first steps

First Steps in Mathematics

Measurement

Understand Units and Direct Measurement;
Understand Indirect Measurement and Estimation

Course Book

PEARSON

Department of
Education and Training

FSIM013 | First Steps in Mathematics: Measurement Course Book

Published in Canada by Pearson Canada Inc.
26 Prince Andrew Place
Don Mills, ON M3C 2T8

Adapted by STEPS Professional Development from the original text produced by the Department of Education and Training, Western Australia.

ISBN-13: 978-0-13-2045223
ISBN-10: 0-13-2045222

Vice-President, Publishing: Mark Cobham
Vice-President, Marketing and Professional Field Services: Anne-Marie Scullion
Publisher: Debbie Davidson
Research and Communications Manager: Chris Allen
Managing Editor: Joanne Close
Copy Editor: Susan Ginsberg
Proofreader: Linda Szostak
Senior Production Coordinator: Zane Kaneps
Art Director: Zena Denchik
Cover Design: Jennifer Stimson and David Cheung
Composition: Computer Composition of Canada

Acknowledgements

Page 12 of the Measurement Course Book is reproduced from *Knowing What They Know: Understanding Students' Mathematical Ideas in Years 6 to 9,* page 14. Copyright © Department of Education and Training, Western Australia. Reproduced with permission.

Page 22. The Broken Ruler Task is reproduced from *Knowing What They Know: Understanding Students' Mathematical Ideas in Years 6 to 9,* page 14. Copyright © Department of Education and Training, Western Australia, 2002.

PEARSON

first steps

Table of Contents

FSIM013 | First Steps in Mathematics: Measurement Course Book

Outcomes of *First Steps in Mathematics:*

- **Enhance teachers' professional decision-making about teaching and learning mathematics.**
- **Improve student learning outcomes in mathematics.**

Probably nothing has more impact on students than the personal, professional growth of their teacher.

Barth, R. (1990). *Improving schools from within*. San Francisco, CA: Jossey-Bass.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Professional Development Messages

These messages underpin *First Steps in Mathematics* Professional Development Resources:

Students may get the right answers for the wrong reasons.

Good questions and tasks provoke students to show us what they know and understand.

Understanding the mathematics helps us make better professional decisions.

Seeing the pattern in students' responses helps us plan effective learning experiences.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Outline of Units

Unit 1 (2 hours) Learning about LENGTH	**Unit 2** (2 hours) LENGTH and DIRECT COMPARISON
Unit 3 (2 hours) Meaningful MASS Measurement	**Unit 4** (2 hours) Constructing CAPACITY Understandings
Unit 5 (2 hours) Advancing AREA Understandings	**Unit 6** (2 hours) Vanquishing VOLUME Misunderstandings
Whole School Planning Setting common goals	

Task Review
Between sessions, small groups review work samples from Diagnostic Tasks.

Task Review
This professional development course is based on a spaced, action-based research learning model. After completing each of the content sessions, teachers are asked to commit to using some of the Diagnostic Tasks with their own students and to work with a small group of colleagues to review them and plan for their students' learning. These Task Review and planning sessions are a critical component of using *First Steps in Mathematics*.

The Task Review and planning sessions will be conducted between each of the content sessions to help teachers use the Planning Cycle for *First Steps in Mathematics* and to support their current classroom practices.

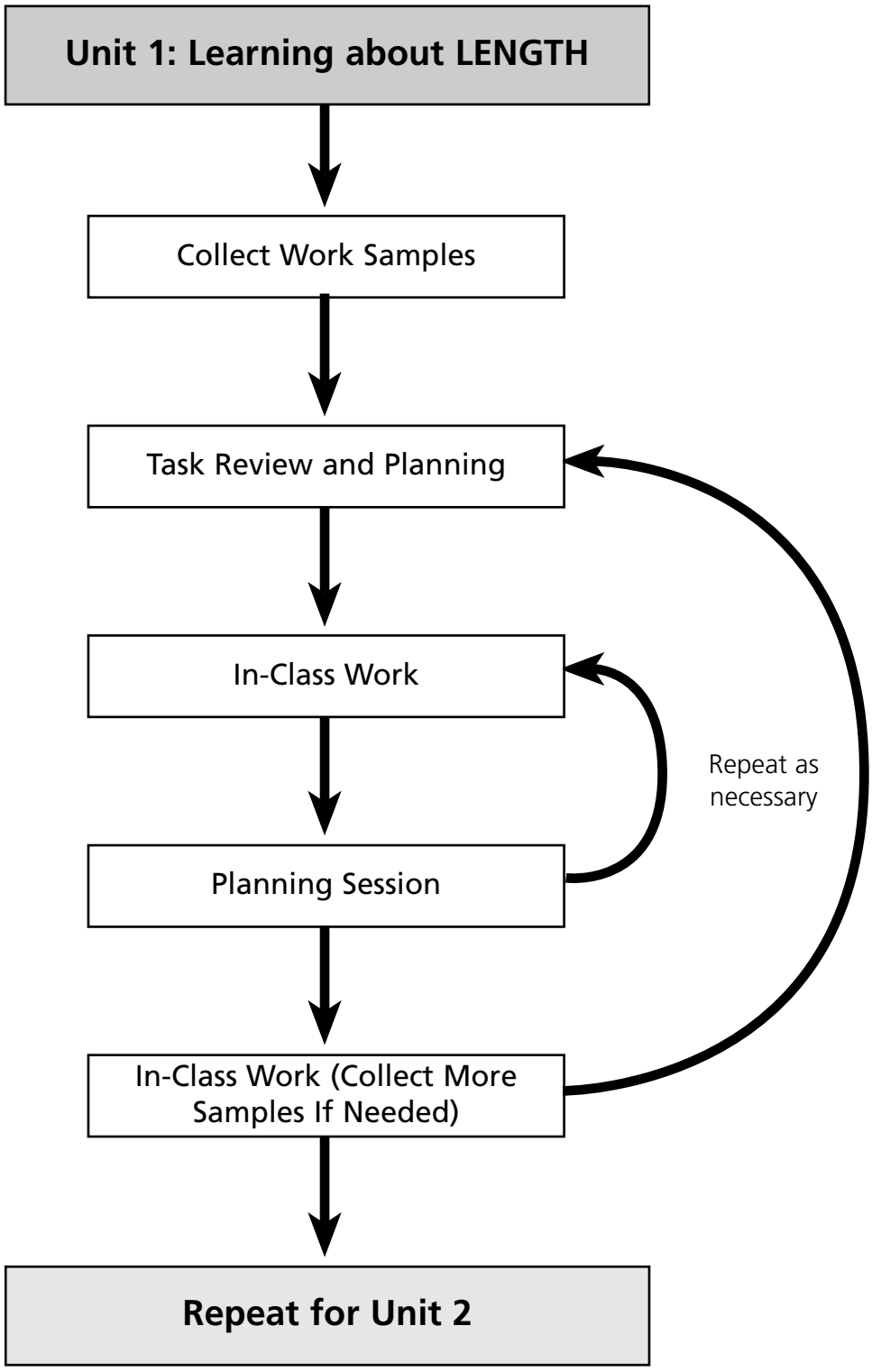
Course Structure
The course includes six compulsory units. Participants must complete all units to complete the course. The timetable should be developed in negotiation with the school or group of teachers.

Ideally, the six content sessions will be presented one at a time, with a Task Review session between each.

Whole School Planning
The Whole School Planning session is a guide to support both the facilitator and the school community to work together to establish common goals for the implementation of curriculum improvement in mathematics using *First Steps in Mathematics*.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Spaced, Action-Based Research Learning Model



Task Review

Task Review Outcomes

You will

- become more familiar with the phases of the Diagnostic Map
- analyze and sort students' work samples
- identify the Key Understandings that your students need to learn
- use *First Steps in Mathematics: Measurement* to plan learning activities and focus questions to move your students forward

Related Reading

Course Book
Diagnostic Tasks and work samples

Diagnostic Map: Measurement

Background Reading
Willis, S. (2005). Making mathematics vital. *Proceedings from the Twentieth Biennial Conference of the Australian Association of Mathematics Teachers*, p. 40.

Collecting Work Samples

Think about the tasks you have seen in the workshops. Some can be used as a whole class activity or with a group of students. Others require an individual interview. Do the same set of tasks with as many students as you are able in the time you have. A minimum of six students will give you a sense of what is happening within your class. Do not prompt or teach students as they do the task. Record what they say and do.

Remember that you will need this information to share with colleagues.

Sorting Work Samples

When you have collected your students' work samples, work with a group of colleagues to sort them into groups of common responses. Think about

- what the students know
- what they need to know (which Key Understanding?)
- which phase of the Diagnostic Map they may be in

Record the Information

Choose how you would like to record the information you have found about your students. This information might be in the form of anecdotal records or checklists, or comparisons with the accepted curriculum outcomes or standards in your province.

Planning for Students' Mathematics Learning

1. Sort your work samples or responses into groups showing common understandings.
2. Decide how to record the information.
3. Work out what mathematics your students know and can do, and then work out what they need to know using
   - Key Understandings
   - Background Notes
   - Diagnostic Map

   Decide which phase
4. Choose which Key Understandings and which part of each Key Understanding to focus on.
5. Decide on activities to move the students along, using
   - Sample Learning Activities
   - Key Understandings
   - other resources
6. Plan your focus questions, using
   - Sample Learning Activities
   - Case Studies

FSIM013 | First Steps in Mathematics: Measurement Course Book

What am I looking for?

Initially the tendency is to "mark" rather than "analyze." It takes some time and practice to get into the habit of looking more closely at what your students' responses are telling you about their thinking.

In marking conventional tests, our central concern is "Should I count this . . . right or wrong?" or "How many marks should I give this answer?" However, this is NOT the way to approach students' responses to the Diagnostic Tasks.

In analyzing Diagnostic Tasks, the question is "What is this telling me about the kinds of mathematical ideas this student is using to tackle the task?" In this case, the student's answer itself is far less important than the reasoning used to arrive at that answer. Associated with each task, you will find information and examples to help you "read" what your students are telling you.

How can I begin to organize the responses?

A useful way to organize a class set of tasks is to categorize or sort students' responses in ways that will help you decide how to act on what you find. Generally, you can sort students into these four broad groups according to the ideas they reveal in their responses.

Students whose responses are complete and accurate, or well beyond expected development

These students may need extra challenges in order to move forward.

(Few students are in this category, though it varies in different schools and classes.)

Students whose ideas are partial or incomplete, but in line with expected development

Though the ideas may not be fully understood, nothing these students write is in conflict with conventional ideas. They are probably making sense of their learning experiences and will continue to move forward, given appropriate whole class teaching strategies.

(Many more students are usually in this category.)

Students who reveal faulty or inappropriate ideas in their responses

These are the students whose ideas will need to be specifically challenged and "unlearned" before the usual mathematics learning experiences will make sense. Depending on their stage of development, these students are unlikely to move forward unless individual attention is given to find the source of their faulty ideas and help them see the inconsistencies in their thinking.

(Usually more students than we would like are in this category—this is the AT RISK group.)

Students who can't or won't do the task, or give minimal response

Where students provide little or no information about their thinking (even though answers may be correct), there is not enough information to draw conclusions about what those students know or don't know. Some may not have heard the instructions, or try to avoid explaining their ideas. Others may not have sufficient writing skills but could give an oral response. Some prefer to say nothing rather than risk an error; others may not have the basic math knowledge needed to begin to make any sense of the task. Whatever the reason, the task is not doing its job, and you will need to gather some more information to find out about these students' thinking.

(For most tasks and in most classes, very few students should be in this category.)

Note: If most of the students in your class do not respond to a task, you need to ask yourself why. It could be for one or more of the following reasons:

- The task itself was not properly understood to begin with.
- The expected mathematics may be too far beyond the class's current knowledge.
- Students may feel the classroom culture is not "safe"—they may be reluctant to expose their thinking in case there are consequences if they get it "wrong."

Department of Education and Training, WA, 2002, *Knowing What They Know: Understanding Students' Mathematical Ideas in Years 6 to 9*, p. 14.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Making Decisions Using Diagnostic Tasks

Task _____

Description of the group

In which phase and why?

Key Understanding

Mathematical focus

Classroom Planner

Week __________ Term __________ Grade __________ Date __________

Key Understanding	Mathematical Focus	Activities	Focus Questions	Observations/Anecdotes

After the Lesson or Series of Lessons

Teachers

What did you learn about the mathematics?

What did you learn about your students?

What did you learn about your teaching?

What would you do differently next time?

FSIM013 | First Steps in Mathematics: Measurement Course Book

After the Lesson or Series of Lessons

Students

Have the students learned what was intended for this lesson?
If not, why not?

How do you know what they have learned?

What else have they learned?

Unit 1: Learning about LENGTH

Unit 1 Outcomes

You will
- become familiar with the layout of the *First Steps in Mathematics* Measurement books
- learn about the layout and content of the Diagnostic Map
- begin to understand how students learn to measure length

Related Reading

***First Steps in Mathematics* Measurement Book 1**
Chapter 2: Understand Units
Key Understanding 3 (pp. 40–53)
Key Understanding 4 (pp. 54–61)
Chapter 3: Direct Measure
Key Understanding 2 (pp. 126–133)
Key Understanding 3 (pp. 134–141)
Key Understanding 4 (pp. 142–155)

Diagnostic Map: Measurement
Matching and Comparing to Quantifying phases

Background Reading
Barrett, J., Jones, G., Thornton, C., & Dickson, S. (2003). Understanding children's developing strategies and concepts for length. In D. Clements & G. Bright (Eds.), *Learning and teaching measurement 2003 yearbook* (pp. 17–30). Reston, VA: National Council of Teachers of Mathematics.

Battista, M.T. (2006). Understanding the development of students' thinking about length. *Teaching Children Mathematics,* October (pp. 140–146).

Nunes, T., & Bryant, P. (1996). Children doing mathematics, chapter 4. In *Measurement systems* (pp. 76–95). Oxford: Blackwell Publishing.

FSIM013 | First Steps in Mathematics: Measurement Course Book

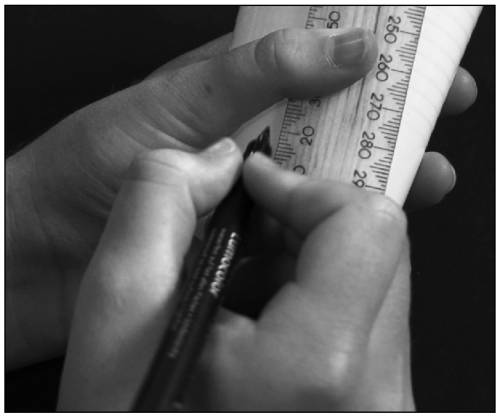
Luke's Story

Luke has just completed Grade 5 and is 10 and a half years old.

He can use a standard measuring cup to measure capacity. For example, when asked, he can use the scale on a beaker to measure out quantities of water and can show where 250 mL would be.

Make a Measuring Cup Task

When completing the task, Luke ignored the medicine glass and the water. Instead, he took the pen and used a ruler to transfer the scale from the measuring cup to the side of the container.



What is wrong with what Luke did?

Why did Luke's teacher take a closer look at his understanding of capacity?

What Luke's teacher knew:

Students may get the right answers for the wrong reasons.

Good questions and tasks provoke students to show us what they know and understand.

Luke's teacher knew a lot about the **mathematics of capacity**, the way **students** develop capacity understanding, and the kinds of **pedagogy** (i.e., the classroom activities and experiences) that would maximize Luke's progress. Luke's teacher exercised well-informed, high-level **professional decision-making**.

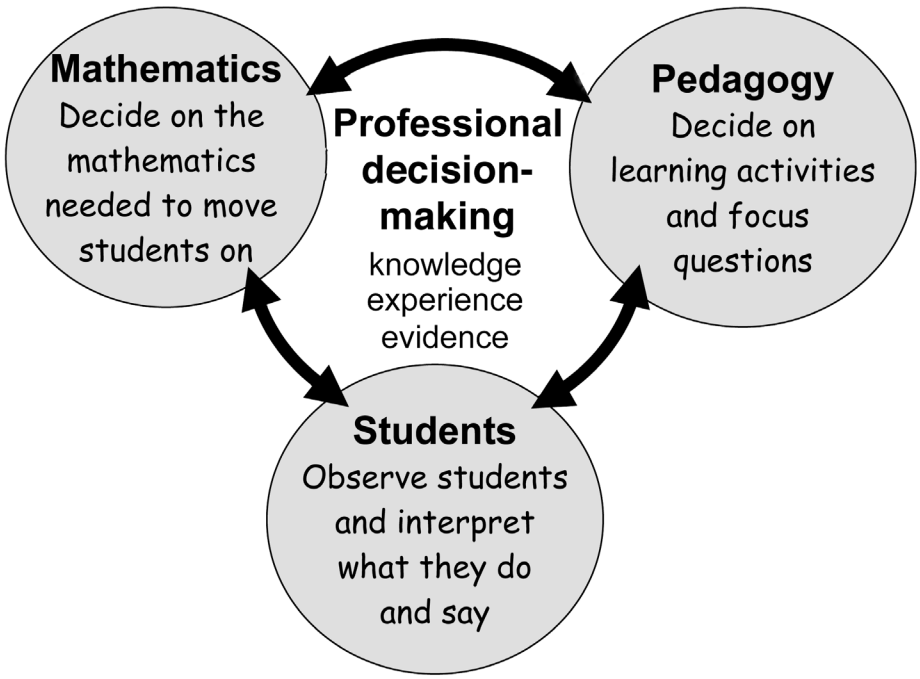
Professional Decision-Making

The *First Steps in Mathematics* Resource Books, and the professional learning programs associated with it, are based on the belief that teachers are in the best position to make decisions about how to help their students achieve the mathematics outcomes. Teachers will base these decisions on knowledge, experience, and evidence.

The process of using professional judgments to make decisions about classroom experiences for students is fluid, is dependent on the situation and context, and varies from teacher to teacher.

This Planning Cycle illustrates how these aspects combine to inform professional judgments about what, when, how, and why particular learning activities and experiences are chosen for particular students or groups of students.

Planning Cycle



Diagnostic TASK

FOCUS
Direct Measure
- Key Understanding 4
- Did You Know? p. 155

Broken Ruler

Grades 3–7

Purpose

To reveal if the student
- can use the marks on the ruler to measure in centimetres
- understands how the number on the scale relates to the units

Materials

For small groups or the whole class, use the attached worksheet.

For individual interviews, use a broken piece of ruler and an object to measure.

Producing Work Samples

It is important that the students use a picture of the broken ruler and **do not** use their own rulers to work out the length of the leaf.

Use this task in an individual interview, or with a small group or whole class.

Individual interview

Provide a broken ruler and ask the student to measure an object that is shorter than the piece of ruler.

Small group or whole class

Read out the problem while the students follow on the sheet. Ask them to write a full explanation of how they worked it out. It may be necessary to do some follow-up interviews to clarify what individual students are thinking.

After the students have found an answer

Ask, *How did you work out the answer?*

Record what the students do and say.

Observe whether they count the spaces or the marks, or use the numbers and any other operation to work out the size of the leaf. Do they count the starting number as zero or as one?

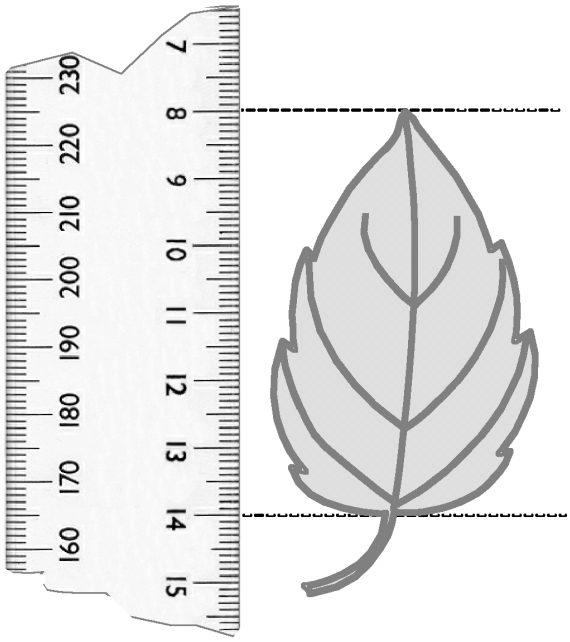
FSIM013 | First Steps in Mathematics: Measurement Course Book

Broken Ruler

Name _____ Grade _____ Date _____

(Do not use another ruler for this task.)

Andrea wanted to measure the length of the leaf she had collected for Science.
All she could find was a broken ruler.
She lined up the ruler and the leaf like this.



Can you say how long the leaf is, using the broken ruler?

The leaf is _____ cm long.

Explain how you got this answer.

Department of Education and Training, WA, 2002, *Knowing What They Know: Understanding Students' Mathematical Ideas in Years 6 to 9*, p. 14.

Broken Ruler

Simon, Grade 6, 11 years old

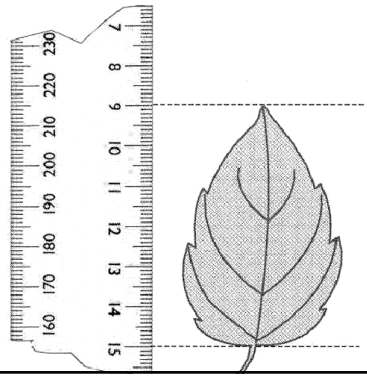
Broken Ruler

(Do not use another ruler for this task.)

Andrea wanted to measure the length of the leaf she had collected for Science.

All she could find was a broken ruler.

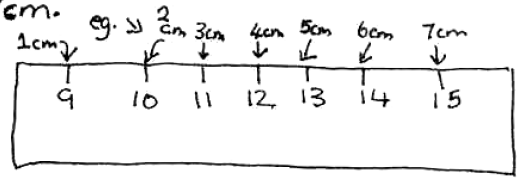
She lined up the ruler and the leaf like this.



Use the broken ruler to say how many centimetres long Andrea's leaf is.

The leaf is 7 cm long.

Explain how you got this answer. I counted the numbers from 9 to 15 and it came to 7cm.



Why did Simon get the wrong answer? What was he thinking?

Would Simon be able to get other ruler or tape measure tasks correct while holding this misunderstanding?

How might this misunderstanding hinder his progress?

Diagnostic Map

When should we expect students to understand the numbers on a ruler?

Planning for Learning

What mathematics does Simon need to learn?

Which Key Understanding would you focus on to plan a program of work for Simon? (pp. 110–111, *Measurement Book 1*)

What mathematics within the Key Understanding does he need to learn? (pp. 142–143, *Measurement Book 1*)

Toothpick Tapes Case Study 2 (pp. 152–154, *Measurement Book 1*):
How has the teacher focused the students' thinking on the mathematics during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

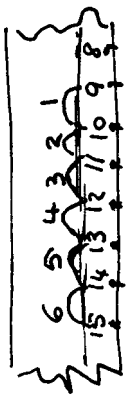
Broken Ruler: Work Samples

Kirri Grade 5

The leaf is 6 **cm long.**

Explain how you got this answer.

I counted from 9 to 15 and got 6.



How did Kirri find the answer?

Which phase is this evidence of?

Why?

Danielle Grade 7

The leaf is 7 **cm long.**

Explain how you got this answer.

All I did was count from 9 to 15 all of the Big lines that indicates the centimetres.

How did Danielle find the answer?

Which phase is this evidence of?

Why?

Jimmy Grade 7

The leaf is 6 **cm long.**

Explain how you got this answer.

I just (added) took 9 away from fifteen and came up with 6.

How did Jimmy find the answer?

Which phase is this evidence of?

Why?

Broken Ruler: Work Samples

Jarrod Grade 6

The leaf is 15 **cm long.**

Explain how you got this answer.

It's 15 because the bottom line is on the number 15

How did Jarrod find the answer?

Which phase is this evidence of?

Why?

Greg Grade 7

The leaf is 6 **cm long.**

Explain how you got this answer.

I started at the 9 wich would be ~~zero~~ and counted 10\1 11\2 12\3 13\4 14\5 15\6 and the fifteen was last so it is six.

How did Greg find the answer?

Which phase is this evidence of?

Why?

Diagnostic TASK

FOCUS

Understand Units
- Key Understanding 3
- Key Understanding 4

Direct Measure
- Key Understanding 2
- Key Understanding 3

Desk through the Doorway K–Grade 5

Purpose

To reveal if the student
- counts units of length to say "how many fit"
- chooses to use units to compare lengths
- chooses appropriate units to compare lengths
- can use units without gaps and overlaps
- recognizes and uses part units

Materials

Desk or table some distance from a doorway

A variety of objects that students can choose to use as units, e.g., felt-tipped pens, Popsicle sticks, paper clips, blocks, marbles, beads, etc.

Teacher Recording Sheet

Procedure

Sit with a student at a desk or table that is a similar width to, but a few metres away from, a doorway, and ask:
- *How many pens do you think will fit across the desk?* If necessary, prompt by saying, *Could you use the pens to check?*
- *How wide is the desk?*

Observe and record what the students say and do:
- Are they careful to avoid gaps and overlaps?
- Do they only fit whole pens, or do they consider part pens as well?

Then ask:
- *If I wanted to push or slide this desk out the door, do you think it would fit?*
- *Could you use something on the desk to check?* (Indicate the pens and the range of other objects.)

Observe and record what the students say and do and whether they
- only consider a direct comparison or non-numerical indirect comparison
- use the pen measurement to say whether the desk will fit through the doorway
- use a different unit for each measurement
- choose a unit that has length
- consider gaps and overlaps
- measure the appropriate part of the doorway
- use fractional numbers for part units

FSIM013 | First Steps in Mathematics: Measurement Course Book

Desk through the Doorway: Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. **How many pens do you think will fit across the desk?** If necessary, prompt with, **Could you use the pens to check?**

2. **How wide is the desk?**

3. **If I wanted to push this desk out the door, do you think it would fit? How could you check?** If the student's response is direct comparison, ask, **Is there something else we could use to work it out?**

FSIM013 | First Steps in Mathematics: Measurement Course Book

Desk through the Doorway

Millie, Grade 2

T: How wide is the desk?
M: *(no answer)*
T: Would you like to use the pens to check?
M: *(places pens across table)* Eight.
T: Is it eight exactly?
M: *(removes the last one)* Seven.
T: So how long is the desk?
M: Seven.
T: OK. Millie, if I wanted to push this desk out the door, do you think it would fit through the doorway?
M: …um…
T: How would you check if you really wanted to know?
M: You could measure it.
T: OK. So how could we measure it?
M: …ummm…
T: I have all these things *(Popsicle sticks, straws, 1-cm blocks, pens, Unifix cubes).* Is there anything you could use?
M: You could use blocks.
T: OK then, give it a try.

(Millie places 1-cm blocks across the desk, close together at first and then farther and farther apart until she reaches the end.)

T: Is that going to help us find out whether it fits through the doorway?
M: Yes.
T: How will it help?
M: I don't know.

What is Millie thinking when she is

- laying out and counting the pens?
- laying out the blocks?

Why doesn't Millie use the number of pens to work out whether the desk will fit through the doorway?

Does Millie

Choose a unit that matches the attribute being measured?

Use units to say how big or how long?

Know that there should be no gaps or overlaps?

Use units to say whether the desk will fit through the doorway?

Diagnostic Map

When would we expect students to use the number of pens to say whether the desk will fit through the doorway?

Planning for Learning

Which Key Understandings would you focus on?

What would be the focus of teaching for Millie?

Choose a Sample Learning Activity.

What focus questions might you use?

Desk through the Doorway

Alice, 9 years old

T: How many pens do you think will fit across the desk?
A: Eight.
T: Would you like to use the pens to check?
A: Five and a half.
T: So can you tell me, how wide is the desk?
A: It is five and a half pens wide.
T: Is it exactly five and a half?
A: Five and three-quarters…?
T: If I wanted to push this desk through the doorway, do you think it would fit?
A: No.
T: How would you know for sure? Is there a way to check?
A: Just by looking. This desk looks fatter than the door.
T: Is there something here you could use to work out if it would fit?
A: I could use the pens.

(Alice places pens across the doorway, on the floor.)

A: Six and a half pens.
T: Six and a half pens. Do you think the desk will fit through?
A: Yes.
T: How do you know?
A: Because that is more pens than that.
T: Do you remember how many pens fit across the desk?
A: Five and three-quarters.
T: And this is...?
A: Six and a half.
T: So is six and a half bigger than five and three-quarters?
A: Yes.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Does Alice

Choose a unit that matches the attribute being measured to do the comparison?

Use units to say how big or how long?

Know that there should be no gaps or overlaps?

Use units to say whether the desk will fit through the doorway?

Which phase is Alice in?

Desk through the Doorway

Rhiannon, 8 years old

T: How many pens do you think will fit across the desk?
R: *(places pens across the desk)* About five and three-quarters.
T: So how wide do you think the desk is then?
R: Five and three-quarters wide.
T: If I wanted to push this desk through the doorway, do you think it would fit?
R: No.
T: No. Is there any way of finding out?
R: Measuring the door and measuring how wide the desk is.

(Rhiannon attempts to place the pens across the middle of the door but has trouble holding them up. She is happy to measure the inner glass section of the door, ignoring the wooden frame.)

R: Five and two-quarters.
T: So do you think the desk will fit through?
R: No. It looks wider than the door.
T: It looks wider. You said the table was five and three-quarters…how wide was the door?
R: Five and two-quarters, but I actually only put five on and then just guessed it was two-quarters.
T: So would that make a difference?
R: Maybe.
T: Maybe…so what else could you do?
R: Could do it with straws. I think I might measure the desk with straws first. *(places straws across the width of the desk)* Three and three-quarters.

FSIM013 | First Steps in Mathematics: Measurement Course Book

(To measure the doorway, Rhiannon holds straws across the opening, in mid-air. She holds up four and estimates the size of the "gap" at the end.)

R: Four and two-quarters.
T: So do you think the desk will fit through the doorway?
R: What was the desk again? Umm…three and three-quarters.
T: So will it fit?
R: Yes.
T: How do you know?
R: Umm…I don't know.
T: Do the numbers help you to know?
R: I don't know…Yes, a little bit.
T: So what was it that told you the desk would fit?
R Probably because it wasn't as wide as the door.
T: So how did you know it wasn't as wide as the door?
R: I measured it with straws.
T: What was it that told you?
R: Umm, I don't know.

Does Rhiannon

Choose a unit that matches the attribute being measured to do the comparison?

Use units to say how big or how long?

Know that there should be no gaps or overlaps?

Use units to say whether the desk will fit through the doorway?

Which phase is Rhiannon in?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Desk through the Doorway

Taylor, 8 years old

T: How many pens do you think will fit across the desk?
Taylor: About six.
T: Would you like to put them on the desk to check?

(Taylor pushes pens into each other to make them into one long line. He places them along the longer side of the desk.)

T: How many?
Taylor: About eight.
T: What about this bit? *(pointing to the part of the last pen that is on the desk; part of the pen was hanging over the edge)*
Taylor: A half.
T: So how long is the desk?
Taylor: Eight and a half pens.
T: Now if I wanted to push this desk out the doorway, do you think it would fit?
Taylor: Yes.
T: How do you know?
Taylor: Because the desk is thinner than the door frame.
T: What could you do to check?
Taylor: Measure it across.
T: OK, you do that.

(Taylor puts pens across the narrow part of the desk top, pushing them together into one long line.)

Taylor: About five and a half.

(He then picks up the five pens in one hand and places them across the doorway.)

Taylor: The desk is shorter.
T: How do you know?
Taylor: Because that one *(meaning the desk)* is that much space, and the door is about that much.

(Taylor indicates with his fingers the length of space after the five pens, showing that the desk is shorter than the doorway by the amount of space between his fingers.)

T: OK, thanks Taylor.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Does Taylor

Choose a unit that matches the attribute being measured to do the comparison?

Use units to say how big or how long?

Know that there should be no gaps or overlaps?

Use units to say whether the desk will fit through the doorway?

Which phase is Taylor in?

Between Now and the Next Session

Create an action plan with your group.

What tasks will you use with at least six of your students to find out what they know?

Task	Class	Page
Broken Ruler	Grades 3–7	21
Desk through the Doorway	K–Grade 5	27

Complete the Action Plan on the next page, noting which tasks you and your colleagues will use and when.

Make decisions about sharing the resources and/or preparation that is needed.

Set the date and time for a Task Review session.

Before you meet, copy the Making Decisions and Classroom Planner masters on Course Book pp. 13 and 14.

Plan for your students' mathematics learning following the steps outlined in the Task Review section on p. 11.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Action Plan

Who?	Which Diagnostic Tasks?	When?	Resource Preparation

Task Review Date: _____ Time: _____ Venue: _____

Unit 2: LENGTH and DIRECT COMPARISON

Unit 2 Outcomes

You will
- understand the mathematics of using units to measure length
- learn about direct comparison and the use of comparative language
- further develop an understanding of the Diagnostic Map, particularly the Emergent, Matching and Comparing, and Quantifying phases

Related Reading

First Steps in Mathematics **Measurement Book 1**
Chapter 2: Understand Units
- Key Understanding 1 (pp. 24–31)
- Key Understanding 2 (pp. 32–39)
- Key Understanding 3 (pp. 40–53)
- Key Understanding 4 (pp. 54–61)

Chapter 3: Direct Measure
- Key Understanding 1 (pp. 112–125)
- Key Understanding 3 (pp. 134–141)

Diagnostic Map: Measurement
Emergent to Quantifying phases

Background Reading
Kamii, C. (2006). Measurement of length: How can we teach it better? *Teaching Children Mathematics,* October (pp. 154–156).

Kribs-Zaleta, C.M., & Bradshaw, D.A. (2003). A case of units. *Teaching Children Mathematics,* March (pp. 397–399).

Willis, S. (2005). Making mathematics vital. *Proceedings from the Twentieth Biennial Conference of the Australian Association of Mathematics Teachers,* p. 40.

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 3, 4

Direct Measure

- Key Understanding 3

Snail Trails

Grades 1–5

Purpose

To reveal if the student

- chooses appropriate objects to represent units of length
- counts to say "how many fit"
- uses objects that have uniform lengths and lines them up without gaps or overlaps
- chooses the same-sized unit for each line
- is not distracted by the numbers (where there are more small units on one line and fewer larger units on the other line)

Materials

A copy of the Snail Trails sheet

Mixed range of materials, e.g., blocks, counters, Unifix cubes, toothpicks, paper clips, marbles, etc. (include broken toothpicks)

Teacher Recording Sheet

Procedure

Individual interviews are appropriate for this task.

Give the student the Snail Trails sheet and ask which trail is longer (NB: the first trail is longer). Prompt the student to use units to measure both lines to say which is longer.

If the student places the same-sized unit on both lines, change the units on one line so that the number of units on the **shorter** line is more than the number of units on the **longer** line. For example, remove the units from the shorter trail and replace them with smaller units or, if the student has already used the smallest unit, substitute larger units on the longer line.

Ask the student to say how long each line is, using the units now on each line, and then ask the student again to say which is the longer line.

Encourage the student to explain: *How do you know?* If the student has changed his or her mind, ask, *Why did you change your mind?*

Record the responses.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Snail Trails: Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. We are going to pretend that two snails left trails on the lawn last night. Can you work out which snail left the longer trail? Use these materials to help you.

If necessary, prompt the student with, **Can you use any of these materials to find out how long this trail is?** (point to one trail) or **How many of these (cubes) fit along this line?**

Prompt the student to measure the other trail if the student does not do so independently.

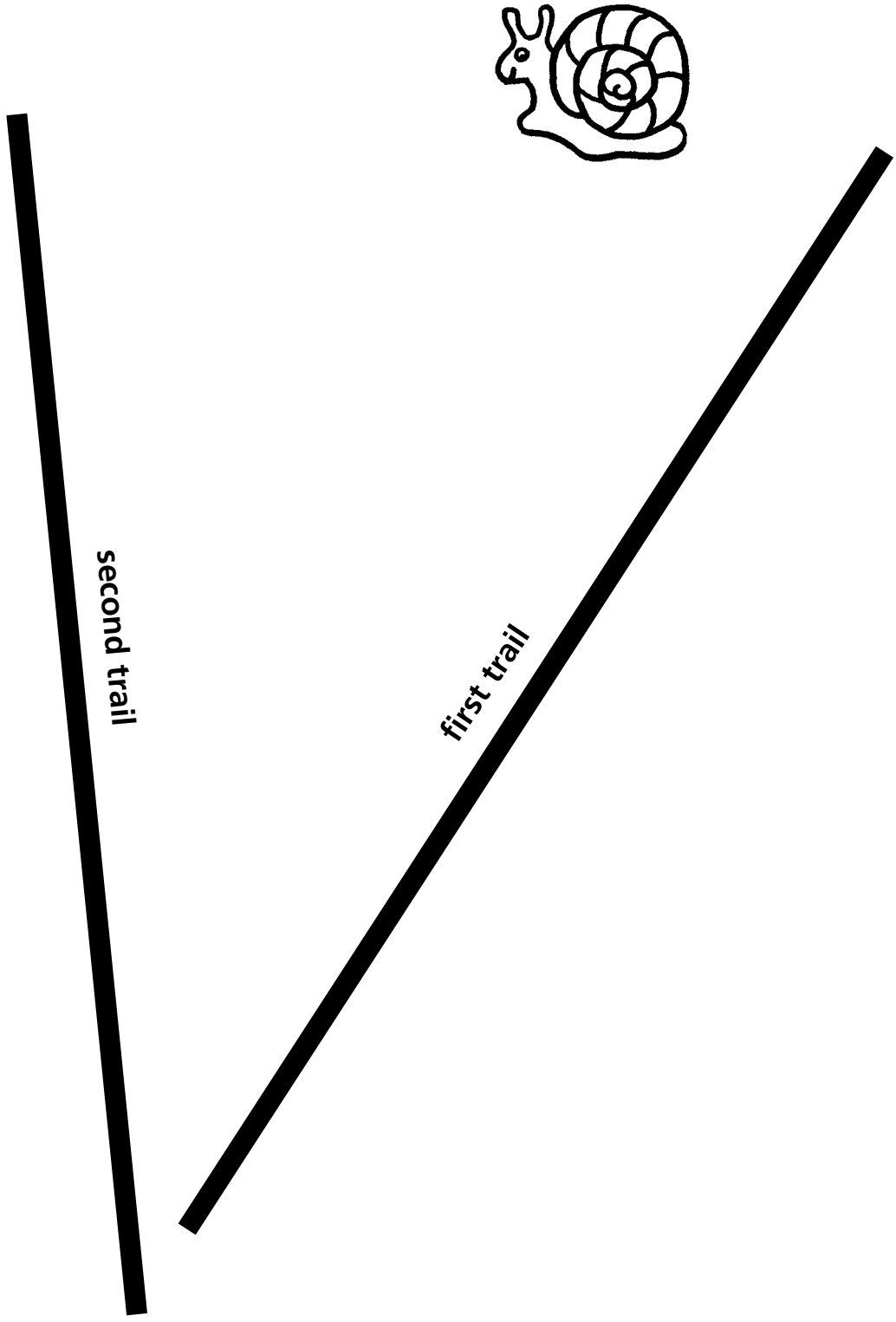
2. If the student does not tell you which trail is longer after placing materials on each line, ask, **Now can you tell me which snail has the longer trail? How do you know?**

If the student chooses the same unit for both lines, remove the units from the *shorter* line and replace them with *smaller* units. If the student has already used the smallest unit, then substitute *larger* units on the *longer* line.

Ask: **How many counters** (or whatever unit used) **fit along here?** (point to the shorter line) **How many toothpicks** (or whatever unit used) **fit along here?** (point to the longer line)
So which trail is longer? How do you know? If the student changes his or her mind, ask, **Why did you change your mind?**

FSIM013 | First Steps in Mathematics: Measurement Course Book

Snail Trails



Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 3, 4

Direct Measure

- Key Understanding 3

Which Line Is Longer? Grades 4–7

Purpose

This variation of the Snail Trails task should be used to interview upper primary students who are not able to show, in the Broken Ruler task, that they are in the Measuring Phase.

To reveal if the student

- chooses appropriate objects to represent units of length
- uses objects that have uniform lengths and lines them up without gaps or overlaps
- chooses the same-sized unit for each line
- is not distracted by the numbers (where there are more small units on one line and fewer larger units on the other line)
- can explain **why** the line with more units is shorter

Materials

A copy of the sheet with lines A and B marked

Mixed range of materials, e.g., blocks, counters, Unifix cubes, toothpicks, paper clips, marbles, etc. (include broken toothpicks)

Teacher Recording Sheet

Procedure

Individual interviews are appropriate for this task.

Give the student the sheet of paper with the two lines and ask which is longer. Prompt the student to use units to measure both lines.

If the student places the same-sized unit on both lines, change the units on one line so that the number of units on the **shorter** line is more than the number of units on the **longer** line. For example, remove the units from the shorter line and replace them with smaller units or, if the student has already used the smallest unit, substitute larger units on the longer line (NB: Line A is longer).

Ask the student to say how long each line is, using the units now on each line, and then ask the student again to say which is the longer line.

Encourage the student to explain: *How do you know?* If the student has changed his or her mind, ask, *Why did you change your mind?*

Record the responses.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Line Is Longer? Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. Two students were arguing over which line was longer. Jane thought line A was longer than line B. Which do you think is longer? Use these materials to help you.

If necessary, prompt with, **Can you use any of these materials to find out how long this line is?** (point to one line) or **How many of these (cubes) fit along this line?**

Prompt the student to measure the other line if the student does not do so independently.

2. If the student does not tell you which line is longer after placing materials on each line, ask, **Now can you tell me which line is longer? How do you know?**

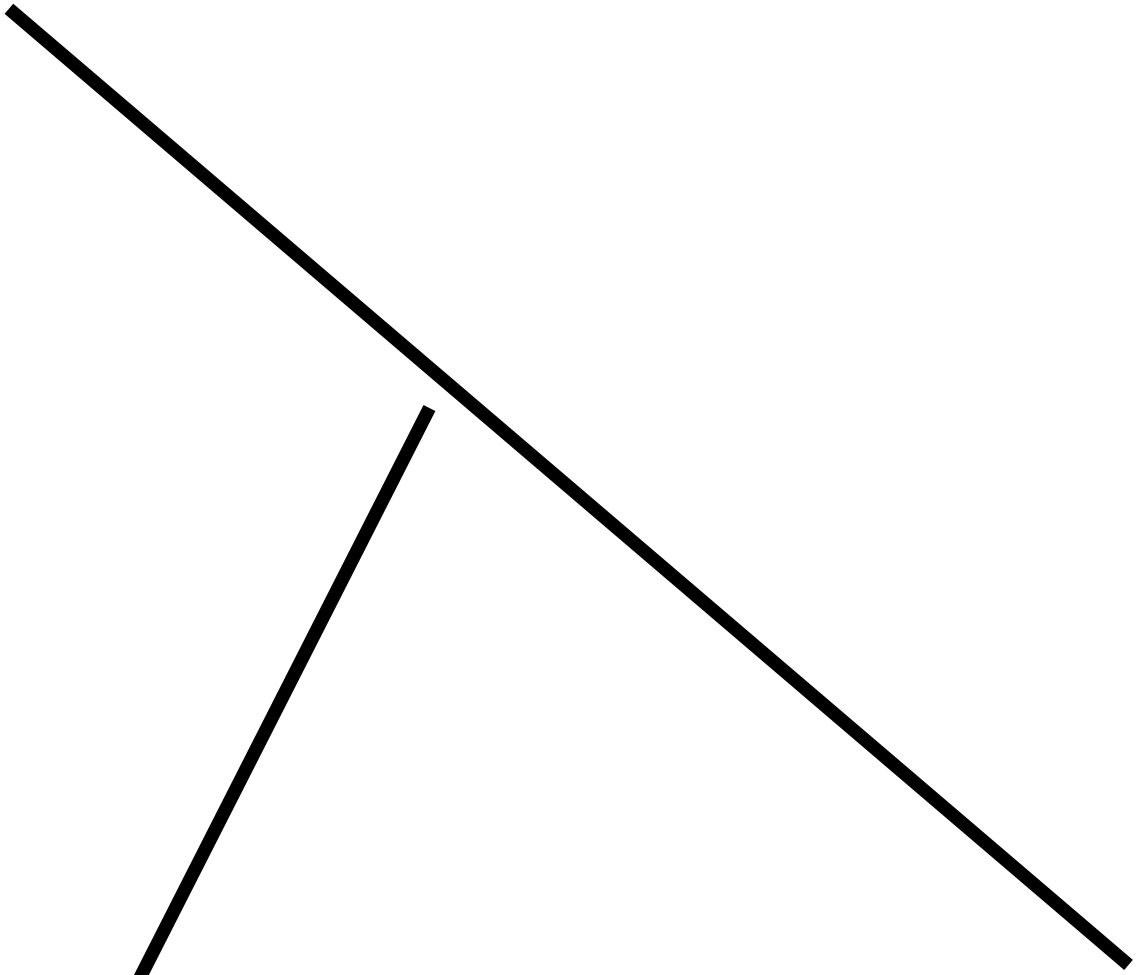
If the student chooses the same unit for both lines, remove the units from the *shorter* line and replace them with *smaller* units. If the student has already used the smallest unit, then substitute *larger* units on the *longer* line.

Ask: **How many counters** (or whatever unit used) **fit along here?** (point to the shorter line) **How many toothpicks** (or whatever unit used) **fit along here?** (point to the longer line)
So which line is longer? How do you know? If the student changes his or her mind, ask, **Why did you change your mind?**

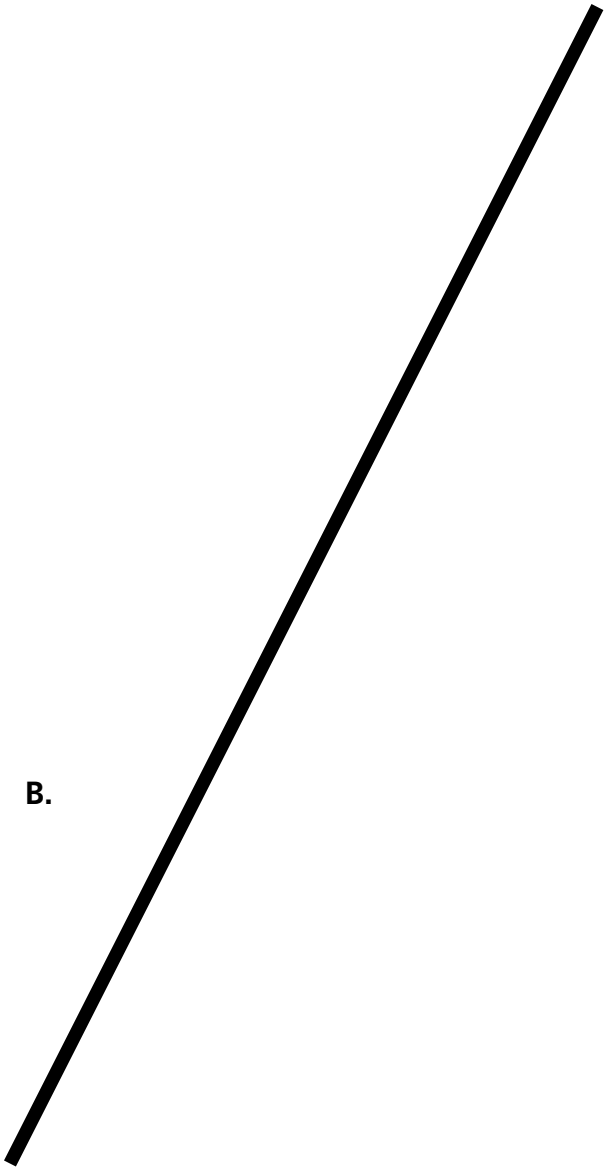
FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Line Is Longer?

A.



B.



Snail Trails

Video of Chris, 10 and a half years old

What does Chris know about using units for measuring length?

Why does Chris change his mind about which line is longer?

Diagnostic Map

In which phase do we expect students to know why they need to choose the same-sized objects as units when comparing two quantities?

Planning for Learning

What mathematics does Chris need to learn in order to be able to trust information about the repetition of units as an indicator of size, i.e., which Key Understanding? (pp. 22–23, *Measurement Book 1*)

What mathematics within the Key Understanding does he need to learn? (pp. 40–41, *Measurement Book 1*)

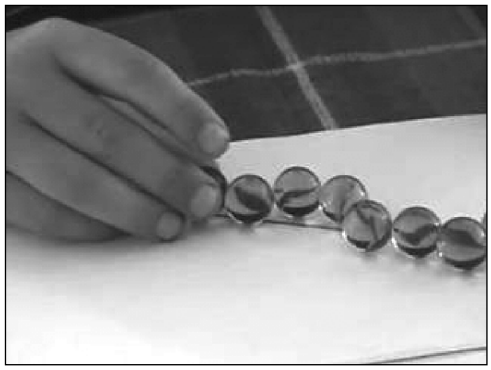
Sample Learning Activities:
Choose suitable learning activities to move Chris forward.

Snail Trails Case Study 1 (pp. 48–50, *Measurement Book 1*):
How has the teacher focused the students' thinking on the mathematics during this lesson?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Choice of Materials

Watch the video of Daniel (6 years old) attempting the Snail Trails task.



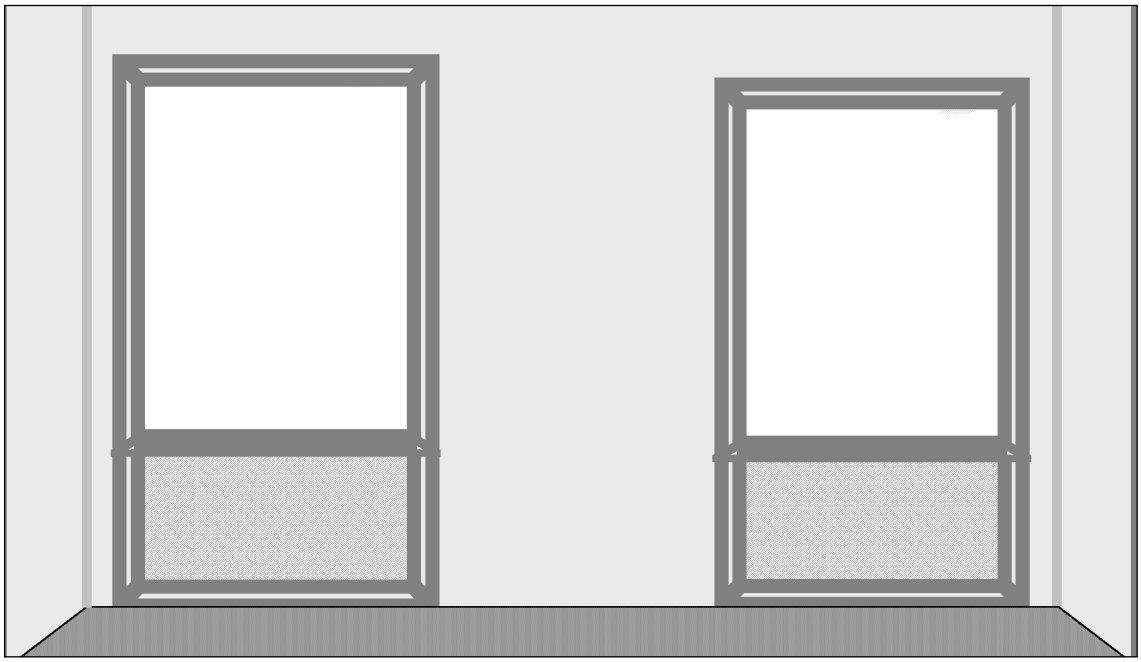
How does the teacher's choice of materials, and the way the materials are arranged, influence what we find out?

Key Understanding 4

The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.

FSIM013 | First Steps in Mathematics: Measurement Course Book

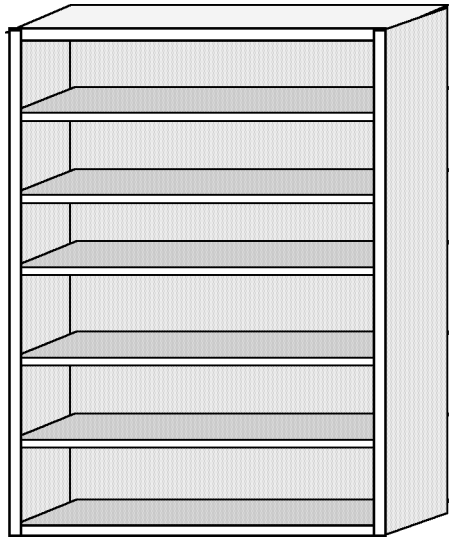
Bookcase Situations



A real-life measurement problem

I'd like to put a bookcase between the two windows in my living room.

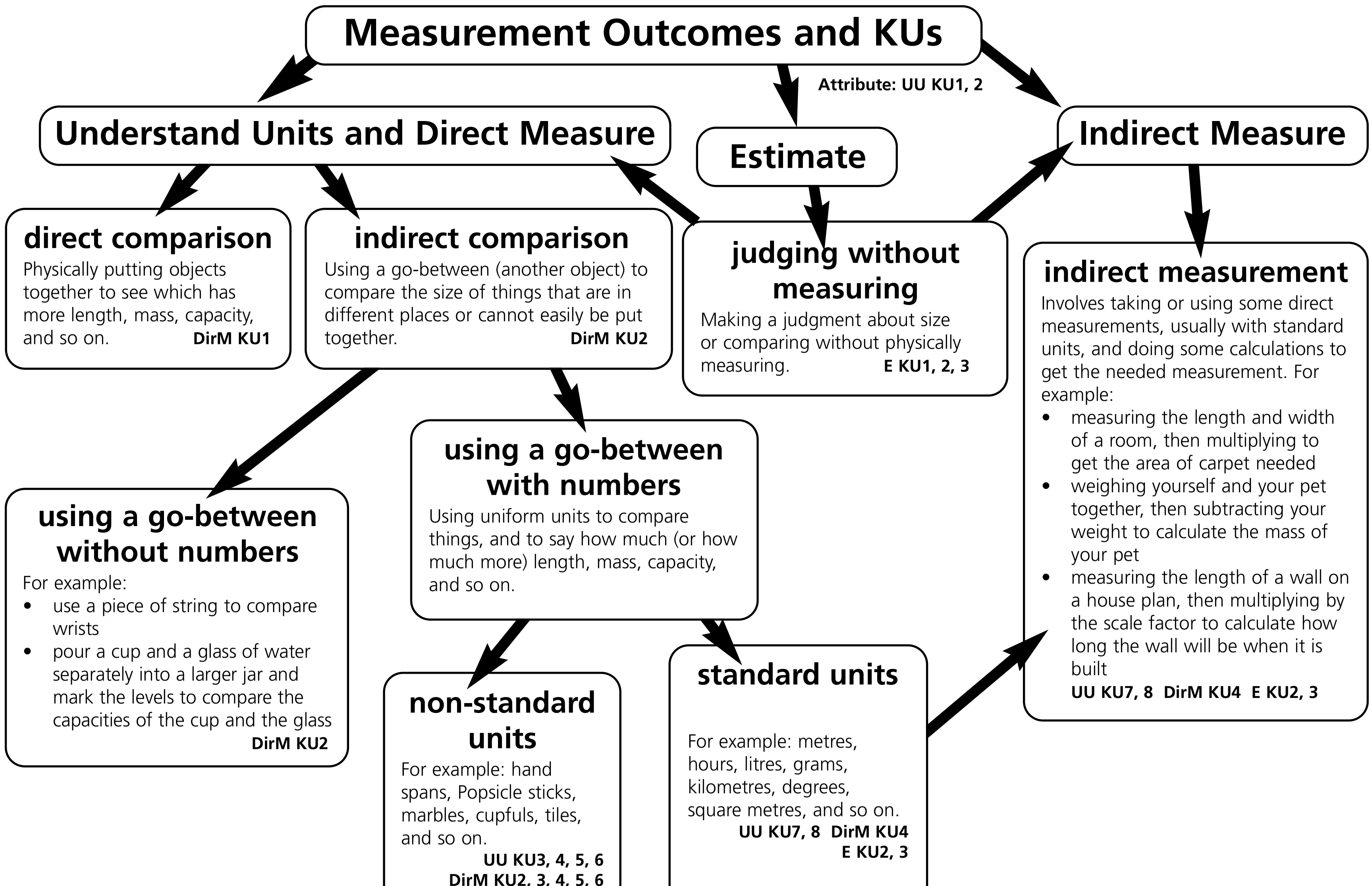
How will I know if it fits?



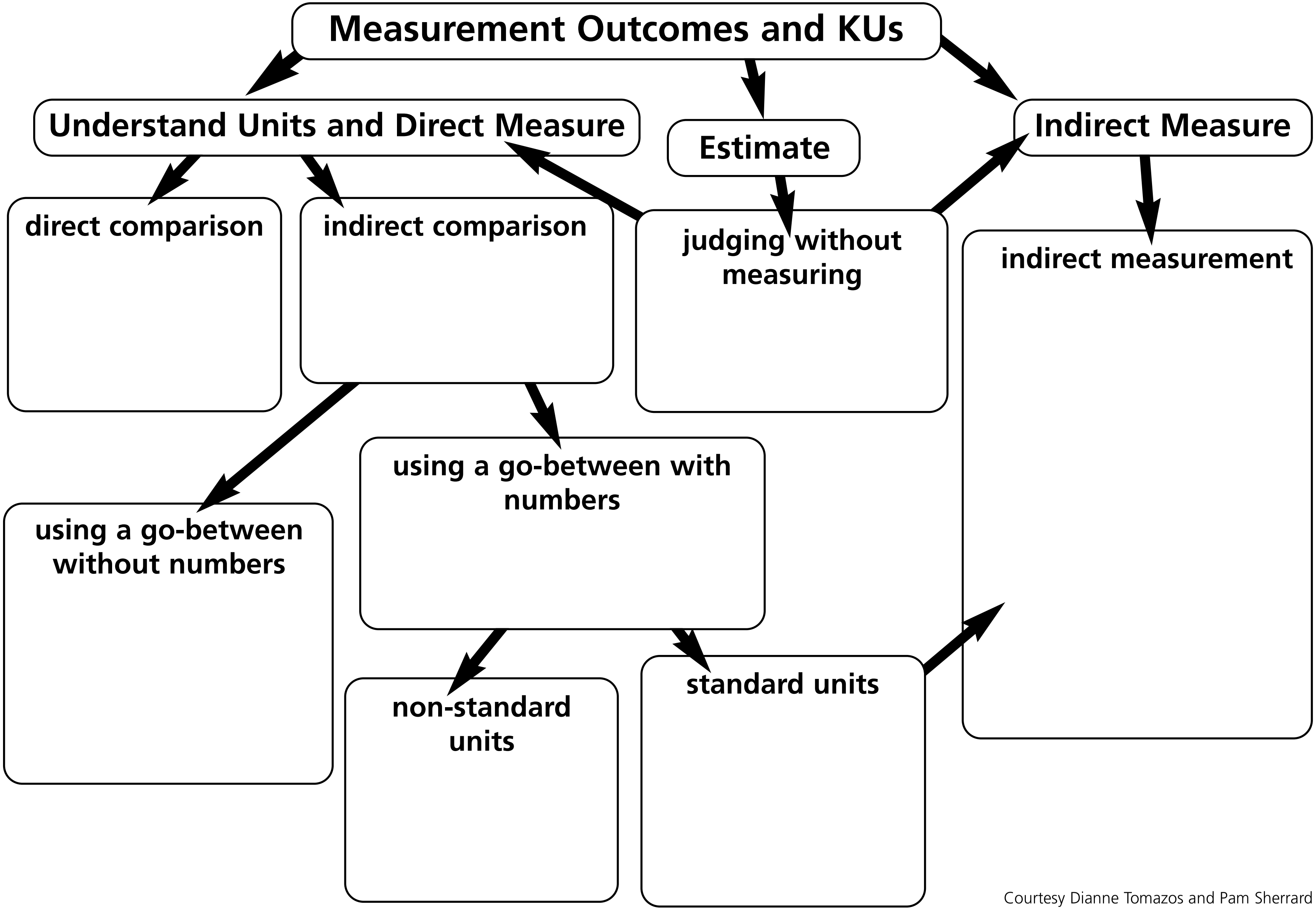
Note: The bookcase is not to scale.

The situation determines if, what, and how we measure.

FSIM013 | First Steps in Mathematics: Measurement Course Book



Courtesy Dianne Tomazos and Pam Sherrard



Courtesy Dianne Tomazos and Pam Sherrard

What I choose to do depends on the circumstances.

What measurement understandings are needed for each situation?

Situation A
I'm rearranging the furniture in my living room, and I can see that the space between the windows is much more than the width of my bookcase.

Situation B
My new bookcase has just been delivered, and the delivery person is asking me where to put it. It looks like it could fit in between the windows in my living room. It's not a very heavy bookcase, so I ask if they'd mind just trying it there to see if it will fit.

Situation C
I'm rearranging my living room and my bookcase looks like it might fit in between the windows. I've got a length of string handy, so I'll stretch it out across the bookcase, mark the length with my fingers, then hold the length of string against the wall between the windows to see if it will fit.

Situation D
I'm rearranging my living room, and my bookcase looks like it might fit in between the windows. I haven't got anything else to measure it with, so I'll use my hand spans to check if the bookcase will fit.

Situation E
In the furniture store, I saw a bookcase that I thought might fit between the two windows in my living room. I asked the salesperson to get me a measuring tape so I could check its width. When I got home, I measured the space between the windows with my own tape measure to see if the bookcase would fit.

Situation F
I'm building a new home and I want to see where I could put my bookcase. I'll use the scale on my house plans to work out how much space there will be between the windows.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Directly Compare Attributes

Use only **direct comparison** to order according to the attribute you have been assigned.
What do students need to consider when making a direct comparison?

Use Direct Measure, Key Understanding 1.

Length
What would students need to know?

Phase:

Mass
What would students need to know?

Phase:

Capacity
What would students need to know?

Phase:

FSIM013 | First Steps in Mathematics: Measurement Course Book

Area
What would students need to know?

Phase:

Volume
What would students need to know?

Phase:

Angle
What would students need to know?

Phase:

Time
What would students need to know?

Phase:

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 2

Direct Measure

- Key Understanding 1

Which Tin?

K–Grade 3

Purpose

To reveal if the student

- focuses on and describes two or three attributes of one object
- compares things with respect to a particular physical attribute
- is able to use comparative language of measurement attributes

Materials

Five tins displaying a range of attributes, i.e., short, thin, wide, tall, heavy, light
Storage box or bag for tins
Coin or similar small object to hide under a tin
Teacher Recording Sheet

Procedure

1. Begin by asking the student to take the tins out of the bag or box and put them on the table, using the opportunity to feel and comment on the mass. If the student does not make comments about the weight of the tins, ask questions such as, *How is that tin different from the other tin?* (point to the heaviest tin). Record the exact words the student uses to compare the tins, e.g., big, not big, heavy, light, not heavy.
2. Tell the student you are going to play a game. Say: *I'm going to hide a coin under one of the tins and then give you some clues to find it. Close your eyes. OK, open your eyes. I have put the coin under the tall, heavy, thin tin. Can you find it?*

   Record which tin the student points to first, which attributes the student focuses on, and how many attributes the student is able to focus on.
3. Exchange roles and ask the student to hide the coin. Say: *Now it is my turn to guess where you hide the coin. You have to give me some clues so I can guess.*

   If necessary, prompt with, *I want you to tell me as much as you can about the tin to make it easy for me to find it. Don't point to it. Keep your hands under the table, because I have to work it out from what you tell me.*

   It may be necessary to prompt the student using three clues. Say, for example, *I can see two tall heavy tins. How will I know which one the coin is under?* Record the attributes and the words the student uses.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Tin? Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. How is that tin different from the other tins? (point to the heaviest tin)

2. I'm going to hide a coin under one of the tins and then give you some clues to find it. Close your eyes. OK, open your eyes. I have put the coin under the tall, heavy, thin tin. Can you find it?

3. Exchange roles.

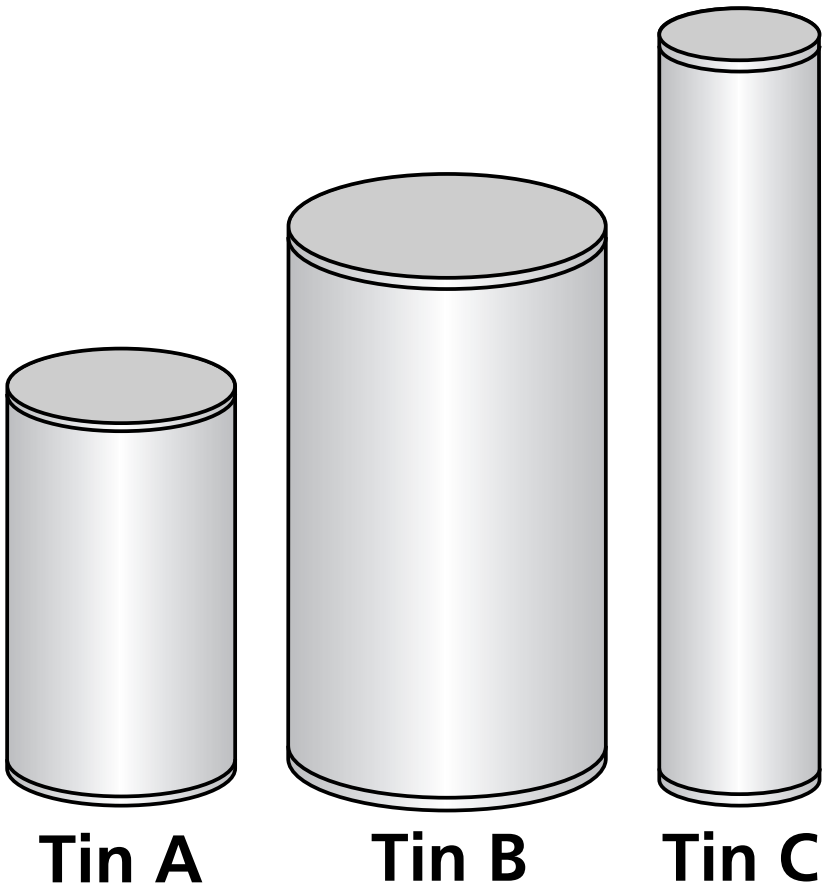
Now it is my turn to guess where you hide the coin. You have to give me some clues so I can guess (remind the student not to point).

After the student places the coin, prompt if necessary with, **I want you to tell me as much as you can about the tin to make it easy for me to find it. I can see two tall, heavy tins. How will I know which one the coin is under?**

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Tin?

Order of tins	Least	Medium	Most
Height:	Tin A	Tin B	Tin C
Weight:	Tin C	Tin A	Tin B
Width:	Tin C	Tin A	Tin B



Which Tin? Work Samples

The teacher hid a coin under Tin B, which was of medium height, the widest, and heaviest. After the student found the coin, the teacher chatted about the sizes of the tins.

Robert, 4 years old

T: Which tin did I hide my money under?
R: The "little bit big" one.
T: *(points to the shortest tin)* How big is this tin?
R: That's a not big tin.
T: Oh…and which tin is this? *(points to the tallest tin)*
R: That's the big, big tin.

Which attributes does Robert refer to?

Can he use more than one attribute at a time?

Does he use different words for length?

What does he do to compare the tins?

Phase:

Gemma, 5 years old

T: Which tin did I hide my money under?
G: The Muva (Mother) one.
T: *(points to the shortest tin)* How big is this tin?
G: That's the baby one.
T: Oh…and which tin is this? *(points to the tallest tin)*
G: That's the Granfarva one.

Which attributes does Gemma refer to?

Can she use more than one attribute at a time?

Does she use different words for length?

What does she do to compare the tins?

Phase:

FSIM013 | First Steps in Mathematics: Measurement Course Book

Beau, 7 years old

T: Which tin did I hide my money under?
B: The middle one. It's not the middle one really but it could be, look! *(orders the tins by height and points to the middle tin)* See, now it's the middle one.
T: What did you notice about that tin when you picked it up?
B: Well…I'll just check. *(picks up the middle tin)* It's a bit heavier than the tall tin. It's actually the heaviest tin. But you couldn't say it's the biggest one. *(points to the tallest tin)* That one is actually the biggest.
T: *(points to the shortest tin)* What do you know about the size of this tin?
B: That's the short tin.
T: Oh, and how is it different from the tin in the middle? *(points to the medium height tin)*
B: That's wider.

Which attributes does Beau refer to?

Can he use more than one attribute at a time?

Does he use different words for length?

What does he do to compare the tins?

Phase:

Comparative terms
General Bigness

Referring to Attributes

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 2

Direct Measure

- Key Understanding 1

Ordering Tins

K–Grade 3

Purpose

To reveal if the student

- knows that ordering objects by different attributes may result in different orders
- can use comparative words to describe order

Materials

Five tins displaying a range of attributes, i.e., short, thin, wide, tall, heavy, light
Teacher Recording Sheet

Procedure

1. Use the tins from the Which Tin? task.

   Ask the student to put the tins in order. Say: *Please put the tins in order in some way*.

   If the student hesitates for too long, a prompt may help, e.g., *Put them in a line from biggest to smallest.*

   Record the attributes the student uses to order the tins.

2. Ask the student to describe the order. Say: *Tell me the idea you used to order them. Tell me about each tin.*

   Record the words the student uses to describe their order.

3. Ask the student to put the tins into a different order. Say: *Could the tins be ordered in a different way?*

   If the student hesitates for too long, prompt with, *Last time you ordered by (e.g., how tall they were). This time can you use something else about the tins?*

   After the student has reordered the tins, ask, *What have you used to order the tins this time?*

   Record the language the student uses.

   Consider and record:

   - Is the student able to reorder the tins?
   - Which attribute does the student use?
   - What words does the student use to describe their order?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ordering Tins: Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. Use tins from the Which Tin? task and ask the student to put them in order. Which attribute does the student use to order the tins?

2. Ask the student to describe the order. Say: **Tell me the idea you used to order them.** If necessary ask, **Tell me about each tin.** If the student hesitates for too long, prompt with, **Put them in a line from biggest to smallest.** If necessary, say: **Start with the biggest one and put it over here.** What words does the student use to describe their order?

3. Ask the student to put the tins into a different order. Say: **Could the tins be ordered in a different way?** If the student hesitates for too long, prompt with, **Last time you ordered by (e.g., how tall they were). This time can you use something else about the tins?** Ask, **What have you used to order the tins this time?**

Is the student able to reorder the tins?

Which attribute does the student use?

What words does the student use to describe their order?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ordering Tins

Video of Chongo, 5 years old

Is Chongo able to order the tins from most to least of any attribute?

Diagnostic Map

Which phases will the Ordering Tins task give information about?

Which phase is Chongo showing evidence of?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Planning for Learning

What mathematics do Robert, Gemma, Beau, and Chongo need to learn?

Understand Units, Key Understanding 1: Attributes

Understand Units, Key Understanding 2: Comparative language

Sample Learning Activities: Animal Stories, p. 34

What focus questions could you use to help these students learn about attributes and comparative language?

Attributes:

Comparative language:

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic Map: Think Pair Share

Refer to the "During" box of each phase.

Emergent

Matching and Comparing

Quantifying

Between Now and the Next Session

Create an action plan with your group.

What tasks will you use with at least six of your students to find out what they know?

Task	Class	Page
Snail Trails	Grades 1–5	42
Which Line Is Longer?	Grades 4–7	45
Which Tin?	K–Grade 3	57
Ordering Tins	K–Grade 3	62

Complete the Action Plan on the next page, noting which tasks you and your colleagues will use and when.

Make decisions about sharing the resources and/or preparation that is needed.

Set the date and time for a Task Review session.

Before you meet, copy the Making Decisions and Classroom Planner masters on Course Book pp. 13 and 14.

Plan for your students' mathematics learning following the steps outlined in the Task Review section on p. 11.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Action Plan

Who?	Which Diagnostic Tasks?	When?	Resource Preparation

Task Review Date: _____ Time: _____ Venue: _____

Unit 3: Meaningful MASS Measurement

Unit 3 Outcomes

You will
- extend the learning about direct comparison to see how it applies to measuring mass
- understand the mathematics of using units and part units to measure mass
- investigate the use of standard units for measuring mass
- further develop an understanding of the Diagnostic Map, including the Measuring Phase

Related Reading

***First Steps in Mathematics* Measurement Book 1**

Chapter 2: Understand Units
Key Understanding 1 (pp. 24–31)
Key Understanding 2 (pp. 32–39)
Key Understanding 3 (pp. 40–53)
Key Understanding 4 (pp. 54–61)
Key Understanding 5 (pp. 62–69)

Chapter 3: Direct Measure
Key Understanding 1 (pp. 112–125)
Key Understanding 2 (pp. 126–133)
Key Understanding 3 (pp. 134–141)
Key Understanding 4 (pp. 142–155)
Key Understanding 5 (pp. 156–167)

Diagnostic Map: Measurement
Matching and Comparing to Measuring phases

Background Reading

Grant, T.J., & Kline, K. (2003). Developing the building blocks of measurement with young children. In D. Clements & G. Bright (Eds.), *Learning and teaching measurement 2003 yearbook* (pp. 46–56). Reston, VA: National Council of Teachers of Mathematics.

Lindsay, M., & Scott, A. (2005). Estimating eggs. *Australian Primary Mathematics Classroom, 10* (4), 4–8.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 2

Direct Measure

- Key Understanding 1

Which Frog Is Heaviest?

K–Grade 4

Purpose

To reveal if the student

- focuses on general bigness and smallness rather than mass
- hefts to directly compare mass
- understands heavier/heaviest and lighter/lightest

Materials

Three toy frogs (or other familiar objects). Ensure that the frogs (or other toys) conform to the following:

- Frog 1: heaviest but smallest
- Frog 2: lightest but largest
- Frog 3: middle sized and mid-weight

A representation of a pond and three lily pads

Teacher Recording Sheet

Producing Work Samples

Set out the three frogs (or familiar objects) on the table next to the representation of the pond and lily pad.

1. Say: *Which of these frogs is the biggest?* Then ask, *Which of these frogs is the heaviest?* Record what the student does.
2. If the student picks up only one frog, prompt to consider all three. Say: *Check to see if you are right.*
3. Say: *Put the heaviest frog on one lily pad.*
4. Then say: *Now put the lightest frog on another lily pad.*
5. To give the student the opportunity to use the language, say: *Tell me about which frog you have put on each of the lily pads.*

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Frog Is Heaviest? Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. Which of these frogs is the biggest? Which of these frogs is the heaviest?

2. Check to see if you are right.

3. Put the heaviest frog on the lily pad.

4. Now put the lightest frog on another lily pad.

5. Tell me about what you have on the lily pads.

Comments

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Frog Is Heaviest?

Video of Jordan, 4 years, 8 months

What does Jordan know about mass?

Which phase does this suggest for Jordan? Why?

Planning for Learning

What mathematics does Jordan need to learn?

Key Understandings

Sample Learning Activities

FSIM013 | First Steps in Mathematics: Measurement Course Book

Why might mass be more difficult to compare than length?

Refer to Sample Learning Activity: Using a Balance Scale, Direct Measure, p. 115
Case Study 1: pp. 122–124

Video of Christen, 10 and a half years old

What does Christen know about mass?

Will this task allow us to decide whether Christen is through the Quantifying Phase?

Measurement Questions

Consider the following questions:

- Which is heaviest?
- How heavy is it?
- How much heavier is it than the other one?

What kind of measurement is each question likely to stimulate?

Which of the questions are more likely to provide information about the Quantifying Phase? Why?

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 3, 4, 5

Direct Measure

- Key Understandings 2, 3, 5

How Heavy Is This Frog/Tin? Grades 5–7

Purpose

To reveal if the student

- uses balance scales accurately to match the mass of an object
- counts how many whole uniform units match the mass of an object
- knows to use the same-size objects to compare two quantities
- lets the number of units override perceptual judgment
- understands and uses the notion of part units when describing the size of an object

Materials

A fabric, weighted frog or Tin One (heavier but smaller); a second frog or Tin Two (lighter but bigger)

Balance scales

Set of washers

Assorted marbles, wooden and plastic blocks, and other objects

Set of weights (e.g., Invicta blue 20 g, 10 g, 5 g)

Teacher Recording Sheet (use as a worksheet for small or whole class assessment or as an observation sheet for individual interview)

Procedure

1. Say: *Which of these two frogs/tins is heavier? How do you know?*
   (To check if the notion of heaviness is dominated by visual perception and to give students the opportunity to self-correct after lifting the frogs/tins.)

2. Point to one of the frogs/tins on the desk. Say: *How heavy do you think this frog/tin is?*
   Wait for an answer, and then say: *Is there any way we could find out for sure?*
   Prompt students to use balance scales, if necessary, and provide various objects.
   Say: *Let's use some of these materials to find out how heavy your frog/tin is.*

FSIM013 | First Steps in Mathematics: Measurement Course Book

3. After the student has weighed the frog/tin ask, *So how heavy is this frog/tin?* After the student answers, say: *Is that exactly how much it weighs?* (To see if students will use smaller units to become more accurate.)

4. If the student is able to use units to weigh one frog/tin, then ask the student to use a different unit to measure the other one. Say: *Can you weigh this frog/tin with* _____ (name of different unit)*?* Suggest lighter objects to weigh the lighter frog/tin or heavier objects to weigh the heavier frog/tin.

5. After the student has weighed the second frog/tin, say: *How heavy is this frog/tin? Which frog/tin is heavier? How do you know?* Then say: *How much heavier is this frog/tin than the other?*

FSIM013 | First Steps in Mathematics: Measurement Course Book

How Heavy Is This Frog/Tin? Teacher Recording Sheet

Name _____ Grade _____ Date _____

Which of these two frogs/tins is heavier?

How do you know?

Choose one frog/tin. How heavy do you think this frog/tin is?

Use the balance scales to weigh this frog/tin.
How heavy is it exactly?

How did you work it out?

Weigh the second frog/tin using _____. How heavy is this one?

So which one is heavier?

How do you know?

How much heavier is this frog/tin than the other frog/tin?

How did you work it out?

FSIM013 | First Steps in Mathematics: Measurement Course Book

How Heavy Is This Frog/Tin?

Video of Kayla, 9 years old

What does Kayla know about using units to measure mass?

Which phase does this suggest for Kayla? Why?

Planning for Learning

Which Key Understanding will help Kayla use units to say how much heavier?

Which of the bullet points within the Key Understanding will be the mathematics focus?

Sample Learning Activities

FSIM013 | First Steps in Mathematics: Measurement Course Book

How Heavy Is This Frog/Tin?

Video of Holli, 11 years old

What does Holli know about using units to measure mass?

Understand Units, Key Understanding 5, p. 62

How can we be more accurate when using balance beams to measure mass?

FSIM013 | First Steps in Mathematics: Measurement Course Book

How Heavy Is This Frog/Tin?

Video of Christen, 10 and a half years old

What does Christen know about using units to measure mass?

Which phase does this suggest for Christen? Why?

Good questions and tasks provoke students to show us what they know and understand.

Diagnostic TASK

FOCUS
Direct Measure
- Key Understanding 4

Kitchen Scales — Grades 5–7

Purpose

To reveal if the student
- understands that the graduations on a kitchen scale mark the end points of gram and kilogram units

Materials

A copy of the worksheets, pp. 82–85

Producing Work Samples

Small group or whole class

Provide every student with a copy of the worksheets.
Read through the worksheets for the students and ensure that they understand the tasks.

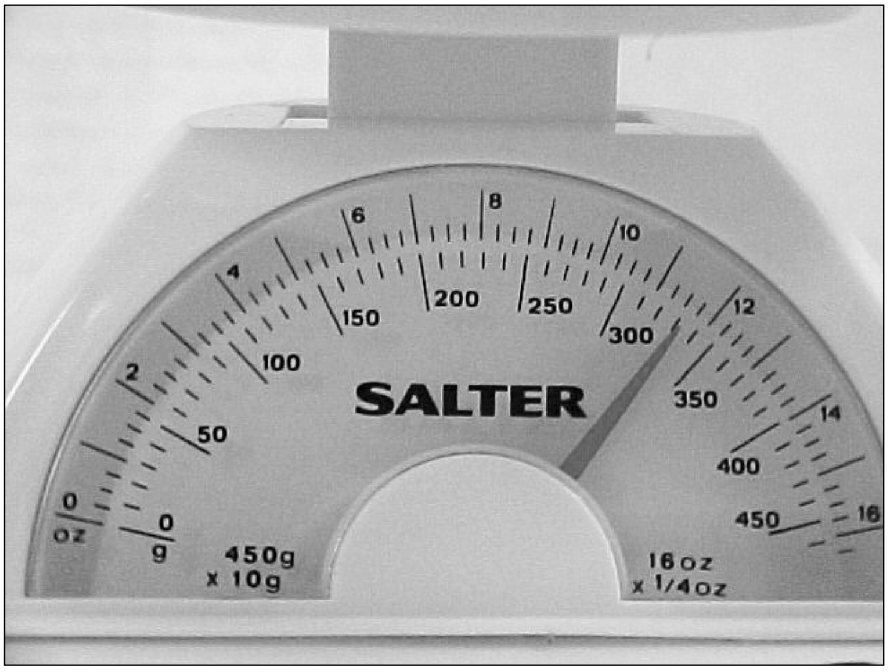
Individual interviews

Interviews may be needed as a follow-up to clarify how some students found their answers.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Kitchen Scales: Student Worksheet

Name _____ Grade _____ Date _____



How much does the jar of coffee weigh? _____

How did you work it out?

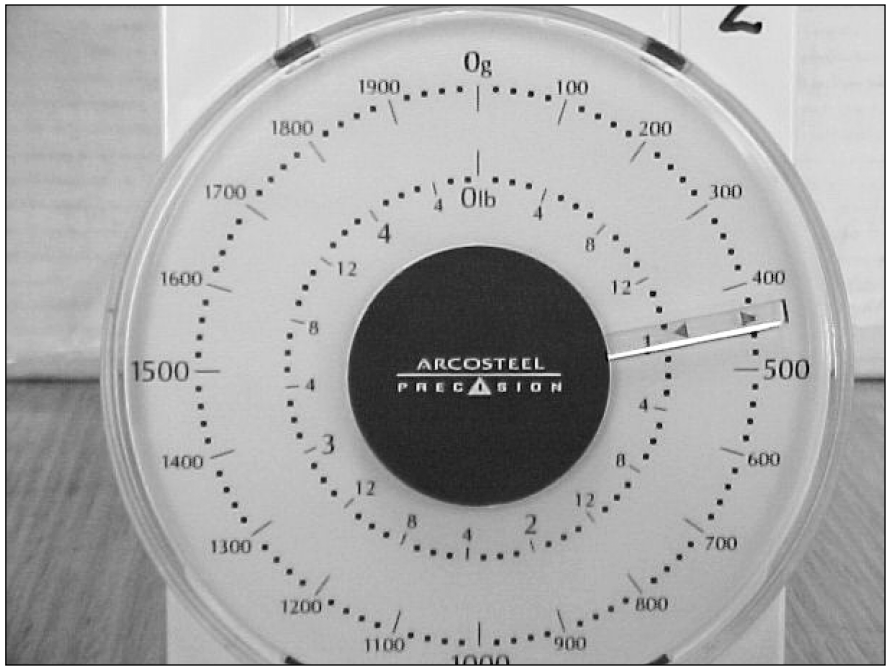
_____

_____

_____

Kitchen Scales: Student Worksheet

Name _____ Grade _____ Date _____



How much does the cup weigh? _____

How did you work it out?

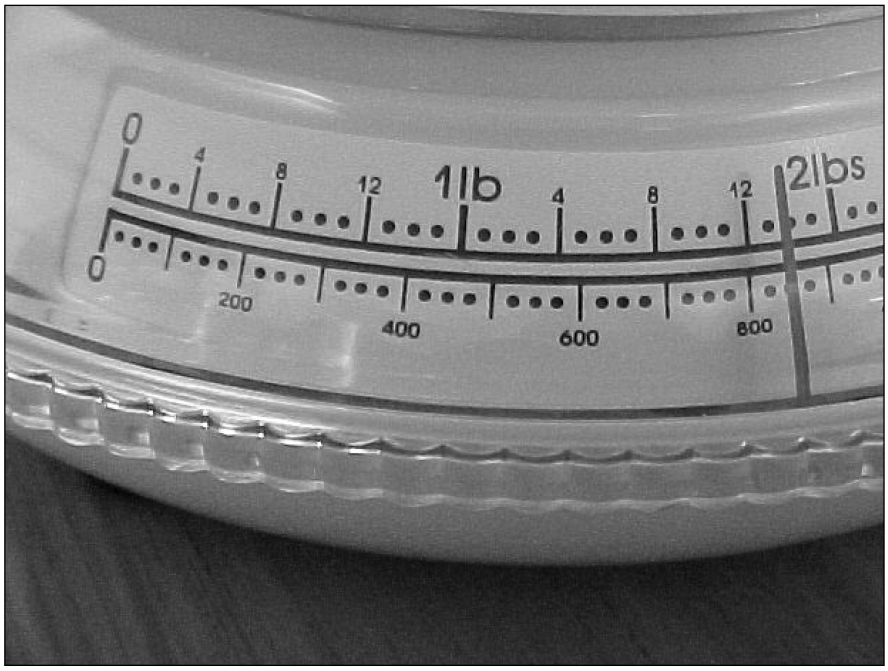
_____

_____

_____

Kitchen Scales: Student Worksheet

Name _____ Grade _____ Date _____



How much does the cup weigh? _____

How did you work it out?

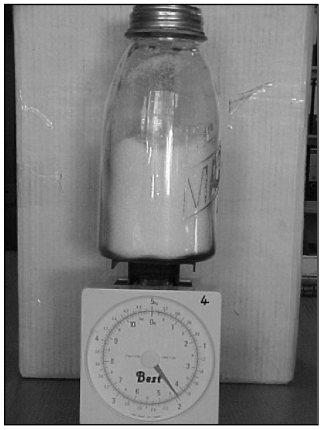
_____

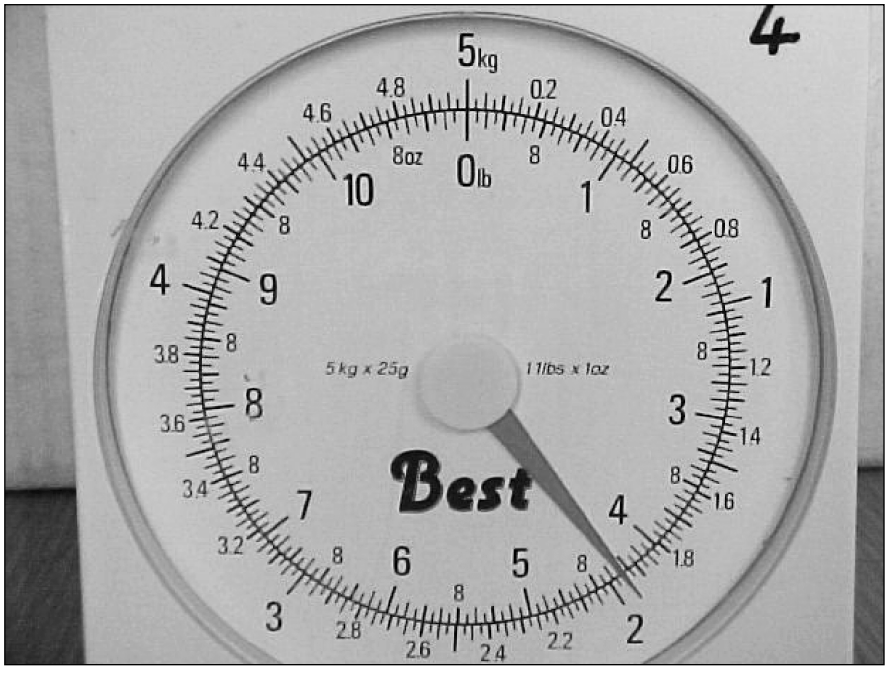
_____

_____

Kitchen Scales: Student Worksheet

Name _____ Grade _____ Date _____





How much does the jar of sugar weigh? _____

How did you work it out?

_____

_____

_____

FSIM013 | First Steps in Mathematics: Measurement Course Book

Kitchen Scales

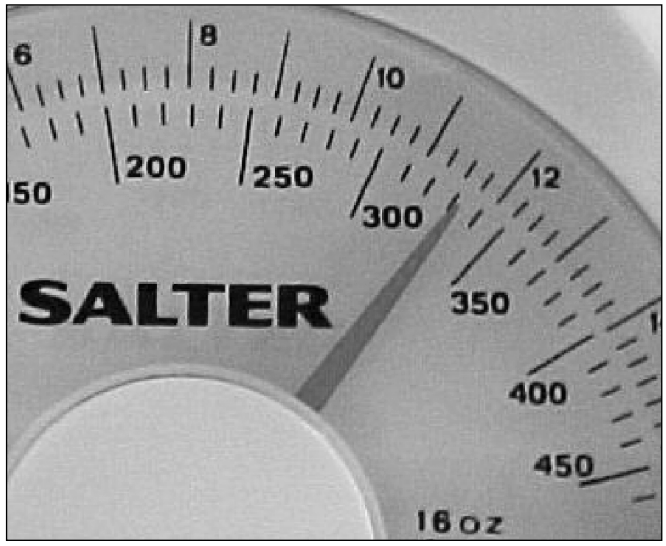
Scale A

What do the lines mean?

What do the dots mean?

Which lines are numbered?

Which lines are not numbered?



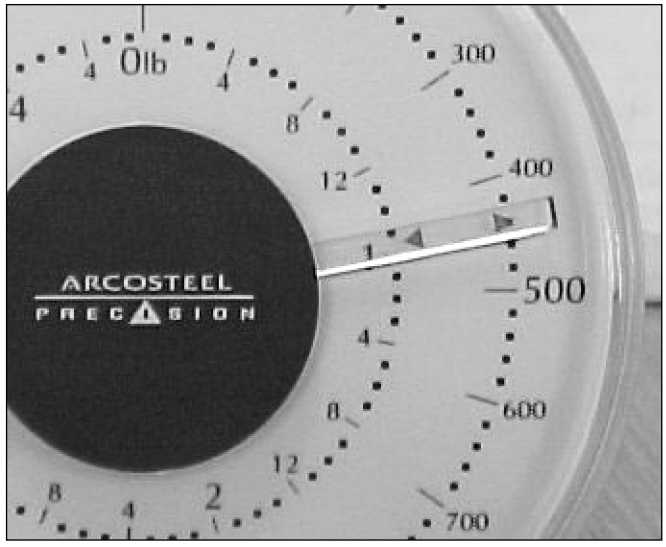
Scale B

What do the lines mean?

What do the dots mean?

Which lines/dots are numbered?

Which lines/dots are not numbered?



Kitchen Scales

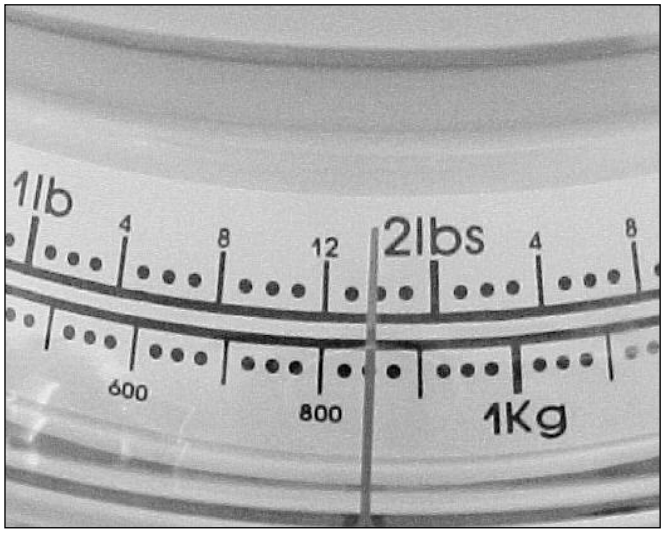
Scale C

What do the larger unnumbered lines mean?

What do the smallest lines mean?

Which lines are numbered?

Which lines are not numbered?



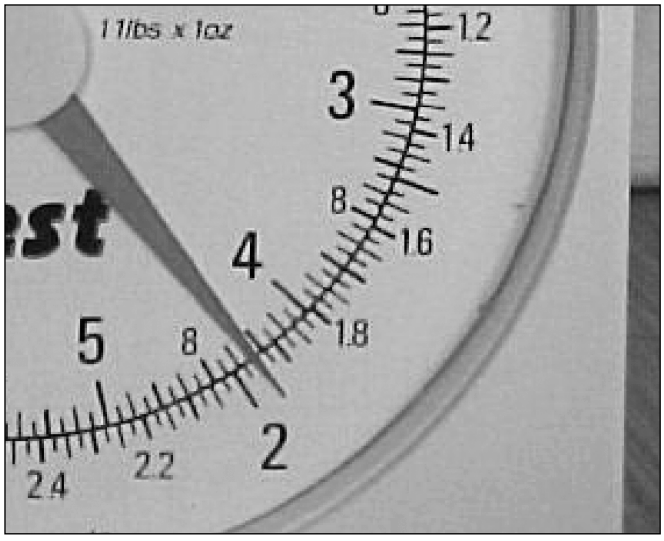
Scale D

What do the larger lines mean?

What do the smaller lines mean?

Which lines are numbered?

Which lines are not numbered?



FSIM013 | First Steps in Mathematics: Measurement Course Book

Kitchen Scales

Put the scales in order from least to most complex, and say why they are ordered that way.

Scale	Reason for the Order
Simplest	
Most complex	

Diagnostic Map
When would we expect students to be able to make sense of the most difficult set of scales?

Video of Rekisha, 12 years old
What does Rekisha need to learn?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Planning for Learning

Refer to Direct Measure, Key Understanding 4, p. 142

Can the same progression be followed for mass? Why or why not?

Find a Sample Learning Activity that will help students see the link between using units in a balance scale and the lines on a kitchen scale.

Mass or Weight

The words *mass* and *weight* refer to two different things. According to *The Canadian Oxford Dictionary*, *mass* is "a coherent body of matter of indefinite shape" (measured in grams, kilograms, and tonnes, and also in ounces, pounds, stone, and tons), whereas *weight* is defined as "the force experienced by a body as a result of the Earth's gravitational pull" (measured in Newtons, after the great mathematician Sir Isaac Newton who developed the laws of gravity). An astronaut on the moon would have the same mass but much less weight than on Earth because the gravitational pull of the much larger Earth is greater than that of the moon.

Teachers should use both words when measuring this attribute, ensuring that students learn to use both words. While young students should **use** both words, we do not expect them to **understand** the difference between them until they are much older.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Linking Phases, Key Understandings, and Diagnostic Tasks

Phase	Key Understandings	Diagnostic Tasks
Emergent Phase		
Matching and Comparing Phase		
Quantifying Phase		
Measuring Phase		
Relating Phase		

Between Now and the Next Session

Create an action plan with your group.

What tasks will you use with at least six of your students to find out what they know?

Task	Class	Page
Which Frog Is Heaviest?	K–Grade 4	70
How Heavy Is This Frog/Tin?	Grades 5–7	75
Kitchen Scales	Grades 5–7	81

Complete the Action Plan on the next page, noting which tasks you and your colleagues will use and when.

Make decisions about sharing the resources and/or preparation that is needed.

Set the date and time for a Task Review session.

Before you meet, copy the Making Decisions and Classroom Planner masters on Course Book pp. 13 and 14.

Plan for your students' mathematics learning following the steps outlined in the Task Review section on p. 11.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Action Plan

Who?	Which Diagnostic Tasks?	When?	Resource Preparation

Task Review Date: _____ Time: _____ Venue: _____

Unit 4: Constructing CAPACITY Understandings

Unit 4 Outcomes

You will
- learn how students develop an understanding of direct comparison of capacity
- plan to develop students' understanding of direct comparison and using units of capacity
- consider the mathematics involved in choosing appropriate units and the practical skills involved in using them well
- further develop an understanding of the Emergent through to Quantifying phases of the Diagnostic Map

Related Reading

***First Steps in Mathematics* Measurement Book 1**

Chapter 2: Understand Units
- Key Understanding 1 (pp. 24–31)
- Key Understanding 4 (pp. 54–61)

Chapter 3: Direct Measure
- Key Understanding 1 (pp. 112–125)
- Key Understanding 2 (pp. 126–133)
- Key Understanding 3 (pp. 134–141)
- Key Understanding 4 (pp. 142–155)

Diagnostic Map: Measurement

Emergent to Quantifying phases

Background Reading

Muir, T. (2005). When near enough is good enough. *Australian Primary Mathematics Classroom, 10* (2), 9–14.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS

Understand Units

- Key Understanding 1

Direct Measure

- Key Understandings 1, 2

Pour to Decide

K–Grade 4

Purpose

To reveal if the student

- responds to and/or uses capacity language
- estimates which of two containers holds more water
- can compare two containers to find out which one holds more by pouring water from one to the other

Materials

Two quite differently shaped, clear plastic drinking glasses
Cloth for spillage
Bucket of water (rice or sand can be used)
Teacher Recording Sheet

Procedure

Individual interview

Ask, *Which of these two glasses will hold the most water? How do you know?* Record exactly what the student says.

If the student fills both containers and makes a judgment just by looking, prompt for direct comparison strategies. Ask, *Is there something else you could do to work out which one holds more?*

If the student does not realize that the water can be used to compare the capacities, fill the larger glass and say, *Pour the water into the other glass to see which one holds the most*. Record what the student says and does.

Small group

This task could be conducted as a small group activity. Provide each student with a set of materials and observe and record the group's actions and conversation. Use the suggested questions from above.

Refer to Direct Measure, Key Understanding 1, Did You Know? on p. 125 to help interpret why some students do not know to stop pouring when the water begins to overflow.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Pour to Decide: Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. Which container will hold the most water? How do you know?

2. If the student fills both containers and makes a judgment by just looking, prompt for direct comparison strategies with, **Is there something else you could do to work out which one holds more?**

Further prompt with, **Will filling one container with water help you to work out which one holds more? Show me.**

Other questions:

FSIM013 | First Steps in Mathematics: Measurement Course Book

Pour to Decide

Video of Emma, 5 years old

What does Emma know about measuring capacity?

Video of Jack, 5 and a half years old

What does Jack know about measuring capacity?

How is Jack's method of measuring different from Emma's?

Diagnostic Map

When should we expect students to

Directly compare capacity by pouring from one container to another?

Use units to say which container holds more?

Planning for Learning

What mathematics does Emma need to learn?
Direct Measure KU1

Did You Know? section, p. 125

Choose a Sample Learning Activity that might help Emma.

What focus questions might you use?

A Note about Capacity and Volume

Volume and capacity are measures of the same attribute—an amount of space—and the same units can be used for either.

Volume is a measure of the amount of space taken up by something, and **capacity** is a measure of the amount of space (or volume) that some things contain.

All objects have volume, but only particular objects, e.g., containers, have capacity.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS
Direct Measure
- Key Understanding 4

Make a Measuring Cup — Grades 4–7

Purpose
To find out if the student understands that the calibrated scale on a measuring cup shows the units of capacity.

Materials
Bucket of water
Range of lids, caps, spoons, toy cups, and a medicine glass
Permanent fine-tipped pen
Ruler
Container that is wider at the top than the base, e.g., a round, transparent take-away food container or plastic cup
Old towel to mop up afterwards

Producing Work Samples
This task could be used in an individual interview or with a small group or whole class.

Individual interview
Ask the student to make the clear plastic cup or container into a measuring cup to use for small quantities of liquid (e.g., paint for an art class).

Provide a range of materials for the student to select from.

Record what the student does and says.

Small group or whole class
Observe all students and take note of whether they choose to use a unit to fill the container and mark the scale on the side accordingly, or whether they use another method. Choose three or four students to talk to in depth, asking them to explain what they are doing and why.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Make a Measuring Cup

Consider Luke's work samples from Unit 1.

Diagnostic Map

Which phase does this suggest for Luke? Why?

Planning for Learning

Luke needs to focus on Direct Measure KU4, p. 142.

Find a Sample Learning Activity to use with Luke and students like him.

What focus questions might you use?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Choosing and Using Capacity Units

Video of Christen (Grade 5)

1. Does Christen choose a suitable unit? What does she base her choice on? (Refer to Understand Units, KU4, p. 54.)

2. What practical skills does she show when using her unit? (Refer to Direct Measure, KU3, p. 134.)

Provide a wide range of materials for students to choose from.

What is the focus of Understand Units KU4?

What is the focus of Direct Measure KU3?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic Map
Which phase does this suggest for Christen? Why?

Using Units Carefully

Students who are through the Quantifying Phase know that they have to use units carefully to make as close a match as possible, i.e., they ensure that the unit size remains constant and that there are no gaps or overlaps.

Refer to Direct Measure KU3 (*Measurement Book 1,* pp. 134–135) and paraphrase the three things that students need to know and be able to do in order to ensure that the unit size remains constant and they fit in as many as possible.

First:

Second:

Third:

FSIM013 | First Steps in Mathematics: Measurement Course Book

Overestimate or Underestimate?

Which of the three things students need to know and be able to do **but was not attended to** caused the measurement error in each situation on p. 103? Sort the situations according to whether an overestimate or an underestimate will occur.

Overestimate

Underestimate

FSIM013 | First Steps in Mathematics: Measurement Course Book

Overestimate or Underestimate

Read each situation below. Decide if the situation will result in an overestimate or an underestimate. Then decide which of the three measurement errors from Direct Measure Key Understanding 3 is at play in the situation. Record your thinking below each situation.

Use the following symbols to record your thinking.

O: Overestimate: The measurement taken will be greater than the actual measurement of the object, event, shape.

U: Underestimate: The measurement taken will be less than the actual measurement of the object, event, shape.

1: The size of the unit must be uniform.

2: The unit must be used in a way that ensures a good match.

3: The object, event or shape must be fully matched and no more.

An electrician does not start with the marker on the trundle wheel at the beginning of the length being measured.

A dressmaker uses a tape measure that is old and stretched.

When butting Popsicle sticks end to end to measure the length of a table, a finger width is placed between each stick.

Paper clips are zigzagged along a line when used to measure its length.

In a "guess the weight of the jelly beans" competition, the container is weighed with the jelly beans.

To determine someone's height, a book is placed on the person's head, and the front of the book is tilted toward the floor.

There are air pockets in the cup of flour for the cake.

Paper squares are overlapped when glued on to show area.

When making a cake, someone uses spoonfuls of ingredients that are level instead of rounded.

When gold nuggets are weighed, all the milligram weights are a tiny bit too light.

During an orienteering activity, the instruction says 25 paces, but you are tired and your steps get progressively smaller.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Between Now and the Next Session

Create an action plan with your group.

What tasks will you use with at least six of your students to find out what they know?

Task	Class	Page
Pour to Decide	K–Grade 4	94
Make a Measuring Cup	Grades 4–7	98
Choosing and Using Capacity Units*	Grades 4–7	100
Overestimate or Underestimate*	Grades 4–7	102

*An activity from this session that could be used as a Diagnostic Task

Complete the Action Plan on the next page, noting which tasks you and your colleagues will use and when.

Make decisions about sharing the resources and/or preparation that is needed.

Set the date and time for a Task Review session.

Before you meet, copy the Making Decisions and Classroom Planner masters on Course Book pp. 13 and 14.

Plan for your students' mathematics learning following the steps outlined in the Task Review section on p. 11.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Action Plan

Who?	Which Diagnostic Tasks?	When?	Resource Preparation

Task Review Date: _____ Time: _____ Venue: _____

Unit 5: Advancing AREA Understandings

Unit 5 Outcomes

You will
- further develop an understanding of the Emergent Phase through to the Relating Phase of the Diagnostic Map
- extend the mathematics knowledge gained for length, mass, and capacity to area
- look at the impact of the various materials and questions used during area activities

Related Reading

***First Steps in Mathematics* Measurement Book 1**

Chapter 2: Understand Units
Key Understandings 1–7 (pp. 24–89)

Measurement Book 2

Chapter 2: Indirect Measure
Key Understanding 1 (pp. 26–43)
Key Understanding 4 (pp. 68–82)

Chapter 3: Direct Measure
Key Understanding 1 (pp. 112–125)
Key Understanding 2 (pp. 126–133)
Key Understanding 3 (pp. 134–141)
Key Understanding 5 (pp. 156–167)
Background Notes (p. 104)

Diagnostic Map: Measurement
Emergent to Relating phases

Background Reading

Battista, M.T. (1999). The importance of spatial structuring in geometric reasoning. *Teaching Children Mathematics,* November (pp. 170–177).

Kribs-Zaleta, C.M., & Bradshaw, D.A. (2003). A case of units. *Teaching Children Mathematics,* March (pp. 397–399).

Outhred, L., & Mitchelmore, M. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education, 31* (2), 144–167.

Schifter, D., & Szymaszek, S. (2003). Structuring a rectangle: Teachers write to learn about their students' thinking. In D. Clements & G. Bright (Eds.), *Learning and teaching measurement 2003 yearbook* (p. 143). Reston, VA: National Council of Teachers of Mathematics.

Willis, S. (2005). Making mathematics vital. *Proceedings from the Twentieth Biennial Conference of the Australian Association of Mathematics Teachers,* p. 40.

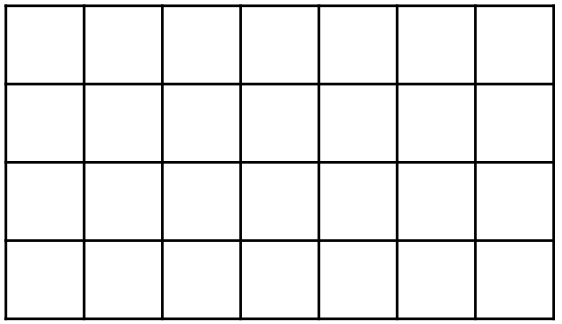
FSIM013 | First Steps in Mathematics: Measurement Course Book

Isaac's Area Work

What Is the Area?

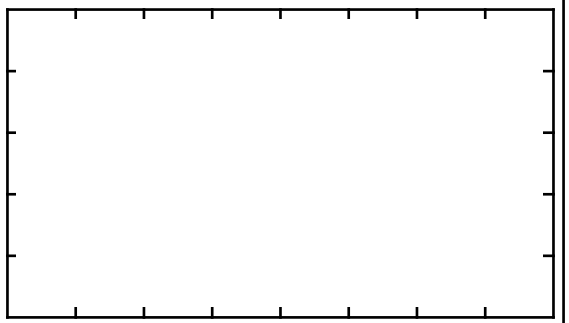
A number of Grade 7 students were interviewed individually and asked to find the area of some shapes. After completing each one, they were asked to explain how they worked it out. Here are Isaac's responses to the tasks.

A



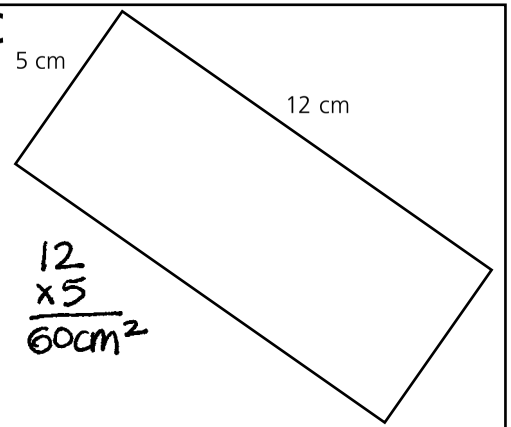
I: 28 squares.
T: How did you work that out?
I: Four down the sides and 7 across the top, and I think that equals 28.

B



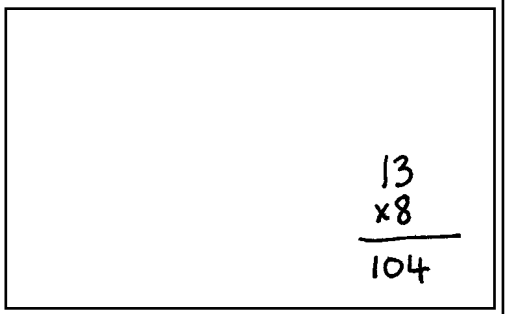
I: 40 squares.
T: How did you work that out?
I: If we joined the lines up, there would be squares. So there's 5 down the sides and there's 8 squares across the top. So 8 times 5 is 40.

C



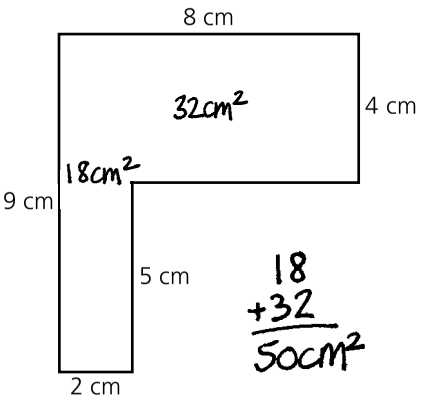
I: I worked out 5 times 12 is 60 centimetres. I mean centimetres squared.
T: Can you tell me why you multiplied the numbers?
I: If I worked it out, it would take a bit longer to count them out. If I know the measurement on each side and multiply, it's quick.

D



T: How did you work it out? 104cm²
I: I measured all the centimetres. There's 13 across the bottom and 8 across the side, so 8 times 13 is 104 centimetres.
T: What did you want to call the measure?
I: Centimetres squared. *(rewrote the answer)*

E



What does Isaac know about measuring area?

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 3, 4

Direct Measure

- Key Understanding 3

Leaf Task

Grades 4–7

Purpose

To reveal if the student

- understands the attribute of area
- can use units to measure the area of the leaf
- considers the gaps and overlaps when using units

Materials

A copy of the Leaf Task sheet

A variety of materials: small and large blocks, pattern blocks, grid paper, counters, cotton balls, paper clips, buttons, Unifix cubes, centicubes, pasta, bread tags, bottle tops, etc.

Procedure

Individual interview

Interviews are appropriate for younger students or for students whom teachers consider may be at risk.

Provide a copy of the Leaf Task sheet and a mixture of materials in one container.

Ask the student: *Can you please use the materials to work out the area of the leaf?*

Note whether the student attends to the gaps between the units. Ask all students to explain their thinking: *Does it matter if you have gaps between your units? Why?*

Note whether the student attempts to keep the units within the leaf outline. Ask all students to explain their thinking: *Does it matter if you have some units overlapping the edge of the leaf? Why?*

Small group

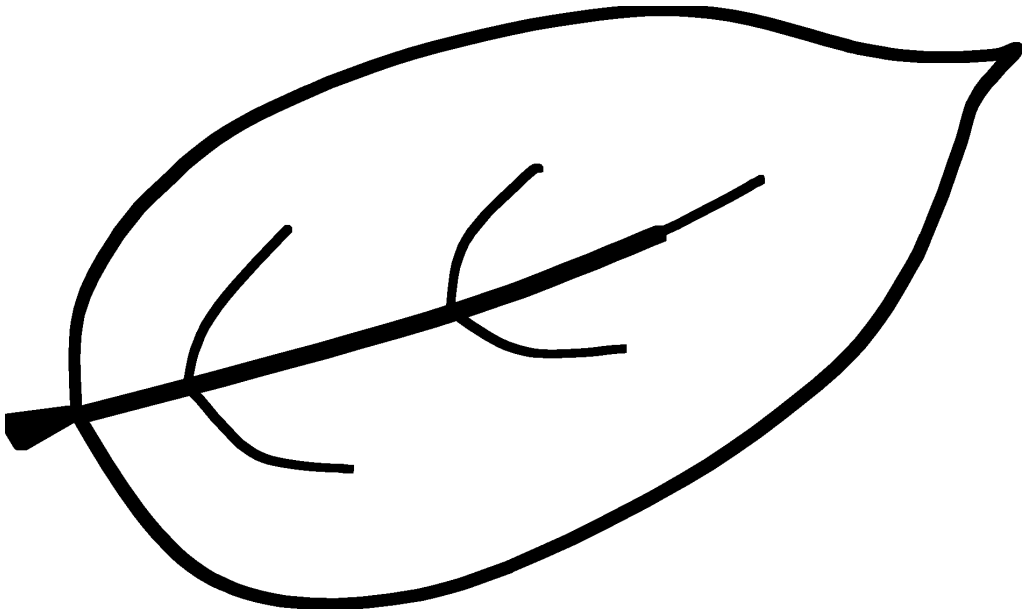
This task may be conducted with a small group, using the instructions above. Take note of which students attend to the gaps and overlaps and then ask them to explain their thinking.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Leaf Task

Name _____ Grade _____ Date _____

Measure the area of the leaf and show or explain how you did it.



FSIM013 | First Steps in Mathematics: Measurement Course Book

Leaf Task: Work Sample

Isaac was asked to find the area of a diagram of a leaf. A variety of materials including counters, pattern blocks, 1-cm and 2-cm cubes, and grid paper were available. He covered the leaf carefully with cubes, but he had many gaps and some overlaps around the edges. The teacher asked if he was happy with some cubes going over the edge.

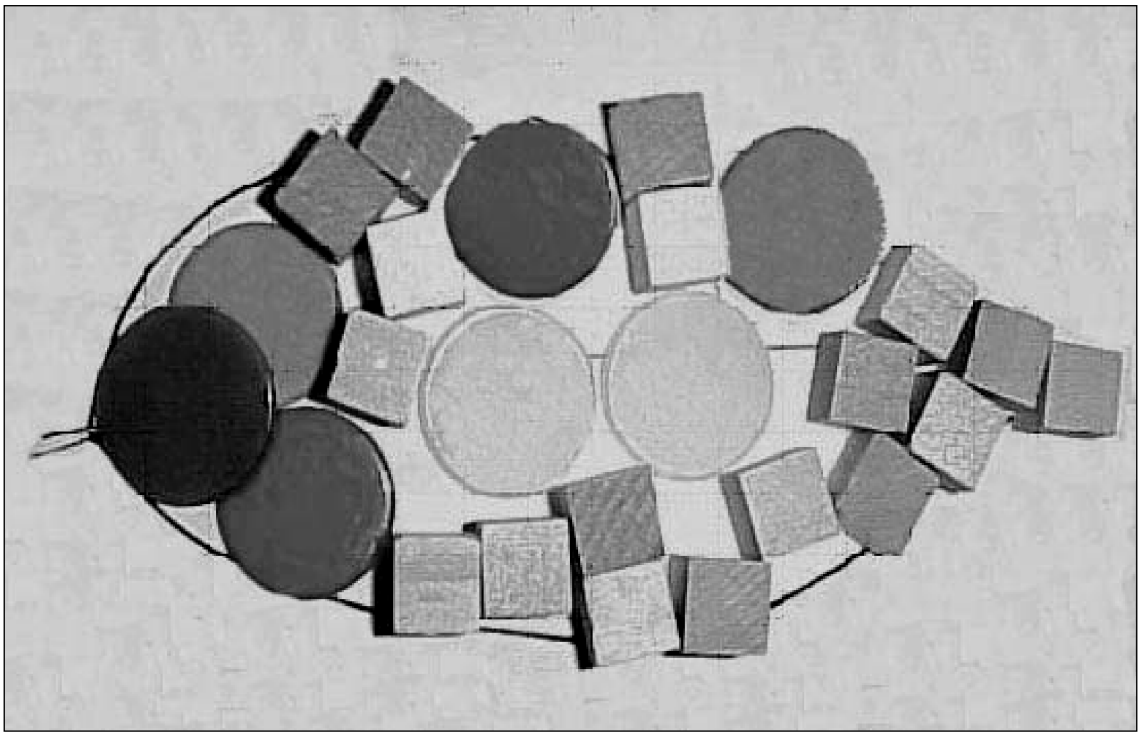
He replied, "No, not really."

The teacher then asked what he could do about it.

He replied, "I could take a few off the edge and put in counters because they're round." He started to do this, then swept them all off the leaf and started again.

The teacher asked Isaac to tell her what he had started with and why he changed his mind.

He said, "I started with the small centimetre cubes and I fitted them in, and there were too many cubes hanging off the leaf, so I started with the counters because they're round and I added the cubes in the empty spaces."



FSIM013 | First Steps in Mathematics: Measurement Course Book

Leaf Task

What new information do we get from Isaac's response to this task?

How can you explain such diverse responses from Isaac?

Direct Measure Background Notes

How is area different from the other attributes? Why is it more difficult to measure?

Refer to Background Notes, pp. 105–106, from "When we use a shape or tile…"

FSIM013 | First Steps in Mathematics: Measurement Course Book

Planning for Learning

What mathematics does Isaac need to learn? Which Key Understandings?

Understand Units Direct Measure

Choose a Sample Learning Activity that would help Isaac use units to measure area.

Understand Units Direct Measure

What focus questions might you use?

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 2, 3, 4, 5, 6

Direct Measure

- Key Understandings 1, 2, 3, 5

Indirect Measure

- Key Understandings 1, 4

Ice Cream Puddles

K–Grade 7

Purpose

To investigate what the student knows about

- the attribute of area
- directly comparing area
- using units to measure and compare areas
- using rectangular arrays and the area formula as a shortcut

Materials

Cardboard cut-out of the two ice cream puddles

A variety of objects for use as units, e.g., 1-cm and 2-cm cubes, tiles, marbles, pattern blocks, round counters

A collection of measuring equipment that extends beyond what is required to measure area, e.g., balance scales, measuring cylinders, string, measuring tape, ruler, pencil, scissors, glue, plain paper, square grid paper; also a container of sand, rice, or water

Teacher Recording Sheet

Producing Work Samples

Individual interview

Hand the student the cut-out of the two puddles and present this scenario: On a very hot day, two children drop their ice creams on the ground. The ice creams melt, making two ice cream puddles.

1. Ask, *Which puddle is bigger? Are you sure? Show me how you know*. Prompt to pick up the puddles if the student does not choose to superimpose.

2. Point to the puddle the student has chosen as the largest and ask, *How big is that puddle?* If necessary, prompt the student to use the materials: *Can you use the materials to work it out?*

FSIM013 | First Steps in Mathematics: Measurement Course Book

3. If the student uses materials that are easy to count, ask the student to compare the two puddles: *How much bigger is that puddle* (the one chosen) *than the other one?* If the student chooses to draw a grid across the puddle, then do not ask for a comparison of the two puddles.

4. If the student does not use the materials to work out the size of the first puddle, or if the student uses materials that are not easy to count, such as sand or string, then prompt again by asking, *How much bigger is that puddle* (the one chosen) *than the other one?*

Ask students to write or draw what they did to work it out. Record observations on the record sheet provided.

Small group

This task may be administered to a small group of middle and upper primary students using the instructions above. Each student needs a copy of the two puddles and easy access to the materials listed above. Ask students to write or draw what they did to work it out. Note what individual students say and do throughout the activity, using the record sheet.

Ice Cream Puddles: Teacher Recording Sheet

Name _____ Grade _____ Date _____

Say: *On a very hot day, two children drop their ice creams on the ground. The ice creams melt, making two ice cream puddles.*

1. Which puddle is bigger? Are you sure? Show me how you know.
If necessary, prompt the student to pick up the puddles.

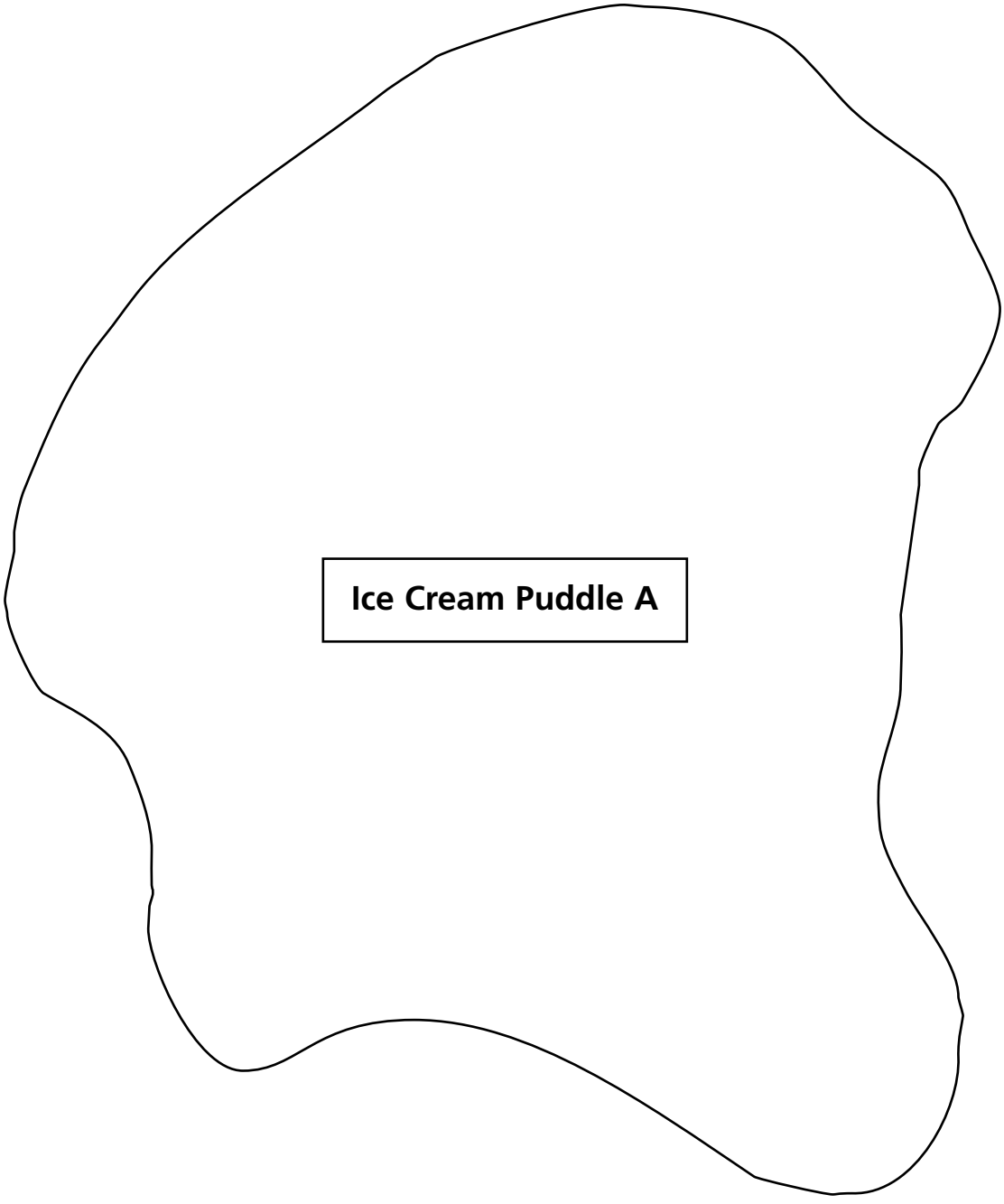
2. Point to the puddle the student has chosen as the largest. **How big is that puddle?**
If necessary, prompt with, **Can you use the materials to work it out?**

3. If the student uses materials that are easy to count, ask, **How much bigger is that puddle** (the one chosen) **than the other one?** If the student chooses to draw a grid across the puddle, then do not ask for a comparison of the puddles.

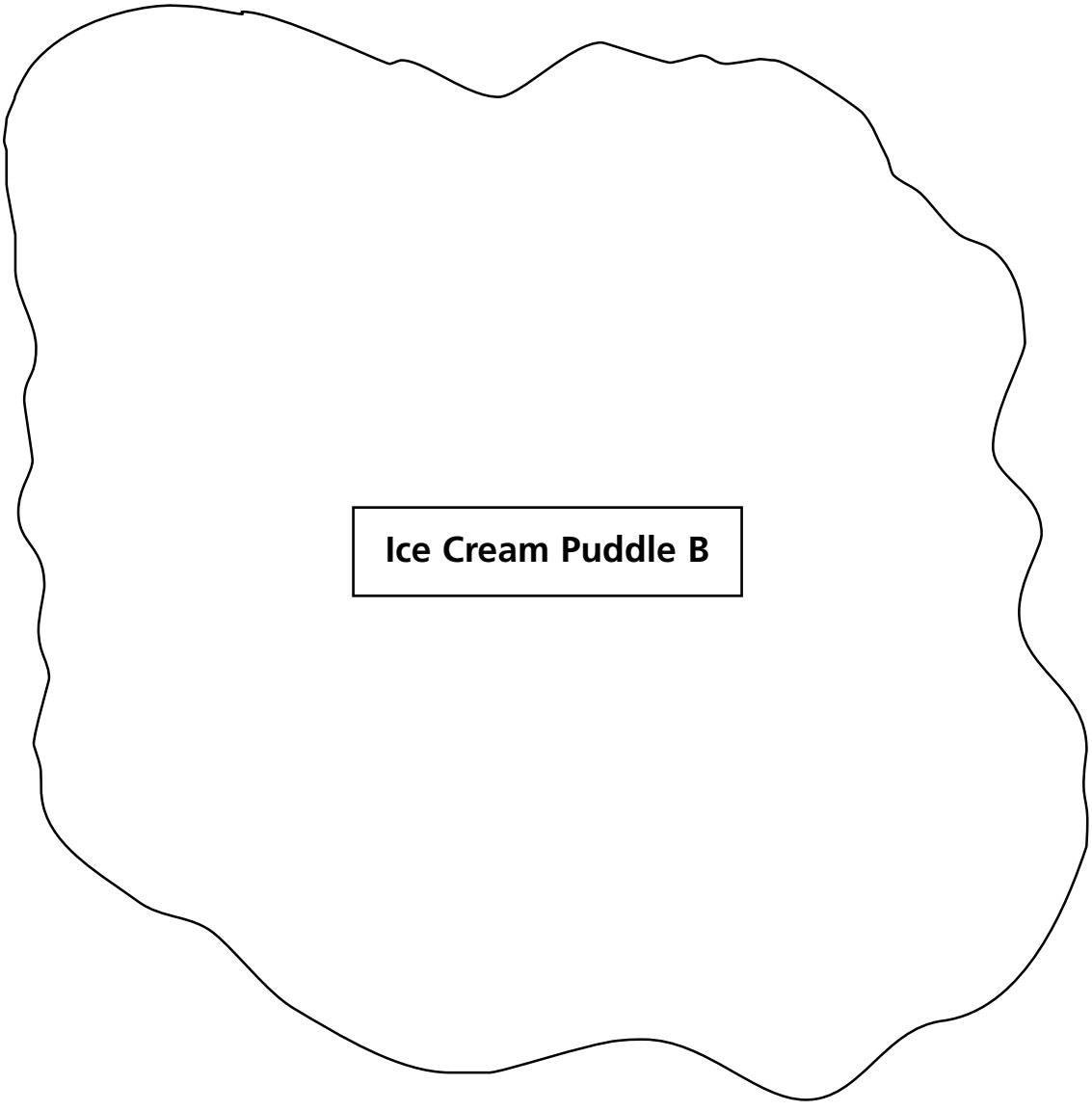
4. If the student does not use the materials to work out the size of the first puddle, or uses materials that are not easy to count, such as sand or string, prompt with, **How much bigger is that puddle** (the one chosen) **than the other one?**

FSIM013 | First Steps in Mathematics: Measurement Course Book

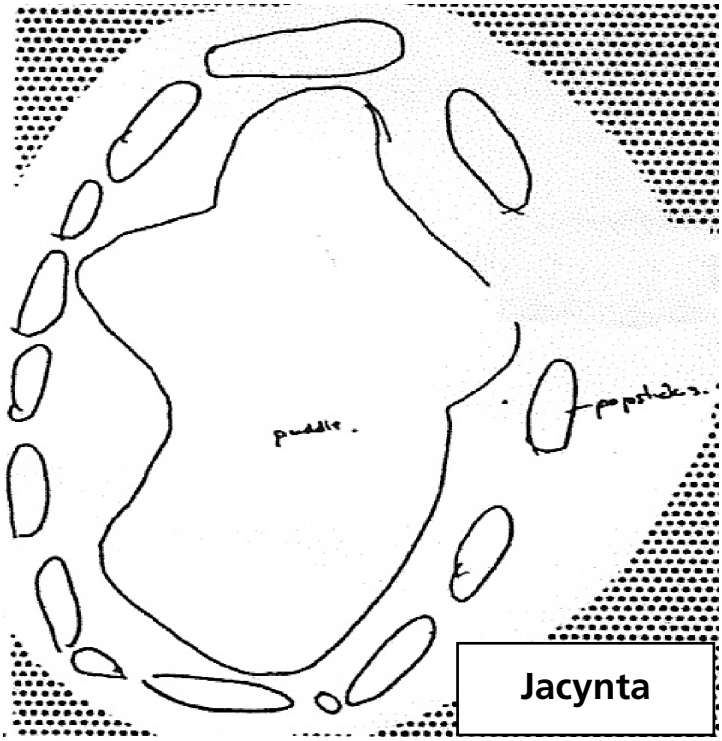
Ice Cream Puddles



Ice Cream Puddles



Ice Cream Puddles: Work Samples



Jacynta (Grade 1)
Prompted to pick up the puddles to compare, she held them in two hands side by side, saying "This is small. This is big." When asked how she knew, she said, "Because I see it."

When asked how big the puddle was, she placed Popsicle sticks around the edge. When the question was repeated, she said, "Very big."

After she drew what she had done, the teacher asked if her drawing was the same as what was on the table, and she replied, "Yes." (Jacynta did not attend to the number or arrangement of the sticks.)

Diagnostic Map
What phase of thinking is Jacynta showing evidence of?
How do you know?

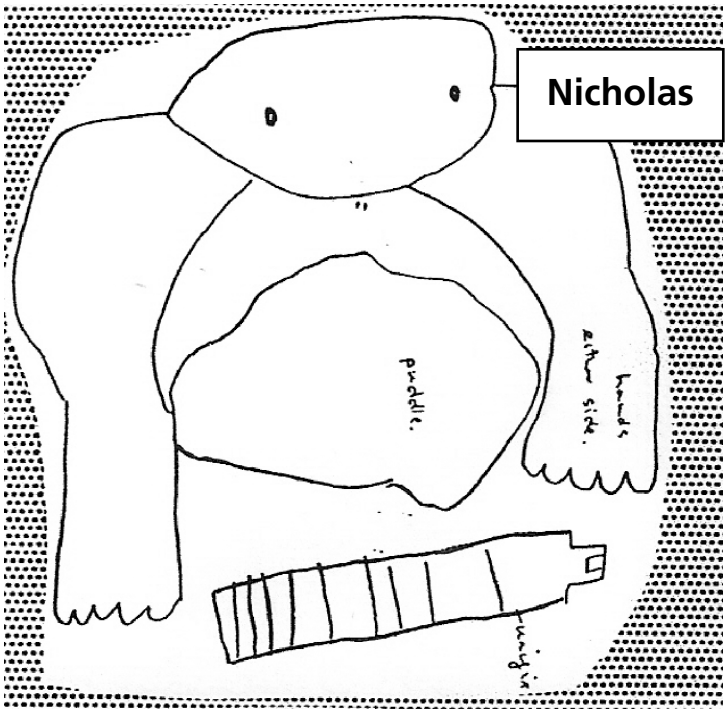
Planning for Learning
Which Key Understandings?

Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ice Cream Puddles: Work Samples



Nicholas (Grade 1)
Nicholas was prompted to superimpose to compare. He noticed the overlap and said, "I think the blue one is bigger than the green one." When asked how big, he lined up some Unifix cubes next to the puddle and then used his hands to show how big.

N: I'm measuring it with my hands.
T: How did the blocks help you to measure?
N: Because I'm putting my hands on them and they were the same.

Diagnostic Map
What phase of thinking is Nicholas showing evidence of?
How do you know?

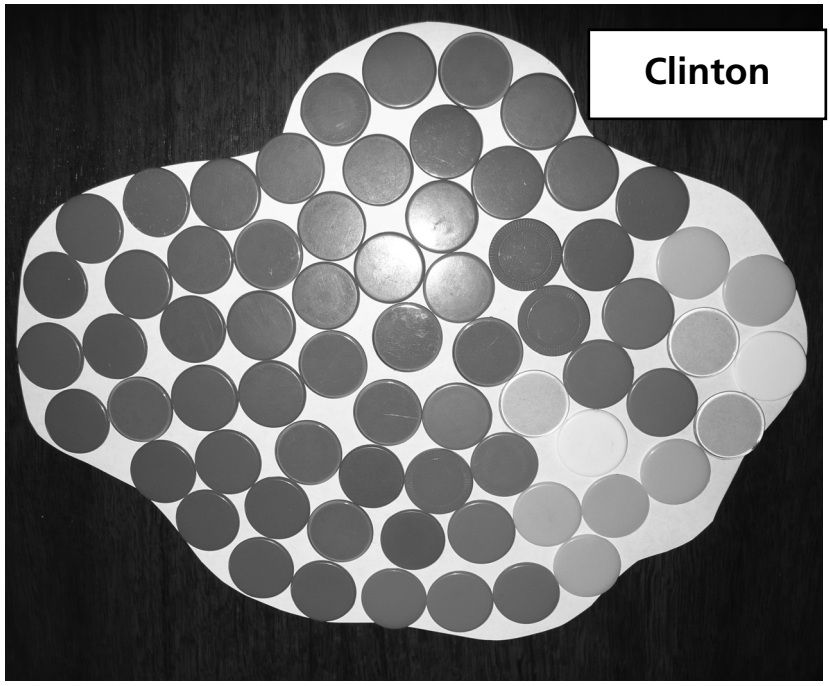
Planning for Learning
Which Key Understandings?

Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ice Cream Puddles: Work Samples



Clinton (Grade 2)
He chose the bigger puddle by superimposing, then said, "You do the area by putting the counters or blocks and making them touch all the same size to find out the area. Count how many counters. The one that's got the most, that's the biggest." When asked how big his puddle was, he carefully covered the puddle with counters. He tried to use all red counters and said they needed to be the same size. He checked the other coloured counters by superimposing to make sure they were all the same size. When asked how big the puddle was he said, "About 50."

Diagnostic Map
What phase of thinking is Clinton showing evidence of?
How do you know?

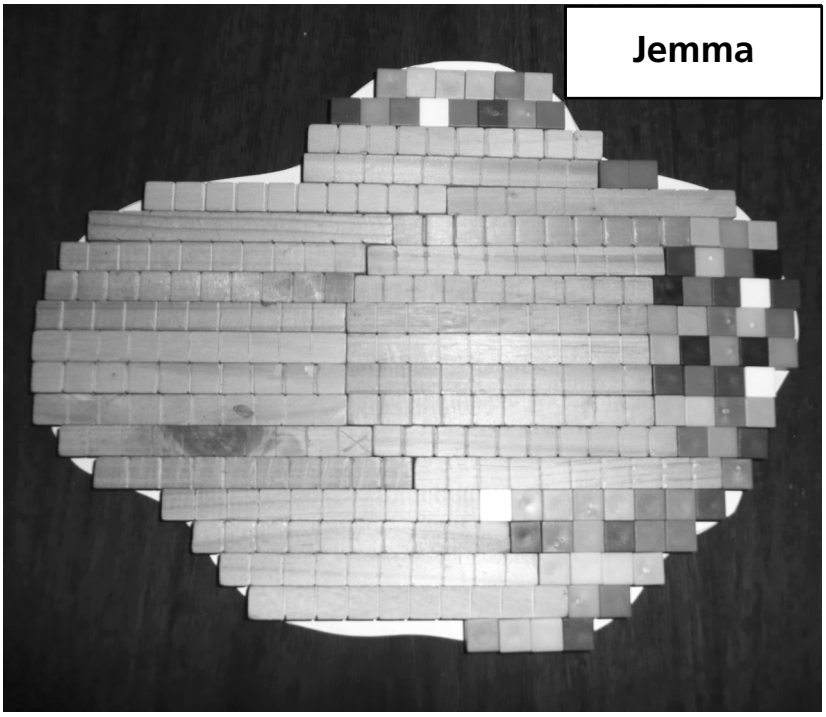
Planning for Learning
Which Key Understandings?

Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ice Cream Puddles: Work Samples



Jemma (Grade 5)
After prompting, she superimposed to compare the puddles. When asked how big the chosen puddle was, she carefully covered the puddle with MAB longs and then filled in the gaps with small blocks. When asked if she had done anything like this before she said, "Yes, you have to put them on to see how many fit on." When she was finished, the teacher repeated the question. "How big is the puddle?" Jemma said, "It won't really tell us how big the puddle is." She did not count the blocks. The teacher asked, "What would we need to do to say how big the puddle is?" She replied, "Use a ruler." She took all the blocks off and placed the ruler across the puddle and then sat puzzled. "It's pretty hard."

Diagnostic Map
What phase of thinking is Jemma showing evidence of?
How do you know?

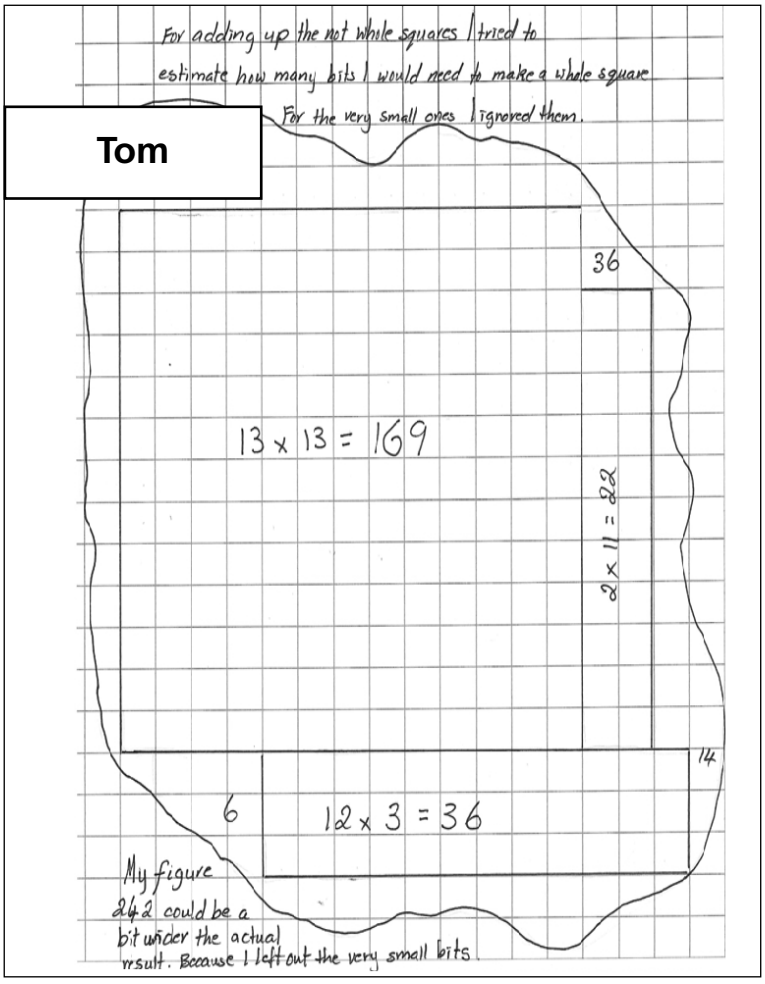
Planning for Learning
Which Key Understandings?

Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ice Cream Puddles: Work Samples



Tom (Grade 7)
Tom chose the large puddle by looking. "It looks bigger because it's more circular." When asked how big the puddle was, he said, "I have to count squares. Am I allowed shortcuts?"

Diagnostic Map
What phase of thinking is Tom showing evidence of?
How do you know?

Planning for Learning
Which Key Understandings?

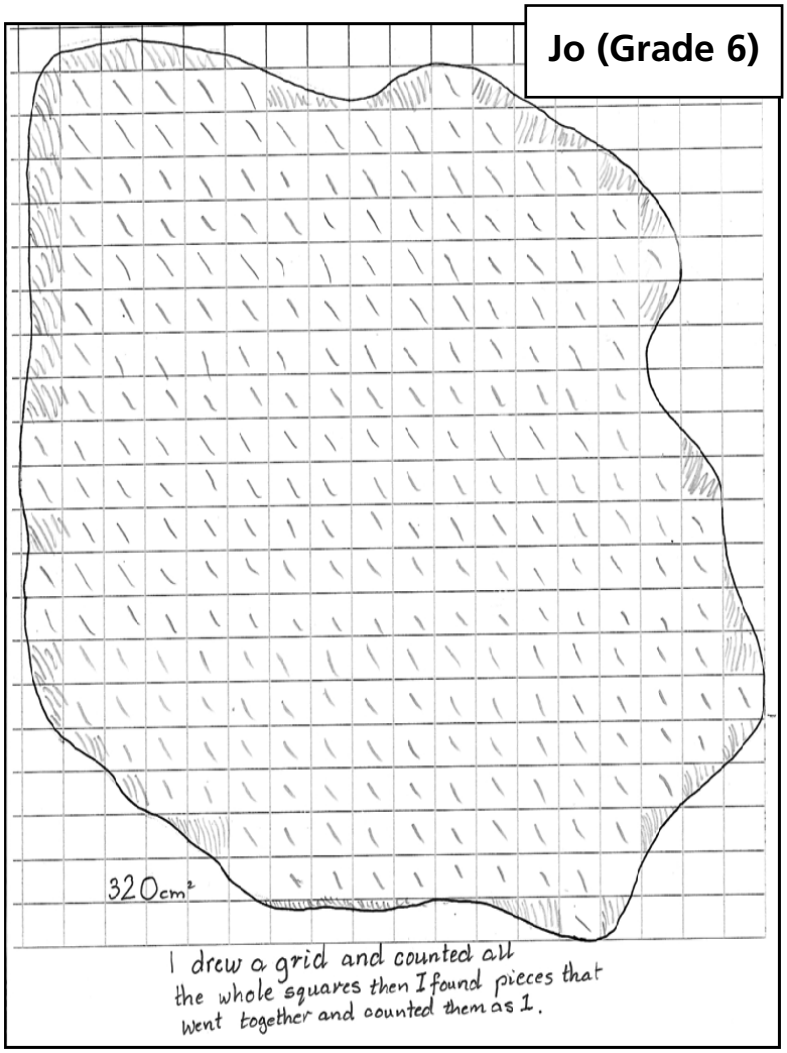
Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ice Cream Puddles: Work Samples

Jo (Grade 6)



Diagnostic Map

What phase of thinking is Jo showing evidence of?
How do you know?

Planning for Learning

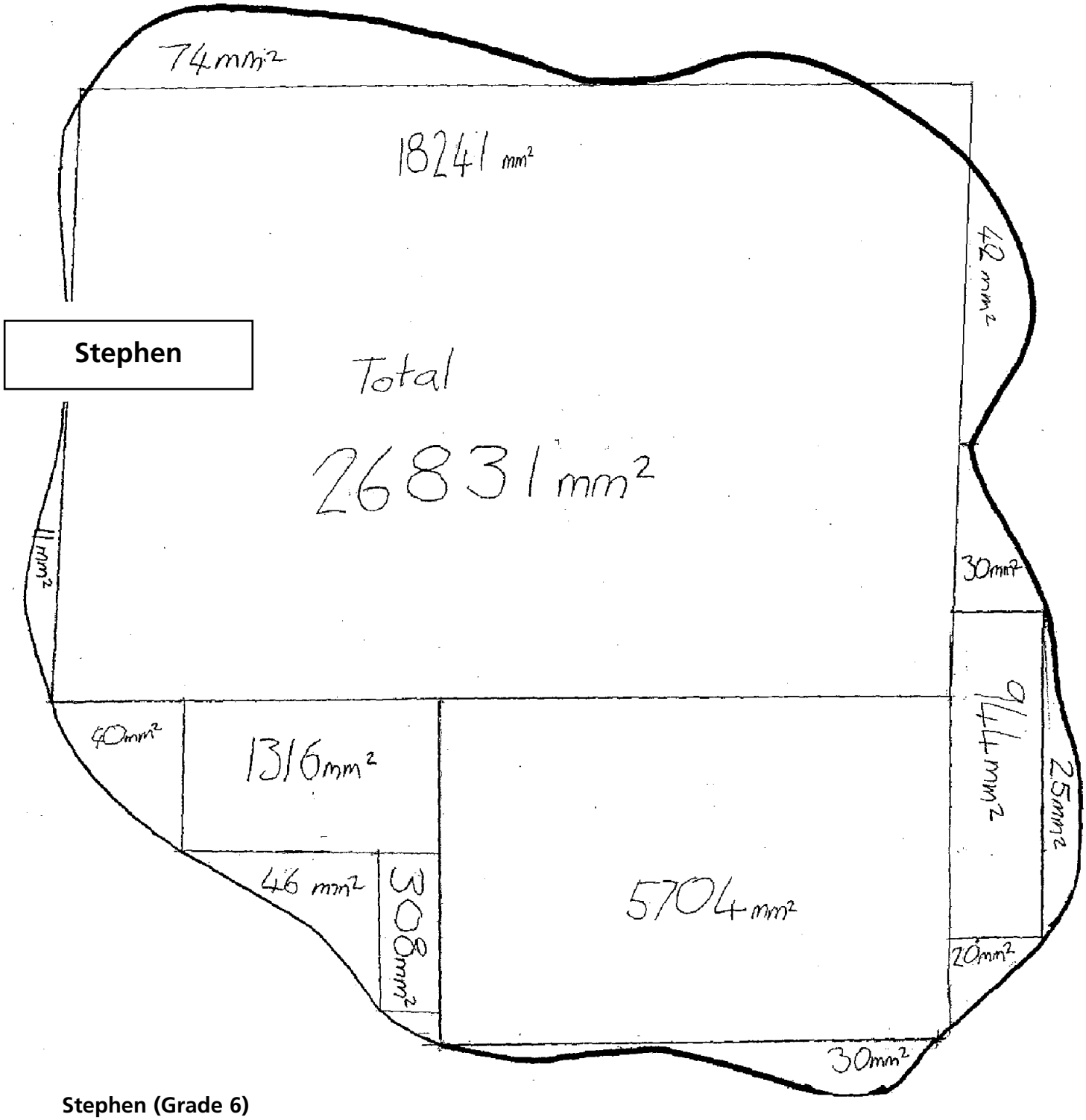
Which Key Understandings?

Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Ice Cream Puddles: Work Samples



Stephen (Grade 6)
When asked how he worked out the area of the puddle, he said, "I cut the puddle into squares and rectangles and added the areas together to reach a total. I worked out the area of the left-over bits with cubes and changed it to square millimetres." (However, Stephen incorrectly multiplied by 10 instead of 100 to change the square centimetres to square millimetres.)

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic Map

What phase of thinking is Stephen showing evidence of?
How do you know?

Planning for Learning

Which Key Understandings?

Choose a Sample Learning Activity that would help.

What focus questions might you use during the activity?

Diagnostic Map

How can the Diagnostic Map help us to identify students at risk?

Is there a student you would be worried about more than others? Why?

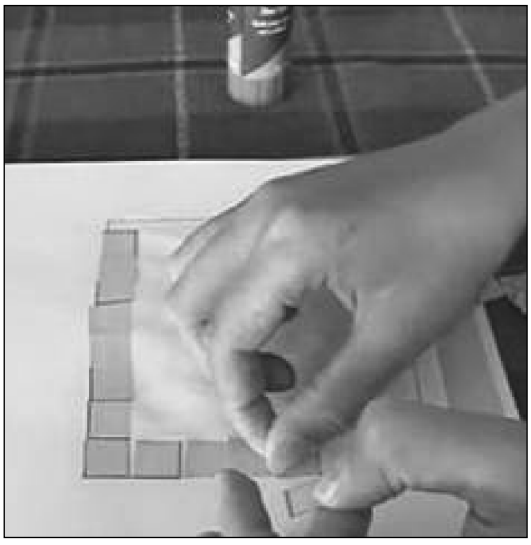
FSIM013 | First Steps in Mathematics: Measurement Course Book

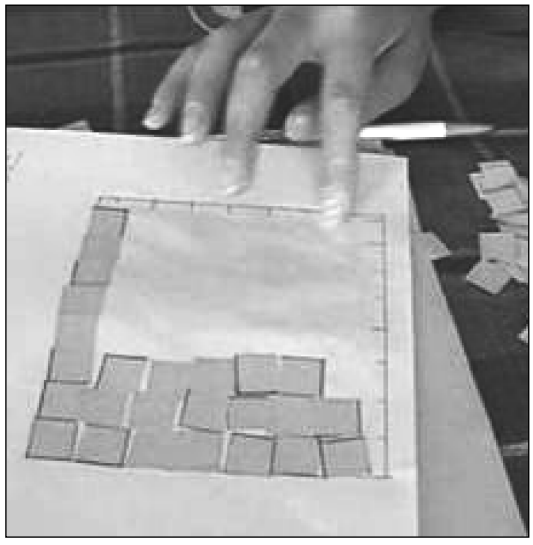
Blake's Area Measurement

How do we move students from "cover and count" to using reliable and practical shortcuts to measure area?

Video of Blake, 12 years old

Clip 1: What focus questions does Blake's teacher use to encourage him to find a shortcut?





At first, Blake appears to be using the array structure to work out how many squares fit the rectangle. However, he is not able to use the array idea to help him know how many part units he will need across the base.

Clip 2: Why does Blake have difficulty working out the area of the part units?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Oil Spills Case Study 4

(*Measurement Book 2*, Indirect Measure, p. 78)

What is the teacher's purpose for the lesson?

What materials are provided for the lesson?

How does this constraint provoke the students to think?

Exploring the Impact of Materials

How do the materials we provide influence the mathematics students choose to use to solve area problems?

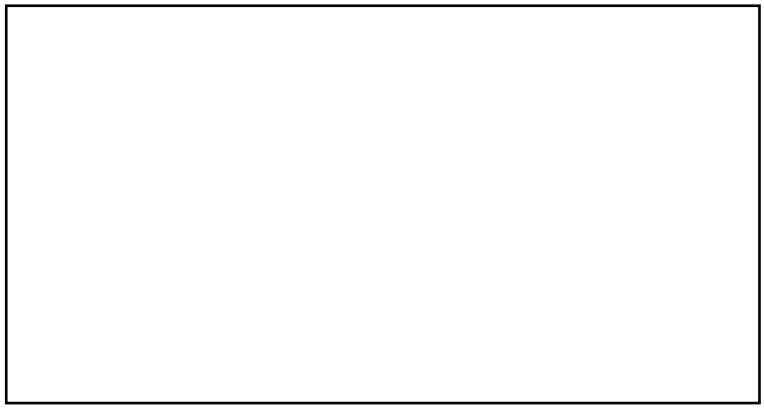
In your group:

- Each take an area task from pp. 130–134.
- Carry out the task with the materials suggested.
- Read the appropriate section of the Background Notes.
- Share your task and your findings with the group.

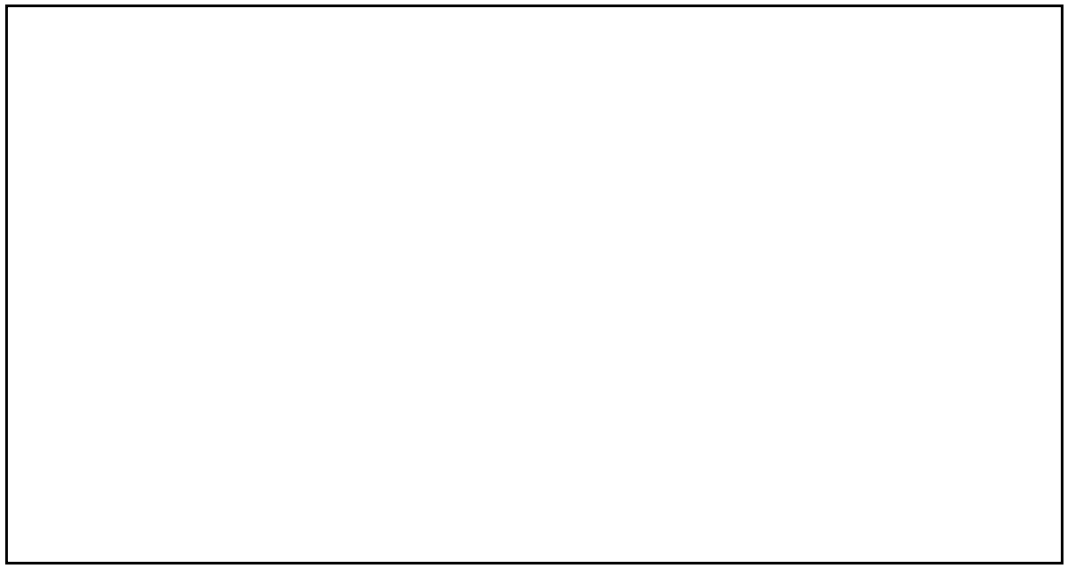
FSIM013 | First Steps in Mathematics: Measurement Course Book

Exploring the Impact of Materials: 1

What is the area of this shape? Use **2-cm cubes** or solid tiles to find out.



What is the area of this shape? Use **paper squares** to find out.



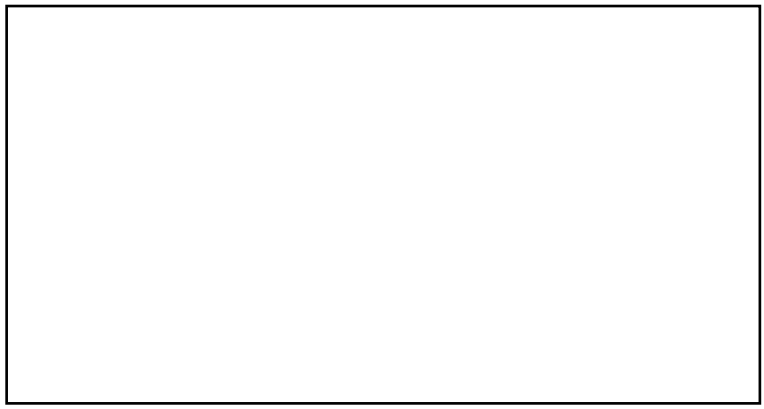
Which part of the task would be more difficult for younger students? Why?

What is different about the mathematics required for the first and second tasks? (Read Direct Measure, p. 107, last paragraph, and p. 108, first two paragraphs.)

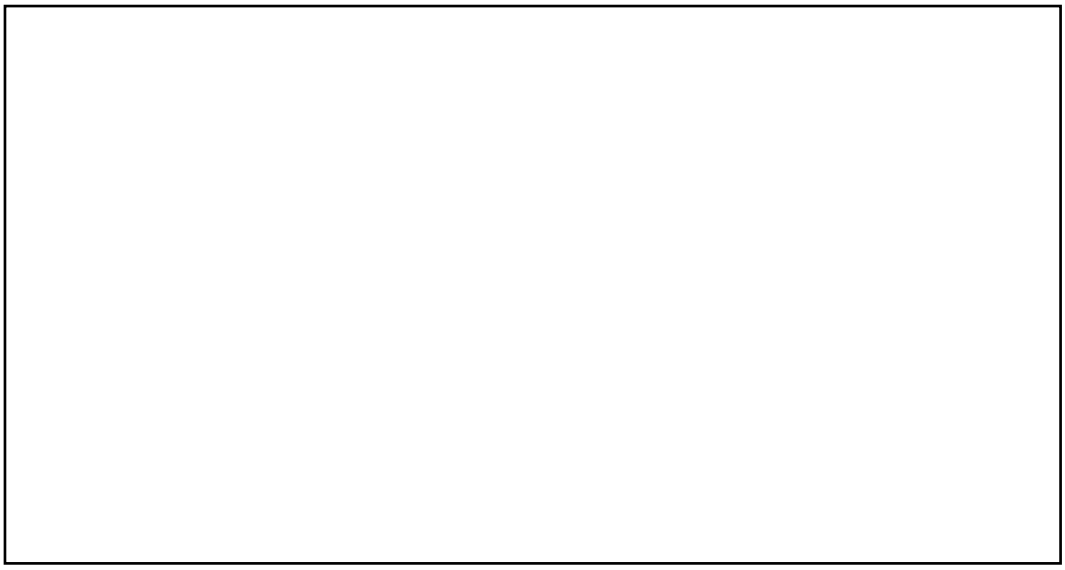
FSIM013 | First Steps in Mathematics: Measurement Course Book

Exploring the Impact of Materials: 2

What is the area of this shape? Use a collection of **2-cm cubes** or solid tiles to find out.



What is the area of this shape? Use only **one 2-cm cube** or tile to find out.
You may also use rulers and pencils/pens.



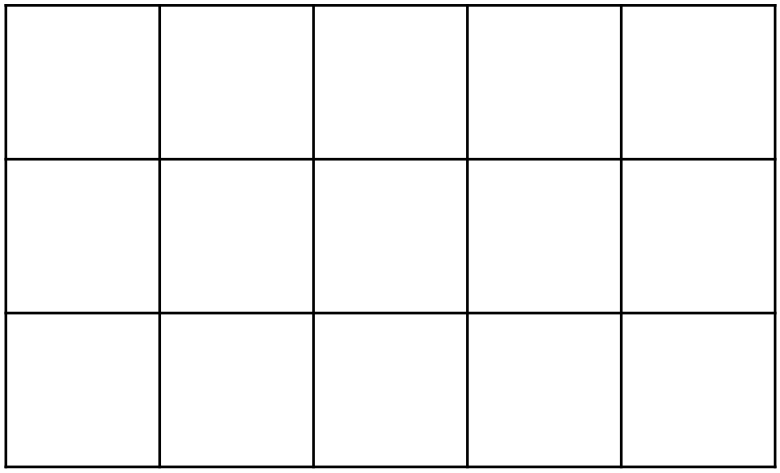
Which part of the task would be more difficult for younger students? Why?

What is different about the mathematics required for the first and second tasks? (Read Direct Measure, p. 104, first and second paragraphs.)

FSIM013 | First Steps in Mathematics: Measurement Course Book

Exploring the Impact of Materials: 3

What is the area of this shape?



What is the area of this shape? Use only **one 2-cm cube** or solid tile to find out. You may also use rulers and pencils/pens.

Which part of the task would be more difficult for younger students? Why?

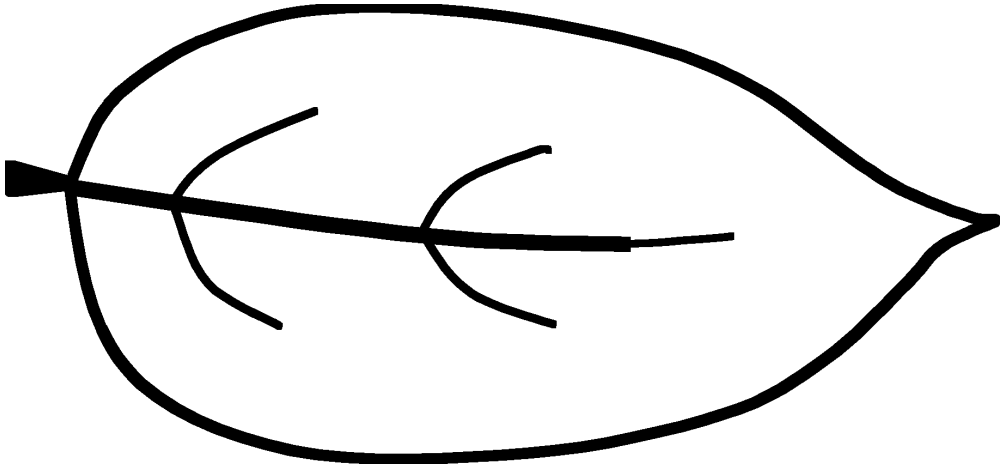
What is different about the mathematics required for the first and second tasks? (Read Direct Measure, p. 108, paragraph 3, "In a similar way…")

FSIM013 | First Steps in Mathematics: Measurement Course Book

Exploring the Impact of Materials: 4

What is the area of this shape? Use **paper squares** to find out. You may also use rulers and pencils/pens.

What is the area of this leaf? Use a collection of **paper squares** to find out.



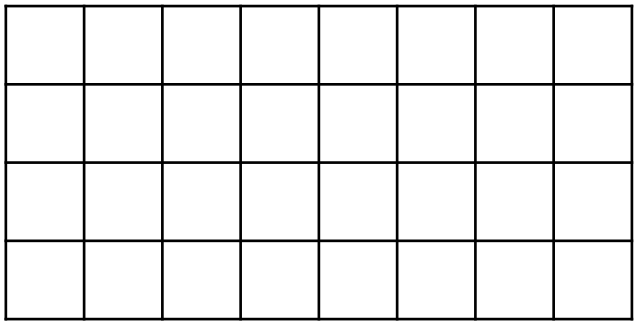
Which part of the task would be more difficult for younger students? Why?

What is different about the mathematics required for the first and second tasks? (Read Direct Measure, p. 105, and p. 106, first paragraph.)

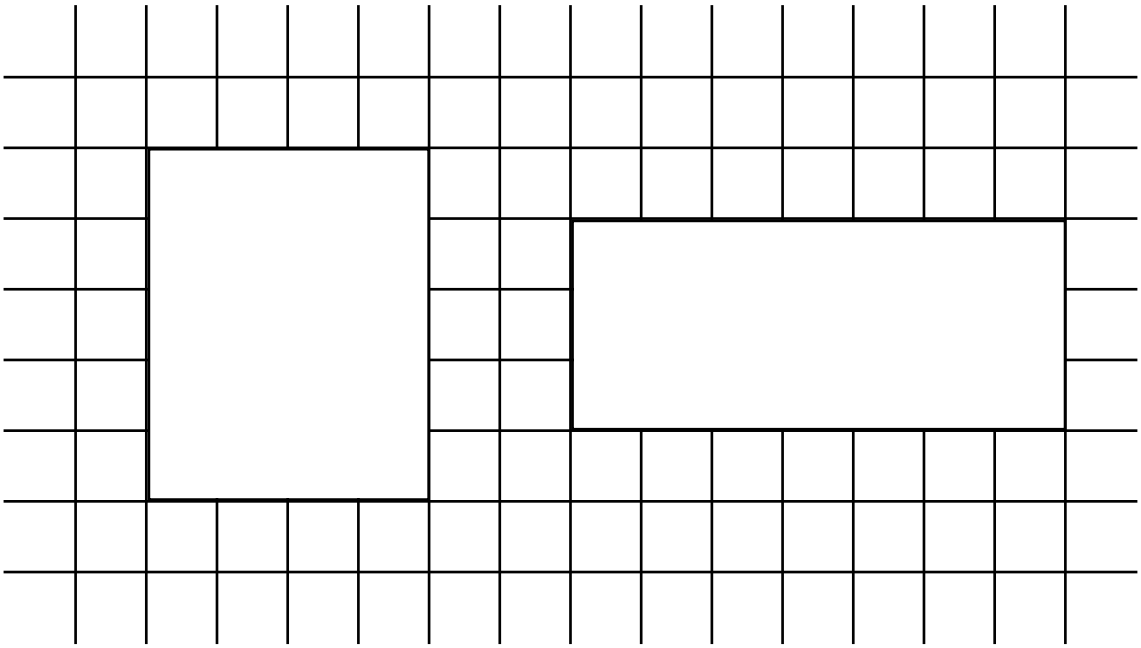
FSIM013 | First Steps in Mathematics: Measurement Course Book

Exploring the Impact of Materials: 5

What is the area of this shape?



What is the area of these shapes?



Which part of the task would be more difficult for younger students? Why?

What is different about the mathematics required for the first and second tasks? (Read Direct Measure, p. 108, third paragraph, and p. 109.)

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 3, 7

Direct Measure

- Key Understanding 3

Indirect Measure

- Key Understanding 1

What Is the Area?

Grades 5–8

Purpose

To investigate what students know about

- using an array structure as a shortcut to counting squares
- using the area formula

Materials

Worksheets

A variety of objects that can be used as units, e.g., 1-cm and 2-cm cubes, tiles, marbles, pattern blocks, round counters

A collection of measuring equipment that extends beyond what is required to measure area, e.g., balance scales, measuring cylinders, string, measuring tape, ruler, pencil, scissors, glue, plain paper, and square grid paper; also a jug of sand, rice, or water

Producing Work Samples

Individual interview

Interviews are appropriate for younger students or for students whom teachers consider may be at risk. Read and familiarize students with the tasks on the worksheets. Students carry out the tasks and record how they worked them out. They may need assistance to record this information.

Small group or whole class

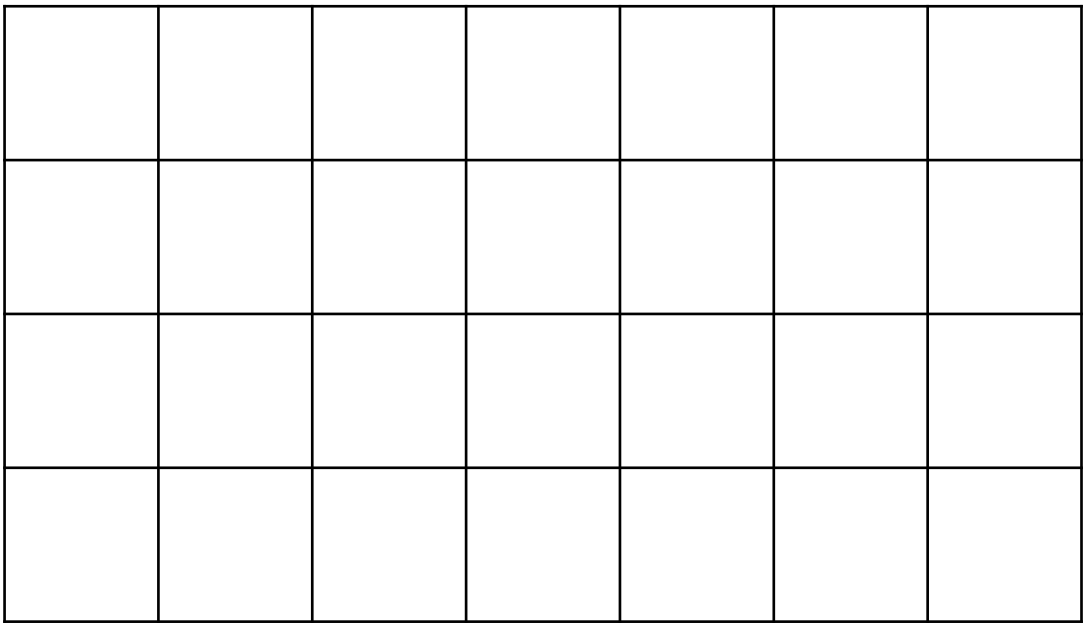
These tasks may be administered in small groups or with the whole class. Read and familiarize students with the tasks on the worksheets. Students carry out the tasks and record how they worked them out. They may need assistance to record this information. Observe and record how the students worked them out and what they refer to as their unit of area.

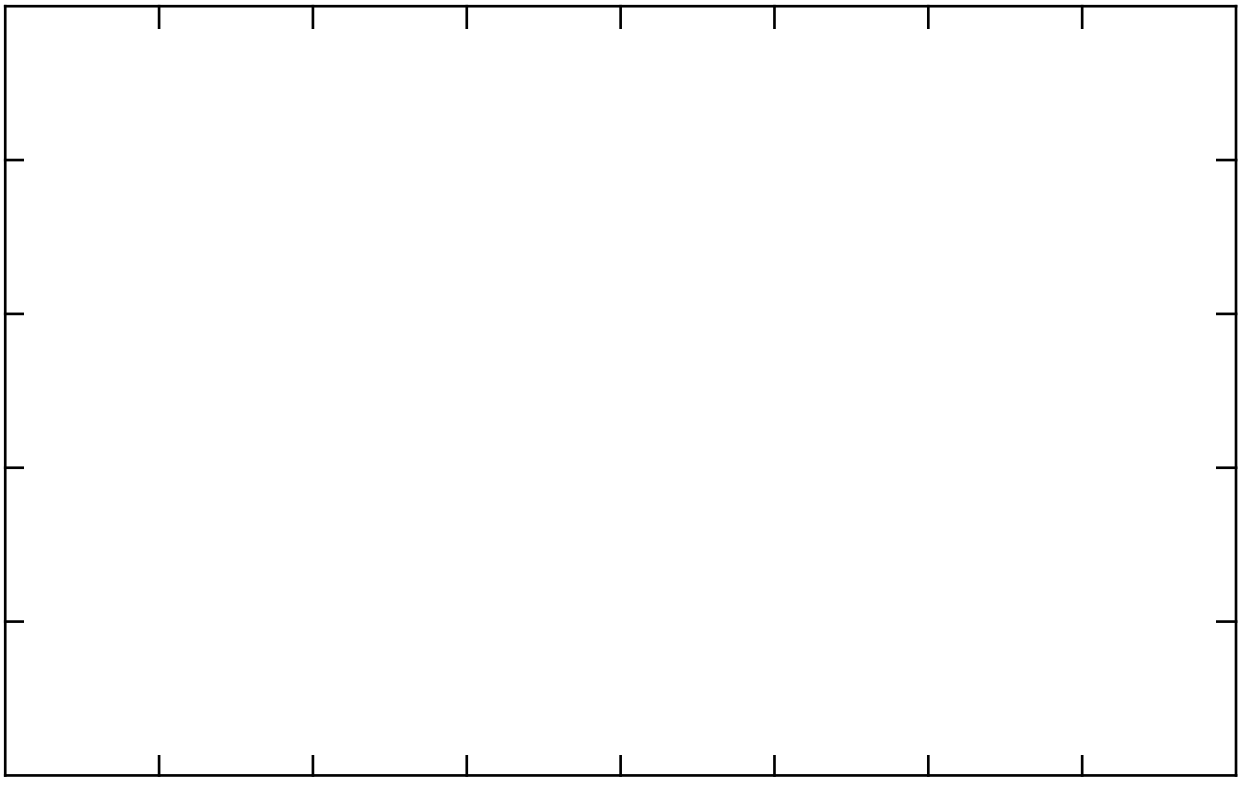
FSIM013 | First Steps in Mathematics: Measurement Course Book

What Is the Area? 1

Name _____ Grade _____ Date _____

What is the area of these shapes? Write how you worked each one out.

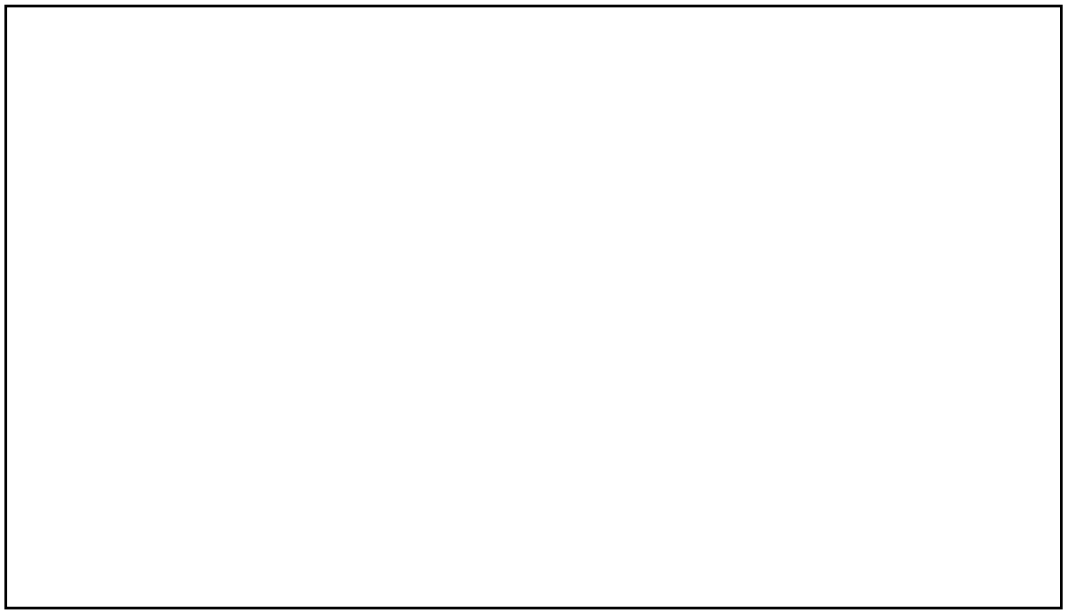


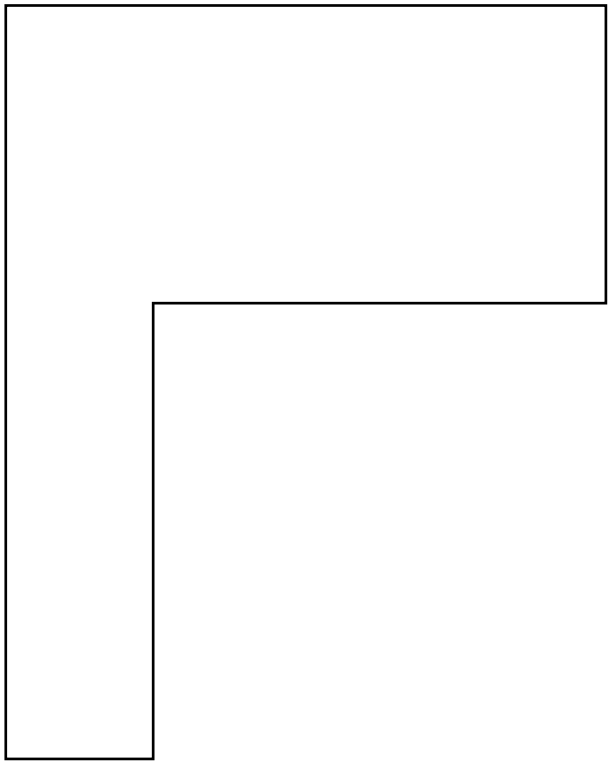


What Is the Area? 2

Name _____ Grade _____ Date _____

What is the area of these shapes? Write how you worked each one out.





Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 3, 7

Direct Measure

- Key Understanding 3

Indirect Measure

- Key Understanding 1

Tiling Problem

K–Grade 4

Purpose

To investigate how and if students use an array structure when working out how many tiles fit in a rectangle.

Materials

One 2-cm tile, cardboard square, or block along with the worksheet

The rectangle may have to be adapted to match the dimensions of the available tile if it is not exactly 2 cm square.

Producing Work Samples

Individual interview

Interviews are appropriate for younger students or for students whom teachers consider may be at risk. Read and familiarize students with the task on the worksheet. Students carry out the task and record how they worked it out. They may need assistance to record this information.

Small group

Administering this task in small groups is appropriate for Grades 3 and 4. It is advantageous for the teacher to observe and record what the students say and do throughout the activity. Read and familiarize students with the task on the worksheet. Students carry out the task and record how they worked it out. They may need assistance to record this information.

Whole class

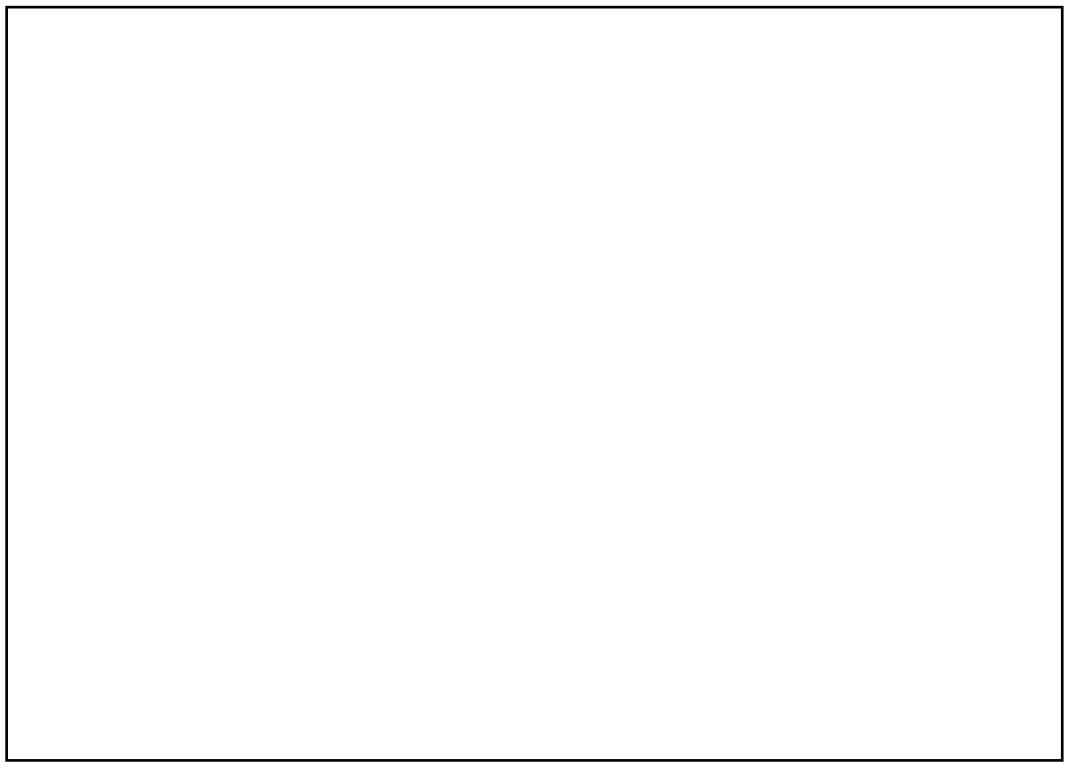
This task is suitable for a whole class for Grades 3 and 4 students who are able to write how they work things out. Students may need help to record this information.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Tiling Problem

Name _____ Grade _____ Date _____

Materials: one 2-cm square of cardboard, tile, or cube, and the worksheet.
How many 2-cm squares or tiles would you need to cover the rectangle?



How did you work it out?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Between Now and the Next Session

Create an action plan with your group.

What tasks will you use with at least six of your students to find out what they know?

Task	Class	Page
Leaf Task	Grades 4–7	110
Ice Cream Puddles	K–Grade 7	115
What Is the Area? 1, 2	Grades 5–8	135
Tiling Problem	K–Grade 4	138

Complete the Action Plan on the next page, noting which tasks you and your colleagues will use and when.

Make decisions about sharing the resources and/or preparation that is needed.

Set the date and time for a Task Review session.

Before you meet, copy the Making Decisions and Classroom Planner masters on Course Book pp. 13 and 14.

Plan for your students' mathematics learning following the steps outlined in the Task Review section on p. 11.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Action Plan

Who?	Which Diagnostic Tasks?	When?	Resource Preparation

Task Review Date: ______________ Time: ______________ Venue: ______________

Unit 6: Vanquishing VOLUME Misunderstandings

Unit 6 Outcomes

You will
- consider the mathematics needed to measure volume
- begin to understand how students learn to measure volume
- further develop an understanding of the Diagnostic Map, particularly the Measuring and Relating phases

Related Reading

***First Steps in Mathematics* Measurement Book 1**

Chapter 2: Understand Units
Key Understanding 1 (pp. 24–31)
Key Understanding 2 (pp. 32–39)
Key Understanding 3 (pp. 40–53)
Key Understanding 7 (pp. 82–89)

Chapter 3: Direct Measure
Key Understanding 1 (pp. 112–125)
Key Understanding 2 (pp. 126–133)
Key Understanding 3 (pp. 134–141)
Key Understanding 5 (pp. 156–167)
Background Notes (p. 104)

***First Steps in Mathematics* Measurement Book 2**

Chapter 2: Indirect Measure
Key Understanding 1 (pp. 26–43)
Key Understanding 4 (pp. 68–82)

Diagnostic Map: Measurement

Emergent to Relating phases

Background Reading

Battista, M.T. (2003). Understanding students' thinking about area and volume measurement. In D. Clements & G. Bright (Eds.), *Learning and teaching measurement 2003 yearbook*. Reston, VA: National Council of Teachers of Mathematics.

Battista, M.T., & Clements, D.H. (2003). Students' understanding of three-dimensional cube arrays: Findings from a research and curriculum development project. In *Designing learning environments for developing understanding of geometry and space* (pp. 227–248). Mahwah, N.J: Erlbaum.

FSIM013 | First Steps in Mathematics: Measurement Course Book

The Connection between Volume and Capacity

The attribute of volume refers to the amount of space occupied by an object.

Capacity and volume measure the same attribute (the capacity of a container being the volume of the material it will hold). (See Direct Measure, p. 106.)

The capacity of a container is the inside volume of the container. (See Indirect Measure, Case Study 1, pp. 40–42.)

Volume as a mathematical concept can be difficult to grasp. When you talk about the volume of something, you need to be clear about what you are attending to. For example, looking at the cup below, you could refer to

- the volume of clay needed to make the cup
- the volume of coffee you could fit into the cup (its capacity)
- the volume of the space the cup takes up in the cupboard

Why is the mathematics of capacity easier for students to understand than volume? Refer to Direct Measure, p. 106.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Has More Volume?

Work in groups to find out which object has more volume. How much more volume does it have in cubic centimetres?

How did you work it out? List the steps in your strategy.

What mathematics did your group know about and use in the course of your strategy?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Has More Volume?

What Key Understandings would you focus on to have students understand the mathematics underpinning your strategy?

Read Understand Units, Did You Know? on p. 102. Did you use this knowledge in the task?

If not, how could this knowledge have been used to work out the volume of the cylinder?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Cubic, Cube, and Cubed

Work in a group of three people.

Using blocks or playdough, each person makes an object that conforms to one of the following descriptions:

- 2 cubic centimetres
- 2 centimetres cubed
- 2 centimetre cubes

What is the difference in these terms? See Direct Measure, Did You Know? on p. 167.

Note: While 1 cubic centimetre (1 cm^3) has the same volume as 1 centimetre cubed $(1\ cm)^3$, 2 cubic centimetres (2 cm^3) does NOT have the same volume as 2 centimetres cubed $(2\ cm)^3$. Can you see why?

cm^3 is always read as "cubic centimetres" NOT "centimetres cubed."

Diagnostic Map

When would we expect students to fully understand and use volume measurement?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 2, 3

Direct Measure

- Key Understandings 1, 2, 5

Indirect Measure

- Key Understandings 1, 4

Which Has More Volume? Grades 5–7

Purpose

To investigate

- what students know about the attribute of volume
- what students know about directly comparing volume
- how they use units to measure and compare volume
- whether they understand the relationship between millilitres and cubic centimetres (Direct Measure, Key Understanding 5)

Materials

A wooden or plastic block (about 5 cm x 5 cm x 5 cm = 125 cm^3)
A lump of playdough or Blu Tac rolled out to look longer and thinner than the block
Assorted objects that can be used as units, e.g., 1-cm and 2-cm cubes, marbles, and pattern blocks
Sand and water, two identical transparent measuring cups, ruler, pencil, measuring cylinders, balance beams, kitchen scales, string, and square grid paper
Teacher Recording Sheet

Producing Work Samples

Individual interview

Appropriate for students whom teachers consider to be at risk, these interviews can also be used to sample a range of ability levels to give the teacher an idea of the students' thinking about volume. Ask questions from the Teacher Recording Sheet, and encourage students to use whatever they need to answer them.

Small group or whole class

This task is appropriate for Grade 6 or 7 students. The objects are placed in a central position and the last two questions are modified as follows:

- *How could you use the materials in front of you to work out which object has more volume?*
- *How could you use the materials to work out how much more volume the larger one has?*

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Has More Volume? Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. What does volume mean?

2. Do the two objects in front of you have the same volume? Explain.

3. Which object has more volume? Use the materials in front of you to work it out. How did you work it out?

4. How much more volume does the larger one have? Use the materials in front of you to work it out. How did you work it out?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Has More Volume?

Video of Chantelle, 12 and a half years old

What does Chantelle know about measuring volume?

- the attributes of volume
- direct comparison of volume
- using units to measure volume
- using shortcuts to work out volume

Diagnostic map

Which phase does this suggest for Chantelle?

Planning for Learning

What mathematics does Chantelle need to learn, i.e., which Key Understanding?

Choose a suitable Sample Learning Activity.

Write some focus questions that you could use to help Chantelle understand volume measurement.

Your Own Students

What do you think your students know about measuring volume?

- the attributes of volume
- direct comparison of volume
- using units to measure volume
- using shortcuts to work out volume

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS

Understand Units

- Key Understandings 1, 2, 3

Direct Measure

- Key Understandings 1, 3

Indirect Measure

- Key Understandings 1, 4

Which Lunch Box Holds More? Grades 3–7

Purpose

To investigate

- what students know about the attribute of volume
- how they use units to measure and compare volumes

Materials

Two small lunch boxes or small boxes that are different in volume but not easy to compare visually (it is important that some of the inside dimensions not be a whole number measure)
Assorted objects that can be used as units, e.g., 1-cm and 2-cm cubes, marbles, or pattern blocks (no rice, sand, or water); have only enough units to fill the larger container
Ruler, pencil, measuring cylinders, measuring cups, balance beams, kitchen scales, string, and square grid paper
Teacher Recording Sheet

Producing Work Samples

Individual interview

Interviews are appropriate for students whom teachers consider to be at risk. They can also be used to sample a range of ability levels in order to give an idea of the students' thinking about volume. Interviewing those students who have carried out diagnostic tasks with other attributes will help build a more complete picture of the understandings of a few individuals.

1. Point to the two lunch boxes and ask, *Which lunch box (or box) holds more? Which lunch box holds more volume?* If the student guesses, ask, *How can you be sure?*
2. If appropriate, ask, *How much more volume does the larger one hold?*

Small group

Small groups are appropriate for Grade 3 or 4 students if they each have two lunch boxes to compare and there are enough materials available. It is useful for the teacher to observe and record what the students say and do, using the Teacher Recording Sheet. Read and familiarize students with the task. Observe and record how students carry it out.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Which Lunch Box Holds More? Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. Which lunch box (or box) holds more lunch? Which holds the largest volume? How can you be sure?

2. How much more volume does it hold? How did you work it out?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS
Indirect Measure
- Key Understandings 1, 4

Block Towers

Grades 4–9

Purpose

- To investigate if and how students use the array structure when counting blocks in three-dimensional arrays
- To provide an idea of the range in students' thinking about three-dimensional arrays

This task is appropriate for students in the later primary years who are unable to correctly carry out the Volume of Prisms (1) task. These students may have had difficulty interpreting the two-dimensional representations of the prisms. Their problem may be spatial and not related to measurement or number.

Materials

Three-dimensional representations (1-cm wooden cubes glued together) of the prisms in the Volume of Prisms (1) task (see below)
Teacher Recording Sheet

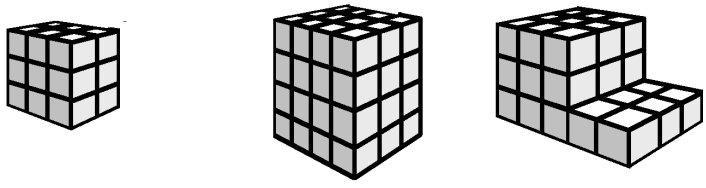
Producing Work Samples

Individual interview

Interviews are appropriate for students whom teachers consider to be at risk, particularly those unable to correctly carry out the Volume of Prisms (1) task. Give the student a block tower to handle and ask, *How many cubes are in this block tower?* Then ask, *How did you work it out?* Observe and record what the student says and does throughout the activity.

Small group

This task is appropriate for older students if there are enough block towers available. Read and familiarize students with the task. Observe and record how they carry it out.



FSIM013 | First Steps in Mathematics: Measurement Course Book

Block Towers: Teacher Recording Sheet

Name _____ Grade _____ Date _____

Give the student the block towers in turn and ask:

1. How many cubes are in this block tower? How did you work it out?

2. How many cubes are in this block tower? How did you work it out?

3. How many cubes are in this block tower? How did you work it out?

FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS
Indirect Measure
- Key Understandings 1, 4

Volume of Prisms (1) Grades 4–9

Purpose
To investigate what students understand about
- using an array to work out the volume of rectangular prisms
- using the *l* x *w* x *h* formula
- working out the volume of more complex prisms

Students who make errors with this task may not be able to interpret the two-dimensional representation of the prisms. They need to carry out the Block Towers task.

Interviewing those students who have carried out diagnostic tasks with other attributes will help build a more complete picture of the understandings of a few individuals.

Materials
Volume of Prisms (1) sheet
Teacher Recording Sheet

Producing Work Samples

Individual interview
Interviews are appropriate for students whom the teacher considers to be at risk. They can also be used to sample a range of ability levels in order to give the teacher an idea of the students' thinking about volume.

Read and familiarize students with the task. Students carry out the task and record how they worked it out. They may need help from the teacher to record their method of working.

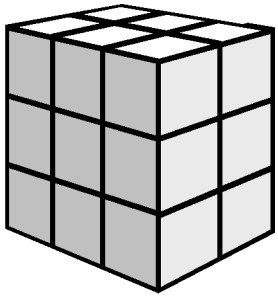
Small group or whole class
Read and familiarize students with the task. Observe and record how the students work out the problem and how they refer to their unit of volume.

FSIM013 | First Steps in Mathematics: Measurement Course Book

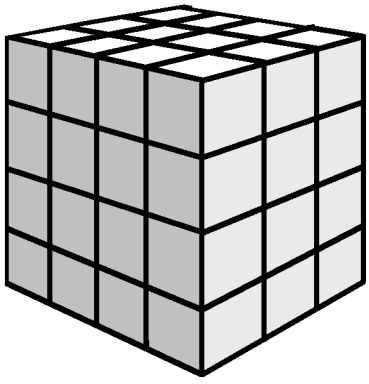
Volume of Prisms (1): Teacher Recording Sheet

Name _____ Grade _____ Date _____

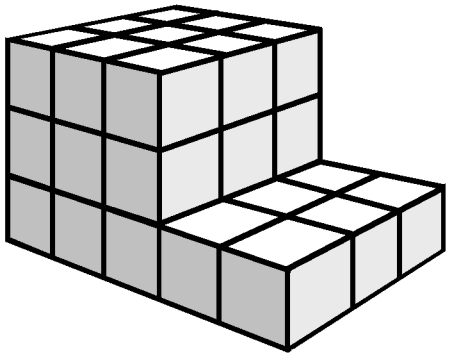
1. What is the volume of this prism? How did you work it out?



2. What is the volume of this prism? How did you work it out?



3. What is the volume of this prism? How did you work it out?



FSIM013 | First Steps in Mathematics: Measurement Course Book

Diagnostic TASK

FOCUS
Indirect Measure
- Key Understandings 1, 4

Volume of Prisms (2) Grades 5–9

Purpose
To investigate what students understand about
- working out the volume of rectangular prisms that have fractional side lengths
- using the *l* x *w* x *h* formula
- working out the volume of more complex prisms

Materials
Volume of Prisms (2) sheet
Teacher Recording Sheet

Producing Work Samples

Individual interview
Interviews are appropriate for students whom the teacher considers to be at risk. They can also be used to sample ability levels in order to give the teacher an idea of the students' thinking about volume.

Read and familiarize students with the task. Students carry out the task and record how they worked it out. They may need help from the teacher to record their method of working.

Small group or whole class
Read and familiarize students with the task. Observe and record how the students work it out and how they refer to their unit of volume.

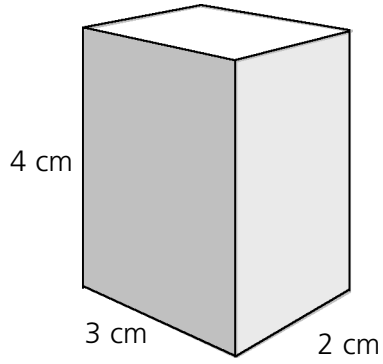
Note: The volume of the prism in the second diagram is 4.5 cm x 3.5 cm x 2.5 cm. It includes fractional lengths but does not indicate this with numbers, to see if students understand the *l* x *w* x *h* formula.

FSIM013 | First Steps in Mathematics: Measurement Course Book

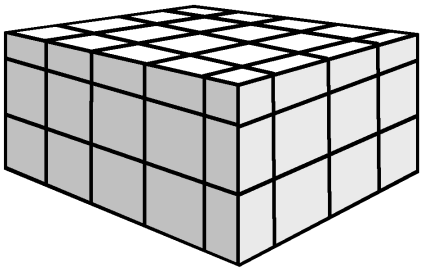
Volume of Prisms (2): Teacher Recording Sheet

Name _____ Grade _____ Date _____

1. What is the volume of this prism? How did you work it out?



2. What is the volume of this prism? How did you work it out?



The Early Years

What would most **young students** think when they hear people talk about volume? (Think about the everyday situations in which they are most likely to hear the word "volume.")

What learning experiences could be provided in the early years to build an understanding of measuring volume?

Which Key Understandings?

Which Sample Learning Activities?

Any other successful activities you have used?

FSIM013 | First Steps in Mathematics: Measurement Course Book

After This Session

Create an action plan with your group.

What tasks will you use with at least six of your students to find out what they know?

Task	Class	Page
Which Has More Volume?	Grades 5–7	148
Which Lunch Box Holds More?	Grades 3–7	152
Block Towers	Grades 4–9	154
Volume of Prisms (1)	Grades 4–9	156
Volume of Prisms (2)	Grades 5–9	158

Complete the Action Plan on the next page, noting which tasks you and your colleagues will use and when.

Make decisions about sharing the resources and/or preparation that is needed.

Set the date and time for a Task Review session.

Before you meet, copy the Making Decisions and Classroom Planner masters on pp. 13 and 14 of the Course Book.

Plan for your students' mathematics learning following the steps outlined in the Task Review section on p. 11.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Action Plan

Who?	Which Diagnostic Tasks?	When?	Resource Preparation

Task Review Date: _____ Time: _____ Venue: _____

Whole School Planning

Outcomes

You will
- create a plan for individuals and the whole school
- work out who will teach what to whom
- become more familiar with the Key Understandings in the *First Steps in Mathematics*, Measurement Resource Books 1 and 2
- explore the beliefs that underpin *First Steps in Mathematics*

Related Reading

First Steps in Mathematics **Measurement Book 1**
Chapter 2: Understand Units
Chapter 3: Direct Measure

First Steps in Mathematics **Measurement Book 2**
Chapter 2: Indirect Measure

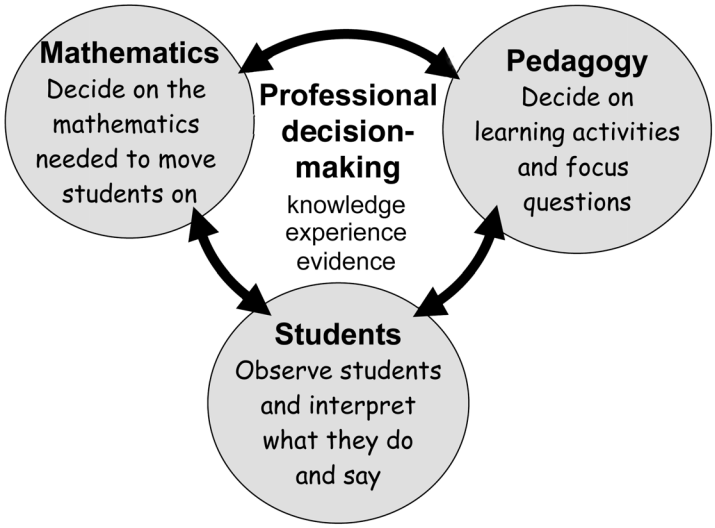
Diagnostic Map: Measurement
Emergent to Relating phases

Background Reading
Darling-Hammond, L. (1994). National standards and assessments: Will they improve education? *American Journal of Education*, *102* (478–471).

Willis, S. (1999). Common outcomes and the curriculum framework. Keynote Paper, Curriculum Framework Seminar, Catholic Education Office, Perth.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Planning Cycle



Planning Questions

What do my students currently know and understand?
(I need to know about my students.)

What mathematics do my students need to learn next?
(I need to know about the mathematics.)

How best can I help them learn it?
(I need to know about the pedagogy.)

Where in the *First Steps in Mathematics* Resource Books can you find help with
Students?

Mathematics?

Pedagogy?

Using Professional Decision-Making to Plan

First Steps in Mathematics supports the belief that teachers are in the best position to make decisions about how to help their students. Teachers will base these decisions on knowledge, experience, and evidence.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Whole School Curriculum Planning

A process for collaborative planning using *First Steps in Mathematics* materials

1. Break into groups according to the grade levels you are teaching.
2. Identify the main Key Understandings that you need to emphasize in your section of the school, i.e., K–Grade 3, Grades 3–5, and Grades 5–8. Use the Key Understandings Overview charts in the Resource Books to help with this process. Record this information on a copy of the **Whole School Curriculum Planner** sheet.
3. Jigsaw (i.e., one or two people from each group move to another group, each of which should now have at least one person from each section of the school). Share your decisions about which Key Understandings to focus on in different sections of the school.
4. Record the consensus decision on an enlarged Whole School Curriculum Planner for inclusion in your school plan and for use as the basis for classroom plans.
5. Working in grade levels, use the **Classroom Overview** sheet to produce a broad plan for the year, and the **Planning Outline** to map out sequences of learning within a Key Understanding.
6. Individual teachers may use the **Classroom Planner** to develop weekly plans of action from the Planning Outline, based on what their students currently know and need to know next.
7. It would be helpful from the perspective of whole school planning to record completed classroom work on the Whole School Curriculum Planner, as a record of the Key Understandings worked on over the year.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Whole School Curriculum Planner: Measurement Overview

Date _____

Understand Units	K–3	3–5	5–8	Direct Measure	K–3	3–5	5–8	Indirect Measure	K–3	3–5	5–8
1. We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.				1. We can directly compare objects or events to say which has more length, mass, capacity, area, volume, angle, or time.				1. For certain types of shapes we can describe the relationship between the lengths of their edges and their perimeters, areas, and volumes.			
2. There are special words and phrases that help us to describe and compare quantity.				2. We can indirectly compare two objects by using other objects as go-betweens or by altering the objects in some way that does not affect the quantity.				2. When two objects have the same shape: • matching angles are equal • matching lengths are proportional • matching areas are related in a predictable way • matching volumes are related in a predictable way			
3. To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.				3. To measure consistently, we need to use our instrument in a way that ensures a good match of the unit with the object to be measured.				3. Scale drawings and models have the same shape as the original object. This can be useful for comparing and calculating dimensions and for making judgments about position.			
4. The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.				4. Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle, and time.				4. We can calculate one measurement from others using relationships between quantities.			
5. Measurements of continuous quantities are always approximate. Measurements can be made more accurate by choosing smaller units, subdividing units, and using other strategies.				5. Units are quantities, so we can use different representations of the same unit as long as we do not change the quantity.				**Estimate**			
6. Our choice of attribute and unit depends on what we are trying to measure and why.				6. We can judge and measure time using both natural cyclical changes and special techniques and tools which people have developed.				1. We can make judgments about order and size without actually measuring. We should think about how confident we can be of our estimate.			
7. Standard units help us to interpret, communicate, and calculate measurements.								2. We can improve our estimates by getting to know the size of common units and by practising judging the size of objects and events.			
8. The relationships among standard units in the metric system help us to judge size, move between units, and do computations.								3. We can use information we know to make and improve estimates. This also helps us to judge whether measurements and results are reasonable.			

Classroom Overview: Measurement

Class _____ Date _____

Understand Units	T1	T2	T3	T4	Direct Measure	T1	T2	T3	T4	Indirect Measure	T1	T2	T3	T4
1. We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.					1. We can directly compare objects or events to say which has more length, mass, capacity, area, volume, angle, or time.					1. For certain types of shapes we can describe the relationship between the lengths of their edges and their perimeters, areas, and volumes.				
2. There are special words and phrases that help us to describe and compare quantity.					2. We can indirectly compare two objects by using other objects as go-betweens or by altering the objects in some way that does not affect the quantity.					2. When two objects have the same shape: • matching angles are equal • matching lengths are proportional • matching areas are related in a predictable way • matching volumes are related in a predictable way				
3. To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.					3. To measure consistently, we need to use our instrument in a way that ensures a good match of the unit with the object to be measured.					3. Scale drawings and models have the same shape as the original object. This can be useful for comparing and calculating dimensions and for making judgments about position.				
4. The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.					4. Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle, and time.					4. We can calculate one measurement from others using relationships between quantities.				
5. Measurements of continuous quantities are always approximate. Measurements can be made more accurate by choosing smaller units, subdividing units, and using other strategies.					5. Units are quantities, so we can use different representations of the same unit as long as we do not change the quantity.					**Estimate**				
6. Our choice of attribute and unit depends on what we are trying to measure and why.					6. We can judge and measure time using both natural cyclical changes and special techniques and tools which people have developed.					1. We can make judgments about order and size without actually measuring. We should think about how confident we can be of our estimate.				
7. Standard units help us to interpret, communicate, and calculate measurements.										2. We can improve our estimates by getting to know the size of common units and by practising judging the size of objects and events.				
8. The relationships among standard units in the metric system help us to judge size, move between units, and do computations.										3. We can use information we know to make and improve estimates. This also helps us to judge whether measurements and results are reasonable.				

FSIM013 | First Steps in Mathematics: Measurement Course Book

Overview of Measurement Key Understandings

	Understand Units Students select what attributes to measure and what units to use.	**Direct Measure** Students measure length, capacity/ volume, mass, area, time, and angle to needed levels of accuracy.	**Indirect Measure** Students select, interpret, and combine measurements, measurement relationships, and formulas to determine other measures indirectly.	**Estimate** Students make sensible direct and indirect estimates of quantities and are alert to the reasonableness of measurements and results.
1	We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.	We can directly compare objects or events to say which has more length, mass, capacity, area, volume, angle, or time.	For certain types of shapes we can describe the relationship between the lengths of their edges and their perimeters, areas, and volumes.	We can make judgments about order and size without actually measuring. We should think about how confident we can be of our estimate.
2	There are special words and phrases that help us to describe and compare quantity.	We can indirectly compare two objects by using other objects as go-betweens or by altering the objects in some way that does not affect the quantity.	When two objects have the same shape: • matching angles are equal • matching lengths are proportional • matching areas are related in a predictable way • matching volumes are related in a predictable way	We can improve our estimates by getting to know the size of common units and by practising judging the size of objects and events.
3	To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.	To measure consistently, we need to use our instrument in a way that ensures a good match of the unit with the object to be measured.	Scale drawings and models have the same shape as the original object. They can be useful for comparing and calculating dimensions and for making judgments about position.	We can use information we know to make and improve estimates. This also helps us to judge whether measurements and results are reasonable.
4	The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.	Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle, and time.	We can calculate one measurement from others using relationships between quantities.	
5	Measurements of continuous quantities are always approximate. Measurements can be made more accurate by choosing smaller units, subdividing units, and using other strategies.	Units are quantities, so we can use different representations of the same unit as long as we do not change the quantity.		
6	Our choice of attribute and unit depends on what we are trying to measure and why.	We can judge and measure time using both natural cyclical changes and special techniques and tools which people have developed.		
7	Standard units help us to interpret, communicate, and calculate measurements.			
8	The relationships among standard units in the metric system help us to judge size, move between units, and do computations.			

Planning Outline: Understand Units

Name _____ Class _____ Date _____

Key Understandings	T1	T2	T3	T4
1. We can compare objects or events by how much of a particular attribute each has. Different attributes may result in different orders.				
2. There are special words and phrases that help us to describe and compare quantity.				
3. To measure something means to say how much of a particular attribute it has. We measure by choosing a unit and working out how many of the unit it takes to match the object or event.				
4. The instrument we choose to represent our unit should relate well to the attribute to be measured and be easy to repeat to match the object or event to be measured.				
5. Measurements of continuous quantities are always approximate. Measurements can be made more accurate by choosing smaller units, subdividing units, and using other strategies.				
6. Our choice of attribute and unit depends on what we are trying to measure and why.				
7. Standard units help us to interpret, communicate, and calculate measurements.				
8. The relationships among standard units in the metric system help us to judge size, move between units, and do computations.				

Planning Outline: Direct Measure

Name _____ **Class** _____ **Date** _____

Key Understandings	T1	T2	T3	T4
1. We can directly compare objects or events to say which has more length, mass, capacity, area, volume, angle, or time.				
2. We can indirectly compare two objects by using other objects as go-betweens or by altering the objects in some way that does not affect the quantity.				
3. To measure consistently, we need to use our instrument in a way that ensures a good match of the unit with the object to be measured.				
4. Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle, and time.				
5. Units are quantities, so we can use different representations of the same unit as long as we do not change the quantity.				
6. We can judge and measure time using both natural cyclical changes and special techniques and tools which people have developed.				

Planning Outline: Indirect Measure

Name _____ Class _____ Date _____

Key Understandings	T1	T2	T3	T4
1. For certain types of shapes we can describe the relationship between the lengths of their edges and their perimeters, areas, and volumes.				
2. When two objects have the same shape: • matching angles are equal • matching lengths are proportional • matching areas are related in a predictable way • matching volumes are related in a predictable way				
3. Scale drawings and models have the same shape as the original object. This can be useful for comparing and calculating dimensions and for making judgments about position.				
4. We can calculate one measurement from others using relationships between quantities.				

Planning Outline: Estimate

Name _____ Class _____ Date _____

Key Understandings	T1	T2	T3	T4
1. We can make judgments about order and size without actually measuring. We should think about how confident we can be of our estimate.				
2. We can improve our estimates by getting to know the size of common units and by practising judging the size of objects and events.				
3. We can use information we know to make and improve estimates. This also helps us to judge whether measurements and results are reasonable.				

Classroom Planner

Week __________ Term __________ Grade __________ Date __________

Key Understanding	Mathematical Focus	Activities	Focus Questions	Observations/Anecdotes

Planning to Advance a Whole Class in One Mathematical Idea (Measurement Key Understanding)

My Grade:
- ○ Kindergarten
- ○ Grade 1
- ○ Grade 2
- ○ Grade 3
- ○ Grade 4

1. What is my teaching goal?

Curriculum Expectation/Outcome

Key Understanding

2. How will I find out what my students know and believe?

Diagnostic Task

3. What do my students know and believe?

Diagnostic Map: Phases of Thinking Summaries

Emergent: The student distinguishes between attributes and knows they can be compared.
At the end of the Emergent Phase for this Key Understanding, students:

Matching and Comparing: The student uses matching and direct comparison to "measure."
At the end of the Matching and Comparing Phase for this Key Understanding, students:

Quantifying: The student trusts the count as an accurate indicator of size and uses it to compare.
At the end of the Quantifying Phase for this Key Understanding, students:

Measuring: The student trusts and uses the measurement as an unchanging property of the object.
At the end of the Measuring Phase for this Key Understanding, students:

5. What evidence of learning do I see and hear?

Observations and Next Steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Page

Focus Questions and Paraphrasing:

... to move students from the Emergent Phase to the Matching and Comparing Phase:

... to move students from the Matching and Comparing Phase to the Quantifying Phase:

... to move students from the Quantifying Phase to the Measuring Phase:

Planning to Advance a Whole Class in One Mathematical Idea (Measurement Key Understanding)

My Grade:
- ○ Grade 4
- ○ Grade 5
- ○ Grade 6
- ○ Grade 7
- ○ Grade 8

1. What is my teaching goal?

Curriculum Expectation/Outcome

Key Understanding

2. How will I find out what my students know and believe?

Diagnostic Task

3. What do my students know and believe?

Diagnostic Map: Phases of Thinking Summaries

Matching and Comparing: The student uses matching and direct comparison to "measure."
At the end of the Matching and Comparing Phase for this Key Understanding, students:

Quantifying: The student trusts the count as an accurate indicator of size and uses it to compare.
At the end of the Quantifying Phase for this Key Understanding, students:

Measuring: The student trusts and uses the measurement as an unchanging property of the object.
At the end of the Measuring Phase for this Key Understanding, students:

Relating: The student uses relationships between attributes to indirectly measure things.
At the end of the Relating Phase for this Key Understanding, students:

5. What evidence of learning do I see and hear?

Observations and Next Steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Page

Focus Questions and Paraphrasing:

... to move students from the Matching and Comparing Phase to the Quantifying Phase:

... to move students from the Quantifying Phase to the Measuring Phase:

... to move students from the Measuring Phase to the Relating Phase:

Planning to Advance a Whole Class in One Mathematical Idea (Measurement Key Understanding)

My Grade:
- ○ Kindergarten
- ○ Grade 1
- ○ Grade 2
- ○ Grade 3
- ○ Grade 4

1. What is my teaching goal?

Curriculum Expectation/Outcome

Key Understanding

2. How will I find out what my students know and believe?

Diagnostic Task

3. What do my students know and believe?

Diagnostic Map: Phases of Thinking Summaries

Emergent: The student distinguishes between attributes and knows they can be compared.
At the end of the Emergent Phase for this Key Understanding, students:

Matching and Comparing: The student uses matching and direct comparison to "measure."
At the end of the Matching and Comparing Phase for this Key Understanding, students:

Quantifying: The student trusts the count as an accurate indicator of size and uses it to compare.
At the end of the Quantifying Phase for this Key Understanding, students:

Measuring: The student trusts and uses the measurement as an unchanging property of the object.
At the end of the Measuring Phase for this Key Understanding, students:

5. What evidence of learning do I see and hear?

Observations and Next Steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Page

Focus Questions and Paraphrasing:

... to move students from the Emergent Phase to the Matching and Comparing Phase:

... to move students from the Matching and Comparing Phase to the Quantifying Phase:

... to move students from the Quantifying Phase to the Measuring Phase:

Planning to Advance a Whole Class in One Mathematical Idea (Measurement Key Understanding)

My Grade:
- ○ Grade 4
- ○ Grade 5
- ○ Grade 6
- ○ Grade 7
- ○ Grade 8

1. What is my teaching goal?

Curriculum Expectation/Outcome

Key Understanding

2. How will I find out what my students know and believe?

Diagnostic Task

3. What do my students know and believe?

Diagnostic Map: Phases of Thinking Summaries

Matching and Comparing: The student uses matching and direct comparison to "measure."
At the end of the Matching and Comparing Phase for this Key Understanding, students:

Quantifying: The student trusts the count as an accurate indicator of size and uses it to compare.
At the end of the Quantifying Phase for this Key Understanding, students:

Measuring: The student trusts and uses the measurement as an unchanging property of the object.
At the end of the Measuring Phase for this Key Understanding, students:

Relating: The student uses relationships between attributes to indirectly measure things.
At the end of the Relating Phase for this Key Understanding, students:

5. What evidence of learning do I see and hear?

Observations and Next Steps:

4. What Tasks and Focus Questions will advance their learning?

Sample Learning Activity:
Page

Focus Questions and Paraphrasing:

... to move students from the Matching and Comparing Phase to the Quantifying Phase:

... to move students from the Quantifying Phase to the Measuring Phase:

... to move students from the Measuring Phase to the Relating Phase:

Linking the Measurement Diagnostic Map and the Outcome Level Statements

Phases	Levels	
	Understand Units	**Direct Measure**
During the Emergent Phase Students initially attend to overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small) rather than describe comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size. As a result, they begin to understand and use the everyday language of attributes and comparison used within their home and school environment, differentiating between attributes that are obviously perceptually different.		
During the Matching and Comparing Phase Students match in a conscious way in order to decide which is bigger by familiar, readily perceived and distinguished attributes such as length, mass, capacity, and time. They also repeat copies of objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. As a result, they learn to directly compare things to decide which is longer, fatter, or heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as "How long (tall, wide or heavy, much time, much does it hold)?" or when explicitly asked to measure something.	1a Students understand everyday comparative language associated with length, mass, capacity, and time.	1b Students directly compare and order "straight" lengths and events in time and count informal units of length, capacity, area, mass, and time to decide "how many fit or match."
During the Quantifying Phase Students connect the two ideas of directly comparing the size of things and of deciding "how many fit" and so come to an understanding that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared without directly matching them. As a result, they trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.	2a Students distinguish the attributes of length, area, capacity, and mass when comparing things and choose things which relate well to the attribute of interest to use as units.	2b Students directly compare and order things by length, area, capacity, mass, and time; indirectly compare lengths and capacities; and use uniform units carefully to measure lengths and capacities.
	3a Students realize that using a uniform unit repeatedly to match an object gives a measure of the size of the object, selecting suitable and uniform things to use as units and a common unit to compare two things.	3b Students directly and indirectly compare and order things by length, area, capacity, mass, time, and angle; measure them by counting uniform units; and use standard scales to measure length and time.
During the Measuring Phase Students come to understand the unit as an amount (rather than an object or a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits. As a result, they see that part units can be combined to form whole units, and they understand and trust the measurement as a property or description of the object being measured, which does not change as a result of the choice or placement of units.	4a Students select appropriate attributes, distinguish perimeter from area and time from elapsed time, and choose units of a sensible size for the descriptions and comparisons to be made.	4b Students measure area by counting uniform units, including where part units are required; and measure length, mass, capacity, time, and angle, reading whole number scales.
During the Relating Phase Students come to trust measurement information even when it is about things they cannot see or handle and to understand measurement relationships, both those between attributes and those between units. As a result, they work with measurement information itself and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.	5a Students take purpose and practicality into account when selecting attributes, units, and instruments for measuring things, and use the relationship between metric prefixes to move between units.	5b Students use a range of whole number and decimal scales for measuring, including making measurements that are more accurate than the available scales allow.

FSIM013 | First Steps in Mathematics: Measurement Course Book

Linking the Number and Measurement Diagnostic Maps