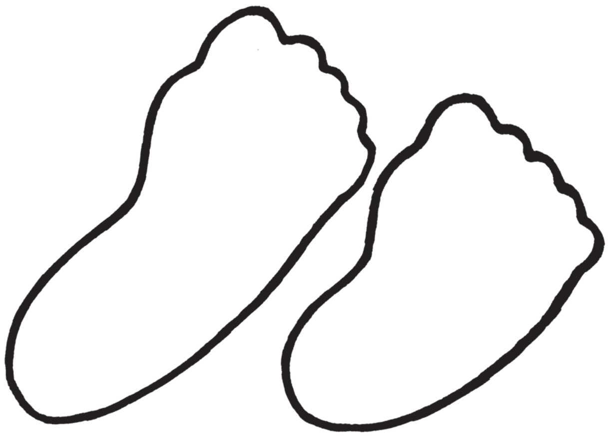
Invite students to play What Am I? using measurement language. For example, say: I am heavier than a pencil, but lighter than a glue stick (wider than a sharpener but narrower than a bucket, hold more than a teaspoon but less than a drink bottle). What am I? Encourage students to ask questions to help their thinking (*Are you narrower than* … ?).

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Bigger Foot

Tell students you heard someone say, "I've got a bigger foot than you." Ask: What measurements do you think they were thinking of? Organize students into pairs and invite them to make a cut-out shape of their foot. Encourage them to talk about the length, width, perimeter, and area of their foot in comparison to their partner's foot. For example, ask: Is your foot longer than Ahmed's? Is it wider? Is it farther around? Does it take up more space?



Comparative Words

Brainstorm and list pairs of comparative words that describe more and less length, area, volume, capacity, time, and mass. Invite students to illustrate a chosen pair. Display the illustrations with the words.

General or Special Words

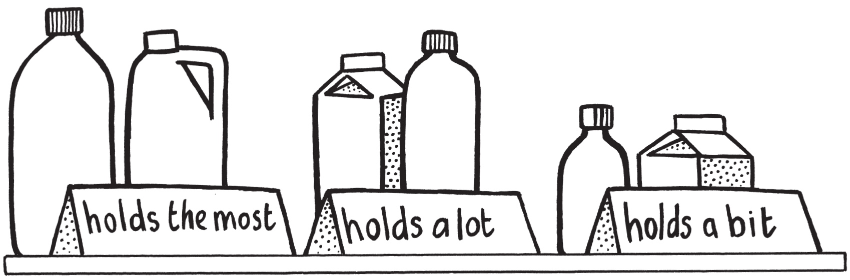
After activities such as Comparative Words, categorize the words according to whether they are "general" or "special" words. For example, "bigger" can refer to lots of different things about lots of different objects, but "taller" is a special word referring to height or how high from the ground. Anything can be "bigger," but only some sorts of things would be "taller."

"What Am I?" Riddles

Invite students to write What Am I? riddles using the pairs of words from the class list of words developed in Comparative Words. For example: *I am shorter than a ruler, but longer than a pencil sharpener. I am heavier than a button but lighter than a book. What am I?*

Organizing Drink Containers

Extend *Organizing Drink Containers*, page 28, by having students order the collection of drink containers and then write a label for each item in the order. For example, when stacked by height, encourage them to use the labels "short," "tall," and "tallest." When stacked by mass, encourage them to use the labels "light," "heavy," and "heaviest." When stacked by capacity, encourage them to use the labels "holds a bit," "holds a lot," and "holds the most." Ask: What does the word "mass" describe? What does the word "capacity" describe?



Science Activities

During science activities involving change (monitoring the growth of plants, animals, crystals), encourage students to select words to describe how each attribute has changed (longer, greater area, wider). Have students use these words in the written reports of the experiments. For example, when students are considering length, they might write: *The plant became taller and each of its leaves became wider*.

Wrapping Presents

Invite students to use newspaper to work out how much material they need to wrap a present. Organize them into pairs and give each pair a shape or box to act as a present. While the students are working, ask: Do you need to know the length measurements or area measurements? How does the volume of the object affect how much wrapping paper you need? How does the surface area affect how much wrapping paper you need? Have students write definitions in their own words for each of the attributes of length, area, volume, and surface area.

Twelve Tiles

Extend *Twelve Tiles*, page 29. During the activity, ask students to use the word "perimeter" to describe the distance around the different shapes.

Sample Learning Activities

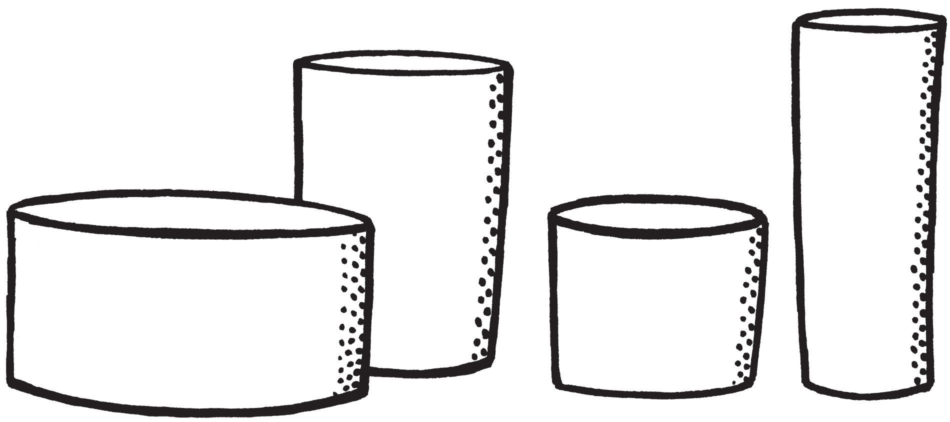
Grades 5–8: ★★ Important Focus

Identikit

Have students use attribute language and the language of the standard units to describe objects. For example, extend Identikit, page 30, by asking students to refer to at least two attributes. For example: *My object has an area of 600 cm^2, but its volume is much less than 600 cm^3*. After the activity, encourage students to reflect on how they arrived at their clues. For example: *Mine was a page. About six Base Ten flats fit on a page, so its area is about 600 cm^2. Its volume is much less than 600 cm^3 because a page is much thinner than a flat.*

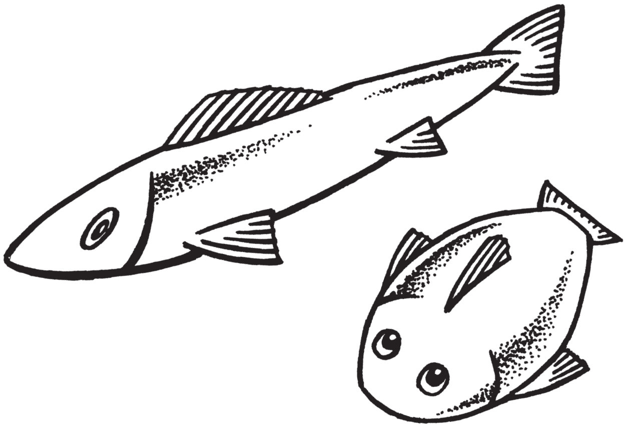
Container Capacity

Encourage students to use terms associated with the attributes of area, volume or capacity in order to make comparisons and discuss relationships between measures. For example, ask students to design and make the largest possible cylindrical-shaped closed container using a single sheet of 8 1/2 × 11 paper. Have them use popcorn or something similar to test the capacity of their container. Invite them to compare and discuss the relative dimensions of their containers and the resulting capacities. For example, *My cylinder is shorter than yours but it has a greater capacity. A smaller circumference does not always mean less capacity*.



Big Fish

Encourage students to collect descriptions of situations in which people use "bigness" to describe relative size or quantity and examine the possible confusion of interpretations that could occur. For example, say: When some people talk about a big fish, they might mean the fish is very long, whereas other people might mean the fish is very heavy. Ask: Can you think of any times when you misunderstood someone because you misunderstood what they meant by a particular size word?



Different Types of Length

Have students brainstorm the different types of lengths that can be measured (height, circumference, depth, width, tallness, thickness). Organize students into groups and assign a type of length to each group. Ask them to list all the objects and situations they can think of that have that type of length (such as thickness: piece of paper, trunk of a tree, sheet of steel). Invite groups to compare lists and identify objects and situations that occur in more than one list. Encourage students to clarify what kind of length they mean for these common situations. Ask: Do thickness and width mean the same thing when referring to the trunk of a tree? What about a sheet of paper?

Key Understanding 3

To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.

We can directly and indirectly compare the size of objects or events and put them in order without using numbers. When we want to describe how big something is or how much bigger one object is than another, however, we use numbers. The process of quantifying attributes is usually called measuring.

To directly measure how heavy a rock is, we might choose the mass of a marble as our unit of mass, count how many marbles it takes to balance the rock (as well as we can) and conclude that the rock is 11 marbles heavy. We might choose the time of one full swing of a locket as our unit of time and use it to measure how much time it took to walk across a room, concluding that, "I took seven full swings, Sam took nine. He took two full swings longer." In fact, we do not always have to actually match an object or event with a unit because we have also developed a range of indirect measurement techniques, but the underlying idea is always to quantify an attribute by finding out how many of the units match or fit the object or event.

Students need to internalize the following ideas if they are to fully understand how measuring works.

- We can use numbers to describe the size of a thing by selecting a unit and counting how many repeats of the unit it takes to match the thing as closely as possible.
- A unit is itself a quantity; that is, it is the mass of the marble that is the unit, not the marble itself.
- The size of something does not change when you use a different-sized unit to measure it, but the number of units taken to match it may change.
- We can say which of two objects is bigger by comparing how many of the same unit match each object.

These ideas develop more slowly than is often recognized. Having developed these ideas, however, students can see why:

- we should generally use the same unit repeatedly to measure an object
- when comparing two objects or events, the same unit should be chosen for each.

Links to the Phases

Phase	Students who are through this phase. . .
Emergent	■ will correctly respond to a request to, for example, count how many pens fit across the table and may have learned to call this "measuring" *For example*: A student may see the task is one of counting to see "how many fit," but even when prompted does not use this information as a measurement to answer questions such as, "Will the table slide through the door?"
Matching and Comparing	■ will, when prompted, use a unit to decide which of two things is bigger, but may be tricked by conflicting information *For example*: A student may believe that the size can change when a different unit is used.
Quantifying	■ understand what it means to measure, i.e., they will unprompted use a measurement to decide whether one object is bigger or smaller than another ■ understand why it helps to use the same size unit repeatedly to measure an object ■ understand why it is necessary to use the same unit for each quantity when comparisons are to be made ■ have developed ideas about comparing by attributes and knowing "how many units fit" and these ideas have come together
Measuring	■ understand the unit as a quantity *For example*: A student will realize that a square with a side of 1 m can be cut and rearranged and still be the same unit, i.e., a square metre.

Sample Learning Activities

K–Grade 3: ★★★ Major Focus

Trains

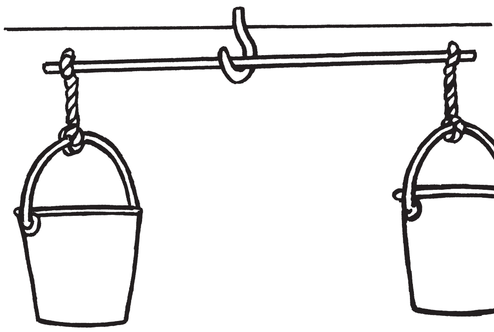
Ask students to make trains from blocks. Invite them to count how many boxcars long their train is. Ask: Is your train longer than Tracey's? She has six boxcars. How many blocks long is that train? Is your train longer or shorter? How did you work that out?

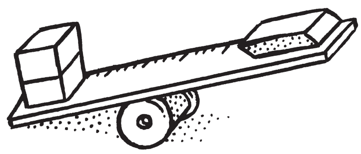
Using a Balance Scale

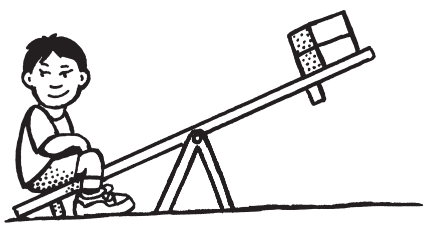
After students have lifted two objects, ask them to place the first object on one side of a balance scale. Invite them to place units (such as marbles, washers) on the other side one at a time and count how many units match the mass of their object to make the scales balance. Encourage them to describe how heavy their object is. (*My bottle weighs the same as 28 marbles*.) (See Case Study 1, page 122.)

Balancing Tools

Extend Using a Balance Scale by making other balancing tools (teeter-totters, buckets on ropes, rulers on spools). Focus on matching mass and balancing by asking: How many marbles (washers) does it take to balance this doll (truck)?







Different Units

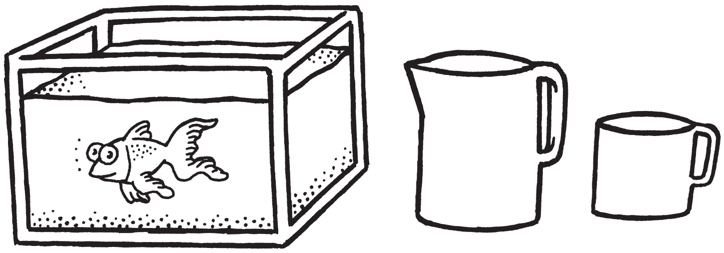
As students use balancing equipment in play or activities, have them use and record how many different units match the mass of their object. Ask: How can your object be the same as 12 washers and the same as only 8 marbles?

Desk Through the Doorway

Present students with length problem situations where measuring rather than directly comparing is required. For example, ask: Will that heavy desk over there slide through this doorway? How do you know? Prompt with questions. Ask: How wide is the desk? How wide is the doorway? How many pens fit across the desk? How many pens fit across the doorway? Does the pen measure help you decide?

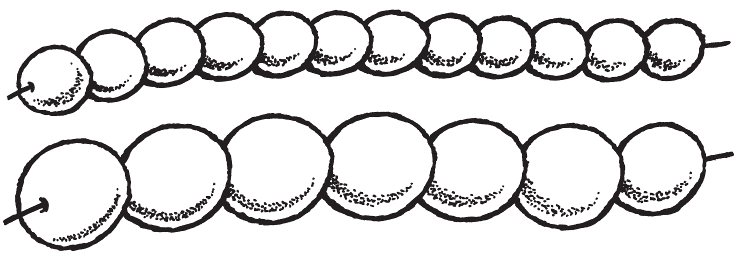
Fish Tank

Give groups of students some large containers and ask them to choose an object (cup, jug) to use as a unit to find the container with the largest capacity or volume to keep a fish in. Ask: How can you use this object to find out the size of your containers? What is the size of your unit? What is the size of the largest container? How do you know it is the largest?



Beads on a String

Invite students to select one length of string from a range of three lengths and one bead size from a range of sizes to make a necklace. When they have made their necklaces, ask them to find a student who has a necklace of the same length but different-sized beads and to count how many beads each necklace has. Ask: How can you have the same length when you have more (fewer) beads?



Covering a Surface

Build on Beads on a String by using same-sized tiles (print shapes, leaves, footprints, pattern blocks) to cover a surface. Invite students to count how many fit and describe the area by the number of units (*This is a 9-tile piece of paper.*)

Sample Learning Activities

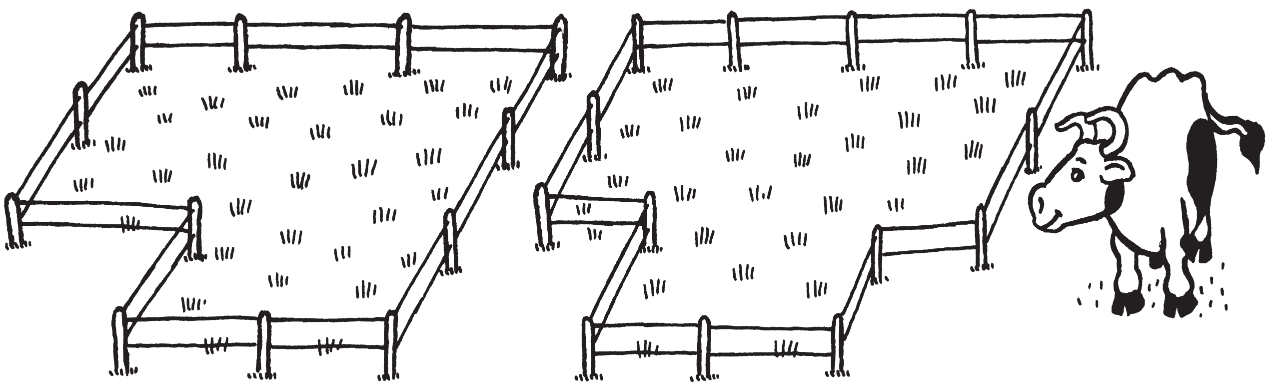
Grades 3–5: ★★★ Major Focus

Snail Trails

Have students draw two "snail trails," one 8 toothpicks long and the other 12 toothpicks long. Ask: Which trail is longer? How do you know? Ask students to remove the 12 toothpicks and replace them with enough craft sticks to cover the line. Ask: How long is this trail now? Which trail is longer? How do you know? Discuss the idea that to compare two things we need to use the same size unit. (See Case Study 1, page 48.)

Hungry Cow

Extend Desk Through the Doorway by having students solve problems involving other attributes where direct comparison is not possible. For example, show students two different, irregular shapes of similar area drawn on one sheet of paper or card. Say: A farmer has these two pastures on his farm. If you were a hungry cow, which would you prefer? Invite them to compare the two shapes. Ask: How can you compare the two pastures? What could you use? Why do you have to use the same-sized object on both in order to compare?



Steps

Have students measure and record distances around the school by walking and counting their steps. Ask: Does it matter whose steps are being counted? What happens if some of your steps are shorter than others? How could you be sure the distance to the office is the same as the distance to the library?

Body Measurements

Have students each choose an object to use as a unit to measure and record information about themselves (height, mass, distance around waist, length of arm, area of footprint). Ask them to compare their measurements with a partner and determine who is taller (has the greater mass, has the longer arm). Ask: What makes you certain you are taller (heavier) even though the number of units is less than your partner's? What have you chosen to use for your unit? How big is it? Be aware that some students may be sensitive about comparing their measurements.

One Craft Stick

Have students use one object repeatedly to measure. For example, to find out if a bookshelf will fit a given space, have students use one craft stick, marking off and counting how many craft sticks long the space is. Then, have them do the same with the bookshelf. Ask: Will the bookshelf fit the space? How do you know? Would you get the same answer if you matched the space with a line of craft sticks? Why? Why not?

One Tile

Repeat One Craft Stick, but have students use one object to measure area. For example, during a science lesson, use one tile to say which of two different-shaped leaves has the larger area. Ask: What do you need to do with the tile to find out? Can you use that information to say how much bigger one is than the other?

Different Containers

Give pairs of students a large container, a selection of small containers, such as cups, jars, and lids, and a bucket of water. Invite them to each choose a different small container to fill the large container. Ask them to take turns filling it with their container and say how many cups (lids, jars) they each took to fill it. When each student has counted and all students have given their results, ask: How can that container be filled by three cups when you fill it and by five jars when your partner fills it?

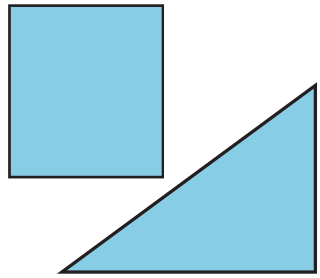
Time Periods

Have pairs of students choose ways to measure short time periods without using a clock (counting claps, ball bounces, pendulum swings). Encourage them to use all the methods to measure the time taken to complete an activity (putting on their shoes, writing their full name, getting a drink). Have students compare results. Ask: How many pendulum swings did it take you to put your shoes on? How many ball bounces did it take? Why do you think you got a different result?

Sample Learning Activities

Grades 5–8: ★★ Important Focus

Garden Plots



Have students decide on the attribute and the unit of measurement, in order to solve problems. For example, a school has two available pieces of land, one a rectangle and one a triangle, and you want to choose the largest one for a garden. Ask: Which one do you think is larger? What do you have to measure? What units of measurement will you use? Why? (See Case Study 2, page 51.)

Comparing Gardens

Build on Garden Plots by asking: How did you decide which garden was larger? Did measuring help? Use an actual example from the class or present a scenario; for example, say: Craig measured the triangular garden as 28 cubes and 6 green triangles and the square garden as 30 cubes and 4 cream rhombuses. He found it difficult to compare the gardens. Why? Why do we have to use the same unit for both shapes?

Citrus Fruit

Have students use units to compare a range of attributes. For example, organize students into groups to decide which citrus fruit has the most juice. Invite each group to choose one type of citrus fruit and measure how much juice it has. Have various containers available for students to use as measures (cups, jugs, eggcups, teaspoons, lids). Encourage groups to compare their results with others to decide which type is the juiciest and by how much. Ask: Was it difficult to compare your results? Why do you think that happened? What decisions need to be made about the unit in order to easily compare the amount of juice?

Identikit

Have students choose suitable units to measure the volumes of different objects. For example, during *Identikit*, pages 30 and 38, encourage them to tell their partner how they measured the volume of objects like a brick, an apple, and a banana. For example, say: Some students used cubic centimetres and some used millilitres for the same object. How would they have done this and who is correct? Draw out the mathematical idea that volume can be measured using cubic units or liquid volume units.

Rectangles

Give pairs of students various rectangles with an area of 1 m^2 (1 m by 1 m, 50 cm by 2 m, 25 cm by 4 m) with which to measure the area of the same region, such as a bulletin board. Tell them that all their shapes have an area of 1 m^2. Have students record and compare measures. Ask: What do you notice? Does the shape of the object matter? Why? Why not? Draw out the mathematical idea that the unit we use to measure the area is a size. As long as that size stays the same, the shape does not matter.

Racing

Organize students into pairs and have them decide on a time unit of some type (claps, ball bounces, counting, pendulum swings) to time each other racing over 50 m. Invite them to record their best time out of three. Display the results on a class chart and stimulate discussion by suggesting the class winner is the one with the lowest number (without accounting for the types of units used). Discuss ways that the given results could be fairly compared to arrive at the 50-m champion. Ask: How could we compare fairly? What would we need to do to find out how much faster the Olympic champion is for that length of race?

Treasure Hunt

Have students plan to hide four or five "treasures" (pennies, buttons) around the school and write a list of measurement instructions for another pair to follow. Ask: What measurement information will you need to give for other students to find the treasure (the next clue)? How will they know how far to walk? How will they know what direction to walk in? How will you tell them how much to turn? Invite students to design their treasure hunt, place their treasures and then swap their clues with another pair to find the treasures.

Sorting Cards

Have students compare measures when the units of measurement are different. For example, prepare some cards showing a range of measures of length, area, volume, capacity and mass, in mostly different sizes (for length use mm, cm, m, and km). Invite students to sort the cards into categories according to the attribute they measure. Then, in turns, have students take two cards from a category pile and say which measure is larger and why (which is larger out of 2 dm and 22 cm). Ask: Did you have to do anything to be able to compare the two measures? Were there measures you could compare without changing to the same unit? How did you order them? Shuffle the cards in each pile and repeat the activity.

CASE STUDY 1

Sample Learning Activity: K–Grade 3—Snail Trails, pages 56, 72
Grades 3–5—Snail Trails, page 44

Key Understanding 3: To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.

Working Towards: Matching and Comparing Phase and the Quantifying Phase

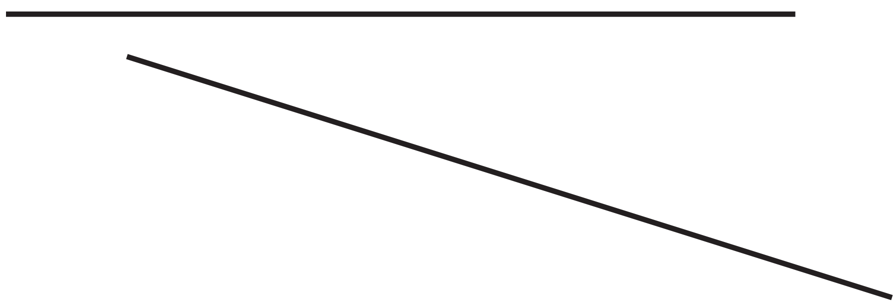
TEACHERS' PURPOSE

Students in my Grade 3 class had begun to use units to measure length and seemed to be able to do it. For example, Roberto chose straws to measure the length of the bookcase and said, "The bookcase is a bit more than 12 straws long."

I was not sure, however, whether they would use this information to compare the length of two different objects and decided that this should be the focus of my next measurement lesson.

MOTIVATION AND PURPOSE

I provided sheets of paper on which I had drawn two straight lines to represent snail trails across a path. Each page was a little different, so students had to work with their own pair of trails.



I gave each pair several different types of materials to use as units from among toothpicks, paper clips, marbles, blocks and beans, but made sure that there was only enough of any one material to measure one of the trails, not both. I had a reason for doing this: I wondered if they really knew that the count was an indication of the length of the object. I did not give them enough objects of the same size to match both lines at the same time because I wanted to create conflict that would provoke the students to think about what to do. My goal was that they would work out that they could use the objects to measure one line, remember the number, and then place the same objects onto the other line.

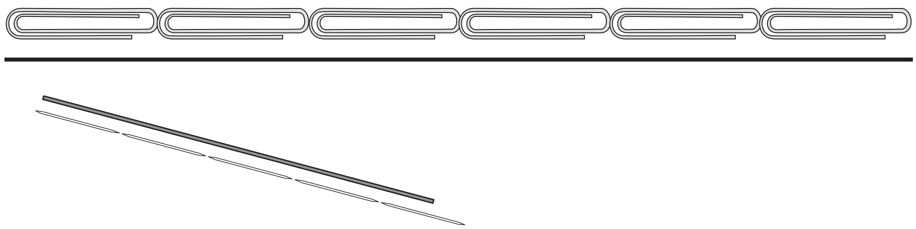
I then asked, "Which snail went farther and how much farther did this snail go than its friend?"

ACTION AND REFLECTION

Almost all students began by choosing one of the units and repeating it by lining up the units end-to-end along the first trail. Many of them, however, happily chose a different-sized object (and hence unit) to measure the second trail. I saw Adam run out of toothpicks and begin using blocks that were a different size halfway along the second line. "There wasn't enough, so I had to use another one." After seeing what others in his group had done, he replaced the toothpicks on the second line with blocks.

Even though I had anticipated that some students would choose to use different units, I was surprised that so few students thought about the need for the same unit and so did not feel any conflict about using different units. Having counted the units, most disregarded the object they had used and simply compared numbers.

I encouraged students to reflect on the sense of what they had found. For example, Kim used paper clips for the horizontal line and toothpicks (which were a different size to the paper clips) for the other.



I asked him why he chose the toothpicks. He said, "They are skinny and they fit on the line better and I can see if they are straight on it."

I then asked him which of his trails was longer and he immediately pointed to the diagonal line. Then, I asked, "How much longer?"

Kim counted the units on each line, frowned, looked a bit confused, counted again and said, "That's six, there's only five there, so it's one more?"

I asked why he had frowned and counted again. He said, "I thought I was wrong because the five one's longer, but I counted again and it is one more there."

I helped Kim notice an anomalous or conflicting result without actually correcting him or simply telling him "how to do it."

CONNECTION AND CHALLENGE

Kim's confusion about what he has found provides an opportunity for him to make a significant leap in his knowledge.

I realized that Kim did not understand how counting the units related to the length. He carefully lined the units up end-to-end as I had taught him, but did not seem aware that the differing lengths of the two units would interfere with the comparison. He also ignored the "part-unit" left on one line. His hesitation, however, suggested he might be ready to move on, so I suggested he try swapping the toothpicks and paper clips.

Kim did this and found that there were seven paper clips and barely five toothpicks. "Ah," he said, "It is two more, that has to be right—the long one's got more this time."

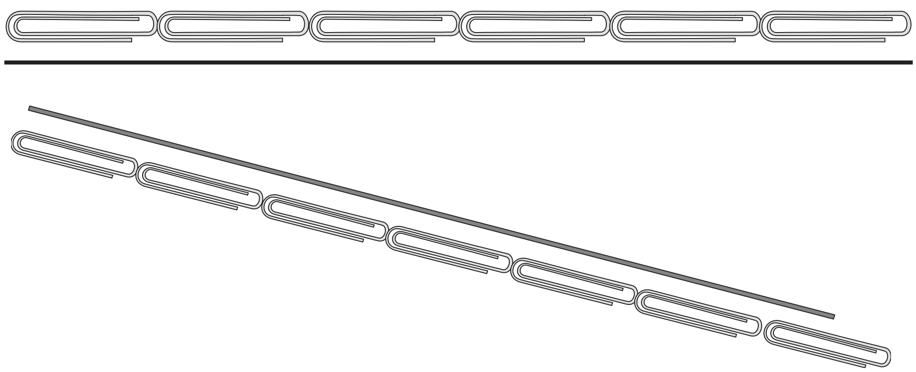
He was clearly more satisfied with this result because the greater number agreed with his knowledge about which line was longer.

At this point, the nature of the teacher's input is crucial.

But I wanted him to go further, and focused back on the type of units. "So there are 7 paper clips on the longer line, but how many toothpicks did you say fitted on the short line when you did it the other way?"

At this point, Kim paused, studying his trails intently, while I waited. At last, he answered, saying, "It was six." But then he added in an excited voice, "You do not need toothpicks, you just need paper clips and then you do not get messed up. It is paper clips on this line and paper clips on that line and then it makes sense. It is 7 paper clips on the long line and this one was 6 paper clips, so the long line is more, it fits one more paper clip."

Catherine had helped Kim begin to make a connection between the length unit and the attribute it measured.



I wanted Kim to see that either the paper clips or the toothpicks could be used to make the comparison, but not both at the same time.

DRAWING OUT THE MATHEMATICAL IDEA

I gave Kim a different coloured felt pen and had him write down what he had found on his sheet of paper

I then paused and said slowly, "So you are saying that this one is 6 paper clips long but this one is 7 paper clips (pointing) and it is the longer one. What about when you measure both with the toothpicks?" I left that with him.

CASE STUDY 2

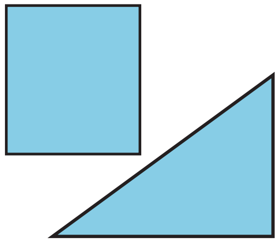
Sample Learning Activity: Grades 3–5—Garden Plots, page 59; Grades 5–8—Garden Plots, page 46, 140

Key Understanding 3: To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.

Working Towards: Quantifying Phase and Measuring Phase

MOTIVATION AND PURPOSE

I began by wanting to get my Grade 4 students thinking about what sorts of things were useful for representing a unit of area. (See Key Understanding 4, Grades 3–5 Sample Learning Activity "Garden Plots.") I asked them to compare two different-shaped garden plots and gave them a range of things to choose from to use to represent their unit, including toothpicks, 1-cm cubes, 2-cm cubes, pattern blocks, rice, counters, and string.



Which garden plot has the most land?Choose the plot with the larger area so we can grow more vegetables on it.

The rectangle was exactly 11 cm by 12 cm so that 2-cm cubes would not fit exactly into it. The right-angled triangle was 17 cm by 15 cm. I chose regions that differed in area by just a few square centimetres so that the students could not say which was larger simply by looking and would have to find another way to compare them.

CONNECTION AND CHALLENGE

When the students began work, I circulated, observing their strategies and asking questions. I noticed that a number of the students made their choice of measuring materials according to the shape of the region they were trying to measure and used different materials on one region than on the other. The idea of "fitting" dominated their thinking. Joshua said, "I have used blocks here, because it is a square and rice here because the blocks do not fit and rice fits better." When I asked him if he had found out which was larger, he just stared at his carefully placed blocks and rice for a few minutes, then looked at me rather helplessly and shook his head.

Other students, like Tilopa, used blocks on both regions, but filled in the gaps around the edges with rice. Asked which was bigger, she counted the blocks but had difficulty when she started to count grains of rice. Thus, some students did not see that using the same material on both regions would help them to make the comparison. Others saw this and started out using the same material for both regions but had difficulty with the gaps around the edges. I decided to focus on using the same material first, rather than the gaps.

ACTION AND REFLECTION

I stopped the students and asked, "So, which region is bigger?"

Most agreed with Joshua, who said you could not tell. "Why not?" I asked.

"Well," said Joshua, "you have to put different things on, blocks for the square and rice for the triangle."

"Why did you have to use different things, Joshua?"

"Because they are different shapes, so some stuff will not fit on," he replied.

"So what can we do to find out which is larger?"

I noticed earlier that Brendon used the same material on both but had difficulty covering the region with the materials provided. "Brendon, did you find out which one was larger?"

"Well, sort of," he said, "I think the square might be bigger."

"What makes you think that?" I asked.

"When I put blocks on both of them I had 30 blocks on the square and only 29 on the triangle. But the problem is that there was lots left over and so I could not tell really."

"You managed to use blocks on both of them." I said this because I wanted the students to begin to think about using the same material for both regions. "Did anyone else cover both regions with the same thing?"

Halimah volunteered that she had used rice on both but could not tell which was bigger because it was too hard to count.

"But if you did count it, would you be able to tell which was the biggest?" I asked.

"Yes," she said.

"So, how would you know which was biggest?"

"Because the one with the biggest number would be the biggest one."

“So, why can’t you tell which is biggest when you count what you have, Joshua?” I wanted the students to think about the importance of using the same unit on both regions.
“Because it is different when you have different stuff,” he said.
“I know,” said Tilopa. “It’s because they are not the same size. You see, rice is much smaller than blocks and so 30 pieces of rice is much smaller than 30 blocks.”

DRAWING OUT THE MATHEMATICAL IDEA

“Yes, that is important,” I said. “You have to use the same thing on both of the shapes, otherwise you can’t use the number to tell you which is bigger.”

I asked them to work with their partner to say what we needed to do, and then asked for a volunteer to say it to the class. I then repeated this for the whole class: “Choose one type of material to use as your unit and cover both shapes with it. Count how many cover each shape. Then the numbers will tell which is bigger.”

They started the task again, all using the one material of their choice. I was not sure that all really understood the importance of using the same unit in order to compare, but they were on the way. I knew that many would experience the problem that Brendon and Tilopa had found, with fitting their units along the edges. This I would deal with later.

Key Understanding 4

The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.

The focus of this Key Understanding is *choosing* appropriate things to use as units. It links closely with Key Understanding 3 in Direct Measure, which deals with the practical skill of *using* units well.

When we measure an object, we choose a unit and compare it with the size of the object, saying how many times as big the object is as the unit. To say that an angle is 60°, for example, is to say that the angle is 60 times the standard angle we call a degree. In practical measurement, we do this by selecting a thing to represent the unit and working out how many match or fit or balance the object. We ask questions such as the following.

- How many of these rods (units of length) fit along the table without gaps or overlaps?
- How many of these hexagons (units of area) cover my mouse pad without gaps or overlaps?
- How many of these cupfuls (units of capacity) fill our container fully without overflowing?
- How many of these marbles (units of mass) balance our teddy bear as well as possible?
- How many of these wedges (units of angle) match this turn as closely as possible?
- How many of these swings of the locket (units of time) does it take to walk across the room?

The thing we choose to represent our unit is our measuring instrument. Although all physical objects have attributes of length, area, volume, mass, angle, etc., they are not all equally helpful to use as units. Our choice can make accurate matching possible or impossible, easier or harder.

First, the thing we choose to represent our unit should relate well to the attribute we are interested in. For measuring length, something long and thin is useful and it should be clear where it begins and ends. String is a useful instrument for measuring length, but it does not work well for measuring area. Paper triangles are good instruments for measuring area but, even though cotton balls have area, they are round and probably too easily squashed to make a good instrument.

Second, to make an accurate comparison, we have to squeeze as many of the units in as possible, but no more. We should, therefore, choose things to use as units that make it easy to match the object without gaps or overlaps. Initially, students will use a wide variety of materials informally to represent their units—rods, leaves, paper clips, craft sticks, marbles, potato prints, for example.

Links to the Phases

Phase	Students who are through this phase. . .
Matching and Comparing	■ will think of the task of measuring as seeing how many of the chosen units fit without much concern for how well they fit ■ will, over time, link their efforts to directly and indirectly compare objects with their capacity to work out "how many fit" ■ will come to see, with appropriate experiences, that if we repeat units to make as close a match as possible to each of two objects, the number of units will tell us the size of each object; and that some of these objects make the matching process easier than others ■ will reject things that do not relate well to the attribute of interest when choosing something to represent the unit
Quantifying	■ understand why gaps and overlaps are a problem and so will try to avoid them *For example*: A student may choose shapes that tile to use as an instrument for area measurement and see why it is important to line up the zero mark on the ruler with the start line to be measured.

Sample Learning Activities

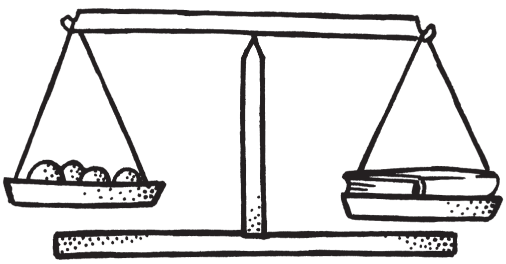
K–Grade 3: ★★ Important Focus

Cupfuls, Pouring, and Scooping

When students use cupfuls and scoops and pour from containers during their activities, focus on their choice of instrument to represent the unit. For example, ask: Will it be easier to use a large cup or a small lid to see how much an ice cream tub holds?

Snail Trails

Give pairs of students a sheet of paper with two different straight lines on it representing snail trails. Invite students to choose a unit and say how many fit along each trail. Then, invite the class to sort out those that make as close a match as possible. Ask: Which unit will closely match the length, toothpicks or pens? Why? (See Case Study 1, page 48.)



Using Balance Scales

Have students use balance scales and choose from different-sized marbles to match the mass of an object (a book, a block, a container). Invite them to work out which marbles make the closest match. Ask: Which marbles will match the mass of the object?

Prints

Invite students to cover a sheet of paper with prints using chosen objects (potato pieces, blocks) as stamps. Ask them to count how many they were able to fit in the region. Ask: Does it matter if they overlap? Can we use the number to say how big your sheet of paper is? At another time, repeat the activity and ask students to see if they can fit more stamps on this time without overlaps. Ask: Does it matter if they overlap? Can we use the number to say how big your sheet of paper is?

Tracking Growth

Have students decide on a way to track the growth of bean shoots (wheat, carrot tops). Invite them to predict how much growth will take place each day and encourage them to choose objects small enough to be used as the unit (soaked dried peas threaded onto sticks, linking centimetre cubes).

Balancing

In small groups, have students measure how heavy four pieces of fruit are. Have them use the balance scales and a unit of their choice (paper clips, cubes, marbles, string, bolts, rice, other pieces of fruit). Invite students to explain why they chose their particular unit. Have them compare what they chose with others. Ask: Which things were easier to get closer to balancing the apple?

Paper Tiles

Have students cover different faces of a packing box with paper tiles to decide which face is bigger. Invite them to choose a unit from a range of different-sized tiles ranging from 8 1/2 × 11 paper to 2-cm squares. After they have counted the whole units, ask: Which sizes of paper meant you could cover more of the face? Which is best to use to say how big each face is?

Farthest Throw

Have students suggest different units to measure how far they throw a large ball. Ask: What do we want to measure? What shall we use? How will we do it? Take groups of students outside and invite them to try the different units. Ask: Of all the things you tried to measure with, which one was the best? Why? Have students record how far they throw the ball over a series of days.

Measuring Everything

Organize students into groups and invite them to measure different things (height of a chair, time taken to do up their shoes, surface area of a desk, capacity of a container, mass of a book). Provide materials for measuring (straws, paper rectangles, paper triangles, cups, pendulums, string, stones, cubes). Encourage them to discuss suitable units for each type of measurement. Ask: What things were better for finding out how long it is (how much it holds, how heavy it is, how much of it is covered, how much time is taken)? Why?

Activity Tapes

Have students make their own tape measures. Ask: Which things will help you to mark off units that are the same length each time? (a marble, a cork, a 1-cm cube, a piece of string). Why should you use one thing over and over?

Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Which Object?
Ask: Which of these objects (length of dowel, carpet square, 2-cm cube) would help you find out who can fit the most in their lunch box (the amount of wallpaper needed to cover a wall, the height of a fence)? Ask students to justify their choices to the rest of the class. Draw out the mathematical idea that different objects are useful for measuring different things and have students explain why.

Suitable Objects
Following activities like Which Object? have students brainstorm a list of objects that could be used to measure short lengths, long lengths, small areas, large areas, small volumes, and large volumes. Ask: What makes an object suitable to use to measure a large area (small volume, long length)?

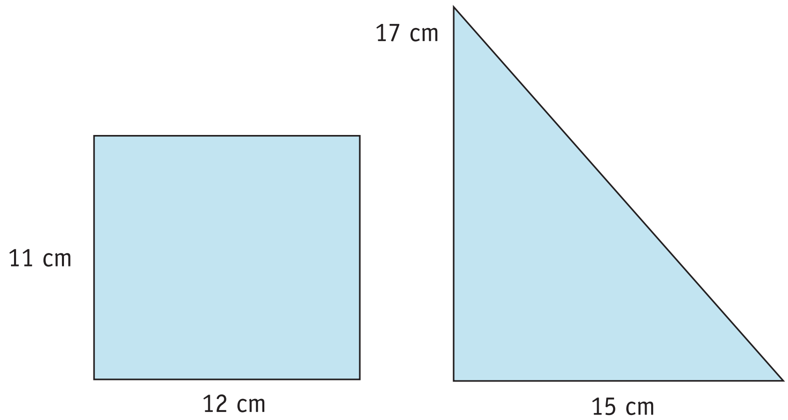
Picture Frame
Have students choose an object to measure the distance around their artwork in order to make a frame. When they have measured their artwork, invite them to compare the objects chosen and explain why their object was useful for the task. Ask: What part of the object did you use to measure your artwork? Why did you use that part of the object? How many of your unit match the length needed for the frame?

Units of Time
Organize students into groups and invite them to investigate which unit of time would be most suitable to measure different lengths of time (the age of a person, time taken to eat your breakfast, time taken for you to blink, length of time you are asleep, time taken to clean your room). Encourage students to discuss reasons for their choices.

Comparing Boxes
Have groups of students work out which box could hold more when comparing two empty boxes with slightly different volumes. Offer an assortment of materials, such as blocks, string, coloured rods, marbles, rice, or beans. Ask: Which objects filled the boxes with no gaps? Which had the biggest gaps? Which objects best show the amount of space in the boxes? Why?

Garden Plots

Tell students that the school gardener wants to choose between two plots to make the school herb garden. The plots look about the same size, but he wants to make sure he picks the one with the larger area. Give students scale drawings of the plots, one triangular and one rectangular. Ask them to find which is larger. Provide a range of materials to choose from to measure with (e.g., pattern blocks, tiles, counters, string, 1- and 2-cm cubes). Ask students to explain why the object they chose was a good unit to use. (See Case Study 2, page 51.)

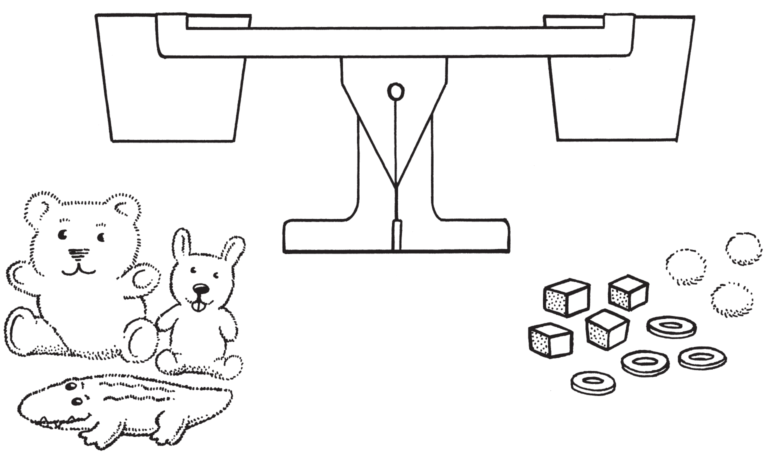


Spoonfuls

Have students use spoonfuls and cupfuls of rice (water, sand) to measure the size of different containers. Ask: Does it matter that we have different measurements for the same-sized container? How has this happened?

Soft Toys

Ask students to use balance scales to find the mass of three soft toys to determine how heavy each one is. Invite them to choose a unit from a selection of materials (marbles, washers, craft sticks, wooden cubes, cotton balls). Ask: Why is it better to use the washers or marbles rather than the craft sticks or cotton balls?

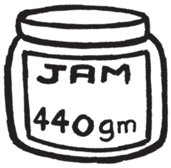


Sample Learning Activities

Grades 5–8: ★★ Important Focus

Different Units

Organize students into groups and provide them with recipes and advertisements that show the units objects are measured in (advertisements from supermarkets and hardware stores). Encourage them to talk about why different ingredients and products are measured using different units. For example, ask: Why is sugar measured in kilograms and jam in grams? Why is a glue stick measured in grams and correction fluid in millilitres? Why is a cassette tape measured in minutes and ribbon measured in metres or centimetres?

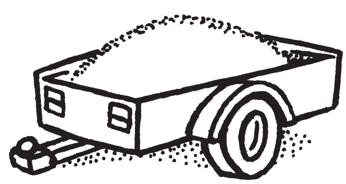


Fruit and Vegetables

Have students compare the volume and surface area of vegetables (potatoes, carrots), using standard units represented by Base Ten materials and 1-cm grid paper. Encourage students to discuss the ways they can transform the objects so that the attributes to be measured can be more easily matched to the relevant units. For example, the potato can be carefully peeled and the pieces laid out on the grid paper to find the skin area. It can be cut and rearranged to approximate prism shapes that can be compared to the volume of Base Ten materials.

Measuring Surface Area and Volume

Ask students to decide on appropriate units to use to measure the surface area of objects that are not rectangular (a circular garden bed) and the volume of objects that are not rectangular prisms (a heaped trailer load of sand). Ask: Which units do you think are best? Why do we use square units for objects that are not square? Why do we use cubic units for objects that are not rectangular prisms?

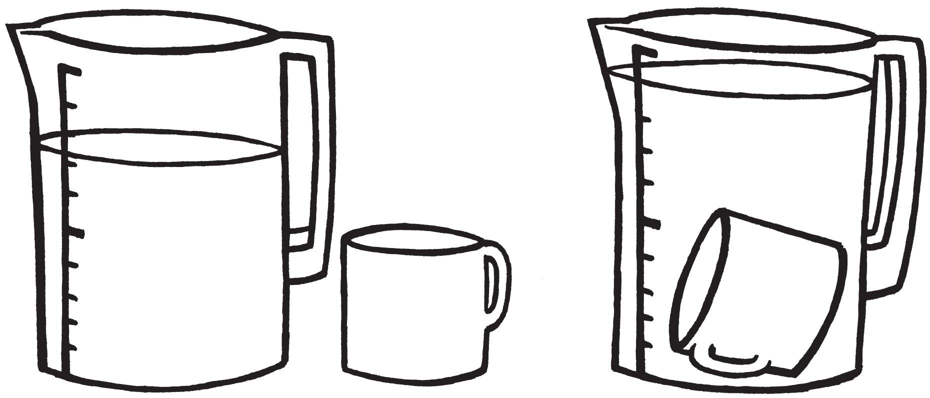


Choosing Units

Have students investigate the units that could be used to measure a variety of attributes (the mass and thickness of a coin, the quantity of water in a bottle, the distance to the city, the capacity of a tablespoon, the capacity of an elevator [usually described in terms of mass], the amount of air inside a room). Encourage students to choose the units they think are most suitable for each measuring task and justify their choices.

Measuring Volume

Ask students to choose the most appropriate instrument to use to measure the volume of different objects. For example, ask students to measure the volume of a mug by immersing it in a measuring jug of water and recording the volume of the displaced water in millilitres. Then have them use a measuring jug to obtain the same volume of play dough and shape it into a cube. Invite them to compare the cube to Base Ten blocks and work out the approximate number of cubic centimetres it contains. Ask: What does this tell you about the amount of clay used to make the mug? How does the volume of 1 cm^3 compare to the volume of 1 mL? Have students measure volumes of other objects, justifying their use of either of the two instruments.



Key Understanding 5

Measurements of continuous quantities are always approximate. Measurements can be made more accurate by choosing smaller units, subdividing units, and using other strategies.

While it is possible to count discrete quantities, such as people in a room or trees in a garden, continuous quantities, such as length, area, and mass, cannot be counted. Instead, we measure such quantities by matching them against repetitions of a unit and counting the number of repetitions. Generally, the measurement is only as accurate as the unit. We might say, for example, that a pencil is 9 cm long. What we really mean by this is that the pencil is closer in length to 9 cm than to 8 cm or to 10 cm; that it is between 8.5 cm and 9.5 cm long. In this case, the possible error of measurement is ± 0.5 cm. If we want to be more accurate, we use a smaller unit or subdivide the unit we have in some way. For example, we might measure the pencil with a ruler marked in millimetres.

First, students should learn to describe their measures using "between" or "to the nearest" statements. They should say, for example, the jug holds between 16 and 17 cups of liquid or the jug holds more than 16 cups but less than 17 cups or the jug holds 16 cups to the nearest cup. Technically, if cups are the unit, "the jug holds 16 cups" means that the jug holds somewhere between 15.5 and 16.5 cups, but in many situations we would interpret 16 cups to mean 16 full cups. Students should discuss these contextual factors in how we describe measurements.

Second, students should develop strategies for making their measurements more accurate. A unit is a size, and the smaller the size, the more accurate the measurement. To choose a unit is, therefore, the same as choosing a level of accuracy. Students can be more accurate by choosing a smaller unit or subdividing a unit. For example, they could begin with a hexagon from a set of pattern blocks as their unit. Having decided they need something more accurate, they might choose the trapezoid, which is half the hexagon.

Third, students should understand that the object being measured does not change when you use a different-sized unit, although the number of units taken to match it will change—the smaller the unit, the more it will take to match the object. Students should therefore expect to find that twice as many trapezoids are needed as hexagons, since they are each half as big.

This understanding is important in helping students to avoid the mistaken idea that a large number implies a large object, such as when they think that 2330 mm is a lot more than 1 km.

Finally, students should subdivide units into parts and describe the parts using fractions and decimals. For example, they might cut a strip of paper as their unit, and then subdivide it into quarters by folding. The strip could then be used to measure accurately to the nearest one-quarter strip. The relationship between the size of the units and the number of repeats needed is the basis for moving between units—that is, the same distance can be matched by 200 cm-units or by 2 m-units—and hence relates closely to Key Understanding 8 about the relationships in the metric system.

Links to the Phases

Phase	Students who are through this phase. . .
Quantifying	■ will refer to the unit size to explain differences in the number of units taken *For example*: A student may explain that the reason Maria found the room was 16 strides wide while Mark found it was 14 strides wide was because Maria's stride was smaller.
Measuring	■ understand the inverse nature of the relationship between the size of a unit and the number taken to match the object or event, and would expect that if you halve the unit size you will double the number of units
Relating	■ understand that choosing a unit is equivalent to choosing a level of accuracy, with the smaller unit producing the greater accuracy

Sample Learning Activities

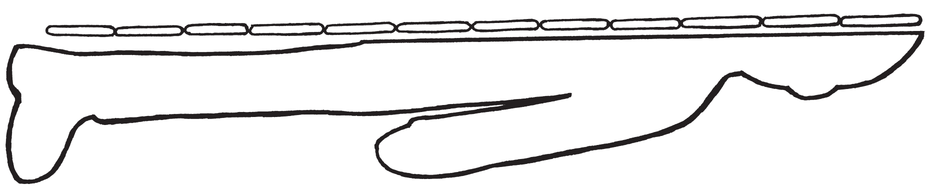
K–Grade 3: ★ Introduction, Consolidation, or Extension

Match Attributes
Ask students to count objects placed along (on, in) a larger object to describe the object in terms of how many things match its length (mass, capacity, area).

How Many?
When students are focused on how many fit, and only use whole uniform units to cover an area with units (match the weight of an object using balance scales, match a given length, use cupfuls to find out how much something holds), ask: Does it match it, or could more fit? How much more does it need? What could we do to make it the same area (weight, length, be full)?

Between
Have students match length, mass, volume, time, and area with whole units and a part of the unit. For example, when students say how many pens fit along a desk, ask: If you could break this pen, how much of it would you need to make the pens match the length of the desk? Invite students to try this with toothpicks. Ask: How many toothpicks long is your desk? How much of a toothpick did you need to fit the gap at the end?

Silhouettes
Have students measure and record their height using craft sticks as units. Ask pairs of students to make paper silhouettes of one another by drawing around one another in a standing pose (feet together) and cutting out the outlines. Ask them to fold the silhouettes in half lengthwise, count how many craft sticks they can lie along the foldline from the baseline to the top of the head, and record their measurement (*I am more than 12 craft sticks and less than 13 craft sticks tall*). Order and display the silhouettes according to actual height. Ask: What do our craft stick measurements tell us about the difference in our heights? What do they not tell us?

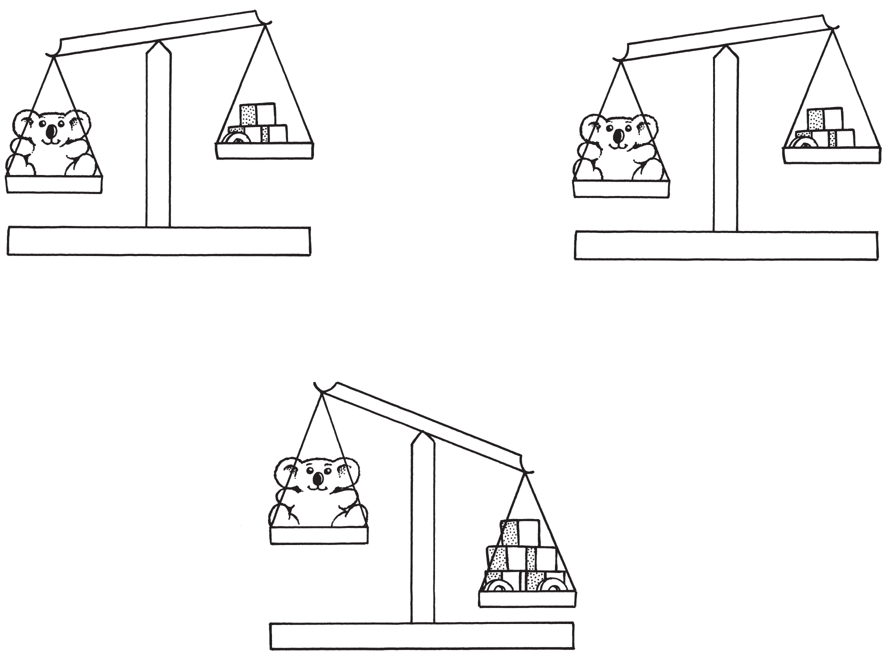


Full to Overflowing

Have students choose a cup from a collection of different-sized cups and use it to fill a clear tub as close to the top as possible. Ask them to record how many cupfuls they fit in before the next cupful will overflow, noticing the amount of space left before each cupful is poured in. Invite students to describe what they found. For example, *I put in two cupfuls and I thought there was lots of room for another one, but it overflowed. So it's more than two cups and less than three cups*.

Using Balance Scales

Have students use units of various mass to try to balance an object on balance scales. Ask them to look at and draw the slope of the balance beam before and after placing the unit that makes the mass of the units go from less than to more than the mass of the object. Ask: Which units changed the slope the most (the least)? Why? Invite students to think of something to use as a unit that would lessen the change in slope.



Tapes

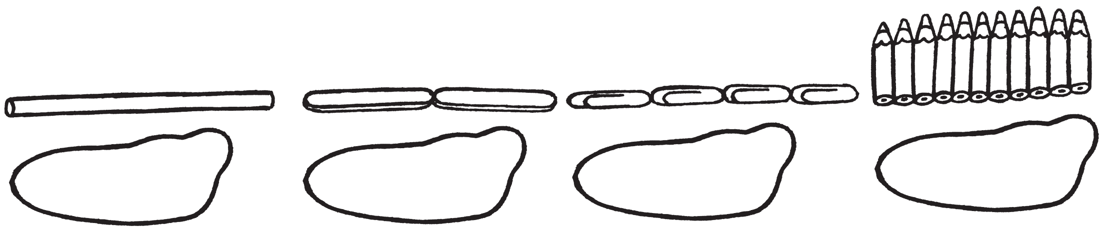
Have students use the tape measures they made in *Activity Tapes*, page 57, to measure lengths around the room. Focus students on using half and quarter measures in order to be accurate. For example, if a students says: It is 7 toothpicks long, ask: Is that exactly right? Encourage students to mark the halfway point between the numbers to help.

Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Feet

Ask students to draw around their foot, then repeatedly measure its length from heel to big toe using progressively shorter units (straws, craft sticks, paper clips, pencil widths). Invite students to record the results using "between" language on separate slips of paper for each unit type (between 1 and 2 straws). In groups, ask students to order their recorded foot sizes for each unit type. Ask: How does the ordering task change as the units get smaller? Why have you found there are fewer feet with the same measure as the unit gets smaller?



Balancing Hexagons

Have students use the hexagons from a set of pattern blocks as a unit of mass to balance with various objects (apple, book, rock, shoe), recording the result in hexagons. Invite them to repeat this, using other pattern block pieces to more closely balance the mass of the objects. Have them compare and discuss their results. Ask: How can this rock weigh both 12 hexagons and 25 trapezoids? What happens to the size of the number when we choose to use a smaller unit? (Make sure students have not mixed plastic and wooden pattern blocks in this activity.)

Smaller Units

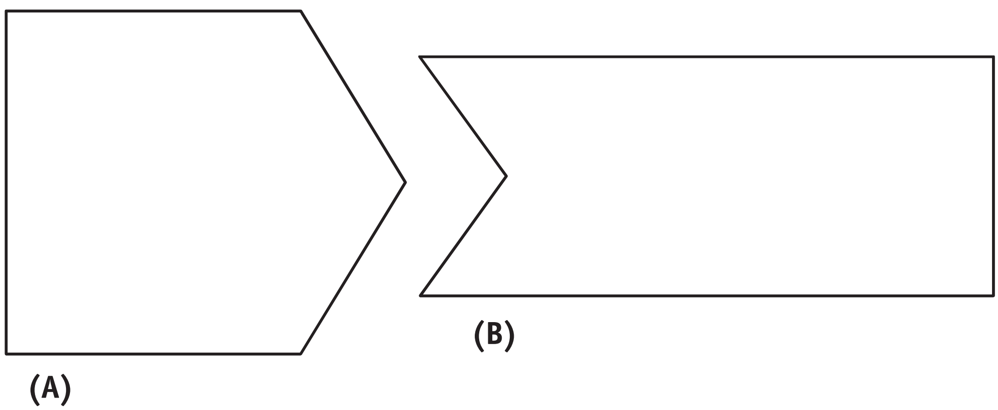
Encourage students to explore the greater accuracy obtained when using smaller units in problem situations. For example, say: Mary used a cup measure and said the water bottle holds 5 cups of liquid. Jimmy used a half-cup measure and said the same water bottle holds 11 half-cups. Jimmy thought he must have made a mistake because 5 cups is equal to 10 half-cups, not 11. Both students were careful with their measurements, so how could this be? Does the volume of the container change when we use different units to measure it? Invite groups of students to use standard cup and half-cup measures to discover what happened.

Throwing Beanbags

Organize students into groups, giving each group a beanbag, a piece of chalk and a metre ruler. Take them outside and have each group measure how far they can throw a beanbag, starting from a baseline (on a basketball court). Ask them to mark their throws with chalk and measure them to the closest metre with the metre ruler. Order the measurements from shortest to longest. Ask: How could we be more accurate? Encourage students to choose either to subdivide the metre ruler or use smaller units, such as decimetres or centimetres. Ask them to remeasure the distances and reorder them. Ask: Why has it been easier to place our throws in order when we have used smaller units?

Hungry Cow

Extend *Hungry Cow*, page 44, by encouraging students to choose from a range of equipment, including cut out 1-cm squares and 2-cm squares, grid paper and clear plastic grids, to compare two different irregular pastures of similar area drawn on one sheet of paper or card. Ask: Why is it easier to be more accurate using 1-cm squares than 2-cm squares?



Note: The two shapes need to be on the same sheet of paper, so that direct comparison is not possible, and it should be very difficult to judge which is larger just by looking. For example, in the above, shape B may look smaller, but is 107 cm^2, while shape A is only 100 cm^2.

Sample Learning Activities

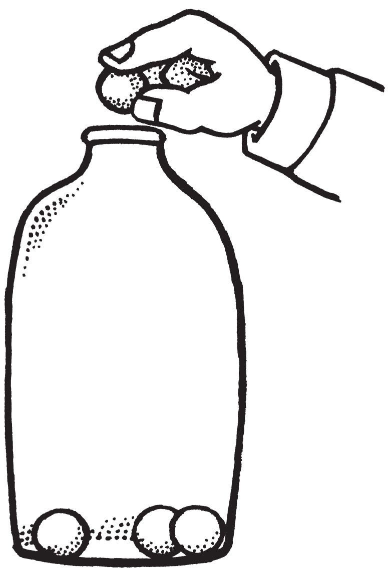
Grades 5–8: ★★★ Major Focus

Body Measurements

Ask students to use equal strips (about 40 cm long) of unmarked paper to measure and record the length of different parts of their body, using the strip as a unit. Ask: Which parts of your body are equal in length? Invite them to talk about the problem this presents and find ways to make and describe part-units for their paper strips. Encourage students to explain and justify their choice of part-units. Ask: Can you measure the lengths accurately enough in part-units to know which parts of your body are equal in length?

Choosing Smaller Units

Have students choose smaller units to make measurements more accurate. Devise a task that takes a few minutes to complete and can be repeated. For example, ask: How quickly can you put 200 marbles, one at a time, into a plastic bottle? Invite students to try the task, timing each other in minutes. Minutes may be sufficient to compare the times at first (nearly 3 min, just over 3 min, 3-and-a-half min), but as students practise and become faster at the task it should become necessary to measure in seconds to differentiate between the fastest times. Draw out why greater accuracy was needed, and how using seconds as a unit provides it.

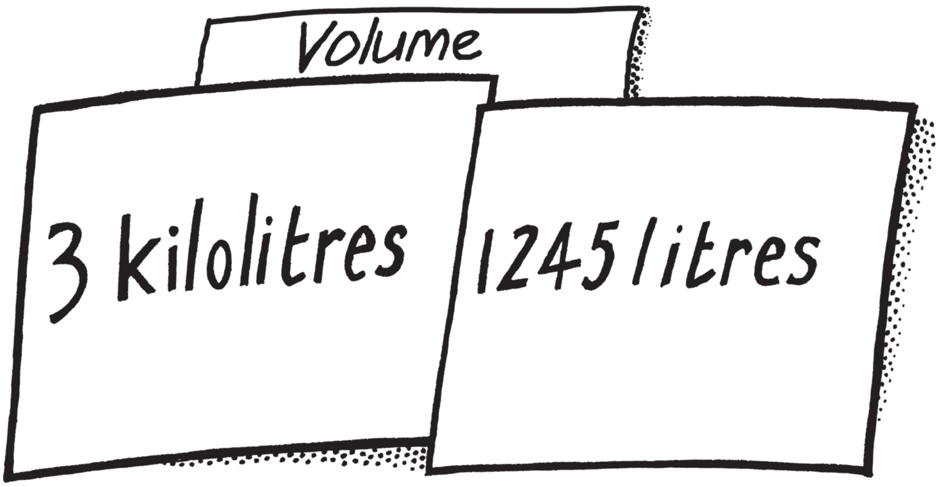


Calibrating Containers

Have students subdivide existing calibrations on a container to make it more accurate. Give groups identical jugs or containers, calibrated every 250 mL. Ask: Can you tell me what these marks mean? Have students use the measuring jug to say how much some smaller containers hold. Ask: Is the scale useful? Why? Why not? Then, have students use materials (small disposable cups, spoons, ruler) to make a more accurate measuring tool.

Sorting Cards

In pairs, ask students to sort cards that show a range of different measures and units into those that measure length, area, volume and mass. Have them take turns to take two cards from a pile and say which measure is larger and why (which is larger of 5 dm and 22 cm, 0.3 L and 0.375 mL, 2 ha and 18 000 m^2, 3 kL and 1245 L). Say: Someone said that 18 000 m^2 was bigger than 2 ha. Is that person correct? Why? Why not? Invite students to shuffle the cards in each pile and repeat the activity.



Overcoming Limitations

Have students explore strategies for obtaining accurate measurements of things that are too small for the measuring tools readily available to them (the thickness of a piece of paper, the mass of a single grain of rice, the volume of a drop from a eye dropper). For example, students could measure the mass of a teaspoon of rice, count the grains of rice, and divide the mass by the number of grains of rice. Ask: What are we doing to the quantity of mass when we divide it by the number of grains? How is this similar to kilograms and grams?

Key Understanding 6

Our choice of attribute and unit depends on what we are trying to measure and why.

If students measure for a reason that is clear and of interest to them, then decisions about attributes and units are more likely to be made thoughtfully and be based on the purpose of the measurement and the practicalities of the situation. During the primary years, they should be encouraged to reflect on and collaborate in making sensible choices about:

- which qualities need to be measured for the task at hand (attribute)
- how accurate they need to be for the purpose (unit)

In order to produce a scale drawing of part of the school garden, for example, students may decide that they need to measure various lengths and angles (attributes) and plan to be accurate to the nearest decimetre (unit) since the scale is to be 10 to 1.

Learning to choose the appropriate attribute links closely to Key Understanding 1. Students need to have a good understanding of what each attribute is and how attributes differ in order to be able to choose which attribute is needed in a particular situation. This is not always an easy task and students may not have the specific knowledge needed to consistently choose the right attribute. Nevertheless, they should experience a wide range of situations that require them to think consciously and carefully about what attributes need to be measured and should reliably choose the attribute for everyday situations.

Having decided what needs to be measured, students should then decide how accurate it needs to be. More accurate is not always better, sometimes it wastes time, over-complicates matters, or makes computations unnecessarily tricky.

The level of accuracy needed depends on the reason for measuring. Students often believe that the unit to be used depends only on the size of the object or event to be measured. Sometimes the unit is related to the size of the object or event to be measured, but often it is not. A building may be measured with considerable precision (in millimetres) for detailed engineering work, but with less precision (metres) for estimating the amount of paint needed. To provide a quote for what carpet will cost for a large hotel, square metres may be sufficiently accurate and anything more accurate inefficient and spurious given the wastage involved. To cut the same carpet to fit a room, even a big room, more accuracy will be needed. If all you need to know is whether the table will fit in the corner, hand spans may be sufficient. Choosing a unit is often iterative: where a bigger unit such as a hand span is used first and fairly quickly, if the things to be compared appear close in size, a smaller unit is then chosen and applied more carefully.

Links to the Phases

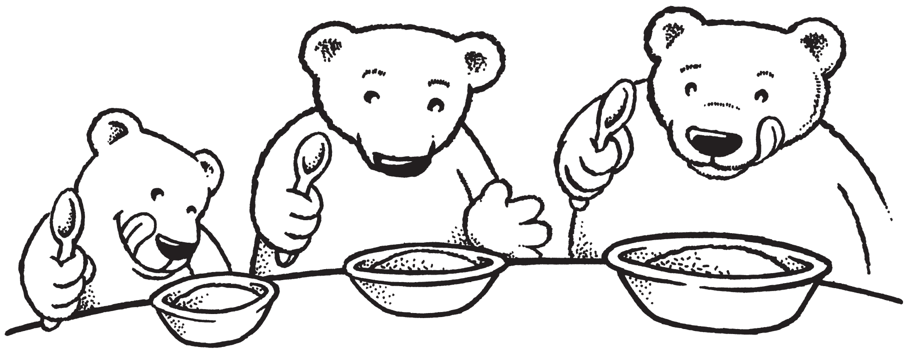
Phase	Students who are through this phase. . .
Matching and Comparing	■ focus on relevant attributes to solve simple comparison problems that are familiar and meaningful to them *For example*: A student may focus on capacity when choosing the drink container that holds more and might show this by choosing a scoop or small container to act as a unit.
Quantifying	■ choose suitable and uniform things to use as units
Measuring	■ choose units of a suitable size for descriptions and comparisons to be made ■ select the attribute that is sensible for the purpose *For example*: A student may realize that even though we might consider the volume of cartons to find out which would hold the most popcorn, dimensions would be more helpful for deciding which would hold the largest number of books.
Relating	■ understand the relationship between unit and accuracy and know that the unit chosen should make sense in terms of judgements about the importance of the measurement and the fineness of the comparison to be made

Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

Porridge for the Three Bears

Have students work in groups to decide on a container to serve porridge for the Three Bears. Baby Bear has a one-container serving, Mother Bear has a two-container serving, and Father Bear has a three-container serving. Invite students to use the unit (container) to find a bowl the right size for each bear's porridge. Ask: Does the size of the container we use to make servings of porridge matter? Why?



Obstacle Course

Invite students to suggest ways to keep track of the time taken to complete an activity (clapping while someone completes an obstacle course). Ask: Do you think this is a fair way to keep track of time? Why? Why not?

Farthest Throw

Have students discuss how to find out who can throw the farthest. Ask: What could you do to find out? What would you need to measure? Would you measure how high or how far you could throw something? What could you use as units to measure that distance? Invite students to choose and try different units (craft sticks, ropes, steps). Ask: Which of the things you tried to measure with made it the easiest to decide who could throw the farthest?

Snail Trails

Have students use glitter glue to make curved snail trails. Ask them to choose from a range of objects to use to measure how far each snail has travelled. Ask: What things can you fit along the curvy part of the trail?
(See Case Study 1, page 48.)

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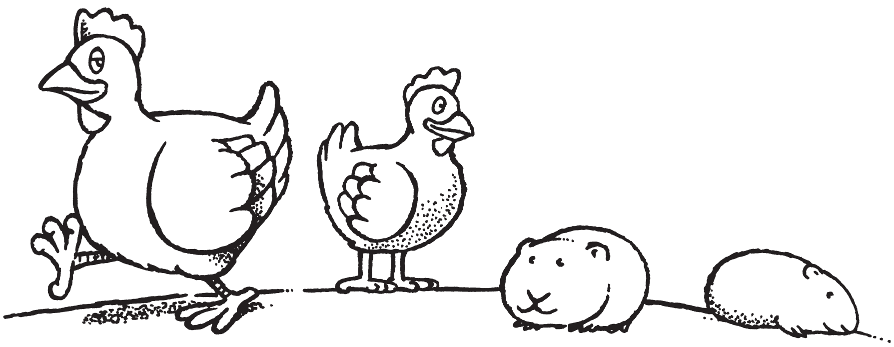
Tracking Growth

Extend *Tracking Growth*, page 56, by asking: What part of the plant are we measuring? Is this its length (area, mass)? How do the peas help you know how much bigger the bean shoot (wheat, carrot top) is?

Weighing Small Animals

Ask students to decide what to measure to work out the size of a small animal (chicken, guinea pig). If they decide to measure its mass, provide a range of materials (blocks, marbles, washers) and ask them to choose what to use as units. Ask: What will we use to find out how heavy the chicken (guinea pig) is this week?



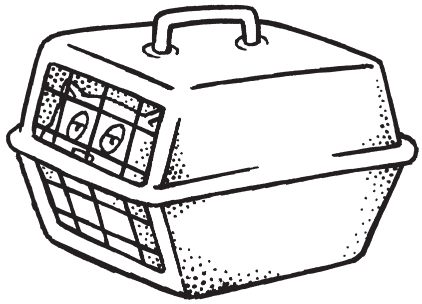
Area of a Puddle

Have students choose from a wide range of materials to use for measuring tasks. For example, when working out the area of a puddle, provide materials unrelated to area as well as materials that match it (rice, counters, blocks, paper tiles, paper clips, toothpicks craft sticks, balance beam, cups, lids, cotton balls). Ask: Which part of the puddle are you measuring? Does your unit help you to measure this part? Would something else be a better unit? Why? Why not?

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Travelling Cage



Say: I need to order a travelling cage for my cat. What will I need to know in order to buy the right cage? What would you measure to help me make my decision? What units would you use? How accurate do we need to be?

Library Bags

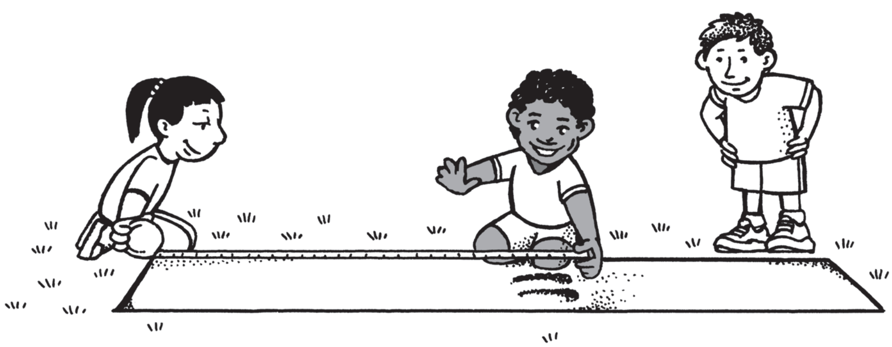
Invite students to design a library bag.
Ask: How big will the bag need to be? What measurements do you need to take? What unit size would be suitable? Why? How accurate do you need to be?

Shadows

Ask students to work with a partner to draw around the outside of their shadow in the morning, at noon, and again later in the day. Invite them to describe how the shadow has changed. Ask: Will measuring the length be enough to decide which is the biggest shadow? Ask students to see if their morning and afternoon shadows have the same area. Ask: What unit size could you use to measure the area? Which one is the most suitable? Why?

Long Jumps

As a class, have students brainstorm all the ways they could measure long jumps for a competition. Ask: How accurate does the measurement need to be? How important is it? Discuss what would be an appropriate unit size (metres, paces) and instrument to represent the unit (metre rule, craft sticks, tape measure). Would it be any different if you were judging the long jump at the Olympics? Could this unit be used to measure the high jump as well?



Juice

Have students decide how much juice will need to be bought for a school excursion. Ask: What will you need to know? What will you need to measure? How accurate do you have to be? What units would be sensible to choose? Will it matter if there is too little or too much?

Paperweight

Have students design a paperweight to hold down paper when the fans are on high (when the doors and windows are open, when they are working outside). Have them decide on a unit of mass that most of the students will be able to use (marbles, washers). Once they have worked out how many washers (marbles) in mass are needed to hold down the paper, ask them to write a memo to other classes to say how heavy the paperweight needs to be.

Units of Time

Ask students to choose an appropriate unit of time to say how long each of a number of activities takes (eating an apple, growing beans to get a pod, walking 1 km, running 500 m at the Olympics, flying to the Moon, a person's lifespan, building a pyramid). Ask: Which instruments are used to measure the different units and why?

Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Lawns and Garden Beds

Say: The gardener needs to put new edging around the garden beds and fertilize the lawns. How will she work out how much edging and fertilizer she needs? What will she measure? Invite students to determine which attributes need to be measured (perimeter and area) and decide on appropriate instruments and units to use. Ask: What unit have you chosen to use for the area? Why? What unit have you chosen for the perimeter? Why? Encourage them to justify their choice of unit size in relation to the level of accuracy required.

Classroom Furniture

Ask students to draw the classroom and its furniture to scale to use as a basis for creating some alternative floor plans. Establish the scale as 1 m equal to 1 cm. Have them measure their desks and work out dimensions for the scaled drawing. Ask: Would you measure your desk to the nearest mm? Would you measure it to the nearest cm? Why? Why not? Which unit would you choose? Why would decimetres be a sensible choice for the purpose of your plan?

Carpet and Paint

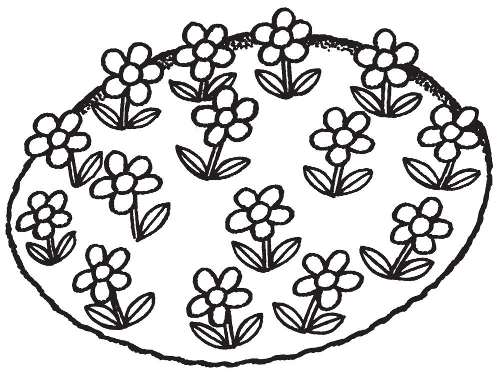
Have students decide which measurements are needed to work out how much carpet and paint would be needed for the classroom. Ask: What should we measure so that sufficient carpet and paint are ordered? What will the carpet layer and the painter need to know? Organize students into groups and give them carpet advertisements and labels from paint cans to help them decide. Ask: Why would the painter need to know the area of wall to be painted, even if the school supplies the paint?

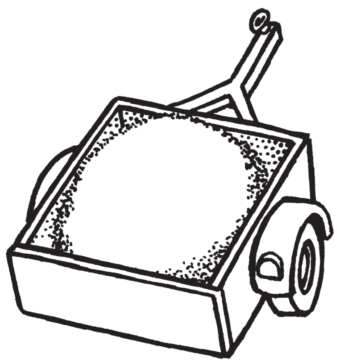
Measurement Situations

Have students build up a list over time of measurement situations; for example, working out how much juice to order for a class picnic (cherries for a fruit salad, bricks for a barbecue area). For each situation, ask students to decide what they will measure and which unit and what tool they will use. Encourage them to justify the size of the unit they have chosen in terms of the accuracy needed. Ask: Why do some situations require a very small unit to measure a large object? (bricklayer building a house)

Measuring Surface Area and Volume

Vary *Measuring Surface Area and Volume*, page 60. Ask students to choose the attribute, unit and instruments to measure the area of objects that are not rectangular (garden bed, orange) and the volume of objects that are not rectangular prisms (trailer load of sand, wet cement). Encourage students to consider a range of purposes. For example, for a circular garden bed, ask: How will your measurements vary if you want to find out how many rose bushes to plant and how much mulch to buy? What about if you want to find out how much fertilizer to spread? Do you need to know the dimensions or the area or volume? Which units are best to use? Why can we use square metres for a garden that is not square? Why can we use cubic units for objects that are not prisms? (because they measure size, not shape)





Treasure Hunt

Extend *Treasure Hunt*, page 47, by asking students to redesign their instructions and adjust their units according to how successful their instructions were. For example, ask: Why might you need to give your measurements in metres rather than steps? How could you describe a turn that is between a quarter turn and a half turn? Would another unit be more useful?

Grades 5–8: ★★★ Major Focus

Track and Field

Invite students to look at track-and-field title holders and their record times or distances for the various events. Discuss the relationships between the distances and the level of accuracy required of the units and the tools needed to measure them. For example, ask: How accurate would you need to be to tell who won a marathon (1500-m race, 100-m race)? Have students simulate the possible "winning margins" (have them touch the wall one hundredth of a second after their partner) using available measuring instruments. Have students research the winning margins for Olympic distance events (shot put, javelin, long jump, high jump). Ask: Why is a tie more likely in high jump than long jump? (the high jump bar is raised in fixed increments)

Birth Mass

Have students bring in their birth mass in grams. Help them to compare their mass to the nearest kilogram by listing them in a table with those headings. Ask: Why does the difference between your birth mass appear to change when larger units are used? Why would babies be measured in grams while older children and adults are measured in kilograms or half kilograms? (See Case Study 3, on the opposite page.)

CASE STUDY 3

Sample Learning Activity: Birth Mass, page 78

Key Understanding 6: Our choice of attribute and unit depends on what we are trying to measure and why.

Working Towards: Quantifying Phase and Measuring Phase

TEACHER'S PURPOSE

My Grade 6 students were using scales to record their mass to the nearest kilogram and comparing the way their mass was recorded at birth with their current mass. I asked why their birth masses seemed to be more than their current mass; for example, "Katie's birth mass was 3254 g, and now she's only 38 kg!"

They laughed and were able to say their mass was actually a lot more now, the "thousands" is because their birth mass was grams and their mass now is kilograms, showing they understood grams were much smaller units than kilograms.

When I asked, "Why weren't you measured in kilograms when you were born?" however, Jemina said, "You can't, because babies are tiny, you have to be bigger to be kilograms."

Others nodded in agreement. Although they were partly right in thinking that the size of the object to be measured was relevant, I was not convinced that they understood that the real issue was how accurate you needed to be. For people of adult size, a kilogram or half kilogram is usually precise enough for practical purposes, but it would not be precise enough to track weight changes in babies.

ACTION ...

The following day, I brought in some spring scales. The scales were calibrated to 100 g and had every 500 g written in, as well as the kilograms. I started by having students make up their birth mass (brought in from home) in Base Ten materials, using either gram mass and balance scales, or the school's digital scales that measured to the nearest gram. The next step was to weigh each child's "birth mass blocks" on the spring scale. Each child took it in turns to place their blocks on the spring scales, while the rest of the students worked out the mass to the nearest 100 g.

The student selected to call out the mass had to justify it by showing how the pointer had to be one side or the other of an imagined halfway mark. The measurements were recorded on a wall chart beside the gram measures.

Students were then asked to weigh their collection of blocks on the bathroom scales in the same way they had weighed themselves before—to the nearest kilogram—and record this as well. All this information was added to another column on the chart.

Name	Birth mass Measured to the nearest		
	g	100 g	kg
Katie	3264	3 kg 300 g	3
Ariel	3235	3 kg 200 g	3

... AND REFLECTION

In a follow-up session, we looked at the chart together and talked about what was different about the measurements of their recorded birth mass and the measurements of the blocks done on the two types of scales.

Ariel observed, “They are all kind of the same thing but sort of different—the first masses are kind of closer to the real thing, but after that the mass is nearly, but not exactly.”

Katrina noticed that by measuring their simulated birth mass (blocks) to the nearest kilogram, different birth masses now looked the same. “If you look at the kilograms when we were born, most everyone was about three kilograms, but if you look at the grams, everyone looks different.”

CONNECTION AND CHALLENGE

I then asked, “If Katie and Ariel actually had been weighed at birth, using the 100-g unit on the spring scale, what would you have thought the difference in their mass was?”

The students responded they would have thought the babies had a difference of 100 g in their mass. I followed on by asking, “Did the babies really have a difference of 100 g at birth?’

The students took a while, but most then thought “no.” Katrina said, “If you look at the column where they are measured in grams, there is only about 30 g difference between them.”

I then asked a series of questions, “Which is the most precise measure? Why do you think so? Which was the least precise measure? Why do you think so?” The students’ responses made me think that they were starting to get the idea that the size of the unit affected the precision of the mass measure.

The smaller the unit, the more accurate the measurement. So if we need to be more accurate we choose a smaller unit.

DRAWING OUT THE MATHEMATICAL IDEA

I then turned to the question of why you might want more precise information for babies than adults. I asked, "So why do you think the babies were measured in grams and not kilograms?"

Quite a few students made comments like, "when you need to be really right," "when it's important to be precise " and "when you need to know the difference between things that weigh nearly the same, a kilogram is too big."

I picked these ideas up and talked a little about how a difference of a few hundred grams can be quite a lot for a new baby. "We often want to check whether the baby is losing or gaining mass and the amounts will be small. For adults, variations of under a kilogram are usually not very significant."

To emphasize that it was not simply the size of the baby that was the issue, I also talked with them about adults who had eating disorders or other illnesses and the need at times to monitor small variations in mass.

They seemed to understand that the need to make fine distinctions (that is, to compare things that are close in size) means you need to be more accurate and hence need to use smaller units.

To decide how accurate we need to be we ask these questions:

- ***How close in size are the objects or events we want to compare?***
- ***How important is it that we get a close match between measurements?***

Did You Know?

In Gulliver's Travels (Jonathan Swift), Gulliver was twelve times as tall and wide as the Lilliputians. The Lilliputians worked out that Gulliver's mass would be related to his volume and that his volume would be 12 3 12 3 12 the volume of a Lilliputian. So they gave him 1728 times as much food. The Lilliputians got their arithmetic right, but not their biology. The amount of food a person needs is related to heat loss, which is why the food value is given in kilojoules (or calories), and heat loss is related to surface area. Hence, Gulliver only needed 144 times as much food as the Lilliputians. Jonathan Swift chose the wrong attribute. Choosing the right attribute requires more than mathematical knowledge, it requires mathematical literary numeracy, that is, the intelligent blending of mathematical and contextual knowledge.

Key Understanding 7

Standard units help us to interpret, communicate, and calculate measurements

Initially, young students will use standard units in much the same way they use non-standard units. As they mature, they should come to understand the usefulness of standardizing some units for recording and communication purposes. They should learn to distinguish between situations in which non-standard units may be used and situations where a standard unit would help. In particular, they should learn the following:

- Standard units are no more precise or accurate than non-standard units. A toothpick will give a more precise measure of the side of a table than will a 1-m strip.
- Non-standard units may be appropriate to and practical for the task at hand. Hand spans may be quite good enough to let you see how far a table will jut into a room.
- Some units are in common use, but are not standardized. These units may be more variable, regional and trade- or craft-specific, but be of local practical use. Three wheelbarrowfuls of sand will communicate well and be reliable within specific contexts.
- When measurements must be recorded for later use, transported, or communicated, it helps to use units that are remembered over time and that other people share. Standard units are useful for this.
- In order to interpret other people's measurements, it is necessary to know their units; for example, we do not know whether 38 is a hot day or a cold day until we know what the unit is.
- To add, subtract, and average measurements, it is necessary to use the same unit. Standard units help when measurements are collected by different people at different times (birth mass).
- Some formulas that show the relationship between quantities depend on the units of measurement. Without standard units, we could not always use each other's formulas.

Students should learn about how various cultures have developed different units of measure, both historically and currently, and that the use of the metric system of measurement is becoming increasingly widespread. The basic metric units are metre, litre, and kilogram, with other units being derived from these.

Links to the Phases

Phase	Students who are through this phase. . .
Quantifying	■ understand that to compare two measurements it is necessary to use the same unit and that the units they choose will often be standard simply because they are readily available in useable form
Measuring	■ use standard units naturally in their own comparisons and communications
Relating	■ can explain the roles of standard units but also recognize situations where a standard unit is not an advantage ■ understand that the fact that a unit is standard does not make it more accurate than other units; accuracy relates to unit size

Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

Naming Units

Informally introduce the names of standard units when the opportunity arises. For example, as students play in a class store, say: That jar weighs 250 g. That tells us how heavy it is. When students read the numbers on a ruler, say: They are centimetres. We can use centimetres to tell us how long something is.

Measuring Jugs

After students have made a measuring jug to use in their activities by cutting the top from a clear plastic bottle and calibrating it in "cups," give them containers showing standard units (medicine and cooking cups and spoons, jugs, litre bottles). Ask: Is your cup measure the same as a cup measure in the recipe books? Does it matter that they are different? Why? Why not? What is a standard cup measure in Canada?

Measuring Lengths

Ask students to measure small lengths (the side of a book, a pencil) by counting centimetre cubes, then by using strips of centimetre-grid paper. Discuss the advantage of writing the numbers on the grid paper, so it is not necessary to count every time. When students are proficient using their grid paper tape, ask them to line up the marks on their ruler with those on the paper. Draw attention to the extra bit of wood or plastic left on each end of the ruler and the zero label at the starting point. Ask: Where do you start measuring from when you use the ruler?

Class Store

Have students set up a class store that will be part of the classroom for several weeks or months. Organize them into groups and invite them to make the products the store will sell. Ask them to label the packages with signs that indicate the size of the containers or the quantity of the contents. Help students find and copy the information from the packaging. Have a variety of different masses of, say, 250 g, 500 g, 1 kg, and 2 kg and different capacities of, say, 250 mL, 500 mL, 1 L, and 2 L. Encourage groups of students to use the store, serving, making purchases, ringing through, or writing down their orders.

Tools for Measuring

Make a class collection of different tools for measuring length (tapes, rulers). Invite students to say what they notice about the size of the gaps between the marks (*They are all the same size. Some are long and some are short.*) Ask: Are there long marks and short marks? What do you notice about where the long marks are? Why aren't all the marks the same lengths? Invite students to choose a measuring tool and measure a given set of small objects (box, block, pencil) by counting the number of units (gaps), then compare their results. Ask: Why do you think you all counted the same number even though you used different tools? Discuss the name of the standard unit on the tool.

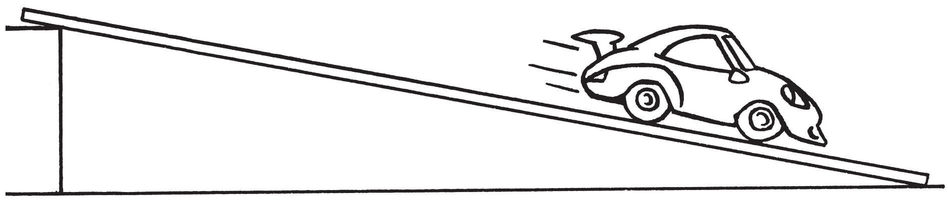
Stories of Units of Measure

Read and tell stories (*Noah's Ark, The Elves and The Shoemaker, The Emperor's New Clothes*) that mention standard units the characters of that time and culture used to measure things. Ask: What did Noah use to measure the ark? (cubits) What could the shoemaker have used to measure the leather? What could the tailor have used to measure the cloth?

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Hardeep's Hand Spans
Present scenarios for students that deal with the reasons for using standard units. For example, say: Hardeep wanted to make a ramp for his toy cars. He phoned the local hardware store and asked if they stocked planks that were 10 hand spans long. What do you think the owner said to Hardeep? If Hardeep went to the store, how could he make sure he got the right-sized plank? What would be another unit for Hardeep to use? Can you think of a situation where hand spans could be suitable? Why?



Units from Other Times and Cultures
Have students use books to research how standard units from other times and other cultures arose and what they were based on. Invite students to re-create these other standard units and use them to measure. Ask: Why do you think units such as the span and cubit are no longer used?

How Much Ice Cream?
Have students work in groups to decide how much ice cream would be needed for a special school lunch, using a non-standard unit such as spoonfuls or cups. Ask: What size container will we need to buy? What unit of measure is on the container? Why are measurements always on things sold in the supermarket?

Tee-Ball
Before students play a game of tee ball, have them research to find out how far apart the bases should be. Ask: Why is this written in metres and not as the number of steps? Invite students to lay out the bases using the measurements and then work out how many steps apart the bases are. Ask: Do we need to use the metre measure each time we play tee ball, or are steps accurate enough? Why?

Cooking

During a cooking activity, have students compare the sizes of different cups before using them. Ask: Does the size of the cup matter? Why? Ask students to find out how much a cup commonly holds in recipe books.

Birth Mass

Have students find out how much they weighed at birth. Record the information and focus on the use of the two different units, pounds and grams. Ask: What is the difference between the two units? Why are both units used for birth mass? Can you think of other times where standard non-metric units are used in the community? (*He's over 6 feet tall, It is miles away.*)

Paperweight

Extend *Paperweight*, page 75, by asking: How can you be sure that the measurements are accurate? Would the mass of the paperweight be the same if gram and kilogram units were used? Why? Why not?

Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Math at Work

Invite speakers (parents) in to talk about the measurements they use at work, or show videos that describe how measurement is used in various occupations. Have students record the types of measures that are used in these different places of work (building sites, weather bureaus, airports, hospitals). Ask: Which of these units are standard and which are non-standard units? When would it be acceptable to use non-standard units? (using wheelbarrows as a measure for the amount of mortar needed in bricklaying) When would using a non-standard unit be inappropriate? (measuring the length of the brick wall to be laid)

Birth Mass

Extend *Birth Mass*, page 87, by asking: How much mass have you gained since birth? Invite students to use scales to determine their current mass in kilograms and compare this to their birth mass to find the difference. Ask: What did you have to do to compare? How easy (difficult) was it? Why?

Comparing Standard and Non-Standard

Have students each make a tape measure using a non-standard unit of their choice (straw, craft sticks, pens). Ask them to make the tape ten units long (a deca-unit) and subdivide it accurately into deci-units and centi-units. Invite students to use their tape to measure and cut a paper streamer to fit across a cupboard or window. Invite them to then use a standard metric tape to measure and cut a second streamer to fit the same space. Ask them to compare the two streamers. Ask: Why is there little or no difference in the two lengths? Why are standard units used if there is no difference in accuracy?

Measuring Time

Have students research how the daily rotations of the Earth, the monthly changes of the Moon, and our planet's yearly orbits about the Sun provided some of the first units for measurement of time. For example, Egyptians were the first to divide the day into 24 units and daylight into 12 units; the Chinese divided the whole day into 12 units; and early Hindu civilizations divided the day into 60 units. After reviewing the different historical methods, invite groups of students to devise a new basis for measuring time, defining the set of units they need, and justifying their choices (e.g., a metric clock).

Overseas

Students research (by looking at catalogues, on the Internet, investigating the packaging of items) the way items are measured in Canada and in other countries (nails, hats, shoes, gloves, shirts, dresses, pants, paper, knitting needles, wire, concrete, rainfall, television screens, bricks, ice cream). Display the items, or pictures of the items, and label them with the measurements as they become known. Encourage students to discover the origin of the various systems and units used and identify those based on the metric system. Explain the difficulties that might arise when the systems are not standardized between countries.

Travelling Overseas

Extend Overseas by having students research standard units used in other countries. Ask: What are some of the problems a traveller might face? What might be confusing? (working out how fast you can drive, following recipes, using electrical appliances)

SI Units

Have students find out which SI (Standard International) units Canada uses and what the standards are for these units. Build up a list of basic units and the units derived from them. Have students research the units used in Canada before the change to metric. Ask: Which units are still used? (miles, feet, inches, pounds, pints) What are their metric equivalents? Why might people continue to use the old units? Provide some American magazines and ask students to list all the measurement words they can find, then compare them to Canadian units for the same attribute, before and after the change to metric. Ask: Which are the same and which are different? Why are communities often reluctant to change their standard measurements?

Units from Other Times and Cultures

Extend Grades 3–5 Sample Learning Activity Units from Other Times and Cultures by asking: Why do you think Canada decided to change to the metric system? What were the advantages (disadvantages) of the change? Have students interview people old enough to remember the change to metric measures. Ask: Was it difficult for them? How? Why?

Birth and Death Dates

Have students research some famous people from over two thousand years ago and list the birth and death dates given. Ask: Would these have been the dates given at the actual time? Have students attempt to discover the way these events could have been recorded in their place and time and the units of time they used.

Key Understanding 8

The relationships among standard units in the metric system help us to judge size, move between units, and do computations.

The metric system has all the features of decimal place value built into it and understanding it relies on understanding the multiplicative relationships among the places in decimal place value. It is this that makes the metric system so useful. Conversions based on tens require only the shifting of the digits to the left or the right of the decimal point (such as changing 34.56 km to 34 560 m) and so are more straightforward than those based on other numbers. The comparison of measurements is also easier. For example, even if we had never seen a litre or millilitre, we can compare 0.4 L with 250 mL because we know that milli always means "one thousandth" and so 250 mL must be 0.25 L which is less than 0.4 L. This also helps us to make judgements of relative size for measures of which we have little personal experience. Knowing how much a litre is, we also know how much a kilolitre is—it is 1000 times as much. This can help us get a sense of measurements outside our direct experience: big distances or very small masses.

Often students learn the relationship between particular units of measure (that 10 mm is equal to 1 cm, and that 100 cm is equal to 1 m) as though they were unrelated "facts." The benefit of the metric system is that the same set of multiplicative relationships is built into all metric measures through the prefixes; having learned the relationship for one attribute, you know them for all attributes and the same decimal structure can be used for all measures.

kilo-unit	hecto-unit	deca-unit	**unit**	deci-unit	centi-unit	milli-unit
1000 units	100 units	10 units	**1 unit**	$\frac{1}{10}$ unit	$\frac{1}{100}$ unit	$\frac{1}{1000}$ unit
thousand	hundred	ten	**one**	one tenth	one hundredth	one thousandth

In the context of helping students understand metric units, it is also important that they recognize which of our units do not use decimals; for example, time and angle (degrees).

(In the Imperial system, conversions between units were unrelated to each other; that is, we had 12 inches in a foot, 3 feet in a yard, 16 ounces in a pound, and so on. This meant that each relationship had to be memorized separately and computations were complex. If we teach students metric relationships and conversions in the same way we would have taught conversions for Imperial units, we have withheld from them the very advantages the metric system brings.)

Did You Know?

The abbreviations for metric units are simple to derive. They are formed by joining a prefix abbreviation to a base unit abbreviation.

For example, 2 *kilograms* is written as 2 *kg* (*k* for *kilo* and *g* for *gram/s*), 2 *decilitres* is written as 2 *dL* (*d* for *deci* and *L* for *litre/s*) and 2 *decametres* as 2 *dam* (*da* for *deca* and *m* for *metre/s*).

For area and volume units, the abbreviation for the length unit is used and ² is added for square units and ³ for cubic units. For example, 2 *square kilometres* is written as 2 km^2 and 2 *cubic centimetres* as 2 cm^3.

Some naming exceptions for metric units that students are likely to encounter are the hectare (ha), which is equal in area to a square with sides measuring 100 m (or 10 000 m^2), and the tonne (t) which has a mass of 1000 kg.

Prefix abbreviations

kilo	k	Derived from Greek for 10, 100, 1000
hecto	h	
deca	da	
deci	d	Derived from Latin for 10, 100, 1000
centi	c	
milli	m	

Base unit abbreviations

length	metre(s)	m
volume	litre(s)	L
mass	gram(s)	g

Sample Learning Activities

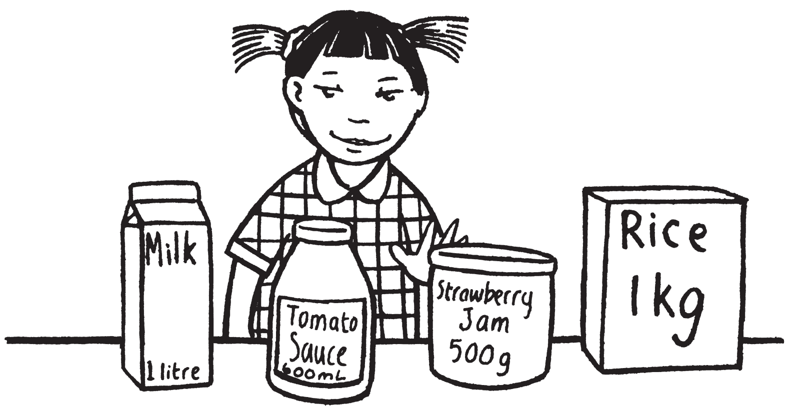
K–Grade 3: ★ Introduction, Consolidation, or Extension

Metric Measurements

Incorporate metric measurement terms such as kilometres, litres, and kilograms into stories and instructions for imitative play (in the sandpit, in the class store). Sometimes refer to milk cartons as litre containers, margarine tubs as 250-g containers, and rulers as 30-cm rulers.

Class Store

Extend *Class Store*, page 84, by discussing standard units as students use the shop to draw out the relationship between standard units in the metric system. For example, say: Sam bought a litre of milk; Carla bought 600 mL. This means Sam has more milk because 1 L (1000 mL) is much bigger than 600 mL.



Deca-Units

Ask pairs of students to choose one of the various units of measurement they have been using (straws, craft sticks, marbles, scoops of rice). Say: Make a unit that is ten times larger to use to measure things in the room. Call this a deca-straw (deca-craft stick, deca-marble, deca-scoop). Encourage them to use the language during measuring in multiples of 10 units. (*This container takes 13 scoops; that means it takes 1 deca-scoop and 3 scoops.*)

Craft Stick Widths

Invite students to use a ruler to find something that is a centimetre wide (width of a craft stick). Ask: How many go together to make a decimetre? Can you find something that is a decimetre wide? How many decimetres go together to make a metre? Help students find out how many craft stick widths go together to make a metre.

Charts 1

Have students gather labels that show measurements and use them to create a chart for each attribute (mass, capacity, length). On their chart, have them order the labels from smallest to biggest. Ask: Are millilitres bigger or smaller than litres?

Charts 2

As students handle and explore units for mass, capacity and length, set up charts that show a gram, a litre, and a metre. Add the *deci*- and *deca*-, the *centi*- and *hecto*-, and the *milli*- and *kilo*- prefixes over time. Have students draw pictures of objects that match each of the measures and add them to the chart. (Students are not expected to remember or learn these names; the purpose is to expose them to the consistencies in the system.)

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Metres to Decimetres

Have groups of students make a metre-length tape and use it to measure how far they can jump. Draw out the limitations in the size of the unit and help students divide the metre into tenths. Name them as decimetres and ask students to use them to compare the distances.

Decimetres to Centimetres

Ask students to use decimetres to measure pencil lengths. Ask: What is the problem with decimetres? What could we do? Help students divide the decimetre into tenths. Ask: What is the name of this unit? How many are in a metre?

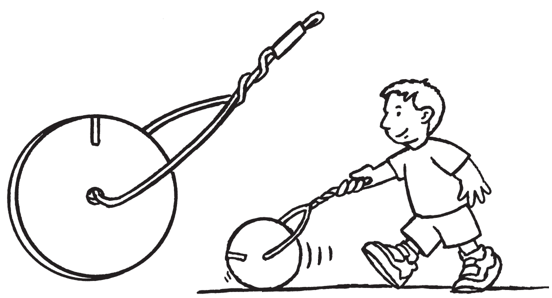
Deca-Jumps

Have students explore the relationship between the *milli* unit, the *centi* unit, the unit, and the *deca* unit. For example, invite them to construct a measuring tape based on a familiar distance (their jump, their stride). First, have them cut a length of tape to show their deca-jump (deca-stride), which is ten times their jump (stride). Then, invite them to mark in the jump unit (stride unit), followed by their deci-jumps (deci-strides) and their centi-jumps (centi-strides). Ask: How would you mark in milli-jumps (milli-strides)? Encourage students to use their tapes for measuring.

Trundle Wheel

Ask groups of students to use string to measure the perimeter of a large round lid and give the unit a name (lids). Help them to make the lid into a trundle wheel and invite them to measure distances around the school by counting the units. When students have recorded their distances, ask them to record them using the name of the unit and the prefixes *deca*-, *hecto*-, and *kilo*-. Ask: What could you call the unit for 10 lids? What could you call the unit for 100 lids? (deca-lids, hecto-lids)

Draw out that 10 units will use the prefix *deca*- and 100 units will use the prefix *hecto*-. Help children to use their calculators to see that the unit gets ten times larger each time.



One Hundred Items

Ask students to use one hundred items (pennies or beads in a pre-sealed plastic bag) as a unit to weigh things in the classroom (a pile of books). Invite them to give the unit a name (pennies, beads). Ask: Is this unit useful to help work out how much one book weighs? Help students to divide the unit into ten small bags of ten which they name using the prefix deci- (deci-pennies, deci-beads) and again into ten single pennies, which they name using the prefix centi- (centi-pennies, centi-beads). Ask: What do *deci*- and *centi*- mean? Why are they used in this way?

Grouping Cubes to Weigh

Following One Hundred Items, invite students to use loose linking centimetre cubes (with a weight of 1 g each) with a balance scale to find the mass of objects. Ask: How can we group these to make counting easier? (in tens, in hundreds) What prefix could we use for the mass of 10 cubes (100 cubes)? (hecto-cubes) After students have constructed decagram, hectogram, and kilogram units, have them weigh the units, using kitchen scales to link the units to those on the dial. Ask: How many cubes did you need for a kilogram?

Medicine

Have students use measuring containers and water to solve the following problem. Say: I need to take 10 mL of medicine a day and I buy it in 100-mL bottles. How many days will it last? If I buy a 1-L bottle, how long will it last? Ask: What information will help you work it out? What do you think the unit names for 10 mL (centilitre) and 100 mL (decilitre) would be? Refer to what they know about length measures and the use of the prefixes centi- and deci-. Encourage students to use a calculator to check their results.

Meaning of Prefixes

Have groups of students find words from dictionaries (computers, items around the room) that begin with the prefixes *milli*-, *centi*-, *deci*-, *deca*-, *hecto*-, and *kilo*-. Encourage students to discuss what they mean and use words and diagrams to illustrate their meaning.

Units of Time

Have students create a chart to show the relationships between units of time (year, month, week, day, hour, minute). Ask: Why do we not use the prefixes *kilo*- and *centi*- when we talk about time? (Time is not metric, although we now use the metric prefixes *milli*-, *micro*-, and *nano*- to divide seconds into smaller units.)

Sample Learning Activities

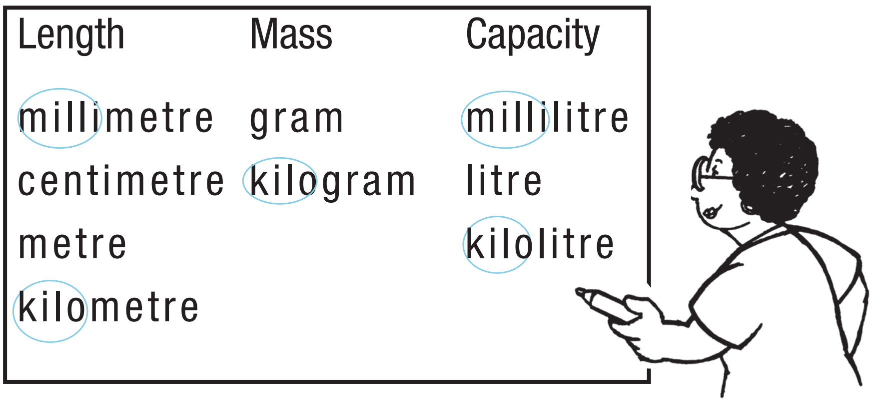
Grades 5–8: ★★★ Major Focus

A Millimetre Thick

Invite students to use a ruler to find something that is a millimetre (thickness of a dime). Ask: How many do you need to put together to make a centimetre (decimetre, metre, decametre)?

Metric Prefixes

Have students collect and categorize the names of known standard units to explore relationships in metric measures. For example, after listing all the units students know, focus on metric units of length, mass, and capacity. Invite students to sort the units from smallest to largest and match up words with the same prefixes before exploring the quantity relationships between units. Ask: How are kilometres and metres similar to kilograms and grams? (Both are 1000 times larger.) What does this tell you about kilolitres? (Must be equal to 1000 litres.) Have we missed any? Could there be a unit called "centigram" or a "centilitre?" (See Case Study 4, page 100.)



Design a Measurement System

Have students select a base unit for mass, volume, and length (the amount of play dough that will fit in a small cubic box might give the base unit for volume and mass, and the height of the box could be the length unit). Invite groups of students to make up a name for each unit and develop larger and smaller units (deca-name, deci-name, centi-name). Ask: How do you know how many of your deci-units of length equals your deca-unit?

Displacement

Give groups of students measures with millilitre calibrations (medicine cups). Have them use displacement of water to find the volume of a rock (marble, washer) in millilitres. Ask them to work together to find how many centimetre cubes displace an equal amount of water. Draw out the observation that 1 cm^3 displaces 1 mL of water.

Mass, Volume, and Capacity

Have students explore the relationships in the metric system between mass, volume and capacity. Invite pairs of students to weigh an empty 1-L milk carton, fill it with water, then weigh it again to find the mass of the water. Ask them to use displacement of water to find the capacity in litres of a decimetre cube (1000 cm^3) and record it. Ask: What does this show? Help them see that a litre equals 1000 cm^3 in volume, which equals 1 kg. Ask: What does this tell us about 1 cm^3 of water? Would you expect a decimetre cube of wood to weigh 1kg? Why? Why not?

Candy Problems

Have students work on problems such as the following. Say: Candies are sold in bulk for $10 per kilogram. How much would it cost to buy 100 g (10 g)? Or, say: One-hundred-gram bags of prepacked candies cost $1.65 each, and 1-kg bags cost $15. Which is the cheapest way to buy 1 kg of candies? Encourage students to use their own methods to calculate the results, then compare strategies. Ask: Were some ways easier than others? Which strategy made it easier to work out? How do the $\times$10 value relationships help you calculate?

Sorting Cards

Extend Grades 5–8 Sample Learning Activity Sorting Cards (Key Understanding 5) by asking questions as the students compare the two measures, such as: What is the relationship between the two units? How many times larger or smaller is one unit than the other?

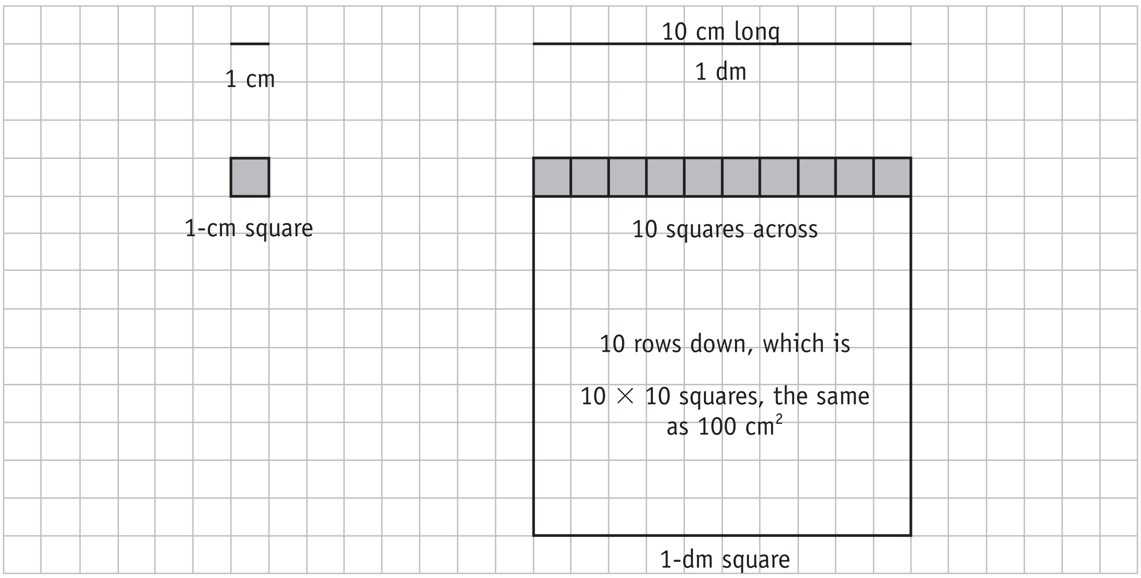
Shadows

Have students use Base Ten flats or decimetre squares of paper to find the area of irregular shapes such as their shadows. Invite them to discuss and justify their strategies for ensuring all of the shape is included in their area measure. Ask: How can the decimetre square be subdivided to help you include all the shape? Would centimetre grid paper help? If you have found 215 cm^2, how many decimetre squares is this? How do you know?

Grades 5–8: ★★★ Major Focus

Length and Square Units

Have students explore the relationship between units of length and units of area. For example, invite them to draw a 1-cm line next to a 1-dm line on grid paper and, underneath, a 1-cm square next to a 1-dm square. Ask: How many 1-cm lengths fit along the 1-dm line? (ten) So, how many 1-cm squares must fit across the top of the 1-dm square (ten) and how many rows of ten 1-cm squares are needed to cover the 1-dm square? (ten) How many cm^2 altogether? (10×10, or 100) Draw out that the area of the decimetre square in square centimetres is equal to the centimetre length squared. Have students explore other length and area unit relationships to show how this is true for any length units. For example, 100 cm equals 1 m, so 100 *squared* (100×100) tells you how many square centimetres in a square metre.



Length and Cubic Units

Extend Length and Square Units by using Base Ten materials to show how length units relate to volume. For example, to show how many cubic centimetres are equal to a cubic decimetre, help students see that ten in a row by ten rows by ten layers (10 cubed) of 1-cm cubes builds a 1-dm cube. Draw out the idea that if 10 cm equals 1 dm, then 10 cubed must be the number of cubic centimetres in a cubic decimetre. Help students see the logical relationships between standard length units (one-dimensional units), standard area units (two-dimensional units), and standard volume units (three-dimensional units).

1 dm cube

1 dm cube

1 dm cube

Ten 1-cm cubes fit across a 1-dm cube.

Ten rows of ten 1-cm cubes make one layer of 100 1-cm cubes.

Ten layers of 100 1-cm cubes make one 1-dm cube made of 1000 1-cm cubes.

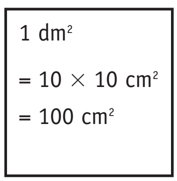
10 cm

1 dm = 10 cm

LENGTH

(one-dimensional unit)

10 cm



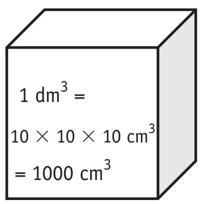
10 cm

AREA

(two-dimensional unit)

10 cm × 10 cm

10 cm



10 cm

10 cm

VOLUME

(three-dimensional unit)

10 cm × 10 cm × 10 cm

CASE STUDY 4

Sample Learning Activity: Grades 5–8—Metric Prefixes, page 96

Key Understanding 3: The relationships between standard units in the metric system help us to judge size, move between units, and do computations.

Working Towards: Measuring Phase and Relating Phase

TEACHER'S PURPOSE

I overheard two of my Grade 7 boys talking about swimming practice for our swim meet.

"I swam a kilometre," boasted David.

"How do you know?" asked Josh.

"Well, the swimming coach told me that the big pool is fifty metres, so I worked out that one lap, that is two lengths, or two fifties—a hundred metres every lap, so ten laps ends up a thousand metres. That's a kilometre," said David.

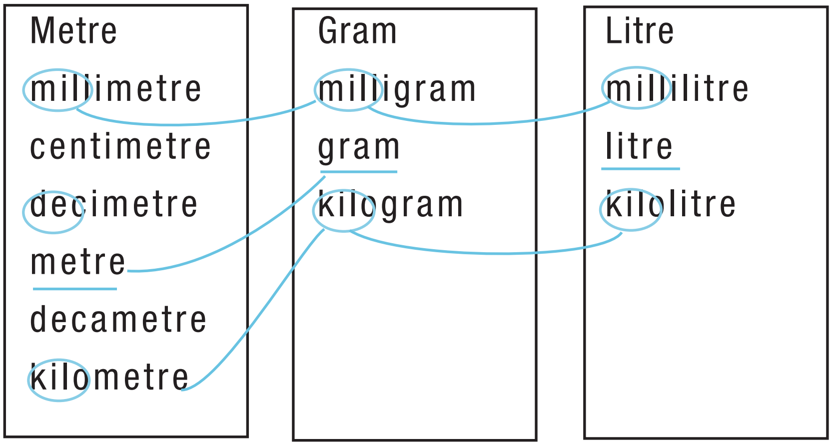
While I was quite impressed by his calculations, Josh was still doubtful. "How do you know a thousand metres is a kilometre?" he asked.

I was expecting David to refer to the fact that *kilo* means "thousand," but there was just a dismissive shrug, "There is!" and the conversation ended there. This made me wonder whether there were others in the class who knew isolated measurement facts, such as a thousand metres equals a kilometre, but did not recognize the underpinning pattern and logic that made metric measures so easy to use.

ACTION AND REFLECTION

We began by having small groups of students brainstorm the names (not symbols) of measurement units (kilograms, centimetres, litres, millimetres) and then sort the units first according to whether they were metric or not and then according to the attributes the units measured (mass, length, area, and volume, and capacity). The non-metric units were put aside for the remainder of this lesson, to be discussed at a later date. The metric units were then listed from smallest to largest for each attribute. When all had finished ordering their lists, we compared them as a class and came to a consensus about the names and the order of units for each attribute. I wanted the students to work from what they knew rather than what I knew, so I did not volunteer any names for units that were missing, nor question how they arrived at the order.

Because I did not want them to become distracted at this stage by the effects of squaring and cubing length units, I suggested we ignore the square and cubic measures for the moment and just focus on the three groups of basic unit words. We matched up parts of words so that we ended up with the following wall display:



Students talked about what they saw:
"None of the lines cross over."
"Metres have got lots more names."
"Milli and kilo are in all the lists."
"Centi, deca, and deci are only in the metres."

At that point, Carrie had her dictionary out. "But you can have a decigram and a decilitre and a decalitre and a decagram; look they're all in here!"

I reminded students about other reference material around the classroom. Students began looking for "centigram" and "centilitre." We added them to the gram and litre charts so that all of the metre prefixes had their equivalent mass and capacity units listed.

Some students researched the prefix naming system and found there were names for even smaller units (micro-, nano-, and pico-units) and larger units (mega-, giga-, and tera-units), which made jumps of a thousand times the value between units rather than just ten times. We talked about Olympic race times and wondered what sorts of measuring situations would need accuracy to a picometre, which is one billionth of a millimetre!

CONNECTION AND CHALLENGE

Once the students thought they had derived all the unit names, I asked them to work out how many of each unit equalled the next-sized units. After studying their lists and dictionaries, it did not take long for several in the class to excitedly exclaim, "It's always ten!"

Kirsten shook her head, "It's not always ten; a kilogram is 100 decagrams, not ten, and that goes for litres and metres as well."

After other students checked to confirm this anomaly, a number of students suggested that there had to be a prefix for "ten times deca." At this point, I conceded that there was a prefix for it, though it was not commonly known, and suggested they look up hecto in their dictionaries. We then completed our charts and added the different values for each unit.

LENGTH	
millimetre	thousandth of a metre
centimetre	hundredth of a metre
decimetre	tenth of a metre
metre	ONE METRE
decametre	ten metres
hectometre	hundred metres
kilometre	thousand metres

MASS	
milligram	thousandth of a gram
centigram	hundredth of a gram
decigram	tenth of a gram
gram	ONE GRAM
decagram	ten grams
hectogram	hundred grams
kilogram	thousand grams

CAPACITY (LITRES)	
millilitre	thousandth of a litre
centilitre	hundredth of a litre
decilitre	tenth of a litre
litre	ONE LITRE
decalitre	ten litres
hectolitre	hundred litres
kilolitre	thousand litres

In the following days, we went on to match the various prefixes to decimal number place values and explored the ease with which we could make conversions based on this knowledge about these commonalities; for example:
1.243 km = 12.43 hm = 124.3 dam = 1243 m = 12 430 dm = 124 300 cm = 1 243 000 mm

Did You Know?

A link between some different attributes has been designed into the metric system:

1 kg is the mass of 1 L of water

and 1 L is the same volume as 1 dm^3

hence 1 dm^3 of water has a mass of 1 kg

but 1 cm^3 is a thousandth of a dm^3

so 1 cm^3 of water has a mass of one thousandth of a kg

hence 1 cm^3 of water has a mass of 1 g

But be careful—we cannot assume it is true of other materials, which might be more or less dense.

Chapter 3

Direct Measure

Carry out measurements of length, capacity, volume, mass, area, time, and angle to needed levels of accuracy.

Overall Description

Students use common measuring equipment and graduated scales such as rulers, clocks, kitchen scales, and grids, choosing equipment or techniques to suit their situation. They understand the importance of making measurements accurately and consistently; for example, making sure the cup is always filled to the same level when measuring capacity, or adjusting for the mass of the container when weighing butter for a recipe. They express measurements in correct units and use their understanding of the common metric prefixes to move flexibly between units and to judge size. They make things using measurement specifications; for example, an open box to hold at least 1 L of orange juice, or marking out the field for a game of softball or tee ball. They judge and measure time, using the various natural cycles in their environment, or available technologies, such as clocks and calendars. They know the difference between time and elapsed time and can calculate the amount of time between 2:05 a.m. and 1:45 p.m. in order to set the video recorder or to prepare and read timetables and programs.

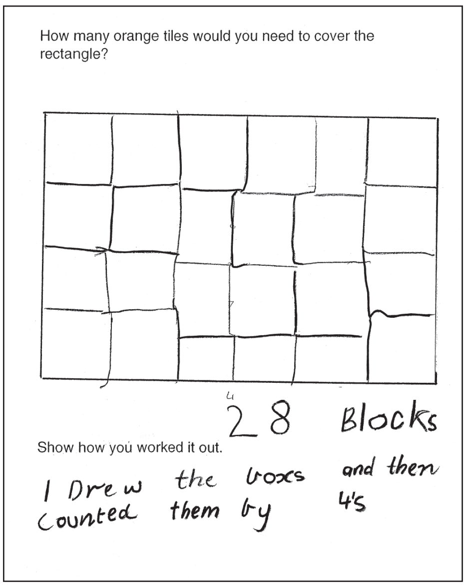
BACKGROUND NOTES

Measuring Area and Volume

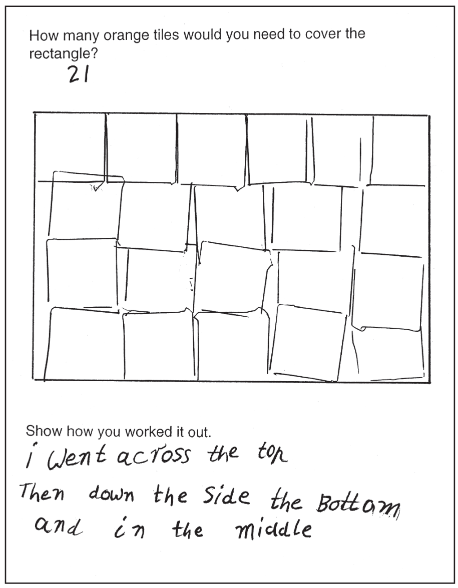
Finding the area or volume of objects directly is more complex than finding length or capacity. Some of the complexity relates to the use of a physical representation of our unit of choice and some to the use of rectangular arrays and grids.

Repeating a unit to match a region or object may be more difficult than repeating units to cover a line segment or to fill a container.

Students have to be able to arrange the things they are using to represent their unit without gaps or overlaps. Squares (and rectangles and triangles) are easier to tile with than are many shapes and that is why we choose them to measure area. Similarly, cubes (and rectangular and triangular prisms) are easier to pack with, which is why we choose them to measure volume. Given multiple copies of a tiling or packing shape, students do not have to do a lot except push them together neatly. They will often do this without thinking about it very much. The challenge is considerably greater if they do not have repeated copies of the unit and have to systematically repeat a single copy of it to cover a region or fill an object. Students generally need quite a lot of experiences where they are provided with a single copy of a tile and have to repeat systematically to cover a region. This focuses their attention on the repeating nature of the area unit.

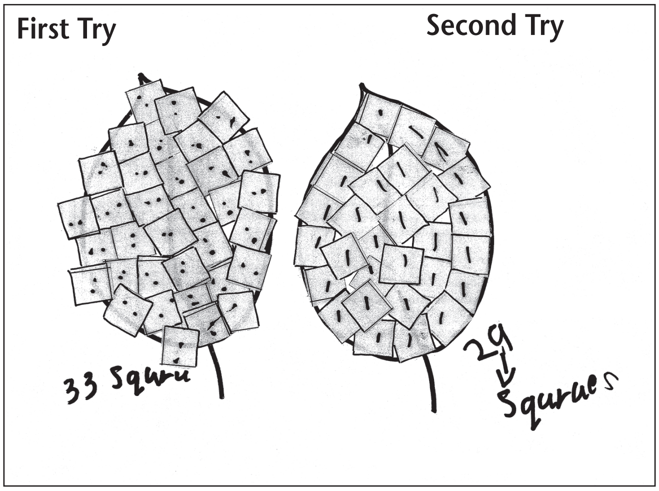


James (Grade 3)



Christina (Grade 3)

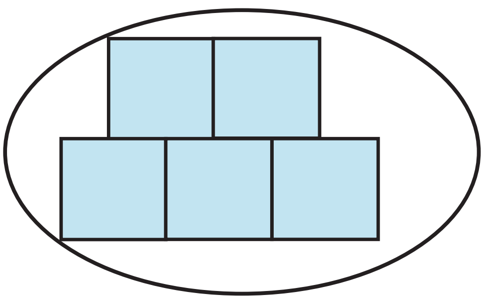
In addition, we can rarely straighten out an irregular region or object as we can a piece of string, or pour the unit into place as we can with a container, and so matching may be more difficult. Students have to be able to work out how many repeats of their unit it takes to closely match the region or object. In practical terms, this means filling inside the border or surface as completely as possible while spilling over the edges as little as possible. This can be difficult conceptually and practically. Students will often be quite disturbed when their 'unit' does not fit snugly within the border.



Tilopa (Grade 3)

When we use a shape or tile to represent a unit, the number of whole tiles that fit inside the border of the region may be quite different from the area of the region measured in that unit.

Students measuring the length of a cupboard using decimetre rods can decide that "the cupboard is between six and seven rods long and closer to seven," simply by finding that six rods will fit in but a seventh will not. Similarly, they can decide that the rock "has a mass of between eight and nine golf balls" by finding that eight is too light and nine too heavy. For the following region, however, only five tiles "fit in" in the everyday sense of the expression, and a sixth tile will not fit in, but the area is not between five and six, it is between nine and ten units.



Giving a more extreme example, the following region has an area of between seven and eight square centimetres but *no* "centimetre squares" fit in.

For this to make sense to students, they have to be able to think of the unit of area as a quantity or amount and not as a particular tile or shape.

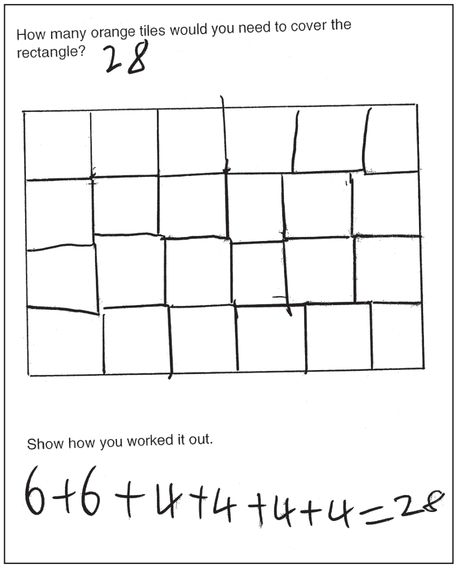
This same issue arises with volume. Although capacity of a container is the volume of the material it will hold, capacity is both conceptually and practically easier to deal with. It is conceptually easier because the idea that a unit is a quantity is more "natural" because of its familiarity and everyday association with fluids (or things that pour). The language we use supports this. Thus we refer to the spoonful as the unit, not the spoon and understand that the spoonful can be spilled or spread and still be a spoonful. It is also easier in practical terms because the materials chosen do pour and naturally fill up the gaps. Even solid materials, such as sugar or sand, because of the relative smallness of the grains relative to the things typically being measured, give the impression of fluidity and appear to fill up the gaps.

Measuring area and volume, therefore, is both conceptually and practically more difficult than measuring length and capacity or even mass. In conceptual terms, to understand the use of a unit to measure area or volume requires that the student think of the unit almost as though it, too, were fluid and could "spill around" to fill up the space tightly. In practical terms, students have to be able to carry out the compensations involved by cutting and rearranging "units" or counting part-units, or by some other strategy for dealing with part-units in order to estimate the number of equivalent whole units.

In short, students have to be able to understand that it makes sense to rearrange the parts of units and combine part-units into whole units (the idea or concept) and also be able to do it (the technique). The very language of "square unit" convinces many students that units of area have to be square shapes or at least that square metres are one-metre-by-one-metre squares. They will need considerable experience in cutting and rearranging shapes to convince themselves that this does not change the area and that a unit of area is an amount of area, not a shape or thing.

We do not have calibrated scales to help us measure area.

It is possible to use calibrated scales to measure length, capacity, and mass, but there is no equivalent for area. The closest we come is to use a square grid with the squares representing the unit. But the square grid does not have a built in starting point or any numerical calibrations to "do the counting" for us. Students have to organize the counting process for themselves and they do not always find this straightforward. For example, Thomas wrote 28 for the total number of tiles, saying that 6 + 6 was for the top and bottom, 4 + 4 was for the sides, and 4 + 4 was for the middle. He counted the corners twice, and also either did not see, or did not see the usefulness of, the array structure.



Thomas (Grade 3)

Note that volume is actually easier than area in this regard. Although we do not have calibrated scales to directly measure volume, we can use "displacement of liquid" to do so. Typically, we place the object into a container of water and use a calibrated scale to measure how much water is displaced. If, for example, 400 mL of water is displaced, then the volume of the object is 400 mL, which is 400 cm^3.

The structure and use of rectangular arrays is not immediately obvious to students.

Although we find the area of many varied regions, most of which are not rectangular, the basis of most measurement of area relates to the efficient use of rectangular arrays. Even for the easily understood task of working out how many square tiles it would take to tile a rectangular room, we do not want to have to purchase the tiles and lay them all out in order to decide how many we need. We do not even want to have to make paper versions of tiles and lay them on the floor. The whole point of using mathematics is to be able to avoid doing this!

One aspect of repeating "units" of area systematically relates to the production of rectangular arrays. We need to think of fitting our representations of the unit together spatially and counting the units in a systematic way. If students are to understand the conceptual basis of the area measurement, they need help, first, to "see" the array structure in rectangles and, second, to use its multiplicative features.

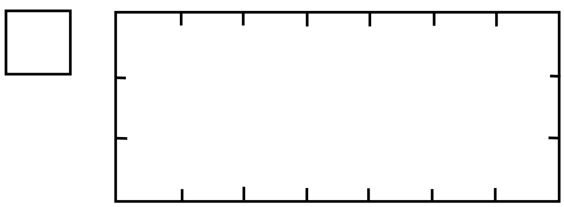
With regard to seeing the array structure, it is often suggested that students should cover rectangular figures with concrete materials in order to develop their understanding of area before proceeding to use formulas, and at first this seems a reasonable suggestion. There is evidence to suggest, however, that this may not be very effective where the materials or tasks are prestructured for the students[1].

[1] See, for example, Outhred, L. and Mitchelmore, M. 2000, Young children's intuitive understanding of rectangular area measurement, *Journal for Research in Mathematics Education*, 31, 2, 144–167.

For example, if students are provided with wooden or rigid square tiles with which to cover a rectangular region, they will be more successful than if they are given paper tiles. Wooden or rigid square tiles fit neatly on the edges and butt firmly against each other, whereas overlaps are more common with paper tiles and more care is needed in arranging the squares.

At first glance, we might conclude that the thicker, more rigid squares would highlight the array structure more and that enabling students to more readily succeed would be a good thing. However, students can get "the right answer" without paying attention to the structure and hence do not learn the significance of the careful spatial placement of the tiles (without gaps or overlaps) or develop a sense of the significance of the rectangular array in enabling us to predict area without laying down every tile or "unit." As suggested above, being able to cover a figure with "units" and count how many there are is not an end in itself, we would rarely do this in everyday life. Rather, the purpose of such activities is to draw out the mathematical features of the situation that enable us in future to predict area without laying down the units or tiles.

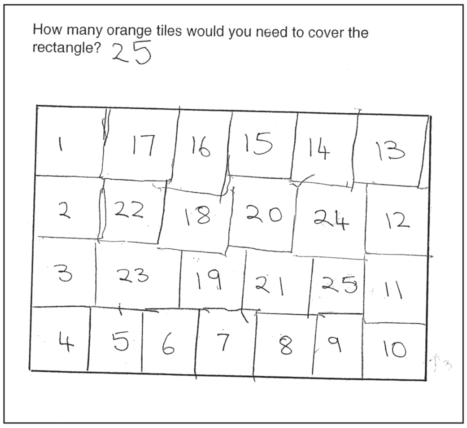
In a similar way, if we always provide a grid on a rectangle, then all the students have to do is count units. They are not learning to produce the spatial structuring of the rectangle needed to see why multiplying works when we find the area of rectangles. For example, in one study by Battista[2] of "above average" students, students were provided with a rectangle marked as shown below, and were shown a square cut-out which exactly matched the markings. Four fifths of 8-year-olds and one fifth of 11-year-olds were unable to predict the number of squares needed.



[2] Battista, M. 1999, The importance of spatial structuring in geometric reasoning, *Teaching Children Mathematics*, November, 170–177.

Although the notion of "covering and counting" to work out area may initially be helpful, it is important that students develop the idea of it as the subdivision of a surface. Thus, during the middle and later elementary years, students need many activities that require them to structure arrays and to reflect on the helpfulness of their structure. That is, *they* need to develop strategies for dealing with regions that are not prestructured by having a grid on them, and where the thing being used as a unit also does not rigidly prestructure the situation.

Students also need to understand and use the multiplicative features of the array structure. Even when students are provided with a filled-in grid, or produce their own, they often ignore the array structure. Some count as did Thomas on the previous page, many count "around" in a spiral fashion as shown below.



Gemma (Grade 3)

This form of counting may be counterproductive if it does not progress to a count that uses the row and column structure, first by ones, then by skip counting or adding the rows. Students need to learn to think of the array in a multiplicative way; that is, they need to recognize the size of the equal groups, as well as the number of times that they occur. For example, "there are 5 in each row, repeated 6 times, that's 5 multiplied by 6." In short, students need to learn to use the array structure to physically (perhaps by drawing) or mentally subdivide the rectangle into a number of equal rows (strips) one unit deep and to multiply the number of rows by the number in each row.

Direct Measure:
Key Understandings Overview

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their grade level.

Key Understanding		Description
KU1	We can directly compare objects or events to say which has more length, mass, capacity, area, volume, angle, or time.	page 112
KU2	We can indirectly compare two objects by using other objects as go-betweens or by altering the objects in some way that does not affect the quantity.	page 126
KU3	To measure consistently we need to use our instrument in a way that ensures a good match of the unit with the object to be measured.	page 134
KU4	Calibrated scales can be used as a substitute for repeating units when measuring length capacity, mass, angle, and time.	page 142
KU5	Units are quantities and so we can use different representations of the same unit so long as we do not change the quantity.	page 156
KU6	We can judge and measure time using both natural cyclical changes and special techniques and tools which peope have developed.	page 168

Grade Levels— Degree of Emphasis			Sample Learning Activities
K-3	3-5	5-8	
★★★	★★	★	K-Grade 3, page 114 Grades 3-5, page 117 Grades 5-8, page 120
★★★	★★	★★	K-Grade 3, page 128 Grades 3-5, page 130 Grades 5-8, page 132
★★	★★★	★★★	K-Grade 3, page 136 Grades 3-5, page 138 Grades 5-8, page 140
★★	★★★	★★★	K-Grade 3, page 144 Grades 3-5, page 146 Grades 5-8, page 149
★	★★	★★★	K-Grade 3, page 158 Grades 3-5, page 160 Grades 5-8, page 162
★	★★	★★★	K-Grade 3, page 170 Grades 3-5, page 173 Grades 5-8, page 176

Key

★★★ **Major Focus**
The development of this Key Understanding is a major focus of planned activities.

★★ **Important Focus**
The development of this Key Understanding is an important focus of planned activities.

★ **Introduction, Consolidation, or Extension**
Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

Key Understanding 1

We can directly compare objects or events to say which has more length, mass, capacity, area, volume, angle, or time.

A great deal can be learned about objects that we can see or hold, and events that are occurring, by directly comparing them. We stand one person next to another to see who is taller, place one pencil alongside another to see which is longer, pour from one container to another to see which holds more, lift two objects in our hands to judge which is heavier, and place one sheet of paper over the other to see which has the greater area. In obvious cases, we can even look and see which object has the greater volume. Sometimes a direct comparison is straightforward and it is relatively easy to reach a correct conclusion so long as we focus upon the right attribute. Often, however, it is neither completely straightforward nor obvious.

In order to make a direct comparison of length, the student must recognize the importance of ensuring that one end of each object is in the same position. With the heights of people, this is usually easy because we stand on the floor and the highest person is the tallest or longest. Were we to lie on the floor, the task would be different and more difficult since we would need to ensure that we matched the position of either our feet or our heads. Matching one end of each object is central in comparing lengths, but is a point that students often miss unless they are provided with learning experiences that draw their attention to it and help them make it explicit. Also, if we need to compare the lengths of curves, a direct comparison of length may be difficult or impossible and an indirect method needed.

Although the basic ideas underlying measurement are the same regardless of what is being measured, the actual process of comparison varies for different attributes, with some attributes being more difficult to compare than others. Students do not always draw the same conclusions as adults might from direct comparison, particularly with measures other than length. For example, students who appear to understand the functioning of a balance beam may allow the appearance of size to override the information provided by the balance and may also be distracted by the pointer and say that the object on the side to which the pointer points is heaviest. Students who happily pour from one container to another may not recognize this as a way of showing which has more. In comparing the area of two shapes, students may try to do it by looking and not superimposing (place one over the other) or making visual adjustments to balance non-overlapping parts. In comparing angles, they may find it difficult to focus on the "amount of turn" between the arms while ignoring the "length" of the arms.

Links to the Phases

Phase	Students who are through this phase. . .
Matching and Comparing	■ can directly compare the length of straight objects, possibly lining up one end where necessary, and use the comparison to say which are the same length, longer, or shorter ■ lift objects, one in each hand, to say which is heavier where the difference in mass is easily discerned but may allow the appearance of size to influence their judgement
Quantifying	■ correctly interpret a balance beam and no longer allow the appearance of size to override the sense of mass ■ superimpose regions as a strategy to decide which of the two regions has the greater amount, although they may focus on some overlapping parts and ignore others ■ will fully fill one container and carefully pour from that container to just fill a second container in order to decide which container holds more ■ understand that to directly compare how long two events take, generally, each has to start (or finish) at the same time ■ are able to directly compare two angles by the amount of turn
Measuring	■ take more care to deal with all non-overlapping parts when superimposing to compare the area of two regions

Sample Learning Activities

K–Grade 3: ★★★ Major Focus

Trains

Vary *Trains*, page 42, by having students line their trains up to compare the lengths. Ask them to make their trains the same length. Ask students to make two or more things that are the same length when they make snakes from play dough, or cut string or tape for collage and straws for beads.

Showing and Telling

When directly comparing quantities, model how you do it, and talk about what you do. For example, when comparing lengths, say: First I need to check that the lengths are lined up at the base, then I look at the tops. When comparing capacity, say: I need to make sure I fill my cup to the same level each time. When comparing mass, say: The one in this hand feels heavier, but I will change hands and see if it still feels heavier in the other hand. When comparing area, say: I think I will put them together to see which one hangs over the other one. For time, say: How will we know which takes longer if we both start at the same time? The one that finishes first is quicker.

Who Is Tallest?

Invite small groups of students to order themselves from shortest to tallest. Ask: If Quentin stood on this box, would he be the tallest? What about if he stood on his toes? What do we need to do to compare our height properly? Could we still find out who is the tallest if we all lie down? How? What would we need to do?

Imitative Play

During imitative play (in the sand pit, in the class store), have students directly compare pieces of equipment ranging in size and number (chairs, cups, plates, bowls, spoons, blankets, bears, dolls, trucks, spades, buckets, buses). Ask: Which is taller? Which is wider? Which is heavier? How can you check?

Sorting Equipment

Have students sort equipment (balls, ropes, bean bags) by size. Discuss which attribute they are using to compare each type. Ask: What are you looking at to decide which one is bigger? Encourage them to describe how they decide which ones are the same size (longer, shorter, heavier, lighter, takes up more/less space). Ask: How did you check them?

Using a Balance Scale

Extend *Using a Balance Scale*, page 42, by asking: How can you make both sides of the scale be halfway up and halfway down? Encourage students to lift the objects to directly compare the mass and experience the fact that they are the same, then place them in the scales and watch the balance. Extend this to comparing objects with different mass and using other weighing tools. (See Case Study 1, page 122.)

Pour to Decide

Have students work over a sink or bucket. Give students two identical clear containers, one of which is full of water. Focus students on the water and ask: If you pour the water into that container, will all of that water fit in or will there be some left over? After students have said what their ideas are, invite them to pour to decide. Ask: What happened? Do the containers hold the same amount of water?

Pour to Decide Again

Extend Pour to Decide by providing two different-sized clear containers. Stop the students pouring if the container is about to overflow and focus them on the water that is left in the first container. Ask: Is there too much water for this container? Does that container hold more water than this one? When students pour from the smaller container, ask: Could that container fit more in? Will that container hold more water? Which container holds the most water? Draw out the idea that the larger container holds more.

Jello Moulds

Have students compare two jello moulds or buckets similar in size. Ask: Can you tell just by looking which one holds the most? How could you check to be sure? Provide students with suitable materials (water, sand, rice) and invite them to try their strategies and consider the effect of filling one mould (bucket) and pouring into the other. Ask: Are there any leftovers? Do they hold the same amount? Does one hold more? Is there enough water (sand, rice)?

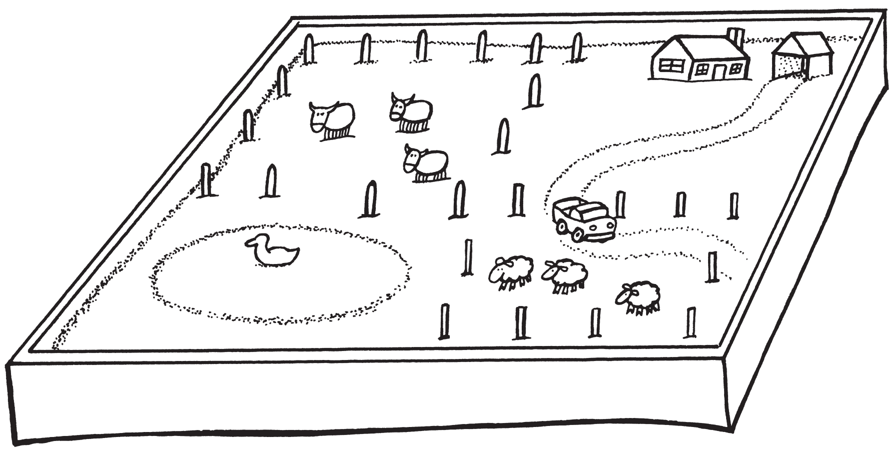
K–Grade 3: ★★★ Major Focus

Cubes and Boxes

Extend *Jello Moulds*, page 115, by providing two boxes similar in size. Ask: Which box is bigger? Have students fill one box with cubes, then transfer the cubes to the second box. Ask: Are any blocks left over? Do you need more blocks to fill the second box? How does this help you say which box is bigger? Does it make a difference if you pack the cubes in the boxes very carefully?

Models

When making models of farms or towns, provide students with transparent shapes that do and do not match the areas of features in the models (parks, ponds, pastures, sandpits). Have students compare the transparent shapes to the features and decide which are larger, smaller or the same size. Ask: How can you find out if this shape is bigger than the pasture (park, pond, sandpit)? Would putting it on top of the pasture help?



Ordering Containers

Give groups of students a range of different-sized containers. Ask: Will the tallest one hold the most? How are these two different? After students have handled the containers and ordered them by height, ask: What other ways could the containers be ordered?

Superimpose

During various classroom activities, find opportunities to have students superimpose to find areas (choose a cover for the computer keyboard, tablecloths for different tables, tangram shapes for pre-drawn pictures, place a foot against the undersole of a shoe, lids on jars).

Sample Learning Activities

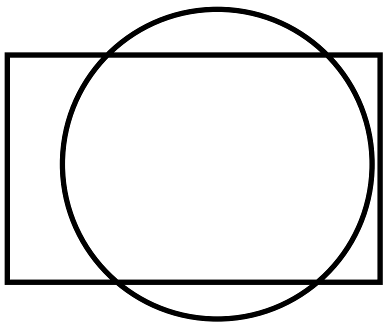
Grades 3–5: ★★ Important Focus

Track and Field Day

For track and field day, cut ribbons into a number of different lengths and mix them together. Have students sort the ribbons by length and put them into different boxes. Ask: How can you be sure all the ribbons in each box are the same length?

Pizza Trays

Show students two pizza baking trays, one a rectangle and the other a circle. Say: The pizza shop charges the same price for both these pizzas. Is there more to eat in the rectangular pizza, or the circular pizza? How would you decide? Provide cut-out models of the trays and suggest students superimpose to decide. Ask: Do you think the bit around the edge of the rectangle will fit into the bit around the edge of the circle?



Smaller but Heavier

Ask students to think about this statement: You cannot tell if one object is heavier than another just by looking. Ask: Is this correct? Provide balance scales and a variety of materials (fruit, vegetables, pencil cases, tissue boxes, tape dispenser, paperweight). Invite students to find objects that look small but weigh more than larger objects. Ask: What is it that makes this small object weigh more than the larger object? Which would weigh more if they were both made of the same material?

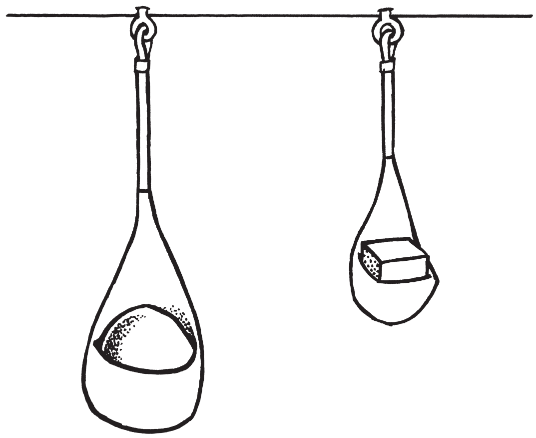
Looking at the Pointer

During mass activities such as Smaller but Heavier, ask: Is the pointer on the balance scales pointing to the side that has the heavy object? How is this different from when we just look at which side is going down?

Grades 3–5: ★★ Important Focus

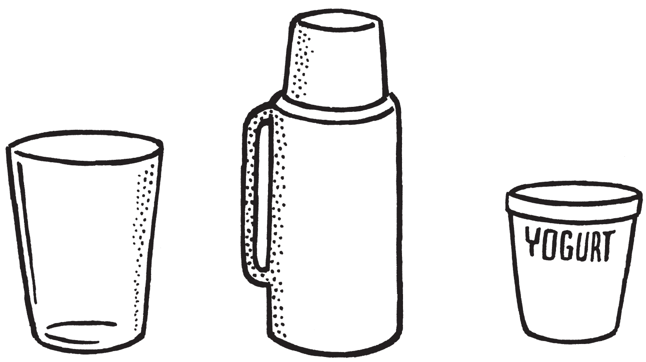
Weighing with Elastic

Have students use two pouches sewn to the end of two strips of elastic to compare the weight of objects. Ask: How is this different from using a balance scale? If both objects stretch the elastic downwards (instead of one going up and one going down) how can we tell which is heavier? Why is it important to hold the tops of both strips of elastic at the same height?



Capacity and Shape

Have students look at several containers that are close in capacity, but different in shape. Ask: Which container would hold the most water? What makes you think so? Invite them to check by pouring water from one to the other.

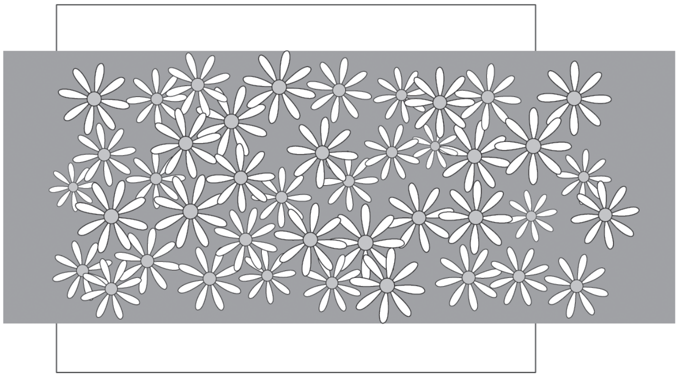


Fitting Boxes

Invite students to fit boxes inside other boxes in order to work out which has the largest volume. Ask: How can you tell which has more volume? What if a box does not fit because its shape is different? How can you find out which is bigger? How would thinking about "which box holds more" help you to find a way to compare volume?

Groundsheet for a Picnic

Invite students to work out which of two big pieces of cloth would make the larger groundsheet for a picnic. Ask: What would be the best way of telling which is the larger? Suggest they put them together (superimpose) to try to judge. Ask: How can we tell which is larger when both have sections overlapping?



Comparing Time Taken

Have students discover the importance of using the same start time when directly comparing the time taken for various classroom activities (sharpening a pencil, putting on shoes, going for a drink of water, writing their name and date on the top of the page). Vary the start times when making comparisons and focus initially on who finishes first. Ask: Is this comparison fair? Why? Why not? How can we find out who really takes the least time? How can we make sure both begin putting on their shoes at the same time?

Book Corners

Have pairs of students use the corners of a book to find corners around the classroom with the same angle. Encourage them to draw each one and label it. Invite students to compare their angles to the corners of a square and introduce the language "right angle" and "90º" to describe the size of the angles. Then, ask students to find angles that are more or less than the right angle (90º). Ask: Why do you think right angles are so common? What would happen if cereal packages were made with angles less than 90º? How would they stack on the shelves?

Sample Learning Activities

Grades 5–8: ★ Introduction, Consolidation, or Extension

Ordering Packages by Volume

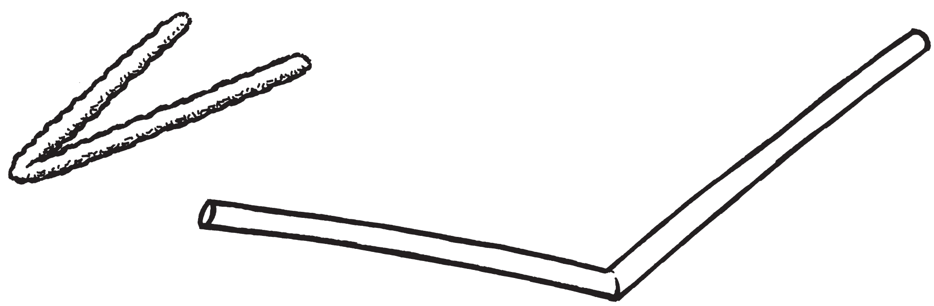
Give groups of students a variety of packages (cereal cartons, tissue boxes, laundry detergent boxes). Ask them to directly compare and order them according to volume. Discuss their strategies. For example, ask: How did you decide which had more volume than another? What parts of the packages did you compare? Is it difficult to be sure that one package has greater volume than another? Why? Why not?

Who Is First?

Focus on directly comparing very short time intervals and the difficulties of ensuring identical starting points. For example, say: We are going to see who can sign their name the fastest (who is first to stand up from sitting cross-legged). How could we make sure that everyone starts at the same time? What would be a fair starting signal? How do we know when someone has finished?

Ordering Angles

Have students use a number of different angles made from varying lengths of straws (pipe cleaners, bent wire) to order the size of some angles. Ask: When you decided which one was the largest angle, what did you compare? What made you put the angle made from the longest straws (pipe cleaners, bent wire) after the one made with the shortest straws (pipe cleaners, bent wire)? Show me two that have the same angle but are made from different length straws (pipe cleaners, wire). How do you know they have the same angle?



Washer in the Drink Carton

Have students use balance scales to directly compare the mass of nine sand-filled small drink cartons. To prepare for the following problem-solving puzzle, add a metal nut or washer to one of the cartons after balancing, then seal them all. Say: The manufacturer knows something fell into one of these nine cartons during packing, making it slightly heavier. You are to find out which carton has the extra mass, but you are only allowed to use the balance scales twice. (Answer: Compare a group of three with another group of three, to find which set of three has the extra, then take the heavier group and compare two of the cartons, to determine which it is. If the scales balance, the heavier carton cannot be on it, so the remaining carton must weigh more than the others.)

Olympic Sports

Have students sort situations into those where direct comparisons are made and those where direct comparisons cannot be made. For example, ask students to list those Olympic sports that are judged by direct comparison (those sports with identical starting times, such as swimming, so that whoever finishes first also takes the least time) and those that are not. Ask: In which sports are all the contestants on the field at once? Can the winning places of these sports all be judged using direct comparison? Why? Why not?

CASE STUDY 1

Sample Learning Activity: K–Grade 3—Using a Balance Scale, page 115

Key Understanding 1: We can directly compare objects and events to say which has more length, mass, capacity, area,volume, angle, or time.

Working Towards: Matching Phase and Comparing Phase

TEACHER'S PURPOSE

I noticed that some of my kindergarten students seemed to have difficulty using the balance scales. Chelsea held down one side containing objects and filled the other side to overflowing, then let go. As the side she held rose, she pushed it back down and tried to make it stay down. She announced, "They didn't work!" and became uninterested in using them.

Later, I gathered the class together. I wanted to teach them to handle the scales properly and to hear one another's ideas as they did so. I asked Chelsea why she thought the scales did not work.

Their replies made me realize they had been focused on the task, but expected the scales to respond the same way their hands did when lifting; that is, both sides go down if they "feel" heavy.

"When you put things in, both of the sides don't stay down. It keeps coming up."

"Yeah," said James, "I push really hard and it will not go down."

Terry nodded.

I drew their attention to the fact that the empty tubs were level and asked them what happened when one side of the scales was pushed down. 'How do the tubs move? Do they always do that?'

Chelsea did not respond.

CONNECTION AND CHALLENGE

I asked James to cup both hands, placed a rock in one of them and asked, "Do your hands feel different, James?"

He shook the hand with the rock in it and said, "This one is heavy."

I asked the class to think what would happen if James put the rock in one side of the scales. Many said, "It will go down!"

The students were buoyed by the result when James tried it. "Why did the side with the rock in it go down?" I asked.

Terry replied, "Because it has got something in it."

I had wanted him to say "because it was heavy." I was getting ready to ask, "Why did the rock make your hand go down?" when Phillipe said, "That one goes down because it's heavier and that one will stay up because it's lighter. The heavy one always goes down. I will show you."

Phillipe chose a larger block and put it in the empty side. The larger block was actually lighter than the small rock and so it stayed up.

He was shocked by this. "The block should be heavier than the rock! That's funny!"

He regained his composure and went on. "Well, heavy things should make the side go down. Well, you see ... well, if that one is lighter, it goes up."

He was unable to explain his thoughts. He had seen something that was in direct conflict with his firm belief that larger objects are heavier.

Later in that day, I gathered the five students who, like Phillipe, were convinced that larger must be heavier around the scales to compare large light objects with small, heavier objects.

I took both objects out and said, "Let's check. Which one is lighter?"

I gave them to Phillipe to help him see that the scales were right. Then I passed them to James. He lifted with both hands and said, "The rock is heavy."

I asked him to place the light object in one side of the scales, then the heavy one in the other and asked Terry to say what had happened. "The rock one's gone down!"

I asked Chelsea, "Did that surprise you?"

She responded, "The block should go down, too."

ACTION AND REFLECTION

I asked everyone to choose two items from a collection of smallish items, to hold an item in each hand and decide which one was heavier. I then asked them to think about what the scales would do when they put the things in. I was not sure whether Amy knew about using the scales and so I asked, "What will the scales do if you put your marble on one side and your lid in the other?"

She shrugged and said, "The marble side will go down?"

I asked Terry, James and Chelsea to lift Amy's objects and say if they all agreed that her marble was heavier. I asked everyone to decide which side would go down, the side with the lid or the side with the marble. As we watched, Amy placed the items in the tubs. I asked Rachel to repeat this using Amy's two items, but changing the sides each time.

A week after this lesson, I focused this group on the idea that the scales would balance if the mass of the objects were the same using a small object and play dough.
When James noticed the scales level, he said, "Look! They're weighing!"

After each turn I asked:

"Which object did Amy say was heavier?"

"Which side is it in?"

"What did that side do?"

"What did the side with the lighter object do?"

After several turns, I asked, "Did the lighter object ever go down instead of the heavier object?"

They all agreed that the heavier object went down each time. I noticed in particular that Terry, James and Chelsea all nodded. I then asked, "Why?"

James said, "This one makes that one stay up."

When I asked why it did that, he said, "Because it is heavy."

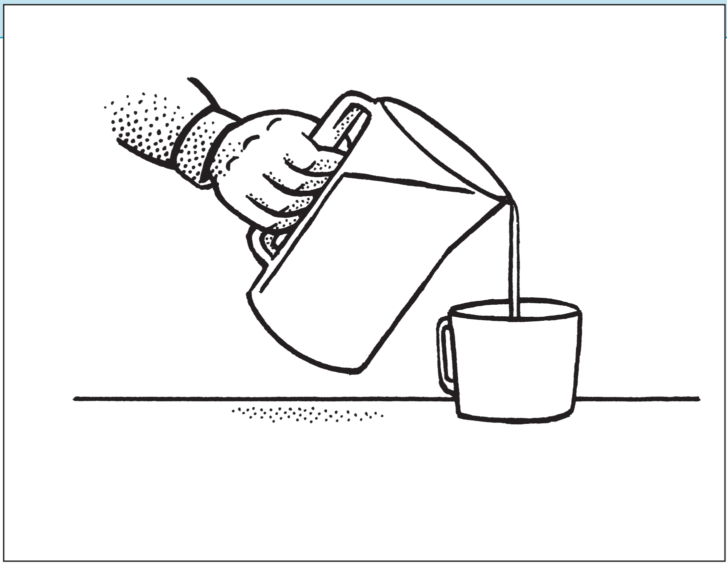
PRACTISING WITH THE NEW IDEAS

One of the groups that I later gathered around the scales was Terry, James, Chelsea, and Amy, to repeat the lifting to compare and notice the positioning of the scales when comparing the objects they lifted. I gave them each two new objects and asked them to use the scales to find out which one was heavier. They could check what the scales told them by lifting afterwards.

I listened to their ideas as they talked together about what would happen and was relieved to know they had begun to predict which side would stay up and which would go down. They had moved on from expecting both sides to go down and stay down if they had something in them, to expecting the heavier side to go down.

Did You Know?

The "logic" of pouring from one container to another to see which holds more seems obvious to most adults. When young students pour, however, some become so focused on the container they are pouring from that they hardly notice what is happening to the other container. Only when the first container is empty, do they shift their focus to the second. If the second container is the smaller, it will now be full, but the students may not realize the significance of the water that overflowed. Thus, the comparison of the capacity of two containers by pouring from one to the other is not obvious to students and experience in pouring from one container to another may be insufficient for the mathematical learning to occur. We need to help students focus on what is happening to the water level in each container and how that relates to the relative capacity of the two containers.



Key Understanding 2

We can indirectly compare two objects by using other objects as go-betweens or by altering the objects in some way that does not affect the quantity.

Often, when a direct comparison of a particular attribute of two objects cannot easily be made, it is possible to indirectly compare them by the use of intermediaries. In essence, the indirect comparison of two objects involves finding or producing alternative objects that can be directly compared. The alternative objects are different from the originals, but share the *relevant quantities* with the originals.

For example, in order to compare the length of two curving paths, we could fit string along the curves, cut and straighten each piece of string, directly compare them, and make an inference about the curved paths. The two pieces of string act as go-betweens. This is not an obvious process to young students. In order to make sense of it, they have to believe that:

- each piece of string is the same length as the curved paths it was fitted along
- straightening the string does not change its length
- lining up one end of the two strings does not change their lengths
- the longer string must come from the longer path.

To compare the width of a doorway with the width of the furniture we want to slide through it, we could cut one piece of string to fit the width of the furniture and compare the string with the door width. This requires that students realize that if the string is less than the door width the furniture will slide through, but if the string is more than the door width the furniture will not slide through. This transitive thinking must be developed through and drawn from carefully structured activities.

To compare the areas of two different shapes of paper, we could alter the paper in a way that leaves the relevant quantity, area, unchanged. We might, for example, cut one piece and rearrange the bits and place them over the other piece. In effect, the altered shape acts as a go-between; it is different from the original in shape, and possibly in perimeter, but has the same area. To compare the volumes of two rocks, we might put each into separate identical containers of water and compare the change in the height of water.

When we use units to compare quantities, we are using the object that represents the unit as a go-between. Each of the above indirect comparison situations could have been dealt with by using an appropriate unit-based strategy. For example, a tape measure could be used as a go-between to measure the girth of two trees, enabling an indirect comparison to be made. Rulers, tape measures, measuring cylinders and squared paper are obvious instruments of indirect comparison.

Note that "Indirect comparison" is not the same as "indirect measurement." Indirect comparison involves comparing two different objects using something as a go-between (the go-between may even be a ruler or measuring cylinder). Indirect measurement involves the calculation of one measurement from other related measurements.

Links to the Phases

Phase	Students who are through this phase. . .
Matching and Comparing	■ indirectly compare lengths such as curved paths
Quantifying	■ indirectly compare capacities ■ can superimpose regions to compare areas, but are unlikely to have a reliable indirect strategy for dealing with non-overlapping parts ■ will attempt, unprompted, to alter one or both of the regions to enable a direct comparison of area *For example*: A student may cut and rearrange the pieces of one to fit one over the other. ■ will elect, unprompted, to use a unit to make a numerical measurement to compare several things *For example*: A student may count how many spoonfuls of rice it takes to fill a container.

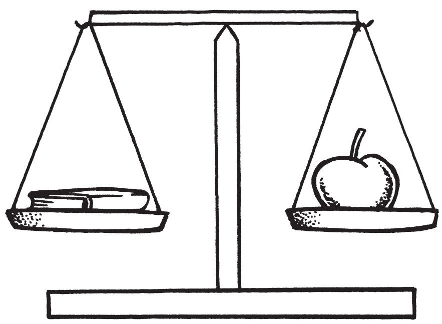
Sample Learning Activities

K–Grade 3: ★★★ Major Focus

Body Parts
Have students use a go-between to measure how far it is around various body parts. For example, invite students to use paper tape to measure around their heads (wrists). Ask: How do you know whose head (wrist) is biggest?

Trains
Vary *Trains*, page 42, by asking students to cut a length of string to match the length of their train. Ask: How many boxcars long is your train? If you make a train tomorrow, will it be longer or shorter than this one? How many blocks will you use to make it? Then, on the next day, have students make a new train and use the string to see if it is shorter or longer than their first train. Ask: How much longer or shorter is your train than yesterday?

Matching Fruit
Ask students to find an object that matches the mass of an apple (orange, banana). The next day, ask them to use the object to find a piece of fruit that matches the mass of yesterday's apple (orange, banana), one that is heavier and one that is lighter. Ask: How did you know the fruit weighed the same?



Pet Rock
Have students solve mass problems where direct comparison is not possible. For example, say: I have a pet rock at home that fits in a closed hand and weighs the same as eight marbles. Do any of you have a pet rock you think weighs more (less)? It has to be small enough to fit in a closed hand. Provide students with rocks of different mass and size and ask them to use marbles and balance scales to see if they can find a rock that weighs more (less) than the one at home.

Pouring and Scooping
Have students use different-sized containers to pour liquids and grains from one to another during play and food preparation. Focus students on both how much a container will hold as well as how much liquid, rice, or sand there is to be held. Ask: Will this container hold all of the drink? How many scoops of sand will your truck hold?

Equal Shares
Have students decide how to make sure each person gets the same amount of water poured from a jug when each student has a different-sized, clear plastic cup. Try out each suggestion for students to say if they think they have as much as the others or if one has more. Extend the activity over two or three days for the students to think about and try many suggestions. Make a list of those suggestions that could and probably would not make equal shares. Encourage students to use one cup as the go-between.

Stamps
Ask students to trace 10-cm-square areas on a large sheet of paper and cover one of the areas with a print using a chosen object as a stamp. Ask: Did your stamp cover the area without leaving gaps? Try another one of your areas to see if you can fit more stamps on. Did your stamps overlap to fit that many on? Try to fit as many stamps on the page as you can without overlapping. Try a different stamp to see how many you can fit with no gaps or overlaps. Which stamps will fit the most prints on without overlaps? Why?

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Oil Spills

Give pairs of students a large copy of a map showing two different oil spills. Ask them to compare the areas of the oil spills. Ask: Would it help to cut off the overlapping bit? Does it change the measurement if we cut up the oil spills?

Oil Spills Again

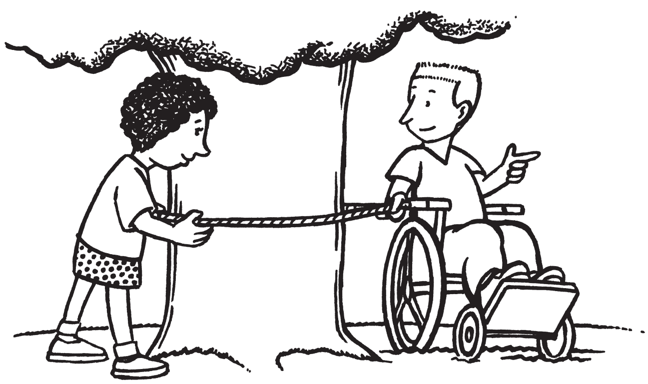
Extend Oil Spills by asking students to use paper tiles to compare the spills. Ask: How can you fit the squares around the edges and into the gaps? Would cutting them up help?

Measuring Drink Containers

Present students with different-shaped drink containers filled with water and an empty, larger-capacity, narrow container. Invite students to take turns pouring the contents into the larger container, marking each level with coloured elastic bands, then pouring the water back again. Ask: How does this show which containers hold more than others? Is this a more useful way of ordering several containers than just pouring from one drink container to another? Why? Why not?

Circumference of Trees

Invite students to find a way to check whether the circumferences of trees in the playground are more or less than the length of the metre rule. Ask: What makes it difficult to use the metre rule itself? What else might you use? Encourage groups of students to make the comparisons, using a method they choose. Discuss the results. Ask: How can you be sure your method works? What would you say to convince someone that the circumference of that tree is more than the length of the metre rule? How is the object you used similar to the metre ruler?

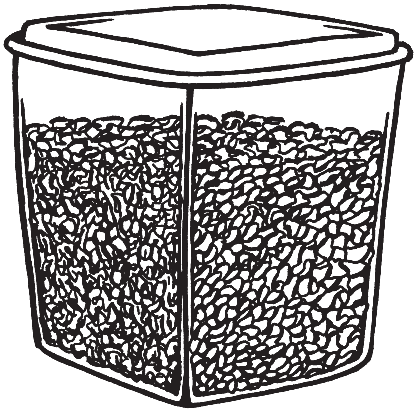


Timing Basketball Scores

Have students suggest and try out strategies for comparing the time taken for activities that cannot be carried out simultaneously. For example, say: If we did not have a clock available, how could we decide who takes the least time to sink five basketball shots when we can only use one basket? Discuss the reliability of various methods (counting hand claps, using a sand timer, using a water clock, measuring the Sun's shadow, measuring the distance a student walks).

Breakfast Cereal

Have students use a go-between to compare capacity. For example, give students a container each. Say: I have a container at home that holds ten cups of breakfast cereal. Will your container hold more or less than mine? Ask: What could you use to work it out? Does it matter if you do not have any breakfast cereal?



Sharing Play Dough

Ask groups of students to use balance scales to share a lump of play dough evenly among three people. Ask: Would using a go-between be helpful? (make each ball of play dough weigh the same as a tennis ball) Does the go-between need to be heavier or lighter? Later, extend the activity by asking: Would using units such as marbles, washers or gram weights be helpful?

Pizza Trays

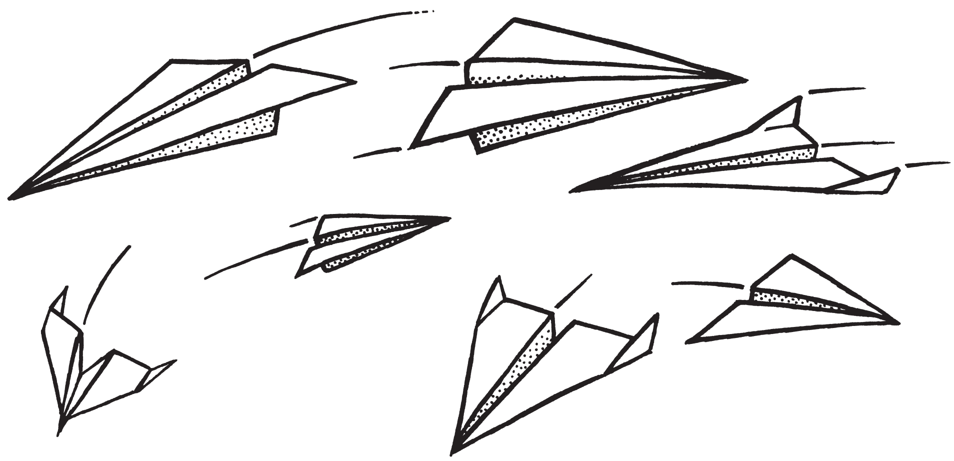
Extend *Pizza Trays*, page 117, by asking: Can you cut and rearrange one of the shapes to find out which one is larger?

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Paper Planes

Have students investigate ways to compare the distances paper planes will travel in a competition. Encourage them to discuss the aspects of distance involved and the difficulties in making comparisons. For example, ask: Should how far it goes up and down count? How can we measure it? What if it travels in a curved path? Give groups of students their choice of materials to make the paper planes and to compare distances during some trial flights. Decide as a class how to define “distance travelled” for the competition and how the distances travelled will be compared.



Surface Area

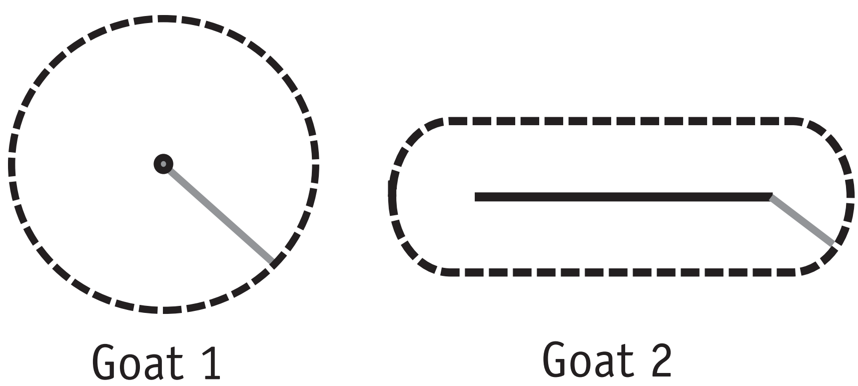
Have groups of students use their own strategies to compare the surface area of two small cardboard boxes without cutting them up. Encourage students to discuss their strategies and their results. Ask: Which box uses more cardboard? How did you work it out? What did you use? Why? Would you have done it differently if I had asked how much more cardboard was used?

Similar Solid Objects

Present students with two solid objects of similar size (small metal toys, china ornaments, drinking mugs). Ask: Which object has the greater volume? That is, which uses more material in its construction? Have students place the objects in a narrow container of water and watch what happens to the water level. Invite students to explore ways of marking or recording the levels to compare the volume of water displaced by each of the objects. Point out to students that it is important that as little water as possible is removed with each object. Ask: What does the higher water level tell you about the amount of material used to make the objects? What would the displaced water be equal to if the object is hollow and the water does not get inside?

Grazing Areas

Say: There are two goats. The first goat is tethered by a rope to a stake in the ground. The second goat is tethered by a rope half as long as the first goat's rope to a sliding rail that is double the length of the first goat's rope. Invite students to use a compass and ruler to draw representations of the two feed areas. Ask: How would you work out which animal has access to the larger area? Encourage students to discuss and justify the method they chose. Ask: How could you convince someone that your method shows which animal has the larger feed area?



Key Understanding 3

To measure consistently we need to use our instrument in a way that ensures a good match of the unit with the object to be measured.

To be able to think of "counting units" as giving a description of the size of an object, we have to believe that if we measure the object a number of times with the same unit we should get the same result each time. This requires that we match the object to be measured with as many units as possible, but no more. Our capacity to do this reliably depends on how well we choose measuring instruments to represent our unit (precision) and how carefully we use them (accuracy). Key Understanding 4 in Understand Units emphasizes choosing appropriate instruments to represent the unit, whereas this Key Understanding emphasizes the practical skill of using them well. This Key Understanding, then, is about the skilful use of our measuring instrument, whether it is a pen repeated end-to-end or a trundle wheel, a plastic tile or a centimetre-square grid, a cup, or a balance beam. The use of calibrated scales as described in Key Understanding 4 is one way of "repeating units" and is therefore closely related to this Key Understanding.

Over time, students should come to see that in order to get reliable results they need to ensure that the unit size remains constant throughout and that they fit in as many as possible, but no more. This requires that they take care in several ways. First, they need to ensure that the size of the things used as units remains uniform throughout the measuring process, checking:

- that the cup is always filled to the same level (capacity)
- that the beans are the same size (mass)
- that the tape measure is not stretched in some places (length)
- that the ruler is not broken on the end (length)

Second, they must use the unit in a way that enables a good match with the object being measured, by:

- putting craft sticks end-to-end (length) and not letting their fingers get in the way
- shaking their container to ensure there are no air pockets (capacity and volume)
- fitting shapes together with no gaps or overlaps (area and volume)

Third, to ensure that the object is fully matched, but no more, they:

- place a book on their head and line up the book with the ruler (height)
- completely fill, but do not overfill, the container being measured (capacity)
- adjust for the container when weighing jelly beans (mass)
- fill as close as possible to the edge when covering regions (area)

Links to the Phases

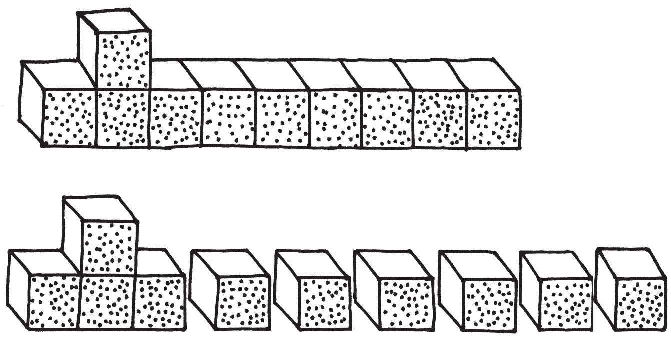
Phase	Students who are through this phase. . .
Matching and Comparing	■ count informal units of capacity, mass, and time, but may see the task literally as a counting task ■ may be casual in using instruments *For example*: A student may not really understand why it matters if part of the spoonfuls are spilled.
Quantifying	■ have begun to understand why it matters if they spill part of their spoonfuls ■ will repeat uniform units of length and capacity carefully, although they may not understand why this is important *For example*: A student may not know why it is important to line up the zero on a ruler. ■ use uniform units consistently and carefully to measure quantities that are uni-dimensional, such as length, capacity, and mass, as well as angle and time ■ use uniform units of area, although they may struggle with what to do along the edges when covering regions ■ may try to use some part-units of area, but may not be successful when it requires them to combine part-units
Measuring	■ are able to use and combine part-units of area and hence can count units and part-units to find the area of any region ■ can count units of volume in straightforward cases
Relating	■ use a wide range of everyday instruments correctly

Sample Learning Activities

K–Grade 3: ★★ Important Focus

Trains

Vary *Trains*, page 42, by helping students notice when two trains with the same number of matching blocks are different lengths. Say: Look at the trains and see if you can work out why this train looks longer. Encourage students to make sure all of the blocks fit end-to-end.

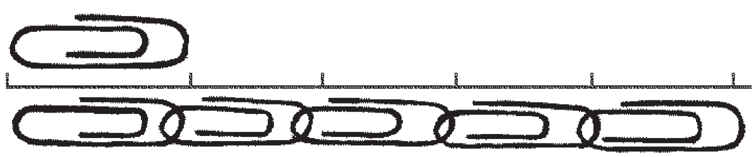


Packing Boxes

Have students pack blocks and other classroom equipment or materials into boxes, trying to fit in as many objects as they can. Ask: Could you fit more in? How could you pack it again so that there are not so many gaps? What makes a good fit? Why are the blocks easier to pack than the toy cars?

Paper Clips

Provide each student with one paper clip and ask them each to make a line segment five paper clips long. Display and compare the line segments. Ask: Should the line segments be the same length or different? Ask a student who has repeated the unit correctly, perhaps using a mark to show where to begin the next unit, to demonstrate how they made their line segment. Focus on the units fitting end-to-end with no gaps or overlaps. Then, ask students to all make a new line segment twelve paper clips long and compare them again. Invite students to thread five paper clips together and match it to the first line. Ask: Why is the first line segment longer? Think about how we joined the clips.



Six-Cup Containers

Provide groups of students with containers that hold six to ten cups. Have students take turns measuring and recording how many cupfuls of water each container holds. Invite them to compare their results. Ask: Why might some of you have found that the same container holds a different amount of water? Encourage students to check their results two or three times. Ask: What are you doing to make the number of cupfuls more (less) than they should be? (spilling some, splashing more in, miscounting the cupfuls)

How Heavy Are Four Pieces of Fruit?

In small groups, have students measure how heavy four pieces of fruit are. Invite them to use the balance scales and a unit of their choice from paper clips, cubes, marbles, string, washers, sand and small cups. Ask students to explain why they chose their units and then compare their measurements with others. Ask: Why should we not use different things as units?

Throwing

Vary *Farthest Throw*, pages 57 and 72, by having pairs of students work together to measure one throw. Invite one student to measure with a chosen unit, then have the other student re-measure using the same unit. Ask: Why might you get different measures? What do you have to do to make sure your measures are both correct?

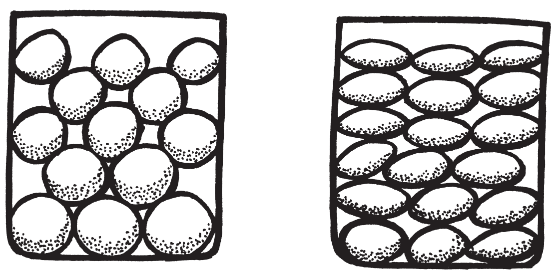
Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Length of a Desk
Ask students to use their pencils to measure and record the length of their desk. Ask: Are the results the same? Why? Why not? Invite students to demonstrate the way they used their pencil to obtain the measurements and to compare the lengths of their pencils. Ask: How would the length of your pencil make a difference? Did the way you measured affect your result?

Play Dough Balls and Mass
Give groups of students play dough and balance scales. Ask them to make small balls out of play dough to use as units for measuring and recording the mass of their toys (books, containers). Ask: Have you checked that the balance scales are level before you begin? How can you be sure each ball weighs exactly the same? Does it matter if they are not exactly the same? Does the pointer on the scales point to the light side or the heavy side? What makes you certain that eight of Carrie's balls will also balance your toy truck?

Play Dough Balls and Capacity
Extend Play Dough Balls and Mass by having students use the same balls to measure the capacity of a range of containers. Ask: What happens if you squash the balls in for some and not for others? Would that give you a different measure? Why? Why not?



Juice for an Excursion
Having decided in advance how many cups of juice are needed for an excursion, have students measure a set of containers to work out which will hold the right amount. Ask: If you do not completely fill each cup as you count, will we have a container that is too small or too big? If you spill a little each time, will that mean that you have counted more or less than you thought?

Spoonfuls of Flour

Have students compare their measurements with others to check for consistency. For example, say: The recipe for pancakes said to put in ten spoonfuls of flour. Ask groups of students to measure out ten spoonfuls of flour in identical, clear containers. Shake the flour level and display the containers in a row. Ask: Is there the same amount of flour in each container? Why? Why not? Compare two students' measures of ten spoonfuls and ask: Why are they different? Ask both students to measure again. Ask: What is different about the spoonfuls?

Recipe Measurements

Provide some recipe books and ask students to find out the size of common measurements used in recipes (tablespoons, teaspoons, cups). Ask: How can you make sure that each tablespoonful is the right size? Ask: Why do recipe books give information about the measures they use?

Area of Identical Shapes

Have groups of students use pattern blocks to measure the area of identical shapes. Invite them to compare the results and the arrangements used. Ask: Does it matter if you have used a different number of blocks? How do the different-shaped blocks affect the area? How can you compare area using different-shaped blocks? Have students suggest measuring "rules" that would enable them to prove the shapes were all the same area; for example, say: Everyone should use the same type of pattern block. Do not overlap the edges. Have students test the rules by measuring again and comparing results. Add more rules if necessary and practise applying the rules for other area comparisons.

Oil Spills

Extend *Oil Spills*, page 130, by asking: Could you fit any more paper tiles onto the shape if you changed the way you have put them on? Do you have any paper tiles hanging over the edges? Are your squares all the same size?

Using Grid Paper

After area activities using repeated units (such as Oil Spills), ask: Would grid paper help? Have students superimpose the shape onto the grid paper. (Transparent grid paper is also very useful for this activity.) Ask: How is this similar to gluing paper tiles onto the shape?

Faster Runner?

Say: By counting handclaps, Jason and Semina measured how long it took each other to run around the building. Semina claimed to be a faster runner because she took 34 claps, while Jason took 40 claps. Is Semina right?

Sample Learning Activities

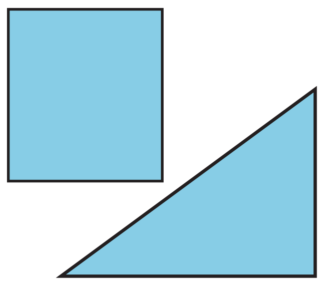
Grades 5–8: ★★★ Major Focus

Postage

Have students see that the instrument representing the unit must be of a consistent size when measuring. For example, say: Postage in an imaginary country is worked out according to how many dried beans it takes to balance the envelope or package that is being sent. Invite groups of students to set up post offices and use the beans to test the mass of sets of postal items (identical for each group). Discuss the reasons for the variation. Ask: Why do some groups end up with a different mass? What problems might there be in using beans? Why might it be unfair to use beans to measure the mass of postal items? What could the person in charge of the post offices do to make sure that all post offices charge the same amount for identical items?

Garden Plots

Ask students to compare two different-shaped garden plots, one a rectangle (11 cm by 12 cm) and the other a right-angled triangle (17 cm by 15 cm). Give them a range of things to choose from to use as their unit, including toothpicks, 1-cm cubes, 2-cm cubes, pattern blocks, rice, counters and string. After they have completed their measurements, have students who have used the same units compare their results. Ask them then to share their work samples and strategies with the class. Ask: Were your measures different? Why? For example, ask students who used 1-cm cubes: Were you able to measure all of the shape with the 1-cm cubes? What did you do with the leftover bits? Ask groups using the same unit to repeat the measuring process, making sure they match as many units as possible, but no more, until they get the same result. (See Case Study 2, Understand Units, page 51.)



Gaps and Overlaps

Extend the Garden Plots activity by asking: If you have gaps between your objects, have you overestimated or underestimated the size of the garden? How do you know? If you have overlapped your objects (or the edge), have you overestimated or underestimated the size of the garden? How do you know? Invite students to look at each other's work. In each case, ask: Has the size of the garden been overestimated? Why? Has it been underestimated? Why?

Fruit and Vegetables
Invite groups of students to choose one fruit or vegetable and agree on a unit to use to measure the surface area. Have students peel the item, spread the peelings as close together as possible on a plain sheet of paper, and trace around them. Ask them to make a copy for each member of the group. Have students use their unit (centimetre grids, dotted grid paper, centimetre squared paper, tiles, cubes) to measure the area, then compare measures within the group. Ask: Were all the results the same? Why? What did you do with the bits that were outside the square? Repeat the measuring in order to get a more accurate measure. Have the group with the smallest range in their measures share their strategies.

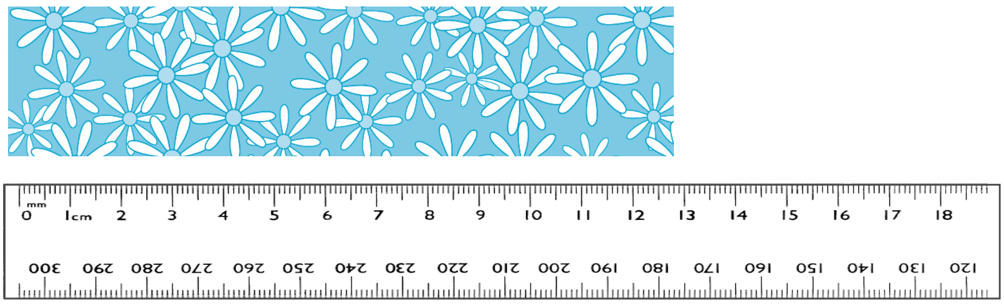
Gaps and Overlaps
Extend Fruit and Vegetables by examining work samples where there are gaps between the shape and the unit. Ask: Would this measure be an underestimate or an overestimate of the measure? Examine work samples where the unit overlaps the edge of the shape. Ask: Would these measures be an underestimate or an overestimate of the measure?

Treasure Hunt
Have students examine their measurement techniques in order to explain errors. For example, ask pairs of students to decide on the units and the appropriate instruments to design a treasure hunt around the school and then write distance and angle measurement instructions for another pair to follow to find the treasure. If the treasure is not easily found, decide if it was an error in the length or the angle measurement. Ask: How would you know which one caused the problem? For example, did you start and stop at the right spot with the trundle wheel? Did you use the correct angle of turn when changing direction?

Key Understanding 4

Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle, and time.

This Key Understanding is about the understanding and skill needed to use graduated scales (rulers, measuring jugs, kitchen scales, protractors, analogue clocks). This needs careful attention during the primary years. Students who are able to place centimetre blocks along a pencil and count "how many," may not be able to use even a simplified ruler marked in centimetres. Also, some students, who can apparently use a simplified ruler may do so by rote and not connect it at all with the process of using centimetre blocks to measure. A child who "forgets" to line up the beginning of an object to be measured with the zero mark on the ruler may well not understand the connection between measuring by repeating units and using a ruler.

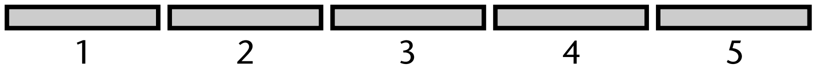


Simply reminding the student to "line up the zero" does not deal with the basic conceptual problem. In essence, what the student is doing is failing to repeat the unit without gaps; in matching the object with the units marked on the ruler, the student has left an unmeasured gap at the beginning of the object. Also, some students believe you have to start at one, as we do in counting, instead of zero.

To link a conventional ruler with units of length, students need first to move from using multiple copies of a unit of length (perhaps a rod) to work out "how many fit" to the idea that you can use one copy of the rod and mark it off along the object to be measured. From this, they should progress to making a tape measure using a unit such as the rod. They have to decide where the starting point is and realize why it is important to give it a label and why we label the ends of the units rather than the middle.

This:

and not this:



Students need to understand that the starting point shows the beginning of the first unit, which means no units used and so is labelled 0. The *end* of the first unit indicates one unit used and so is marked 1; the end of each unit marks the number of units long the object is. These are complex ideas, but they must be developed if students are to use calibrated scales effectively. Making their own ruler or tape measure should assist students to understand how calibrated scales are made and used. They should also make their own calibrated scales for capacity, mass, angle, and time. Calibrated scales do not have to be in standard units, but it may be helpful for students to graduate their own measuring equipment in standard units before using purchased equipment. For example, a 2-m height scale marked in centimetres and decimetres or a 10-m string marked at 10-cm intervals can be prepared by the students themselves, as can calibrated containers.

Links to the Phases

Phase	Students who are through this phase. . .
Quantifying	■ given a practical unit of length, capacity, or mass, can use it to construct their own ruler, calibrated container, or spring balance ■ can read graduated scales with every mark labelled, e.g., 1, 2, 3, 4, 5 . . .
Measuring	■ can read graduated scales with some of the marks not labelled, but each mark is one unit, e.g., 5, 10, 15, 20. . . ■ can read scales involving multiples of ten and simple decimals, e.g., nine marks are placed between each whole but are not labelled
Relating	■ can read whole and decimal number scales where the number of marks is fewer or greater than the number of units, e.g., every fifth mark is labelled 10, 20, 30. . . or every fifth mark is labelled 1, 2, 3 . . .

FSIM010 | First Steps in Mathematics: Measurement Book 1

Sample Learning Activities

K–Grade 3: ★★ Important Focus

Straw Lengths

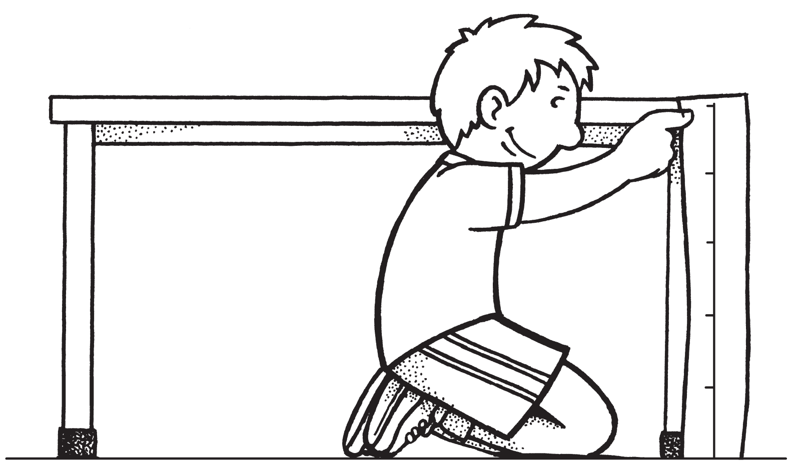
Have students each use one straw as a unit to measure paper strips that have been pre-cut to identical lengths. Compare any differences in results and talk about the various strategies used. Ask students to measure again, but this time have them mark the tape after each straw length to improve accuracy. Compare and discuss the consistency of the results. Ask: Why is it easier to check how many straws long it is when you mark each straw length as you go?

Stamps

Have students use the edge of an item (a block, an eraser, the end of a wooden ruler) to make "stamp tapes" for measuring length. Invite them to stamp a row of the units along a paper strip, making sure the stamps are just touching each other. Ask: Why do you need to be careful to have no gaps or overlaps if you are going to use the tape to measure how far you can jump? Organize students into groups and ask them to measure and describe lengths using their tapes (*I can jump nearly 30 block stamps and just over 46 eraser stamps*).

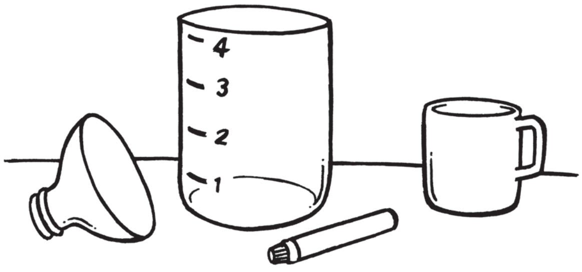
Measuring Vertical Lengths

Ask: How could you use craft sticks to measure vertical lengths like the height of the desk? (by gluing craft sticks onto a strip of card) Lead students to the idea that craft stick lengths can be marked on the card and numbered to save repeated lining up and counting, and have them make their own craft stick tape measures from strips of paper. Invite students to use their tape measure to measure lengths in the classroom and compare results. Ask: What makes it easier to use the card than the craft sticks? How do the numbers help?



Make a Measuring Jug

Have students each make a measuring jug by cutting the top from a clear plastic bottle and calibrating it in "cups." Invite them to pour full cups of water into the jug, marking the level after each cupful. Encourage them to name the jugs (e.g., the 2-cup jug). Ask students to use their jug to measure the capacity of containers, reading the scale. (*This jar holds a bit more than 3 cups.*)



Compare Measuring Jugs

As students use their own measuring jug in Make a Measuring Jug, focus on variations in the measures when two jugs have been made using the same cup. Ask: How could this happen? Invite students to watch each other make a new jug each and explain the variation. Encourage all students to recheck their use of units with a partner. Ask: Why might the marks not match when you put the two jugs together? How could the marks have ended up farther apart on one of the jugs? What do you need to be very careful about when using the cup as your unit?

Calibrate Containers in Litres

Extend Make a Measuring Jug and Compare Measuring Jugs to calibrating larger containers in litres. For example, ask students to mark the inside of a bowl with a permanent marker after each litre is poured in. Compare the students' version with buckets and pans that are marked in litres on their internal surface. Ask: How are the markings different? How are they the same?

Comparing Spring Scales and Balance Scales

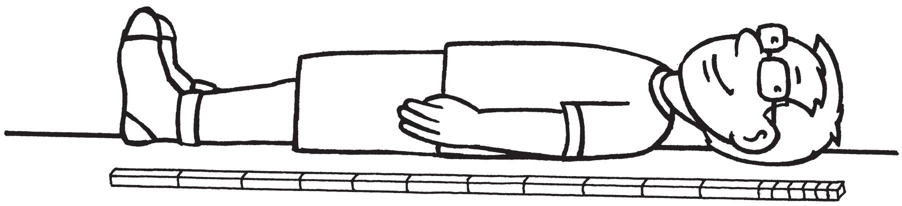
Ask students to place various packages or cans of food marked in gram weights (200 g, 250 g, 500 g) in opaque bags. Give groups of students different kitchen scales and spring scales and invite them to work out what the mass of the bag is and then to look in the bag to check their accuracy. Ask: How is the spring scale the same as the kitchen scale? How is it different?

Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Measuring Height

Have students measure their height by lying down and placing Base Ten longs and small cubes along their length. Ask them to mark each piece used on a strip of card and stick the strip to a wall or door. Invite students to measure and record their height in longs and small cubes again, this time using the marked scale on their vertical strip of card. Ask: Is there a difference in the two measurements? Why? Why not? What makes the vertical scale easier to use than the longs and cubes on the floor?



Using Different Tapes

Extend Measuring Height by asking students to measure their height using a different tape measure (i.e., another student's strip of card marked in longs and small cubes). Ask: Are your measures the same amount? Why? Why not? Where the two tapes vary in accuracy, ask: If you measured yourself on your tape this month and then on someone else's tape next month, how would you know if you have grown? How can you check the tapes to make them all measure the same height in longs and small cubes?

Toothpick Tapes

Have students use an informal unit (craft sticks, toothpicks) to construct a measuring tape, gluing multiple units onto a paper strip. Ask them to use this to measure each other and objects around the classroom, including curves and awkward measurements. (length of their arm, waist measurement, circumference of the garbage can, height of desks, height of shelves). Discuss the advantages and disadvantages of using this type of tape measure. Have students then make a second tape by marking off the length of each craft stick (toothpick). Invite them to decide where to put the numbers onto the tape. Draw out the conventional way to number the units from a zero starting point. Ask: What would happen if you wrote the number in the space rather than at the end of the unit? How is the second tape easier to use than the first? Encourage students to use their new tape to re-measure their objects. (See Case Study 2, page 152.)

Varying Measurements

Extend Toothpick Tapes by having pairs of students use their toothpick (craft stick) tapes to measure the same set of objects. Ask: Why do the measurements vary? Will this happen if we use a ruler or a dressmaker's tape measure instead? How are these measuring tools the same as (different from) your toothpick (craft stick) tapes?

Picture Frame

Ask students to use a ruler to measure around the edge of a picture in order to cut a length of card to frame it. Ask: Where do you start and finish each measurement? Which part of the ruler goes on the place that you start? How do you know how long each section is? Do the numbers label the lines or the spaces on the ruler? What do the numbers tell you? What do the small lines between the numbered lines mean?

Nails and Elastic

Ask small groups of students to weigh ten pennies by placing them in a bag on the end of a piece of elastic attached to the wall or bulletin board. Have them mark the mass on a strip of card taped up behind the "scales," then weigh 20 pennies (30 pennies, 40 pennies), mark the different positions on the card and use this calibrated scale to say how much other things weigh in "pennies" (the cup weighs the same as 20 pennies). Ask: What makes the calibrated scale easier to use than balance scales? What if you lost the pennies? Could you still use the balance scales to weigh in pennies? Could you still use your calibrated scale? What would you need to use if your object weighed much more than 40 pennies? What would be the limits of your calibrated scale compared to the balance scale?

Broken Ruler

Have students use a paper representation of a broken piece of ruler with only the 6-cm to 10-cm section to measure several small items. Invite students to discuss the technique they used. Ask: Which part of the scale represents the centimetre lengths? (the spaces, not the marks). What would be the longest object we could measure without needing to move the ruler? What about if we included the millimetres?

Grades 3–5: ★★★ Major Focus

Measuring Bottles

Ask students to make their own fluid measuring bottle from a plastic drink bottle using cupfuls as the unit and marking off the height of each one using an elastic band. Invite them to mark lines on a paper strip to match the elastic bands, place the strip on a different container and test whether the lines work for that container. Ask: Do the lines on the strip work for the new container? Why? Why not? What difference does it make if one of the bottles is wider than the other?

Labelling Litres

Have students calibrate unmarked 10-L plastic buckets in litres by pouring in the bottom 10 cm of a 2-L milk carton (approximately 1-L) and marking the level inside the bucket after each one. Discuss whether or not every litre mark needs to be numbered. Ask: What if we only labelled every 5 L, how would you measure out 7 L of water?

Did You Know?

You may not want to use a 1-L container for labelling litres because the container may actually hold more than one litre of liquid. Instead you may wish to cut the bottom 10 cm from a 2-L container so that measurement is close to 10 × 10 × 10. It is important for students to make the connection between volume (1 dm^3) and capacity (1L). Later they can weigh the litre of water to find its mass (1 kg).

Sample Learning Activities

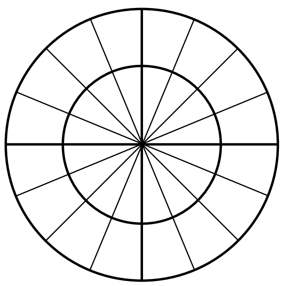
Grades 5–8: ★★★ Major Focus

Calibrating Containers

Have students make a container to measure small amounts of liquid. Show students a container calibrated every 250 mL. Ask: Can you tell me what these marks mean? Would it help you to accurately measure how much this small container holds? Why? Why not? Invite students to use materials (small cone medicine measurer, medicine spoon, ruler, disposable cups) to calibrate the container to make it a more suitable measuring tool.

Angles

After comparing the size of angles directly (by placing the corner of a book on the corner of a desk to check they are the same angle, opening a compass to match how far the door is open), ask students to construct a cardboard "protractor" using a paper plate. Have them fold their plate in half, then repeatedly in half to form 16 equal angles in the circle. Invite them to label the "angle units" (i.e., the 16 equal sections of the circle) and use the instrument to measure and draw representations of angles of various numbers of angle units (e.g., 15 angle units, 2 angle units). Ask: How can you label your protractor so that the turn can be measured in either direction?



Two-Metre Tape

Have students construct a 2-m measuring tape using Base Ten longs to calibrate the tape in metres (ten longs), decimetres (one long), and centimetres (tenth of a long or small cube). Invite students to choose a labelling system to use on their tape. Ask: Is it necessary to label every marking? Where will you write the numbers for the decimetre lengths? What numbers will you write at the 1-m and 2-m markings? Invite pairs of students to measure various distances and circumferences, comparing the results obtained by their two tapes. Ask: Did you get the same measurement on each tape? Why? Why not? Why might your measurement differ? How can you check that your measuring tapes match each other?

Grades 5–8: ★★★ Major Focus

Marks not Numbered

Have students read graduated scales where the marks are not all numbered. For example, give students measuring cups, jugs, or cylinders and quantities of liquid to measure that will end up on the un-numbered calibrations. Ask: What do the marks between the numbers represent on your container? How do you know? How did you use the marks to work out how much water was in your container?

Reading Between the Lines

Extend Marks not Numbered by having students attempt to measure out quantities of liquids that come between the marks on their measuring instrument. Ask: How do you know it is 325 mL when the beaker is calibrated every 100 mL? (It is a quarter of the way up the space between the 300-mL and 400-mL marks and I know 25 is a quarter of 100, so it is 325 mL.) How can you use your medicine cup to measure out 7 mL when there are no marks between 5 and 10 mL? Why does the cone shape of the medicine cup make it harder to judge the "in between" measures accurately?

It Needs Fixing

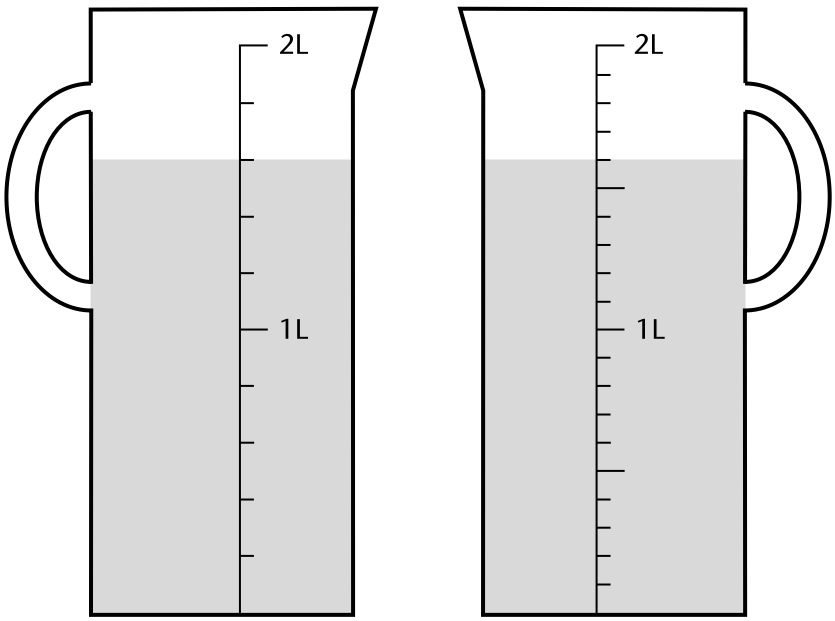
Have students calculate to address inaccuracies in equipment. For example, say: Our tape measure has stretched so that when it says 1 m, the object is really 1.2 cm longer. Ask: What would be the real length of a table that the stretched tape shows as 1 m (the room that measures 4 m, the chair that measures 50 cm, my desk that measures 1.5 m)? After students have worked out their responses, invite them to work together to find a strategy for checking the correct lengths. For example, they might cut a length of paper tape 1.2 cm longer than a metre to use as a stretched metre tape and measure out the incorrect lengths and then check them with a standard tape measure to find the true lengths. Ask: Does the stretched tape overestimate or underestimate the true length?

Slow Clock and Heavy Scales

Extend It Needs Fixing by asking students to make adjustments for other calibrated scales. For example, say: Our clock is 5 min slow (fast). Ask: What should we do to work out what the right time is? Draw out that the situation requires addition (subtraction). Then, say: The bowl on the kitchen scales adds 100 g to the mass. What should we do to work out the mass of what is in the bowl? Draw out that subtraction would be needed.

Calibrated Scales

Use an overhead projector to show a drawing of a measuring jug containing liquid. Place a scale on it, showing five calibrations between each whole number of litres. Ask students to record how much liquid there is. Then, remove the scale and replace it with one that has ten calibrations between each litre. Ask students again to record how much liquid there is. Some students are likely to have written different numbers (1.3 for five gradations and 1.6 for ten gradations). Ask: Can both answers be right? Use the conflict between answers to generate discussion of the meaning of the gradations. Have students draw similar measuring jugs calibrated in different scales, then exchange the drawings and show the liquid levels on the jugs. Have students exchange again and work out the amount of liquid in litres. Ask: How did you work out the parts of a litre? Which scales were easiest to read? How can you be sure your reading is correct?



CASE STUDY 2

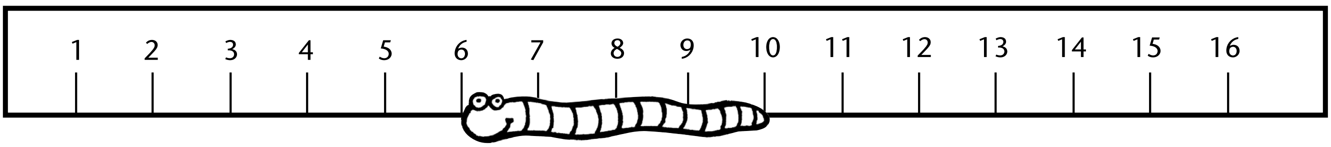
Sample Learning Activity: Grades 3–5—Toothpick Tapes, page 146

Key Understanding 4: Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle, and time.

Working Towards: Matching and Comparing Phase, and Quantifying Phase

TEACHER'S PURPOSE

My Grade 4 students found the following drawing in an old math book.



They all agreed the worm was 5 cm long! Probing revealed that the students were counting the marks, not the spaces between the marks. None thought to find the difference between 6 cm and 10 cm. They had not connected the calibrations on their rulers with the way we use physical units during length measuring activities. I decided I needed to help them make the link.

The probing questions included:
How did you work it out?
What did you count?
Where did you start your count from?

ACTION AND REFLECTION

I asked students to make a tape measure from paper tape using their own choice of a unit. I provided craft sticks, straws, toothpicks, glue, and paper tape.

They began gluing units along the tape, lining up their chosen units end-to-end without leaving spaces or overlaps. After a few minutes, Alex and Yenchee came and asked, "Couldn't we just use a pen to mark where each craft stick comes to?"

I was pleased to agree to this and thought it might be useful later in the lesson. When the tapes were dry, the students tried them out, measuring many objects in the room and around the school.

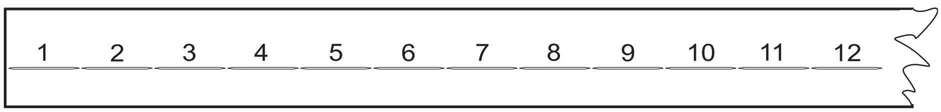
As a class, we then talked about the effectiveness of the measuring tapes. I was able to draw out the many disadvantages of having the actual units glued to the tape. Students made comments like, "Using craft sticks was good for straight things, but they would not bend and the paper tore when we tried to measure around the tree" and "Some of the sticks fell off when we used ours."

Then, Sharla said, "It did not really matter, because we could see where they had been stuck on and we just counted where they had been."

This seemed the perfect opportunity to ask Alex and Yenchee to show how their tape was marked. Everyone thought this was much better than using the units themselves. I asked if anyone had difficulty counting the units as they were measuring.

Daniel said, "Ours was toothpicks and Ben kept losing count, so, in the end, what we did was write the numbers next to the toothpicks so we did not have to count."

Several others had also done this, but I noticed that they had written the numbers next to the units.



I decided that this needed to be the focus of my next lesson.

Drawing out the disadvantages:
What problems did you have using your tape for measuring curved things?
Did you have the same problems when you were measuring straight things?
Did it matter that your craft sticks fell off?
What did you do when your tape ripped?
Did that change the length of the craft stick unit?

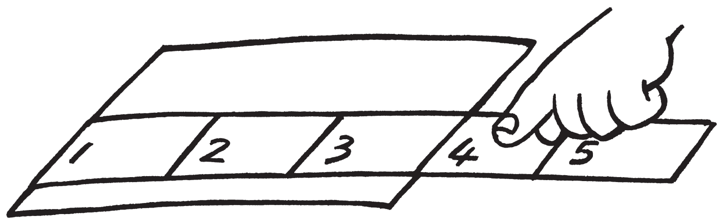
CONNECTION AND CHALLENGE

The following day, I asked everyone to use what they had found out and create a new "improved" measuring tape using an agreed unit. We decided to make a "toothpick measuring tape." All took care to accurately mark the length of each toothpick on their tape, showing they had understood how the markings represented the units. When it came to numbering their scales, however, many wrote the numbers next to the space where the toothpick had been, not close to the end marks. Several students had written "1" at the very beginning of the tape. There were a few who numbered their tapes in the conventional way.

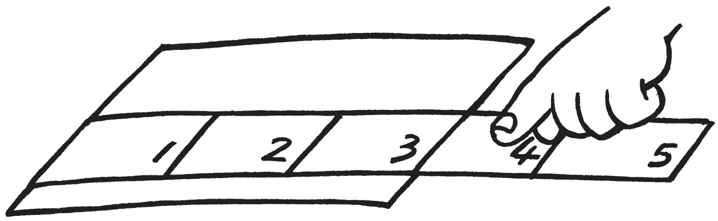
Because I wanted to help students see for themselves how their unconventional markings could cause difficulties, I asked students in pairs to use their tapes to measure the same objects and talk about any differences they found. To maximize the chance for conflicting results, I purposely paired students who used different numbering methods.

This also helped me to see how individuals interpreted their own tape markings. I overheard Angie complain to Mohammed that he had written four where the three should be. Mohammed argued, "But that is for that toothpick, that four is for that toothpick" (pointing to the fourth space).

To facilitate the discussion I asked:
Show us how you used your tape to measure this book.
How did you know how big the book is?
Did the number help?
If the end of the book was not on a mark, how did you know which matchstick to finish your count on?
Can we say seven-and-a-half matchsticks long?



Mohammed's Ruler



Angie's Ruler

Angie was not satisfied and continued to insist it was misleading to do it that way. I called the class together at that stage and talked about everyone's experiences. I asked Angie to explain how her tape worked.

"I just line it up and look at the mark and the number is just there. See, it is three toothpicks long."

DRAWING OUT THE MATHEMATICAL IDEA

I asked other students to demonstrate how their tapes worked, including Mohammed. It soon became obvious to all the students that positioning the numbers at the right-hand end of each unit space (rather than within it or at the beginning of it) made it easier to keep track of the number of units and eventually everyone agreed this was the best strategy.

I was now confident that the students knew the value of marking off the length of their unit and using a number for each mark, but I was not sure that they had connected this with the way a standard ruler works. I decided that this would be the focus of tomorrow's lesson.

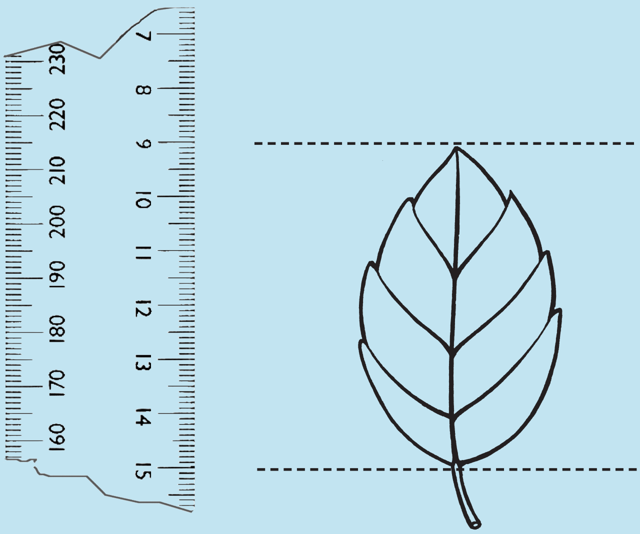
Did You Know?

A diagnostic activity for the Grades 3–8 students and beyond

Seven hundred students in Grades 5–10 were asked if they could tell how long the leaf is from the broken ruler. The illustration was accurate; that is, the distance between the centimetre marks was actually one centimetre on the diagram.

A small minority of students said 15 cm. Almost half in Grade 5, one third in Grade 6, and one quarter in Grades 7 and 8 said the leaf was 7 cm long.

One wrote: Starting from 9 cm (since it's a broken ruler), I counted 9 cm as 1 cm, 10 cm as 2 cm, 11 cm as ...



Another said: *You look at the 9 as a one and 15 is 7.*

Some even set up a table showing 9 = 1, 10 = 2, 11 = 3, and so on, to "prove" their answer.

Most who correctly said 6 cm counted the centimetres or the spaces.

I got the answer by counting in between 9 and 10 and then 10 and 11 until I got to 14 and 15 and got the answer 6.

Very few were able to think of the 9 as a zero point or use the difference between 9 and 15 to find the length.

Many students experienced conflict, saying: *It could be 6 or it could be 7 depending on how you do it.*

For example, one Grade 7 student came back to his teacher three times in the space of half an hour, asking to change his answer between 6 and 7. The conflict helped this student. Finally, he came back triumphantly and said: *It has to be 6 because you need to count 10 as 1, not the 9, 9 is just where the first centimetre starts.*

Key Understanding 5

Units are quantities and so we can use different representations of the same unit so long as we do not change the quantity.

As suggested in Key Understanding 3 in Understand Units, units are quantities; thus, 1 cm is an *amount* of length and 1 m^2 is an *amount* of area. The same unit of area might be represented by a leaf, a triangular tile, or square grid paper. In classroom activities, however, we often refer informally to the object chosen to represent a unit as though it was itself the unit. We talk about craft sticks and hexagons as though they are units, when we really mean the length of the craft stick or the area of the hexagon. When students use blocks as measuring instruments, for example, they may use the mass of the block as the unit for deciding how heavy something is, the length of its side for deciding how far away something is, and the area of a face for comparing the area of two leaves.

This distinction seems subtle, and the language needed to make it explicit is quite complex, so the informal way we refer to objects as though they are units is understandable, especially in the early years. Nevertheless, when we ask students to make sure that they choose a unit that will tile without gaps and overlaps, it is not surprising that they could think that two different shapes are two different units. If they have the same area, however, the two shapes are the *same* unit in different forms. Some *forms* are easier to use because they enable us to make a good match with the object to be measured. If students persist for too long with the idea that the particular objects or shapes are the units, then they may have difficulty progressing in their understanding of measurement.

For example, a length unit may be made from rubber and curved as needed; an area unit might be cut and rearranged. In order for students to make sense of this, they need to understand that units are really quantities and not objects. The form or look of the thing being used as a unit can change so long as it does not alter the relevant quantity. Students need to understand *why* a square metre can be cut to fit it into a shape when measuring area and a bit of play dough can be rolled and still be used as the same unit of mass, but not as the same unit of area. As indicated in the Background Notes on page 104, finding the area or volume of objects by direct measurement is more complex than finding length or capacity, both practically and conceptually, and needs careful development.

Links to the Phases

Phase	Students who are through this phase. . .
Quantifying	■ use uniform units to measure quantities that are uni-dimensional, such as length, capacity, and mass, as well as angle and time ■ use uniform units of area, although they may struggle with what to do along the edges when covering regions ■ may try to use some part-units of area, but may not be successful when it requires them to combine part-units
Measuring	■ use and combine part-units of area and can therefore count units and part-units to find the area of any region ■ count units of volume in straightforward cases

Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, and Extension

Pliable Units

Have students investigate situations where it helps to use pliable units such as pipe cleaners to measure length. Invite them to choose objects to use as units to measure lengths (distance around a ball, distance around a picture frame, distance around their shoe or footprint). Ask: What changes when you bend the pipe cleaner? What stays the same? How can you check that the bent pipe cleaner is just as long as the straight pipe cleaner?

Trundle Wheel and Metre Rule

Ask students to compare the length around a trundle wheel to a metre rule. Use each to make a 2-m chalk line on the playground and compare the two lines. Organize students into groups and ask some to use a metre rule and some to use a trundle wheel to measure the markings on the basketball court. Ask: What was easy (difficult) about using a metre rule? What was easy (difficult) about using a trundle wheel? What do you need to be careful about when using the trundle wheel? How did you keep track of where to begin the next metre when using the metre rule?

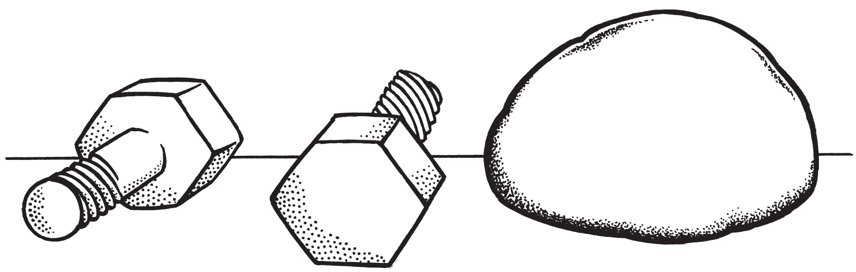
Play Dough Balls

Invite students to make play dough balls of equal size and use them to compare the capacity of margarine and yogurt containers. Ask: How many balls fit into each container? Does it make a difference if you squash the balls to fit them in? Why do more fit in when you squash them up? Which container will fit the most balls? How do you know? What changed about the balls when you squashed them? What has stayed the same?

Covering the Desk
Ask students to find out how many whole pages of 8 1/2 × 11 paper fit onto their desks without any spaces between or overlaps at the edges. Ask: Are there any parts of the desk still not covered? If you cut up sheets of paper, how many more sheets do you think you would need to fill in the gaps? Invite them to take that number of extra sheets and cut them to fit one at a time to test how many they really need. Ask: How many pages altogether did you need to cover your desk? What changed and what stayed the same when you cut up pages to fit? (The shape changed, but the quantity of paper was the same.)

Leaves
Invite students to trace around a leaf and arrange whole 2-cm-squared tiles in ways that get the most into the area of the leaf. Ask: How could you fit another tile in? Will cutting it up help? Encourage students to keep track of how many tiles they have used with the fewest gaps, no overlaps and filled as close as possible to the edge of the outline of the leaf. Later, when students have become very careful about leaving no gaps or overlaps, draw attention to the way some have arranged tiles to make the job easier (e.g., rows and columns).

Bolts and Play Dough
Have students make collections of uniform units using different objects. For example, provide two bolts and a large ball of play dough for students to use as units to match the mass of objects that are heavier than two bolts. Ask: How can we use the play dough to make another thing to use that will be the same mass as a bolt? Encourage students to use the balance scales to make balls of play dough equal to a bolt.



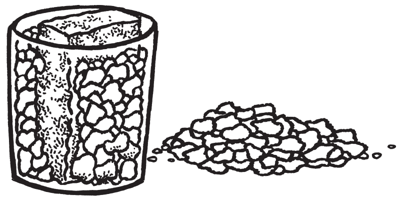
Craft Sticks
Extend Bolts and Play Dough by asking students to find objects equal in length to a craft stick and make a collection of them to use as representations of the same unit. Ask: To find out how wide your desk is, would it matter if you started measuring with a craft stick and then changed to one of the other objects? Why? Why not?

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Breakfast Cereal Biscuits

Have students decide how many units of shredded wheat breakfast cereal fit into a container without leaving spaces. Ask: What did you need to do to some of the shredded wheat to fill the spaces? What changes when you crush one to fit it in and what stays the same? Is it okay to count the crushed one as "one shredded wheat?" Why? Are we still finding out how many shredded wheat biscuits will fit into the container?



Awkward Lengths

Ask students to use their rulers to measure some awkward lengths (e.g., distance around the table leg). Then have them copy the ruler (its centimetre unit markings) onto a paper tape and measure again. Ask: How is the "tape ruler" the same as (different from) your ordinary ruler? Where would it be useful to have a flexible ruler? Compare this tape to dressmakers' and builders' tapes.

Toy Wheel

Ask students to use a paper strip to measure the distance around a toy wheel. Have them straighten out the paper strip to establish the circumference of the wheel as a unit of length measure. Then, invite students to use the two forms of the same unit to measure a length and compare the results. Ask: What was different (stayed the same)? Have students compare a trundle wheel to a metre rule and a metre tape measure. Ask: Where would each one be used?

Teaspoons in a Jug

Give students water, teaspoons, cups, and jugs and ask them to work out how many teaspoonfuls of water fill a cup and how many cupfuls fill a jug. Ask: How many teaspoonfuls fill the jug? How does the cup help you to know how many teaspoonfuls fill the jug?

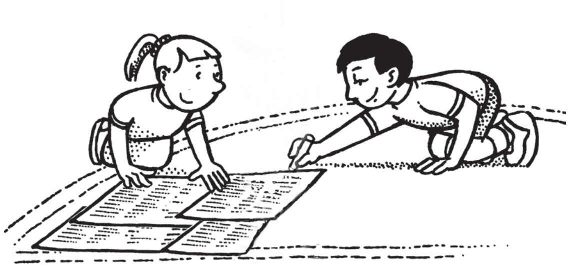
Play Dough Balls

Extend *Play Dough Balls*, page 158, by asking: If you squash them into the container have you changed the size of your unit? How do you know?

Mass Units

Give students balance scales and pattern block hexagons as mass units to weigh a variety of objects. Ask: What could you do if you ran out of hexagons? Could you use triangles or blue rhombuses instead? How many triangles would you need to match the mass of one hexagon? Encourage students to explore the relationship between the mass of hexagons and the other pattern block pieces. Ask: What if we mixed plastic and wooden pattern blocks? How can you check if that would make a difference to your measurements?

Centre Circle



Have students use chalk and a metre square made of newspaper to find the area in square metres of the centre circle on a basketball court. Encourage them to mark out the full units that will fit inside the shape and then discuss how they will measure the remaining spaces (make more metre squares and cut them up to fit, or count imagined cut-up shapes). Ask: How can you be sure the part pieces put together can be counted as whole pieces?

Roll-On Lawn

Say: The gardener wants to plant "roll-on" lawn over a rectangular area 11 m by 15 m, but the turf only comes in 2-m squares. How many should she buy? Give students 2 cm^2 paper tiles and a rectangular piece of paper 11 cm by 15 cm to make the calculations for the gardener. Ask: Is a strip of turf 1 cm wide by 4 cm long the same size as 2 cm^2? Explain how you know.

Swimming Pools

Have students draw different shapes that have the same area. Say: A swimming pool company has advertised a rectangular-shaped swimming pool that has a water surface area of 18 m^2, and has said they would build it for the same price in any shape. Give students a three-by-six grid drawing of the original pool design, with each square representing 1 m^2. Ask them to design the pool. Ask: How do you know your pool is equal in area to the original rectangular pool?

Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Food Packages

Provide students with a range of empty food packages and bags of products that are sold by mass. Ask them to fill the packages so that they weigh the indicated amount. Ask: There are not enough gram weights, so what objects could you use to represent units of measure? Invite students to make up a range of measuring instruments that represent 100 g, 50 g, and 5 g. Ask: How is it that such a small bag of play dough represents the same mass as a large bag of toothpicks? What are the different ways you can measure 750 g of rice into the package?

Covering One Square Metre

Ask: What different shapes can you make that cover one square metre? Can you make a circular square metre, a triangular square metre, a diamond square metre? Organize students into groups and have a competition to see which group can make the longest square metre. Ask: How can you prove that your shapes cover exactly one square metre?

Shadows

Have students see that it makes sense to combine part-units into whole units when measuring area. Ask pairs of students to draw around their shadows and use Base Ten flats or decimetre squares of paper to measure the area in square decimetres. Ask: How did you deal with the parts of the shadow not covered by the decimetre squares? What did you do with the part-units? Did the area of the shadow change when you combined the part-units? Why? Why not? How do you know you have measured all of the area of the shadow?

Measuring the Spaces Leftover

Have students use 10 × 10 × 10 cubes cut from 2-L milk cartons to find the capacity of large boxes. Discuss strategies for finding how much space is left along the sides. Ask: How can you find out how much space is left along the sides? Could you use materials that pour, like wheat or rice, to measure the spaces left? How would you then work out the capacity of the box?

Irregular Bottles

Ask students to mark a scale on an irregular clear plastic container (e.g., window cleaner bottle) by pouring in equal measures of water. Invite them to compare the distances between the marks. Ask: Why are they different? What changes and what stays the same? Show students a range of standard capacity measuring equipment of different shapes and sizes. Invite them to compare the distances between units on the scale markings. Ask: Why are they different? How can the scales for capacity vary so much while the scales for length always have the same distance between units?

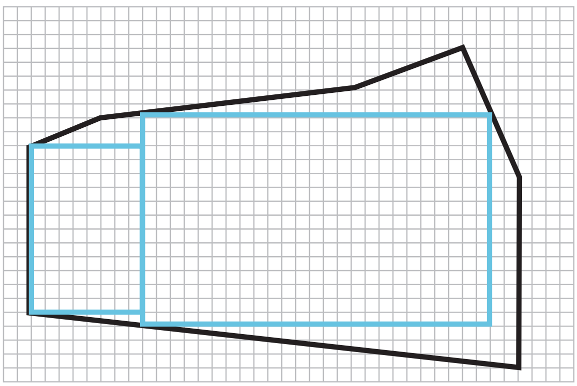
Same or Different Measures

Have students say whether different representations of measures describe the same measure or different measures. For example, ask: Which of these pairs describe the same measure?

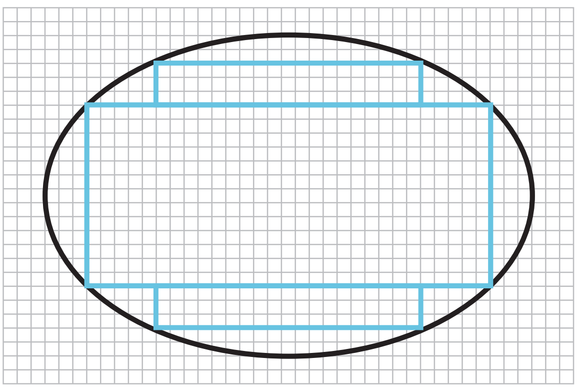
34 km	34 000 m
3 kL	300 L
2.5 cm	25 mm

Irregular Areas

Have students use arrays within irregular shapes in order to work out the area. For example, say: The farmer needs to calculate the area of the pasture in order to work out the amount of seed required. The council needs to calculate the area of the swamp to calculate the amount of spray for mosquitoes. Invite students to work out the areas using a centimetre grid overlay or grid paper to help them see arrays within the irregular shapes. Ask: Can you see any arrays in the shapes? How can the arrays help you to work out the area? How did you deal with the parts outside of the arrays?



Farmer's pasture



Council's swamp

Grades 5–8: ★★★ Major Focus

Decimetre Squares

Provide students with decimetre squares of coloured paper and have them choose items around the room that have a surface area more than (less than, the same as) one square decimetre. Encourage students to cut up the decimetre squares and rearrange the pieces to match objects that are near to a decimetre square, but very different in shape (a ruler, a card, a pencil case, the sole of a shoe). Ask: Is the surface on the card the same amount as your decimetre square, or is it larger? What changes and what stays the same when you cut up the square? What shapes can measure a square decimetre? (See Case Study 3, on the opposite page.)

Same Volume

Have students represent the same volume of material in different ways. Invite them to make a cubic decimetre of play dough using a Base Ten cube as a model, then pack the play dough into a 1-L milk carton or into 10 × 10 × 10 bottoms of 2-L drink containers. Ask: What do you notice? Have them repeat the activity, making different volumes of play dough (e.g., 200 cm^3 using two Base Ten flats as a model) and a calibrated measuring cup to find out how many millilitres it measures. Work back the other way so students work out the cubic measure from a certain mL measure. Ask: What links have you found between volume (cm^3) and capacity (L and mL)? Does it matter whether you use cubic centimetres or millilitres to say how much play dough? Does it make sense to describe the capacity of a cup or an amount of liquid in cubic centimetres? (Yes, because it is the same measure of volume. However, it is important to note that we usually use capacity measures for liquid volume.)

CASE STUDY 3

Sample Learning Activity: Grades 5–8—Decimetre Squares, page 164

Key Understanding 5: Units are quantities and so we can use different representations of the same unit so long as we do not change the quantity.

Working Towards: Quantifying Phase and Measuring Phase

TEACHER'S PURPOSE

The students in my Grade 5 class had grouped objects by whether their surface areas were more than, less than, or about equal to a square decimetre. They used 10 cm by 10 cm coloured paper squares and filled in a chart as they made their decisions.

Less Than	About Equal	More Than
sticky label eraser pencil roll of sticky tape	pencil case	my desk dinosaur book worksheet

They tended to choose things like erasers, which fitted within the paper square, to put in their *less than* column and obviously large things to put in their *more than* column. The *about equal* column was practically empty. Not surprisingly, they had difficulty deciding when surfaces were about the same size, but they also had difficulty with some shapes. My concern was that they might have been thinking that a square decimetre of area had to actually be square or be able to contain a square of side one decimetre. That is, they might think of it as a shape, not a size. I decided to provoke some conflict about this.

CONNECTION AND CHALLENGE

I asked the students to categorize things closer in size to a square decimetre, but in a range of shapes.

I was not really surprised when Elliot said, "How are you supposed to tell? It will not fit in. There are bits sticking out."

"Hmm, I wonder what we can do about that," I responded.

"That bit of my yellow square might fit on that bit of the lid, but I am not sure."

"Why don't you cut up your decimetre square to find out?"

That seemed to help Elliot and others. There was a hive of activity and most were able to classify the objects, once they started cutting and rearranging their squares. Hannah had cut a decimetre square and placed it neatly on a birthday invitation, but had not written anything. I asked, "So, is that card more than, or less than, or the same as a square decimetre?"

"Well, I can't tell because that's not a square decimetre any more."

"Why is that?" I asked.

"Well, it is not square any more."

I wondered how many other students might be thinking that because the decimetre square no longer looked like a square it was not the same size as the original square or was no longer the same unit of measure.

REFLECTION

I asked the students to select one of their objects and explain what they had found. Jason said his plastic ruler was more than 1 dm^2. He used an uncut decimetre square to show us how he cut it up to place it on his ruler. There was still some ruler showing. "This ruler is more than one square decimetre."

I asked the other students what they thought. Most agreed, but not Kelly.

"But the ruler is too small and no square decimetres fit in."

"But it is still the same amount of space," claimed Jason.

"Well, maybe, but how can you say it is a square decimetre when it is not square any more?" said Kelly, with an air of having the winning argument. "It is more like a rectangle now. Maybe we should call it a deci rectangle."

I arranged the pieces that Jason had arranged on his ruler alongside the original square, "So, what is different?"

"Well," said Belinda, "it is not the same shape. Before, it was square, and now, it is a rectangle."

Dane said, "The lengths of the edges have changed. The edges on the decimetre square are 10 cm, but now it is … (he measured Jason's long rectangle) it is 25 cm long and 4 cm wide."

"But it is still the same square. It is just cut up and put together differently," said Jason. "It takes up the same room. It is just moved around."

"So, what is the same about the two shapes, then?" I asked.

Students started saying things like "the surface," "the space," and "the area."

DRAWING OUT THE MATHEMATICAL IDEA

To help students understand that changing the shape does not change quantity, I got a piece of string and showed the students that I was cutting off a 20-cm length. I stretched it into a straight length and asked how long my string was. They answered appropriately.

I made it into a square shape. "How long is my string now?"

I cut it up and asked, "Is there still 20 cm of string?"

They all agreed that there was. I then said, "The size of the string stays the same no matter how you arrange it."

"Yeah," said Jason, "that's what happed to my yellow square. It started out as square, but it has just changed the shape. It is still the same size."

After that, we continued around the group with students saying whether their surface was less than, more than, or about a square decimetre and why. I focused them on the fact that a unit was an amount or quantity of area and so a square decimetre was an amount: "Is that surface on the card the same amount as your decimetre square or is it larger? How many square decimetres is it?

Did You Know?

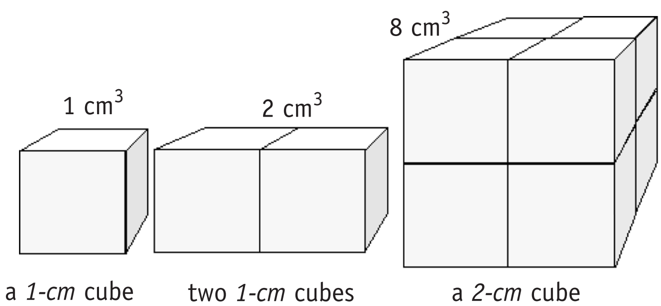
Students may be confused by "cubic," "cube," and "cubed." This isn't surprising, given the complexity of the language.

2 cubic centimetres (2 cm^3) or *2 centimetres cubed* is a quantity, so it may be any shape and does not have to involve actual cubes

2-cm cube refers to a cube measuring 2 cm on each side

2 centimetre cubes is ambiguous, with a slightly different voice emphasis it might mean *2 centimetre-cubes* or *2-cm cubes*.

These subtleties need to be made explicit over time and students given plenty of practice in using the language for themselves. They should be encouraged to seek clarification when meaning is ambiguous.



Key Understanding 6

We can judge and measure time using both natural cyclical changes and special techniques and tools which people have developed.

The measurement of time has always been of concern to people. Communities past and present have used a combination of cycles in natural events (the passage of the Sun, tides, plants) and technologies (a variety of forms of clocks and calendars) to enable them to determine both "what time (season, of the day) it is" and "how much time has passed."

During the primary years, students should develop the basic ideas and skills associated with measuring time (e.g., what time it is) and elapsed time (how long it took). The standard Canadian method of measuring time is based on the idea that each moment has a particular unique time associated with it. We indicate when we were born by stating the year, month, and day and sometimes also the hour and minute of day. In order to be able to specify time uniquely, we choose an arbitrary starting point and units that are used to mark off time from that starting point. The units we use for describing what time it is, was or will be, are related to the recurring or cyclical nature of physical phenomena that produce day and night, the waxing and waning of the Moon, and the seasons. Many social and cultural practices relate to these recurring physical phenomena. So, within particular communities, it is possible to guess what time of day, week, or year it is from what people are doing. Conversely, it is also possible to say what people are likely to be doing from the time of day, week, or year. While students will not develop a sophisticated understanding of the cause of these cycles during their primary schooling, they should develop a sense of the recurring nature of phenomena that are familiar within their local and extended communities.

Links to the Phases

Phase	Students who are through this phase. . .
Emergent	■ use words that relate to time like before, now, after, day, and night, and link the passing of time with repeating events in their own lives (getting light, the trains start, saying prayers) ■ place regular activities, such as getting dressed, in typical orders ■ know clocks are used to tell the time of day and that the hands move or the digits change as time passes
Matching and Comparing	■ order familiar events in their lives into typical sequences and use regularly occurring events as cues to the time of day or year (the position of the Sun, the noise chickens are making, the number of cars in the parking lot) ■ have begun to use calendars and to read the time on clocks, although they may not have fully developed the clock-reading skills ■ recognize key times on an analogue clock and can link it to typical activities
Quantifying	■ understand the difference between "what time it is" and "how much time it took," although they may still believe that the person who arrives latest is also the one who took longest to get there ■ read the time on clocks (digital and analogue), calendars, and straightforward timetables and schedules (the television guide, the program for athletic events)
Measuring	■ prepare feasible timetables and use timetables and programs that may involve 12- or 24-hour time (e.g., to set a video recorder in advance) ■ investigate different ways people have measured and recorded time *For example*: A student may describe the calendars and timepieces of the cultures of origin of various students in the class.
Relating	■ interpret even complex timetables and schedules as long as they are familiar and relevant to their own lives

Sample Learning Activities

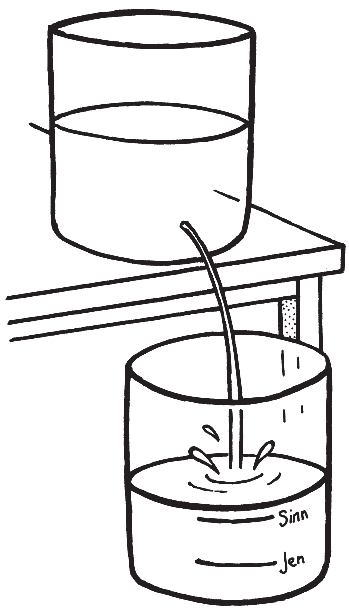
K–Grade 3: ★ Introduction, Consolidation, or Extension

Races

Involve students in different types of races in order to directly compare the duration of time for various events (egg-and-spoon race, three-legged race, skipping race, hopping race). Draw out any rules that may be necessary to ensure fair comparison of time (the same starting point, the same finishing point). Ask: Do you all have to start at the same time? What about the timer? Focus on the language of comparison (less time, more time, took a shorter amount of time, took a longer amount of time).

Water Timer

Invite students to make and use a water timer to compare the time they take to do a task that cannot be performed simultaneously. For example, ask: Who is the fastest to collect the flags from the buckets and place them in this bucket? Invite a student to complete the task. Begin the timer when the student starts and mark the level in the bottom container when the student finishes. Write the student's name next to the mark and invite another student to try, starting the timer again and repeating the process. Ask: How do we decide who is faster from the two marks?

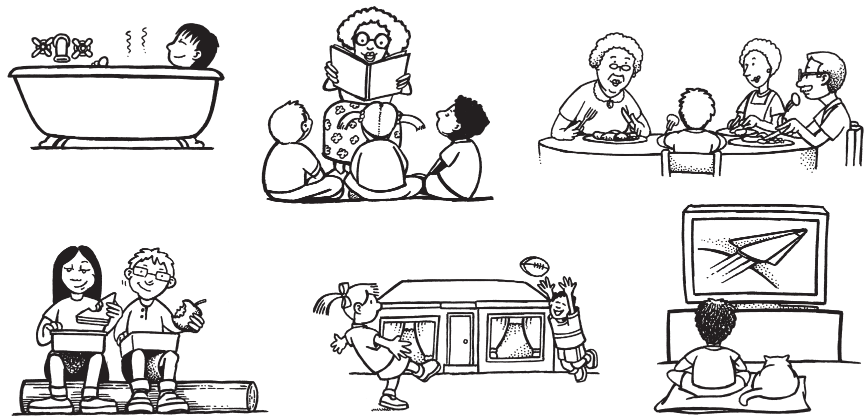


Timing a Run

Have students compare times. For example, have one student run to an agreed spot while a partner counts to 50. Before they begin, ask: Do the runner and the counter need to start at the same time? Invite the rest of the students to choose a partner and try it out. Talk about variations and consistencies of results. Repeat the activity, using other lengths of time (singing songs, eating an apple, building a tower). Ask: What difference did your speed of counting make? How did you make sure you both began at the same time? Why is that important? How do you know who took less time?

Daily Events

Have students sort illustrations of daily events according to the part of the day in which they occur in their families. (*I have put the picture of children in the bath at night because I have a bath before I go to bed, or I have put it in the afternoon because I have my bath before supper and it's not even dark then.*) Invite students to identify activities that most of them do at similar times of the day and those that vary a lot. Ask: Are there activities that everyone does at the same time? Why do you think that is?



How Long to Go

Incidentally discuss time measurement during other activities. For example, say: It is 5 min to recess, do you think we have enough time for five more questions? How many could we do? Or, say: There are 10 min to go until lunchtime and we have eight more pages to read. Can we finish the book? Invite students to monitor progress using the clock (e.g., *Five minutes have gone past and you have read five pages. I am sure you will finish now because you have got 5 min to go and only three pages left.*)

K–Grade 3: ★ Introduction, Consolidation, or Extension

How Long it Takes

Have students find out how long it takes to do certain things (taking their shoes off, getting a drink). Use different units to measure the time (hand claps, counting, making tally marks). Ask: Which methods were best for short time intervals? Which methods were best for long time intervals? Were some better for making comparisons than others? Why might counting be better than hand claps?

Time Line

Have students make a time line of events that happen on a particular day in class, taking special notice of the order of events, and the start time and finish time for each. Use the time line to compare elapsed time between events. For example, ask: Is it longer between the start of school and recess than between recess and lunch? Did it take more time to eat your lunch than it took to have silent reading?

Environmental Factors

Have students identify environmental factors by which we tell the time of day and year. For example, say: Shut your eyes and listen. What do you hear that suggests what time of day it is? Look around. What do you see that suggests what season of the year it is? Help students find out how Native Canadians used natural phenomena to make judgements about time (e.g., invite a local Native elder to talk to students, or find a relevant book, video or website).

Sample Learning Activities

Grades 3–5: ★★ Important Focus

Walking Clock Face

Have students make a large clock face on the floor, using small tiles for minute marks and numbered cards for the hours. Ask one student to represent the hour and another to represent the minute hand. Invite them to walk around the inside of the clock face to show the hour- and minute-hand positions for given time periods. For example, say: Show how the hands will move if you begin at 1:00 and stop at 2:00. Now, begin at 2:30 and stop again at 4:45. Where will the hour hand have to be when the minute hand stops?

Egg Timer

Have students make an egg timer using two small empty bottles, one with a screw cap in which a small hole is made. Have them add dry clean sand and tape the bottles together so that the sand flows from one to another. Encourage them to test the duration of time the sand takes to flow through and change the amount of sand to measure specific time durations (e.g., 3 min, 5 min). Ask: What is the greatest length of time your timer could be made to measure? What about if you used larger bottles? What if you made the hole larger? Use the timer to record passing of time during daily events. For example, say: You have 3 min to finish that piece of work.

Grades 3–5: ★★ Important Focus

Analogue and Digital Clocks

Ask students to draw two analogue clock faces and two digital clocks. Ask: What time did you get out of bed? What time did you leave for school? Invite them to show both times on both clocks and then to choose one type of clock to work out the length of time between rising and leaving for school. Have students discuss the reasons for their choice and the differences between each type of clock. For example, ask: How is "five to" shown on each clock? Why does the hour hand not point to 6 when the time is 6:55? How can you work out the length of time between 6:55 a.m. and 8:00 a.m.? Have the two types of clock in the classroom so that students can use both.

Clocks and Stopwatches

Incidentally refer to the clock during the course of the school day; for example, say: Let's finish this activity at 10:00. Then, later say: Look, the clock says 10:00, so it is time to finish. Have students use a stopwatch to compare the movement of the minute hand to short periods of time. Ask: How long until 10:00? If we start the stopwatch now and stop it at 10:00, what will it show?

Pictures of Time

Give students magazines and storybooks to look at and ask: Do the pictures tell you anything about the time of day (time of year, historical time)? What clues can you see? Encourage students to use time clues to decide where some pictures are from (*I can tell it is Christmas time by the tree and presents, but it cannot be Christmas in Canada because they are swimming outside so it must be Christmas in a place like Australia*).

Calendars

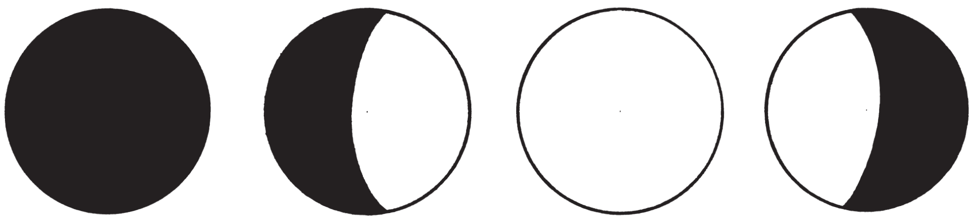
Examine and compare different calendars to find the various ways in which the months, the days and weeks are shown and written. Discuss the reasons for the different formats. Ask: Why do you think some calendars have larger spaces for Mondays to Fridays? Which of these calendars are easiest to read? How quickly can you find your birthday on this calendar (compared to another)? Look at calendars from different years. Ask: What do you notice about your birthday in each of these three years? Does this happen for all dates? How is 2007 different from 2008? How does looking at February in each year help you explain the difference?

Different Ways of Seeing Time

Have students investigate different ways people deal with time. For example, invite a Native elder to answer questions like: How did you know when it was time to eat (when to sleep, when it was time for a particular ceremony, when to meet with others, when to fish, where the tides would be)? Invite a retired farmer to answer similar questions: How did you know when it was time to eat (when to sleep, when the local agricultural show was on, when to meet with others, when to plant crops, when to begin harvesting)? Record and compare the answers. Ask: What differences can you see between the way the two speakers view time? How does the way each tells the time give information about what is important in their lives?

Phases of the Moon

Ask students to record the phases of the Moon by drawing what they can see over a two-month period. Ask: Is there one full Moon each month? How many days is it between one full Moon and the next? Why does the length of months vary from between 28 and 31 days? Is this related to the phases of the Moon? How?



Sample Learning Activities

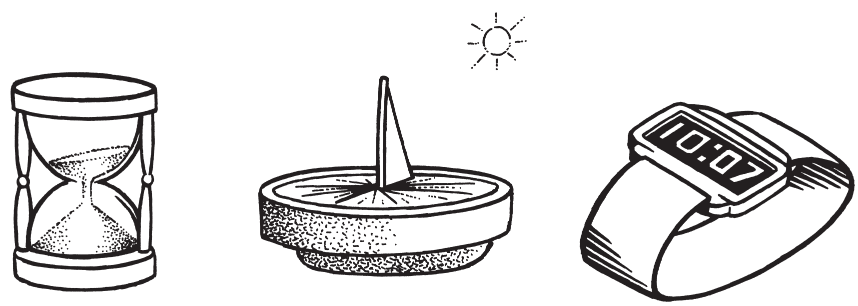
Grades 5–8: ★★★ Major Focus

Candle Timer

Ask students to calibrate tall candles by marking them clearly in equal sections and numbering the sections. Begin burning one candle at the first bell and record in candles and parts of candles the main divisions of the day. On the next day, cover the clock and use the candle clocks to monitor time during the day. For example, say: You have just two candle parts to finish your work. Ask: What is exactly the same about the candle clock and the wall clock? Why do you think people decided to use mechanical or electronic clocks rather than things like candles? Be sure to take care when burning candles in the classroom.

Clocks Over Time

Collect and display time measuring instruments from different places and times (wind-up alarm clocks, electric clocks, different kinds of watches, travelling clocks, sundials, hourglasses). Discuss any unusual features. Ask: What is different? What is the same? What can you see that has improved over time?



Ancient Measures of Time

Have students research the origins of our current system of measuring time and compare it to the Egyptians, who were the first to divide the day up into 24 units (daylight into 12 units and dark into 12 units), and the Hindu civilization, which divided it into 60 units. Ask: How are the systems similar? How are they different? How could our system be improved? What makes it difficult for a government to change the system? Invite students to devise an alternative time system. Encourage them to justify its usefulness.

Days and Months

Have students research the origins of the names of the days and months. Ask: Why do you think weeks do not have names? How can we track which week it is? Why are months about 30 days long, and weeks seven days long?

Time Lines of Settlement

When constructing and comparing time lines in other learning areas, such as a time line of European settlement of Canada with a time line of Native people's settlement of Canada, ask: How could we construct one time line which included both? Why is it difficult to draw a time line of Native Canadian settlement? Construct a time line of human habitation of a part of Europe that covers the same time period as Native Canadian settlement. Ask: What makes it easier to do this for European history? How could we find out how Native Canadians represent time in their stories about the past?

Stopwatch

Have students use a stopwatch to investigate the measurement of very short time intervals (less than a few seconds). Ask: Can you start and stop the stopwatch in less than one second? What does the stopwatch show? How do you know it took less than a second? How are seconds divided into smaller units? How is this different from the way minutes and hours are divided? Encourage students to try to time things that can be done in less than a second. For example, ask: How long does it take to say your name?

Comparing Elapsed Times

Have students read, write and compare elapsed times. For example, ask them to research record times of sports events, such as the marathon or the 1500-m freestyle, using the Guinness Book of Records, Olympic and Paralympic websites, and other sources. Ask: Would 2:12:36 be the record time for the marathon or the 1500-m freestyle? Would 14:56.4 be the record for the 1500-m freestyle or the men's 100-m race? How do you know? What do the numbers mean? Invite students to use a stopwatch to experience the lengths of time represented by the different record times.

Grades 5–8: ★★★ Major Focus

Timetables

Present groups of students with different types of timetables (classroom timetable, TV guide, bus timetable, train timetable, airline schedule, meeting agenda, movie times). Encourage students to discuss common features. Ask: What is the same in all of the timetables? In what ways are the bus and train timetables different from the classroom timetable and TV guide? Ask further questions that can be answered using the timetables. For example, say: The movies I want to see are in different theatres. Can I see them both during the same afternoon? How long would I need to wait between sessions? Or, say: I want to watch as many movies as possible on television this Saturday. Make me a "movie watching" timetable to show me which channels to turn to and when.

Appendix

Name: ____________________ Date: ____________________

Line Master 1 **2-mm Grid Paper**

FSIM010 | First Steps in Mathematics: Measurement Book 1

Name: ____________________ Date: ____________

Line Master 2 **1000 Grids**

FSIM010 | First Steps in Mathematics: Measurement Book 1

Name: ______________________________ Date: ____________________

Line Master 3 **Ten-Squared Grid Paper**

Name: ______________________________ Date: ____________________

Line Master 4 **10 × 10 Array**

Name: ______________________________ Date: ______________________

Line Master 5 **1-mm Grid Paper**

Planning Master

Classroom Plan for Week ________, Term ________ Grade Level: ____________

Curricular Goal/Key Understanding	Mathematical Focus	Activities	Focus Questions	Observations/ Anecdotes

Emergent Phase

During the Emergent Phase

Students initially attend to overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small) rather than to describe comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgements of size.

As a result, they begin to understand and use the everyday language of attributes and comparison used within their home and school environment, differentiating between attributes that are obviously perceptually different.

By the end of the Emergent phase, students typically:

- distinguish tallness, heaviness, fatness, and how much things hold
- start to distinguish different forms of length and to use common contextual length distinctions; e.g., distinguish wide from tall
- use different bipolar pairs to describe things; e.g., thin–fat, heavy–light, tall–short
- describe two or three obvious measurement attributes of the same thing; e.g., tall, thin, and heavy
- describe something as having more or less of an attribute than something else, e.g., as being taller than or as being fatter than.

These students recognize that numbers may be used to signify quantity.

Matching and Comparing Phase

Most students will enter the Matching and Comparing phase between 5 and 7 years of age.

As students move from the Emergent phase to the Matching and Comparing phase, they:

- may not "conserve" measures; e.g., thinking that moving a rod changes its length, pouring changes "how much," cutting up paper makes more surface
- may visually compare the size of two things, but make no effort to match; e.g., saying which stick is longer without lining up the bases or which sheet of paper is bigger without superimposing
- compare time spans but may not take into account different starting times; e.g., deciding that the TV program that finished latest was on longest
- use bipolar pairs but may have difficulty with some comparative terms; e.g., lift to decide which is heavier but say both are heavy because both hands go down
- may distinguish two attributes (such as tallness and mass) but not understand that the two attributes may lead to different orders of size for a collection, expecting the order for tallness and the order for mass to be the same
- while describing different attributes of the same thing (tall, thin, and heavy) may be confused by a request to compare two things by different attributes, particularly if the comparisons lead to different orders
- often do not think to use counting to say how big or how much bigger; e.g., they may "weigh" something by putting it into one side of a balance and smaller objects into the other side but not count the objects

During the Matching and Comparing Phase

Students match in a conscious way in order to decide which is bigger by familiar, readily perceived, and distinguished attributes such as length, mass, capacity and time. They also repeat copies of objects, amounts, and actions to decide how many fit (balance or match) a provided object or event.

As a result, they learn to directly compare things to decide which is longer, fatter, heavier, holds more or took longer. They also learn what people expect them to do in response to questions such as "How long (tall, wide or heavy, much time, much does it hold)?" or when explicitly asked to measure something.

By the end of the Matching and Comparing phase, students typically:

- attempt to focus on a particular attribute to compare two objects or events; e.g., how much the jar holds
- know that several objects or events may be in different orders when compared by different attributes
- line up the base of two sticks when comparing their lengths and fit regions on top of each other to compare area
- use the everyday notion of "how many fit" and count how many repeats of an object fit into or match another; e.g., How many pens fit along the table? How many potato prints cover the sheet? How many blocks fit in the box?
- count units and call it "measuring;" e.g., *I measured and found the jar holds a bit more than 7 scoops*.
- refer informally to part-units when measuring uni-dimensional quantities; e.g., *Our room is 6 and a bit metres long*.

Quantifying Phase

Most students will enter the Quantifying phase between 7 and 9 years of age.

As students move from the Matching and Comparing phase to the Quantifying phase, they:

- while knowing that ordering objects by different attributes may lead to different orders, may still be influenced by the more dominant perceptual features; e.g., they may still think the tallest container holds the most
- may count “units” in order to compare two things but be fairly casual in their repetition of units, not noticing gaps or overlaps; e.g., placing the first “unit” away from the end when measuring length, not worrying about spills when measuring how much a container holds, not stopping their claps immediately the music stops
- do not necessarily expect the same “answer” each time when deciding how many fit
- many not think to use unit information to answer questions such as: Which cup holds more? Will the table slide through the door?
- may not see the significance of using a common unit to compare two things and, when using different units, let the resulting number override their perceptual judgement
- while many will have learned to use the centimetre marks on a conventional rule to “measure” lengths, they often do not see the connection between the process and the repetition of units

During the Quantifying Phase

Students connect the two ideas of directly comparing the size of things and of deciding “how many fit” and so come to an understanding that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared without directly matching them.

As a result, they trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Quantifying Phase cont.

By the end of the Quantifying phase, students typically:

- attempt to ensure uniformity of representations of the unit; e.g., check that the cup is always full, the pencil does not change length, the balls are the same size
- use the representations of their unit carefully to make as close a match as possible, avoiding gaps and overlaps; e.g., choose a flexible tape to measure the perimeter of a curved shape
- know why they need to choose the same size objects to use as units when comparing two quantities
- see repeating one representation of the unit over and over as equivalent to filling or matching with multiple copies of it
- connect the repetition of a "unit" with the numbers on a whole-number calibrated scale
- make things to a specified length in uniform units (including centimetres and metres)
- use provided measurements to make a decision about comparative size; e.g., use the fact that a friend's frog weighs 7 marbles to decide whether their own frog is heavier or lighter
- count units as a strategy to solve comparison problems such as: Whose frog is heavier? Put the jars in order from the one that holds the most to the one that holds the least.
- are prepared to say which is longer (heavier) based on information about the number of units matching each object
- think of different things having the same "size;" e.g., use grid paper to draw different shapes with the same perimeter
- add measurements that they can readily think of in terms of repetitions of units; e.g., find the perimeter of a shape by measuring the sides and adding

Measuring Phase

Most students will enter the Measuring phase between 9 and 11 years of age.

As students move from the Quantifying phase to the Measuring phase, they:

- while trying to make as close a match as possible to the thing to be measured, may find the desire to match closely overriding the need for consistency of unit; e.g., they may resort to "filling" a region with a variety of different objects in order to cover it as closely as possible
- may not understand that the significance of having no gaps and overlaps is that the "true" measurement is independent of the placement of the units
- may still think of the unit as an object and of measuring as "fitting" in the social sense of the word (How many people fit in the elevator? How many beans in the jar?) and so have difficulty with the idea of combining part-units as is often needed in order to find the area of a region
- many confuse the unit (a quantity) with the instrument (or object) used to represent it; e.g., they may think a square metre has to be a square with sides of 1 m, may count cubes for area and not think of the face of each as the unit
- may interpret whole numbered marks on a calibrated scale as units but may not interpret the meaning of unlabelled graduations

During the Measuring Phase

Students come to understand the unit as an amount (rather than an object or a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and trust the measurement as a property or description of the object being measured that does not change as a result of the choice or placement of units.

By the end of the Measuring phase, students typically:

- expect the same number of copies of the representation of their unit to match the object being measured regardless of how they arrange or place the copies
- understand that the smaller the unit the greater the number; e.g., are able to say which is the longer of a 1-km walk and a 1400-m walk.
- compose "part-units" into wholes, understanding, for example, that a narrow garden bed may have an area of 5 or 6 m^2 even though no whole "metre squares" fit into the bed
- can themselves partition a rectangle into appropriate squares and use the array structure to work out how many squares are in the rectangle
- interpret the unnumbered graduations on a familiar whole-number scale
- understand the relationship between "part-units" and the common metric prefixes; e.g., know that a unit can be broken into one hundred parts and each part will be a centi-unit
- work with provided measurement information alone; e.g., order measurements of capacity provided in different standard units, make things which meet measurement specifications

Relating Phase

Most students will enter the Relating phase between 11 and 13 years of age.

As students move from the Measuring phase to the Relating phase, they:

- while partitioning a rectangle into appropriate squares and using the array structure to find its area, may not connect this with multiplying the lengths of the sides of a rectangle to find its area
- while understanding the inverse relationship between the unit and the number of units needed, may still be distracted by the numbers in measurements and ignore the units; e.g., say that 350 g is more than 2 kg
- while converting between known standard units, may treat related metric measures just as they would any other unit, not seeing the significance of the decimal structure built into all metric measures

During the Relating Phase

Students come to trust measurement information even when it is about things they cannot see or handle and to understand measurement relationships, both those between attributes and those between units.

As a result, they work with measurement information itself and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

By the end of the Relating phase, students typically:

- understand that known relationships between attributes can be used to find measurements that cannot be found directly; e.g., understand that we can use length measurements to work out area
- know that for figures of the same shape (that is, similar) the greater the length measures the greater the area measures, but this is not so if the figures are different shapes
- understand why the area of a rectangle and the volume of a rectangular prism can be found by multiplying its length dimensions and can use this for fractional side lengths
- think of the part-units themselves as units; e.g., a particular unit can be divided into one hundred parts and each part is then a centi-unit
- subdivide units to make measurements more accurate
- choose units that are sufficiently small (that is, accurate) to make the needed comparisons
- use their understanding of the multiplicative structure built into the metric system to move flexibly between related standard units; e.g., they interpret the 0.2 kg mark on a scale as 200 g
- notice and reject unrealistic estimates and measurements, including of objects or events they have not actually seen or experienced
- use relationships between measurements to find measures indirectly; e.g., knowing that 1 mL = 1 cm^3 they can find the volume of an irregular solid in cubic centimetres by finding how many millilitres of water it displaces using a capacity cylinder

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