Malke Rosenfeld

Math ON THE Move

Engaging Students in ***Whole Body Learning***

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This book is dedicated to Dr. Patrick Hill (1939–2008), who oversaw two of my independent projects during my time at The Evergreen State College. His steadfast belief in my line of inquiry and his enthusiasm for my endless questioning about what it means to know, think, and learn in community have propelled me forward to this day.

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Foreword

Maybe you are standing at a conference, or library, or your computer reading this foreword as you decide whether you're going to read this book. I would strongly suggest that on your way home with the book, or added to your online shopping cart, you pick up a roll of painter's tape.

This is a book that deserves to be not only read, but experienced. As I read, I taped out a square and a grid on the floor, and stepped and slid and jumped my way through this book, and my understanding of mathematics and student learning is richer for it.

During the process of writing the foreword for this book, I got the chance to join Malke Rosenfeld as a fellow learner in a mathematics workshop about tiling and symmetry. We were both challenged by the task of using only paper and pencil and visualization to imagine what different sets of five square tiles (pentominoes) would look like if they were rotated, reflected, and translated.

Inspired by what I had learned from *Math on the Move*, I began to wonder: would this task be different if we changed the scale from paper-size to floor-size? Like the students who participated in the "Scaling Up" (p. 42) and "Collaborative Rope Polygons" (p. 55) activities, would we have new insights (and new challenges) if we explored the shapes by taping them out on the floor, or trying to use our bodies to make them?

I also wondered what it would feel like to be embodying the shape, instead of looking at it. I thought of how in all of the Math in Your Feet lessons, there's a period where students are in the observer role, and a period when they are in the maker or doer role. How might my experience of these tiling shapes be different as a doer rather than observer?

So we went out into the hallway, taped down a grid, and instead of visualizing and drawing, we figured out how to step the paths of our pentominos. I wasn't sure what would happen if we made these changes, but we were richly rewarded. Suddenly, not only did the shapes make more sense, but it became easier to transfer our learning back to paper and pencil as well! We made sense of the pentominoes, got better at drawing them, and learned new ways to think about reflection (changing every "left" to a "right" and vice versa in our paths made the reflection of our pentominoes) and rotation (rotating our bodies before we made the path made a rotation of our pentominoes). We generated new questions to explore and had new insights into our original questions!

This experience turned me into an evangelist for *Math on the Move*. I encourage you to spend some quality time reading and rereading Chapter 3, in which Malke lays out a "zero-entry pool" for getting your feet wet with body-scale moving mathematics. What I love about that chapter is that the experiences she describes, like making figures bigger than a sheet of paper, or experiencing lines and grids that are big enough to walk around on, can become a regular habit when students are struggling with any concept related to space and/or patterning. I now routinely ask myself, "How might students think about this differently if they were in it, walking around, rather than outside it, drawing it on paper?" That question never would have occurred to me as a mathematical thinking question without *Math on the Move*.

I feel a need to confess, before we get any further, that I thought of myself, before reading this book and getting a chance to dance with Malke, as someone who is clumsy and better at cerebral pursuits like math than embodied pursuits like dancing. Through reading and dancing I came to two big realizations.

First of all, I realized than in *Math on the Move*, the math is the dance and the dance is the math. Dancing is not just a trick to help me remember a math idea that I could access another way. Dancing the same as my partner *is* congruence. Dancing in the opposite way from my partner *is* reflection. For students who are great athletes or dancers but don't see themselves as great mathematicians, letting moving math come into the classroom gives them a chance to be the math experts. Are they the best at spotting or describing congruence in shapes they look at? Maybe not. But they can be the best in moving congruently with their partners.

Second, I realized that my fear of dancing was analogous to many peoples' fear of mathematics. I had never given myself permission to take dancing risks, make mistakes, and learn! The warm inviting tone of the book, and my experiences getting to work in person with Malke, helped me enter into a mental space where I could test out new dance ideas, and if I made mistakes at first, keep working until I got better . . . or change my ideas and try something I could master.

Whether you are a confident mover who is still learning how to bring that joy to your mathematical explorations, or a confident mathematician still learning how to be fluent in your precise movements, or someone for whom dance and math both sound intimidating, I think this book will inspire you to try something new. Malke shares stories of students learning about how to work in a group, how to have calm control over their bodies and their ideas in order to make new, creative inventions, and how to use mathematical and movement creativity to find joy in thinking, making, mapping, explaining, and sharing.

I would encourage every reader to pick up some tape, make some lines, grids, or shapes on the floor, find some kids, and get moving. Keep these questions in mind:

- How does being inside this body-scale space give me new insights and questions?
- How am I taking risks, trying new things, making mistakes, and learning?
- How am I supporting others to help them take risks and learn?

You'll find a new joy and creativity, and rigor and precision, in your moving and math that will help you see the world in unexpected ways!

Introduction

> Most people feel that they have no "personal" involvement with mathematics, yet as children they constructed it for themselves. Jean Piaget's work … teaches us that from the first days of life a child is engaged in an enterprise of extracting mathematical knowledge from the intersection of body with environment. The point is that, whether we intend it or not, the teaching of mathematics, as it is traditionally done in our schools, is a process by which we ask the child to forget the natural experience of mathematics in order to learn a new set of rules.
>
> —Seymour Papert, *Mindstorms,* 206–7

I am quite aware, when describing the work I have done since 2004 to help children make math and dance at the same time, that the Math in Your Feet program is the perfect blend of the two most anxiety-inducing disciplines for most of us raised in American society (and I'm only half joking). For one thing, both math and dance have a lot in common in their apparent ability to invoke fright and a flight response. They also share the deep-seated myth that we as a culture hold about learning and knowledge: that you are either good at math or dance or you're not. I think this is partly a case of self-fulfilling prophecy but more likely a result of how both subjects have historically been taught in a "Here's how you do it; now do it on your own" kind of approach.

Despite these deep-seated cultural attitudes, I know from experience it is possible to move beyond the fears related to both math and movement. In school I was always a happy geometry student but never numerically literate. Until recently, my numeracy was about on par with every other American's; that is to say, every time numbers came up in conversation, I'd get this funny tight feeling in my chest and change the subject. So, it was a little alarming to me when, after developing the Math in Your Feet program, which at the time focused mostly on geometry topics, I began to realize there might be more math in the dancing the children were creating—and that not only did I not know what it was, but I had no idea how to start figuring it out. It was an uncomfortable feeling, but one that I ultimately could no longer ignore, spurring me into an ongoing inquiry into what it means to "do" mathematics, and specifically what it means to learn and make math.

Although I used to think that math was some kind of inaccessible, abstract magic trick, a sort of in-joke that excluded us common folk, these days I appreciate that math is not that at all. The reality of math as most of us know it is like that story where three men are standing in a dark room touching different parts of an elephant. None of them has the full picture because each one perceives only isolated, individual elements of the whole animal. The reality, I'm discovering, is that math is just like that elephant: a large, expansive, multidimensional, intelligent, sensitive, expressive creature. The problem is that most of us have been standing around in that dark room since about kindergarten, grasping only its tail and thinking, *This is what math is and, personally, I don't think it's for me.* We've been unable to see the larger, incredibly beautiful picture that would emerge if only we would turn on the lights. Or, in reality, if we could find someone who would help us understand what we are looking at once those lights go on. I aspire to communicate this larger picture of math as engaging, beautiful, and personally relevant in all my teaching work.

But my work is not just about the math. As a visiting teaching artist, I provide children experiences with percussive dance (styles like tap dance, step dance, and clogging) in both performance and workshop settings. In workshops and longer-term school residencies, my goal is to provide students with enough of a feel for the art form and its basic movement vocabulary and skills so that they can quickly begin to experience the core of the discipline—the process of making their own foot-based dance patterns. I do this in the way I structure the learning space and the learning and making processes themselves. Every activity, student demonstration, and emergent skill builds on the one before it to illustrate the following tenets:

> We can create completely original ideas using the language of rhythm and patterns in our feet.
>
> There is a process to making something new.
>
> We can be musical, mathematical, and physically expressive at the same time.
>
> Little ideas as expressed through the rhythm and pattern we make with our feet can go together to make bigger ideas.
>
> We can relish and celebrate the effort it takes to do this.

In this book I illustrate and explain how children can become both dance makers and math makers at the same time and, more generally, what makes math and movement such effective partners.

A Different Approach to Learning

It is the partnership itself that influences and defines the learning that happens at the intersection of math and dance. Sometimes this intersection is called *arts integration*, but in my experience this is a somewhat inaccurate term. To me, the word *integration* implies that there is some kind of merging between the arts content and the other discipline, when, in reality, efforts in this direction are often like a sixth-grade dance: yes, the boys and girls are all in the same room, but they stick to their own sides of that space, and never the twain shall meet. Arts integration is challenging to do in a meaningful way, no matter what subjects are being brought together. In her post "STEAM-Roller" on the *Education Week* blog *Teacher in a Strange Land*, Nancy Flanagan (2013) wrote: "I like a snappy multiplication-facts rap as much as the next guy, but the fact is—it's not easy to integrate rich arts practice or content into science and math instruction. Especially when the assumption is that good curriculum begins with 'core subjects,' the arts acting as a kind of color commentary." When paired with math learning, the whole moving, dancing body definitely needs to be more than color commentary or a wallflower at that sixth-grade dance.

These days I prefer to call what I have done to bring math and percussive dance together simply *interdisciplinary learning* because that is the most accurate description. Both math and dance are discrete disciplines that require students to gain content knowledge, develop skills, and cultivate thinking and reasoning fluency in order to create meaning within their respective systems. However, as I illustrate throughout this book, the process of bringing math and a moving body together, whether within the system of dance or at *moving scale* (which is explained and illustrated in more detail in Chapter 1 and Chapter 3) is not as straightforward as one might think. First of all, we need to take a close look at our current practices of how we engage children's bodies in the classroom. Too often the moving body is used primarily as an object for literal interpretation, illustration, and memorization of math concepts. Conceptualizing the body in this way, as a drawing or mnemonic tool, severely limits its potential in a learning setting.

Deborah Ball has said that math pedagogy "must be based on the structures of the discipline in order to avoid corrupting or distorting its content" (1990, 4). This statement rings true for the discipline of dance as well and, most important, has implications for how we conceptualize, create, and execute interdisciplinary curricula. We can't make up the dance to fit the math, nor can we make up the math to fit the dance. This book is, in part, about how we can harness our students' inherent *body knowledge* (a term coined by Seymour Papert, 1993) to help them develop new understanding and facility with mathematical ideas that often seem remote and impenetrable to our learners.

This new body-based understanding can be developed and supported through a making process. I see some parallels between arts-based learning and the maker movement and maker education; in both settings the focus is on the design and construction of objects in response to a perceived need, which allow for high learner agency, experiential learning, and a chance to tinker—to play around with a set of ideas until you land on an answer that makes sense to you, creating your own understanding and meaning in the process. The results of math-and-dance making, however, are a little more temporal, a little less tangible, and they require us to look more closely for evidence of learners' growth during the process. Regardless of the product developed, both the maker movement and I have been highly inspired by Papert's work and thinking about design environments in which kids learn the language of mathematics in the context of *using* math to make something new. There is meaning in the making, and by that I mean the making of tangible things and also the making of a larger understanding about how ideas work, how they are connected to other ideas, and how these ideas become useful objects with which to create a place for ourselves in the world. In my view, the territory of learning needs to include experiences where learners get really messy, dig in, try things out, make mistakes, and arrive at multiple right answers as well as learning in more formal contexts. It's the real-life experiences that are essential for children as they work hard to learn how to communicate their own and others' ideas.

Why We Should Use the Body in Math Learning

In this book I endeavor to explain why we should use the whole, moving body in math learning by pulling from both research and practice to build a framework for meaningful, body-based math learning, but the short answer is that when children harness their innate body knowledge for mathematical sense making, they also harness their whole selves in the pursuit of new ideas and understanding. They develop, communicate, and reason about mathematical ideas both nonverbally and verbally. Teachers regularly report to me during the math-and-dance work I do with their students that they cannot believe how much the children are "talking math" while they endeavor to meet the physical and mathematical challenges presented to them and during other parts of the day. To me this makes perfect sense because this is how we learn the meaning of words in the first place—in context. Children can make good sense of the world when they get a chance to interact with it, and children are also well able to reason with and about things they observe and do. But they can do this only if they get the chance to do, make, investigate, converse, wonder, build, express, and reflect. Without these kinds of interactions they might still be able to memorize math facts, but memorization would not necessarily mean they would know, for themselves, that something was true.

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About This Book

In this book I focus on what it means and looks like to bring meaningful movement and math learning together in the classroom. It is completely understandable if you initially have reluctance, doubts, or questions about using movement in math class. This reluctance could be, in part, related to the fact that lots of us have really learned math only as it's presented *on* the page or as a series of rules, facts, and procedures to memorize. Math learning using the whole body can feel and look very different than what we're used to thinking of as math. But experiencing math this way can become a potent opportunity to create new insights about the math ideas not necessarily or immediately accessible to us as represented visually on the page. We want math to make sense to our students, and the moving body is a wonderful partner toward that goal. Of course, trying out any new approach for the first time may induce a little trepidation, but this book (and its online video companion) is filled with everything you need to get started: the whats, the whys, and the hows of helping learners make math meaningful through purposeful, whole-body-based math investigations and problem solving.

What's in the Book?

Chapter 1: The Body as an "Object to Think With"

In this chapter I provide an overview of what meaningful whole-body math learning looks like in my own and others' math and dance classrooms. I also provide a conceptual framework and pedagogical base for teachers wishing to engage their students' whole bodies in mathematical sense making.

Chapter 2: How Is This Math?

I provide the answer to the common question "How is this math?" through illustration and description of the kinds of mathematics that can be explored, learned, and expressed by the moving body, including spatial reasoning, dynamic geometry, part-whole relationships, equivalence relations, and mathematical thinking and sense making.

Chapter 3: Beyond Mnemonics: Getting Started with Moving-Scale Math

This chapter provides a number of options for getting started with non-dance, whole-body, moving-scale math learning in your classroom. It also functions much like a zero-entry pool in that you can get your feet wet by adding students' whole bodies into the math you are already doing and then, as you build experience and confidence in this new mode of teaching and learning, venture in a little deeper.

Chapters 4 and 5: Implementing Math in Your Feet

In Chapter 4 I detail the first three stages (understand, experiment, and create) of the Math in Your Feet lesson progression for grades 3 through 6, which can also be adjusted for second graders in the second half of the school year. In Chapter 5 I lay out the final three stages of math-and-dance making (combine, transform, and communicate).

Chapter 6: Facilitating the Math-and-Dance Classroom

In this chapter I lay out the three main strategies for facilitating a moving, math-and-dance-making classroom. I also discuss considerations for learners with special needs.

Chapter 7: Adapting Math in Your Feet to the Primary Grades

The work primary students do in Math in Your Feet looks similar to the work intermediate students do, but the pace and tone of that work are necessarily different. In this chapter I provide a lesson progression specifically tailored to this age group and highlight how the process of creating maps of their math-and-dance patterns is integrally intertwined with the math-and-dance making itself.

Chapter 8: Assess, Extend, and Connect

How do you assess the work and learning of moving students? This assessment is tied to the ways in which you help students extend and connect their moving math learning in the form of mapmaking, written reflections, and word studies. In this chapter I detail what this looks like and also provide suggestions for where and how to connect the moving math to other, more familiar math-learning contexts.

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Chapter One

The Body as “An Object to Think With”

As teachers we know kids love to move—in fact, there’s a developmental imperative at play that cannot be ignored. But how can we harness this innate and essential playfulness in our students in a way that moves them, literally, toward conceptual understanding of elementary mathematics? Using the moving body in math class is about more than getting kids out of their seats to get the wiggles out or to memorize math facts. Instead, we need to treat the movement as a partner in the learning process, not a break from it.

Meaningful Connections Between Math and Dance

When I first wondered if there might be math in the dance making I had been doing with elementary students, I knew I didn’t know enough math (yet!) to pair the two in a meaningful way and that I needed an interpreter to help me connect the two. I was very lucky to be introduced to elementary math specialist Jane Cooney, who became a generous resource and mentor. In my first meeting with Jane I described how I dance on my small square dance board (more traditionally called a *step-a-tune*) and the many ways that I supported children in becoming dance makers using the elements of percussive dance. Neither Jane nor I wanted to make up the math to fit the dance or to change the nature of my traditional dance form to fit the math. Considering this balance between the math and the movement is possibly the most important issue to attend to when bringing movement of any kind into math learning. The challenge lies in navigating the palpable tension between the physical, kinetic existence of the body in time and space and the often static nature of mathematical concepts we encounter on the page. At the beginning of my journey, I had experienced math only as fixed and as a series of facts already decided, composed primarily of two-dimensional images and immutable right answers. But I also suspected that some of those math ideas could be explored in the percussive dance making I did with kids. So Jane and I set about looking for useful connections between the two disciplines. After some experimentation during my two pilot years of working with fourth and fifth graders in a large school district in Indianapolis, Indiana, I established five themes around which all our work revolves:

- *Patterns:* Students create, observe, descre, and compare patterns; the creation aspect has similarities to a language arts writing-and-revision cycle.
- *Problem Solving:* Students solve problems with a focus on mathematical thinking and sense making and relevant math practices.
- *Combination:* Students engage in a highly interesting mathematical activity that also ups the physical thinking and problem-solving challenges for students.
- *Transformation:* Students work as a team to reflect the original math-and-dance pattern over a line of reflection.
- *Communication:* Students speak about and reflect fluently on their ideas and the decisions they've made in the process, in both math and dance, through written, spoken, body-based, rhythmic, and symbolic language.

Each theme represents and makes explicit a shared idea found in both math and dance and points to the ways we use the math to build and describe our dance patterns to the mutual benefit of both subject areas.

My biggest takeaway when thinking about bringing the math and percussive dance together was that both dance and math are discrete disciplines; both are meaning-making systems in their own right with their own aesthetics, traditions, content, and pedagogy. It's really not reasonable to expect that there will be a one-to-one correspondence between every idea in both disciplines. There are, however, places where the two overlap in gorgeous ways, and this book is full of what I've discovered at the intersection of percussive dance, mathematics, and learning; how we can translate these insights to nondance movement; and how you can bring that into your own classroom. Still, as I continue my inquiry into this topic I am continually reminded that *not all of dance is mathematical and not all math is danceable*;

in particular, the moving body is not always the most useful tool for expressing and representing every mathematical idea. (I'll dig more deeply into the math itself in the next chapter.)

Criteria for Meaningful Moving-Scale Math Learning

My own search for meaningful connections between math and percussive dance started with a question; the answers I've found since then also have relevance for learning math at moving scale (also called walking-scale or body-scale), outside a dance setting. These phrases indicate that the scale of the learning activity now allows enough room to freely use the whole moving body in a mathematical investigation. To create a meaningful context in which children can think deeply and engage in mathematical sense making with their whole bodies, the following elements must be present. I'll discuss these in more depth in the rest of this chapter and in the chapters that follow.

- *The math-and-movement lesson provides a structure in which students make choices, converse, collaborate, and reflect verbally on what they did, how they did it, and what they noticed while they were engaged in whole-body-based activity.*
- *The body activity is focused on mathematical sense making, not mnemonics, often through efforts to solve a physical or moving-scale challenge of some kind, not on illustrating a math idea as it is typically represented on the page.* The most appropriate mathematics for whole-body investigations are, as described in the next chapter, found on the action side of math, something the moving body is perfectly situated to express and explore.
- *The teacher is the facilitator of the activity, pacing, and discussion.* The teacher's role in this learning-while-moving approach is not about instruction or acting as the ultimate expert. Instead, the teacher is more of a facilitator, supporting learners and their collaborative relationships as they work toward addressing and reflecting on the challenge in question and, later, connecting the mathematics as represented and experienced in the moving challenge to new modes or contexts. The teacher questions, makes observations, and moderates discussions, all of which help children understand both the math ideas and the activity in which they're engaged from multiple perspectives.

- *The activity explores one or more mathematical ideas at a new scale.* Typically, math investigations in K–6 classrooms happen at hand scale or paper-and-pencil scale, meaning through the manipulation of hand-sized objects or within the confines of a piece of standard notebook paper. Creating a math task at moving scale means you can be engaged in one of two different but related approaches. The first approach is making a familiar math object, like a hundred chart, bigger so that it is big enough for a whole body (or bodies) to explore. The second approach involves focusing on the scale of the *activity*. When a student stands up to move her whole body to interact mathematically with her peers and the environment, she will experience and notice different aspects of the math ideas under investigation.
- *Students experience the activity as both doers and observers.* We can create new perspectives and new meaning when we are literally at the center of the action. We also need a chance to observe the action from outside of the experience; observing others work and/or listening and watching as they explain their reasoning has the potential to give us insights into our own work, exposing things we might not notice while we are engaged in the activity.
- *In partnership with the change of scale, the math-and-movement activity should be explicitly connected with the same math idea as it is experienced in other contexts, scales, or modes.* Providing multiple opportunities and modes in which to investigate a particular mathematical idea helps students become more flexible problem solvers and better understand how the idea can be used and applied in different contexts. This means, in part, that during or after a moving activity there should be an opportunity for verbal or written reflection on the experience (both the activity and the math); a chance to gather and analyze data created by the movement challenge; or time to create a map or a more formal record of the activity.

Bringing Meaningful Movement into Math Learning

How can we engage the moving body in a way that is pedagogically sound and useful for both disciplines (not to mention beneficial for our students)? Here are three dance-based examples that exemplify a positive balance between the math and movement and reflect the previous criteria.

Kindergarten: The Quality of Numbers One Through Five

With the guidance of a teacher, a class of kindergarten students have created a dance using the numbers one through five and are onstage, performing for their families. Although we think of numbers as something to count, the children are not counting out the beats to the music or counting the numbers while they dance. Instead, the children embody the quality of each number, starting with one child at a time crossing the stage. As the dance begins there is energetic Irish fiddle music playing and, as the teacher calls out, "One!" individual students begin to cross the stage with scarves waving, moving and dancing as the music moves them. Some walk forward, others backward; some skip, some jump, and some turn as they go. It's clear they've had a chance to explore ways to move through space including a variety of locomotor movements and are able to make their own in-the-moment choices for this dance. After all the students have crossed, the teacher calls out "Two!" and the whole class returns to the stage, each looking around for a partner with whom to dance. When they find someone to dance with they face each other, which makes each grouping of two clear to both the students themselves and their audience. When the teacher calls out, "Three!" the students must hold scarves in a ring of three and then turn in a circle. When they hear, "Four!" they make groups by putting their hands on the shoulders of the kid in front of them and walk around the stage as four-person units. They end their dance with "Five!" by creating five-person frozen sculptures (Ruggery 2012).

The dance has a clear sequence and is developmentally appropriate for the age group. By this I mean that the dancing has not been overly prescribed and is not reliant on technical skills beyond the ability of this age group; the work is also obviously personally relevant to the individual children. In terms of the math, the children express their understanding of the numerical groupings by actually putting themselves in the groups with no prompting from the teacher. Their dancing also illustrates how we can use movement to express dynamic mathematical ideas, in this case the real meaning behind the numerals 1 through 5, and emerging understanding of groups, which is an important core concept for multiplicative thinking later on down the line. Most importantly, the children are operating in a moving and changing system within the dancing itself and are able, with little prompting, to navigate their way through both the math and the movement. In a sense, it's hard to tell the difference between the math and the dance. Being able to do both at the same time is what brings the excitement and meaning of ideas expressed through both disciplines to the stage.

Second Grade: Contrasts

Teaching artist Kimberli Boyd has second-grade students start a math-and-movement lesson by creating a body shape while standing in place and then finding a contrasting shape. For example, if one shape means stretching tall, the contrasting shape would be something that is close to the ground. They continue the lesson by performing contrasting motions (fast and slow, smooth and sharp). A child demonstrates a movement that Kimberli describes as "controlled and kind of slow"; then Kimberli asks the class, "What is the opposite motion?" The class replies, "Fast." In another contrast, children are now in pairs; one partner is still while the moving partner travels around him or her. To close the lesson, Kimberli has groups demonstrate what they've learned during the activity; the class watches as two groups of students demonstrate tall and short, fast and slow, smooth and sharp, moving and still and, finally, "one final shape to end the dance" (DetroitYouthVideos 2013).

In this lesson (which is appropriate for first and third graders as well), the language of dance is expressed verbally and through the movements of the body; the language and movement together provide fertile ground for exploring the mathematical ideas of logical opposites and difference. It's also a great way to support kids in exploring new ways to move their bodies. By blending basic movement concepts of what the body can do in time and space (move or be still, grow or shrink, perform sharp or smooth movements, and so on) with the idea of *contrast*, students have the opportunity to engage personally and viscerally in understanding the ideas of difference and opposites.

Fifth Grade: Symmetry

A class of fifth graders is exploring the dynamic ideas embedded in reflection symmetry. The teacher reviews reflection symmetry visually in two-dimensional space on an overhead projector before the students review how reflection can be created using three-dimensional body shapes and movements. The teacher asks the fifth graders to find words to describe why they think another student has created a symmetrical shape with his or her body and then engages the whole class in brainstorming revisions to make the pose more symmetrical. These are key skills needed for analysis in both math and dance modes; in particular, using specific language and identifying properties in the context of dance making make students' dances stronger and more interesting. The lesson progresses from individuals demonstrating symmetry in their own bodies to teams of students reflecting their partners' movements, as different groups bend, twist, and turn their bodies. One student illustrates what it means to put children literally at the center of their own learning when she says: "When you do math worksheets you usually do it for a teacher for homework, but when you dance it you're doing it for yourself" (Caspersz n.d.).

In all three of these moving-scale and math-and-dance activities, what stands out the most to me is the equal time and attention paid to both the math and the dance, that the body is not relegated to the role of illustration, that learners have agency over their moving bodies in a way that they wouldn't in a teacher-created, student-memorized dance, and that neither the math nor the dance is singled out as the sole reason for the activity. In a moving-scale approach the goal is the same: the activity is both the mode and the context in which students work to access the mathematics.

Avoiding Limited Uses of Movement in Math Class

While the classroom vignettes in the previous section illustrate useful examples of what meaningful math and movement can look like from the outside, we also need to consider nonexamples. What does *less meaningful* math and movement look like? There is an image that gets forwarded to me every six months or so by people who know I am interested in both math and dance. The original picture and its variations are titled "Beautiful Dance Moves" or "Function Calisthenics" or something related. In the picture a stick figure poses its arms in various graph shapes, with the graph's equation written below each figure. Although this picture, which gets passed around so often, must feel meaningful to many people, re-representing the original two-dimensional representations of graphs with your body to memorize them is not what I would consider mathematical sense making.

Here are some other examples of classroom activities I've seen that illustrate using the body as a tool for learning math in limited ways. Despite these limitations, most of the following activities are helpful for some aspects of learning math; there are times when we need multiple strategies for memorizing our multiplication tables so we can engage more freely with other math ideas and challenges. However, from the standpoint of *combining* math and a whole, thinking, expressive, moving body, especially if it's within a dance system, I find these activities narrowly conceived—not just in the math and the dance but in the learner's role as well:

- using arms to create symbols for operations, like $+$, $-$, and $=$ (focusing on creating representations of the symbols, not expressing their meaning)
- using hand movements in a song about memorizing a procedure

- putting arms into the static shape of lines, graphs, or angles as they are represented on the page
- jumping on the beat in a predetermined pattern while skip counting
- bouncing on an exercise ball while reciting multiplication facts
- singing a song with an accompanying dance about finding the area of a circle, using movements that bear no relationship to the properties of a circle
- exploring a math concept such as high versus low in isolation, removed from a narrative context (such as retelling a story) or the larger context of dance learning and making
- having multiple students become the sides of a triangle by lying on the floor

The issue for me lies not only in the fact that each of these examples treats math as simply a set of rules and facts to be learned but also in the fact that the teacher has provided specific instructions for movement and physical patterning in each activity. Even if the math-learning goal stayed the same (memorizing facts or procedures, for example), most of these activities would become immediately more meaningful if the children had a role in developing the movement choreography. Starting with the question "How can we think about or show this math *idea* with our bodies?" and offering multiple opportunities for children to contribute their own ideas and build something new as a group creates a more significant experience for the learners.

The Body as a Thinking Tool

The three more meaningful activities described on pages 5–7 are strong examples of purposeful movement in math class. They also exemplify what Seymour Papert, in the preface to his seminal book *Mindstorms* (1993), identified as *body knowledge*. In these examples, the learners' existing body knowledge is both harnessed and deepened in an environment that provides thoughtful, math-informed, movement-activity sequences facilitated by the teacher.

The Power of Body Knowledge

The phrase *body knowledge* is such a potent one. It is a challenge to our collective conception of what knowledge is and where it resides; it also places the student in the very center of the learning process. Body knowledge, also referred to as *embodied cognition*, helps us understand the processes of thinking and learning with our bodies. This is something we begin developing from birth. Developmental psychologists have shown that in babies, "cognition is literally acquired from the outside in" (Smith and Gasser 2005, 13). This means that the way babies physically interact with their surroundings "enables the developing system [the baby!] to educate [herself]—without defined external tasks or teachers—just by perceiving and acting in the world" (13). Ultimately, "starting as a baby [as we all did] grounded in a physical, social, and linguistic world is crucial to the development of the flexible and inventive intelligence that characterizes humankind" (13).

Understanding what embodied cognition is and what embodied learning looks like is the focus of a multidisciplinary group of cognitive scientists, psychologists, gesture researchers, artificial intelligence scientists, and math education researchers, all of whom are working to develop a picture of how we learn and think with our bodies. Some of these researchers are focused more specifically on a variety of ways the body is involved in learning and expressing mathematical ideas. Much of their focus has been concentrated on the way our hands express ideas in concert with speech; this kind of research has shown clearly that gestural expression plays a large role in thinking through and expressing mathematical ideas (Alibali and Nathan 2012). Gesture research has become a strong platform on which to develop even more insights into a thinking, moving body. From here, other researchers have concentrated on the use and benefits of hand-scale tools and manipulatives; still others are focusing on the whole body as the primary actor in embodied mathematical activity (Hall, Ma, and Nemirovsky 2015)—quite literally as the "object to think with" (another enduring phrase coined by Seymour Papert) within a mathematical investigation. In general, the research over the past few decades has resulted in a general acceptance that it is impossible to ignore the body's role in the creation of mind and thought, going so far as to agree that that there would likely be no mind or thinking or memory without the reality of our human form living in and interacting with the world around us.

Sense Making with the Body

Since the publication of *Mindstorms*, the phrase *body knowledge* has often been used to describe what is happening in situations where children are primarily using their hands or eyes (often in front of computer screens) or in *any* situation in which a child is out of her seat and moving. The fact is we all use our bodies to interact in, through, and with the world, all the time, every day. Given the inevitability of our bodies being present in some way in everyday learning, the questions then become *What does it mean to use and develop body knowledge?* and *How can we incorporate or harness this idea in the pursuit of rich conceptual understanding of mathematics?* My ongoing concern has been about the common assumption that body knowledge is created or used simply by moving a part of the body or being on one's feet. This is what is assumed when, for example, a student is shown how to make a static representation of a right angle using her arms. This activity is unnecessary because this piece of math can be explored and identified in many different contexts and is already quite familiar to upper-elementary students if they've had a chance to explore it on the page. If you follow up with a question about the measurement of a right angle, kids will be able to tell you that answer, too, because it is a math fact they have memorized, often without struggle. The real potential for using the moving body in this situation is investigating questions like these:

What are angles, really?

What do they do?

What is the relationship between a right angle and other kinds of angles?

What might happen if we combine two or more right angles?

These questions are interesting in and of themselves and also because they instigate some interesting new thinking and question asking, as well as new opportunities for application. Investigating what angles really are is also a question that a moving body is perfectly primed to explore. An angle is an amount from *here* to *there*; ninety degrees is the result of a rotation from *here* to *there*; and the moving body is the perfect tool for exploring *here* to *there*.

While we can *see* children's bodies moving, it is not always an easy or intuitive process to understand the mathematical learning happening and the knowledge expressed while their bodies are in motion. The challenge lies in navigating the palpable tension between the physical, kinetic existence of the body in time and space and the often static nature of mathematical concepts we encounter on the page. However, while it might appear that the images of mathematical ideas are frozen on a page in time and space, much of math is, as I detail in Chapter 2, actually about action, movement, and relationships.

In a sense, the moving body is best conceived as a tool for *thinking through* or making sense of a mathematical question to find an answer or create new insights not accessible through conventional means. From the very beginning of the Math in Your Feet work, we present children with the challenge of creating their own dance steps while simultaneously using math ideas to inform and describe what they are doing and why. They respond by thinking through this challenge with their bodies in a way that is at once dance, physical thinking, and mathematical problem solving and sense making. This kind of activity is similar to how math manipulatives or other thinking tools can be used in math class. Considering the role of hand-based manipulatives in math class can help us create a conceptual frame around the work we might want to do with whole, moving bodies in mathematical investigations.

The Purpose of Thinking Tools in Math Class

Math manipulatives include, among many others, pattern blocks, snap cubes, Dienes (base ten) blocks, and counters. They have the potential to help make the sometimes elusive ideas of quantity, pattern, transformation, and structure more visible to learners. Most often used in K–2 classrooms, their potential does not stop there; physical models and other handheld objects are useful at all levels of math learning. In my own recent experience I explored symmetry groups by turning and flipping a paper square and triangle in various sequences with my hands, writing down my results, and looking for patterns in the table I created. It was the movement inherent in this activity that strengthened my understanding of the rotations and reflections and their relationship to each other in a way I could have not done through internal visualization alone. The action involved in *moving* (flipping, turning, sliding, building, constructing, drawing) while working through mathematical inquiry and problem solving is, I believe, one of the most important aspects of using math manipulatives and other handheld tools in math class.

What does it mean that students are thinking through math ideas while using these kinds of hand-scale tools? The answer to this question can inform our efforts to put the moving body to good use in pursuit of a strong conceptual understanding of mathematics. Master math educator Henri Picciotto (n.d.) makes clear that tools are generally used in two ways: either prescriptively or as an opportunity for guided exploration:

> Tools can be used poorly, and often are. Manipulatives can be "taught" in a "this is how you do it, now practice" style that works so poorly with so many students. Electronic tools are often used in a "do this, do that, what do you notice?" style which suffers from the misconception that students will notice the thing we want them to notice.

> Intelligent lesson design using learning tools is often based on a reversal of traditional teaching practice, and the creation of "no threshold, no ceiling" problems. . . . The problems need to be engaging and accessible, but at the same time they should be worthy of reflection, collaboration, and discussion.

A thinking tool in math class, then, is not the object itself but the context in which it is used; it is a context that requires learners to engage with the materials, the math ideas, and classmates to create mathematical meaning, whether they are investigating number patterns or geometric attributes or making sense of information in a problem-solving context. Other distinguishing factors of objects as thinking tools include high learner agency, a chance to learn through dialogue with peers, and an understanding that there might be multiple paths to the end goal even if, ultimately, everyone gets to the same answer.

As an example, let's think about pattern blocks, a collection of wooden or plastic shapes (equilateral triangles, two different rhombi, hexagons, squares, and trapezoids) that are used to create geometric designs and patterns and offer the opportunity for exploring symmetry and noticing relationships between the shapes. Pattern blocks can be used in many ways, often through guided inquiry, both geometric and numerical. Elementary teacher Simon Gregg gives his third- and fourth-grade students the challenge of coming up with multiple versions of a dodecagon using pattern blocks; you can also challenge students to see how many different hexagons they can make using the same materials. Both scenarios are examples of identifying a learning goal and allowing for learner agency and exploration within the task itself. Chris Hunter is a math coach who has used pattern blocks in a different but still guided approach, this time focused on numbers, geometry, and algebraic thinking. He asks his students to engage with questions like these:

- *You have 6 hexagons and 4 trapezoids. What is the greatest and least perimeter you can make?*
- *How many ways can you find to show me $\frac{1}{2}$? Can you describe it another way?*
- (When given an increasing pattern) *How many in figure 100? Figure* n?

If you can create this kind of structure around mathematical exploration with handheld objects, you can also do this with the moving body in the classroom.

The Body as a Thinking Tool

What do we want the mover to do and to learn? This is a critical question for anyone set on using physical objects (including the body) in math learning. Just because a child has an object in hand (or is up out of his seat) does not mean he is learning math. It is not the object itself that supports the student in coming to know and understand but rather the context in which that tool is used. Deborah Ball has also written about the importance of creating context when using manipulatives. "Although kinesthetic experience can enhance perception and thinking," she writes, "understanding does not travel through the fingertips and up the arm. And children also clearly learn from many other sources—even highly verbal and abstract, imaginary contexts. Although concrete materials can offer students contexts and tools for making sense of the content, mathematical ideas really do not reside in cardboard and plastic materials" (1992, 47).

Put another way, using tangible, moveable objects (including the moving body) can be useful in math learning as long as attention is paid to the math *ideas* as well as what you do with the object. Being on your feet is not a guarantee of either developing or using body knowledge in mathematics—and it goes the other way, too. Just because you're a skilled mover does not necessarily guarantee you will be good at math. The meaning is created through thoughtful construction of activities or even whole sequences of investigations where each mode, mathematical and movement, comes to influence the understanding of the other.

Here is another useful excerpt from Deborah Ball on the topic of manipulatives, but this is also where my analogy between a moving body and inanimate objects used in math class diverges. The moving, learning body is more than an object—it is the whole child interacting in and with the larger environment (classroom culture, peers, teachers, and the physical space in which the activity is occurring.) So, where Ball uses the words *manipulative* and *material*, I have substituted *body* and ask you to instead imagine the animated, curious children who populate your classroom:

> Unfortunately, creating effective vehicles for learning mathematics requires more than just a catalog of promising [bodies]. The context in which any [body] . . . is used is as important as the [body] itself. By context, I mean the ways in which students work with the [body], toward what purposes, with what kinds of talk and interaction. The creation of a shared learning context is a joint enterprise between teacher and students and evolves during the course of instruction.

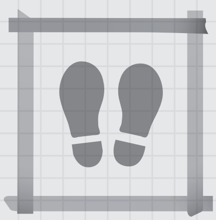
> Developing this broader context is a crucial part of working with any [body]. (1992, 18)

This passage perfectly describes the dynamic of teaching and learning in Math in Your Feet and at moving scale.

***Try It Yourself!* Part 1**

Before you go further, get a roll of masking tape or painter's tape. Measure out four equal lengths of tape, each two feet long. Use the individual pieces of tape to make a square on the floor.

Start with your feet together in the center of your square. How many ways can you think of to split your feet apart (using a jump to get there)? **Hint:** You can rotate your body to face different directions in your square. Does that give you some new options?



What "Thinking Through" Looks Like with the Whole Body

The moving body is truly a multidimensional object to think with and needs to be treated as such—a true partner in learning. What does it look like to think through a math idea while dancing or, indeed, to think through making a dance step using math ideas? Here is an example within Math in Your Feet, where children are thinking through a math-and-dance challenge with their whole bodies.

Thinking Through "Not Quite the Same"

Fairly soon after students start making their first four-beat dance pattern (pattern A), I introduce the idea of *sameness*, as in "How can you dance the same as your teammate?" or "How can you and your partner dance the same as each other?" When dancers do the same moves at the same time, this kind of sameness is called unison. Here, two boys are working together, using their bodies and conversation to clarify some issues that are keeping them from dancing in exactly the same way. These boys have facility with the dancing but are still working to understand and clarify the direction of one specific turn in their pattern. In the following exchange, my role is one of observer as they work to clarify sameness in their dancing.

Me: [Watching them dance together and noticing some differences in the last two beats] *OK, let's do it beat-by-beat, OK?*

[The boys dance one beat at a time, at a slower tempo, counting together.]

Me: [Talking to Juan] *So, you know what I noticed? I noticed you were turning your body a little differently in that corner. Let's try again. Look at each other's feet and see what happens.*

[They do the dance again. Their last two beats are still different from each other.]

Me: *And also, on those last two beats, I noticed you're turning right and you're turning left. So why don't you work on . . .*

Juan: [Quickly gesturing left with his right hand] *We'll turn left.*

[The boys dance but turn on the second beat instead of the fourth.]

Me: *Try one more time? One, two, three . . . no, no! It was on the fourth beat you were turning.*

Boys: *Ohhh!*

[Juan gestures and demonstrates to Dalton turning right with his body; then they dance.]

Me: *Oh, hold on!* [Talking and gesturing to Dalton] *You need to go that way. You're turning right on the first one and then a right turn again. Aha! There you go, that's it. So try that again. . . . One, two, three. . . . Yay, that's great!*

In the actual experience the boys' movements were more intense and kinetic than I've transcribed it here. I appear to be talking a lot in this segment, but every time I noticed something that was not the same, the boys immediately started moving again to figure out how to correct it. Their bodies did the thinking; my role was to observe their work and provide some outside perspective about and language for the relevant properties that were not yet being danced the same. In math, the idea of sameness can also include discussions and analyses of differences, similarities, and change and we need shared language and awareness to do this. Being physically involved in trying to *make* sameness and difference moves the conversation from the more familiar activity of identification to a brand-new experience of literally embodying the concept.

Thinking through specific challenges like sameness and difference (illustrated in detail in other parts of this book) while engaged in making dance and math at the same time is a collaboration between the body and all its senses (Hall and Nemirovsky 2011), the influence of one's partner or collaborator, the kinetic energy distributed around the room as everyone works, and the words we use to talk to ourselves and communicate as precisely as possible to others. Thinking through mathematical and movement ideas at the same time positions the body as a dynamic object to think with and allows learners the opportunity to think deeply about core mathematical ideas in new and novel ways, which I explore in much more depth in the next chapter.

Try It Yourself! Part 2

The boys in the example were clarifying the direction of the turn on the fourth beat of their pattern. Here is a turn challenge of your own to try!

Put your **feet together** and **stand** in the center of your square dance space. **Turn** your body right, toward the first side. **Turn (while jumping)** toward the next side. **Turn** toward the third side. **Turn** toward the fourth side. How many times did you have to **turn** to get all the way around? How much, in fractions, did you **jump** on each turn? If you turned one-fourth of the way around each time you jumped, how much was each rotation in degrees?

If you **jump** and **turn** halfway around your square, how much have you rotated? How many jumps will you need to get all the way around? If you start your movements facing forward, **jump** and **turn** ninety degrees to the right and then **jump** and **turn** 180 degrees to the left, where will you end up?