"These materials are like *Reading Strategies* for math! My only complaint: Where was this when I was a classroom teacher?"
—**Jennifer Serravallo**, author of *The Reading Strategies Book*

Digital Sample

MATH in PRACTICE

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What's inside?

"Sue has written a great primer on what we need to know and be able to do in our elementary classrooms to be far more effective teachers of mathematics. What more could a teacher of K–5 math ask for?"

—Steve Leinwand, American Institutes for Research, author of *Accessible Mathematics*

"I always felt stupid in math class."

"Math was boring."

"I just don't get math."

Have you heard comments like this from adults?

Math today

This is a product of "the way we've always done math"—endless worksheets, one way to get an answer, and memorization without understanding. Some students get it, many others don't. Some students excel, many others flounder.

Today, effective teaching is being redefined. At all grade levels, our students must understand what they are doing and apply their knowledge and skills to solve real problems. If mathematicians solve problems, reason, communicate, model, and make connections, we should be actively doing this in our classrooms.

The challenge

Do you remember this saying?

"Ours is not to reason why; just invert and multiply."

Many of us learned tricks like this to remember how to—for example—divide fractions. But many of us also have no idea why doing this actually works:

$$\frac{1}{4} \div \frac{2}{3} \longrightarrow \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$$

Memorizing facts and formulas is entirely different from the problem solving, modeling, and reasoning your students are asked to do in class and on tests now. Often, we're re-learning math content at the same time we're being asked to teach it in this new way—and this shift can be challenging!

"Why should I change how I teach math?"

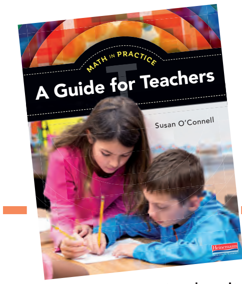
"How can I teach math confidently and effectively when I don't really understand it myself?"

"I need some new strategies and approaches to use in my classroom."

Why *Math in Practice*? Who is it for?

Math in Practice is designed to support teachers. Rather than providing another sequence of lessons and units to take students from the beginning to the end of the year, *Math in Practice* focuses on developing deep content knowledge, understanding why certain strategies and approaches are most effective, and rethinking our beliefs about what math teaching should be.

MATH in PRACTICE



Guide for Teachers

The ***Guide for Teachers*** is the linchpin of the *Math in Practice* series. It lays out big ideas in best-practice math instruction, providing a foundation for the accompanying grade-level books. Throughout the Guide for Teachers, you'll find what standards and research say about these topics, extensive support for effectively incorporating these strategies into your everyday instruction, and opportunities to reflect on your teaching.

Explore instructional strategies such as:

- asking questions that stimulate student thinking
- exploring math concepts through modeling
- using formative assessment to guide instruction.

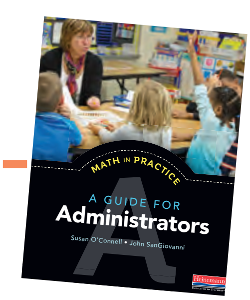
See inside for a sample of modules included in each grade-level book.

Grade-Level Books (K–5)

Each ***grade-level book*** is organized into modules that carefully unpack the specific math content taught at each grade, K–5. Rather than being used in order, teachers can select modules as needed based on their curriculum map, areas of instructional focus, or reteaching needs.

Every module provides insights about the key math ideas and a wealth of related tasks, activities, and teacher notes—all with the goal of helping every teacher develop a deeper understanding of what they are teaching. An extensive collection of corresponding online resources provides a wealth of classroom-ready tools to enhance instruction.

Guide for Administrators



The ***Guide for Administrators*** looks at the shift in math education through the eyes of math coaches, school principals, and district administrators. This book was written specifically with leaders in mind—to support them as they endeavor to improve the teaching and learning of math throughout their school and district.

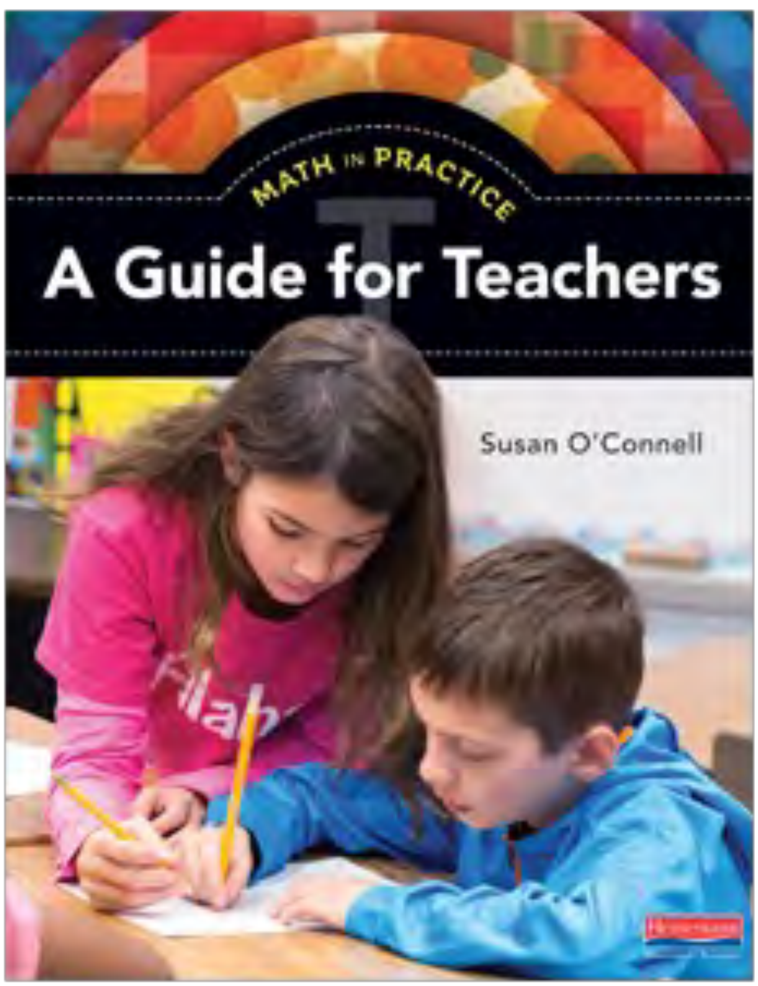
In concert with the ideas discussed in the Guide for Teachers, you'll find support for topics such as:

- understanding what to look for in effective math classrooms
- helping parents understand how their child is learning math
- developing consistent approaches to intervention, differentiation, and assessment.

MATH IN PRACTICE

A Guide for Teachers

Sample



Contents

CHAPTER 1

Step Back and Let Them Think

Letting Students Do the Thinking

How much thinking did you do in math class? Were you asked to think for yourself or were you asked to memorize shortcuts or find answers based on a specific procedure taught to you? For many people, math class was more about doing than thinking, yet a major focus in today's math classrooms is the development of mathematical thinkers. In this chapter, we explore ways to get our students doing the thinking, including these ideas:

- organizing classroom tasks that focus on discovery and insight rather than the teacher telling students how to do math
- using just the right questions to stimulate and stretch students' mathematical thinking
- orchestrating problem-solving experiences that develop the thinking skills and dispositions of a problem solver.

We build mathematical thinkers as we take a step back from telling how to do math and instead guide our students to think about, model, talk about, discover, and make sense of math.

What the Research, Standards, and Experts Say About Students Doing the Thinking

The National Council of Teachers of Mathematics (NCTM) (2000) emphasizes learning mathematics through problem solving, reasoning, communication, representations, and making connections. Learning math is an active process. In *Principles to Actions* (2014), NCTM recommends research-informed teaching practices that include:

- supporting productive struggle in learning mathematics
- building procedural fluency from conceptual understanding.

Through productive struggle, students are given opportunities to grapple with math ideas. They work together to observe and discuss the ideas to make sense of them. And we shift from simply having students remember the steps of a procedure to developing understanding before introducing procedures. Understanding does not develop simply by telling someone to understand. Explorations, observations, modeling, and discussions become key ways in which our students build a solid foundation of understanding, allowing them to make sense of math ideas, which has been found to be an important component of math proficiency (Bransford, Brown, and Cocking 1999). When students memorize procedures without understanding how they work, they are less likely to be able to use them flexibly and may have less retention of the procedures (Hiebert 2003).

The Common Core State Standards, through their Standards for Mathematical Practice, also emphasize the importance of developing mathematical thinkers. These standards highlight the importance of using models, discussions, and problem-based tasks to build student understanding. Standard 8 (Look for and express regularity in repeated reasoning), in particular, highlights the important role of discovery in our math classrooms. Standard 8 focuses on the importance of students seeing the repetition that occurs in mathematics, and using that repetition to make conjectures and develop rules and generalizations (Common Core State Standards 2010). We set up investigations so students can better understand math content, but in addition, our students develop the important skills of investigating, observing for repetition, making sense of their observations, hypothesizing, testing conjectures, and developing generalizations. They are learning the skills of being a mathematician.

Arousing Discovery

For many of us, classroom time was spent doing procedures. We spent little time talking about what we did and rarely dissected the steps of the procedures. Rather than making sense of them, we simply memorized rules, or shortcuts, without understanding what we were memorizing. We found correct answers, but it wasn't an understanding of math that led us to those answers. Our brains were filled with lots of rules and shortcuts that were easily misunderstood, confused, and forgotten.

There is nothing wrong with shortcuts, rules, and algorithms. Many of them can be quite helpful for getting to an answer efficiently. Mathematics values efficiency and we want our students to be able to find solutions in the most efficient ways. But we also want to be certain that our students understand these shortcuts. For that to happen:

1. We want students to be involved in *finding* the shortcuts, not just using them.
2. We want students to be able to understand, explain, and justify why they work.

When students discover rules, procedures, and concepts, they are building an understanding of how math works. Telling them how it all works robs them of the opportunity to make sense of the math they are learning. Rather than teaching lessons that focus on only the mechanics of math processes, we develop lessons that are solidly grounded in and directed toward understanding those processes. We ask students to visualize math ideas, to talk about those ideas, to put the ideas in a context, and to answer deep questions about the math. (We'll address all of these in more detail in the chapters to come.) We focus on the math in a way that allows our students to make sense of the ideas and uncover the rules for themselves. Rather than *telling* them, we help them *discover* math ideas!

DISCOVERING VERSUS BEING TOLD

In one classroom, students are told that when multiplying a number by 10, they should simply add a 0 to the number. The students practice the skill with a series of computations.

$2 \times 10 = \underline{20}$

$4 \times 10 = \underline{40}$

$5 \times 10 = \underline{50}$

$7 \times 10 = \underline{70}$

Their teacher is pleased that their computations are correct.

In another classroom, the teacher begins by posing the same computations, asking students to use base-ten blocks to find the products:

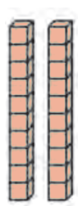
$2 \times 10 = \_\_\_$

$4 \times 10 = \_\_\_$

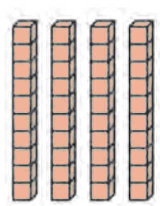
$5 \times 10 = \_\_\_$

$7 \times 10 = \_\_\_$

Students show 2×10 with 2 rods of 10 and see 20 as the total. They model 4×10 with 4 rods of 10 and count the rods, 10, 20, 30, 40, to find the total.



$2 \times 10 = 20$



$4 \times 10 = 40$

As they record the products on the board, the teacher asks them to observe the data.

$2 \times 10 = 20$

$4 \times 10 = 40$

$5 \times 10 = 50$

$7 \times 10 = 70$

What do they notice? Why is it happening? Does it make sense? She asks them to talk with a partner about their observations and insights and then has students share their thoughts with the class.

> There is a 0 in all of the products like 20, 40, 50, 70.
>
> All of the products start with the number that is one of the factors, like 2 is in 20 and 4 is in 40.

As the teacher challenges them to explain the interesting repetition they are observing, they begin to analyze what they are seeing:

> When we did 2×10, we showed it with 2 rods, but that is 2 tens because each rod is 10, so that's why it's 20.
>
> When we had 4 tens, we just counted the 4 rods, 10, 20, 30 40. It's just counting by tens. 7 tens is 70, like 10, 20, 30, 40, 50, 60, 70.

The teacher asks students to predict the product of 8×10, and then has them check their prediction with the base-ten blocks. Were they right? How did they know what to predict? A student might say that she just knew to put a 0 behind it. The teacher probes more and asks students to explain their thinking.

> *Why would you do that? Will that always work?* (Yes, because it worked every time we did it.)
>
> *But, why?* (Because it is just 8 tens and that's 80.)
>
> *So, 8×10 is the same as 8 tens or 80?* (Yes, because we just moved the 8 to the tens place. It's 8 tens not 8 ones anymore, so we needed a 0 to show it was 8 tens, that's how you write 8 tens.)
>
> *Yes, the 8 is now in the tens place showing that it is 8 tens. 8 tens is 10 times more than 8 ones. That is how we record numbers using our understanding of place value. 10 times more moves the digit one place to the left!*

So which group of students gained more from the lesson? In both cases, the students were able to find the solutions. But the students who explored and conjectured throughout the task were being mathematicians. As they investigated the patterns, rather than being told a shortcut, they developed important understandings about multiplication and place value. They made connections to previous understandings (e.g., skip counting and the value of digits) and used their mathematical reasoning to make sense of their observations and develop and justify rules. And they will bring this deeper understanding to the next math task they encounter.

A FOCUS ON INVESTIGATIONS

Helping our students discover math ideas doesn't necessarily take longer than telling them how to do math, but it does take a different mindset—the mindset that students can come up with insightful observations and valid conjectures, that students can be mathematical thinkers.

Rather than *direct* teaching in which we tell our students facts and formulas and show the steps for algorithms, we focus on planning meaningful tasks, considering the classroom experiences and line of questioning that will guide our students as they explore, observe, analyze, and conjecture about the math. It is about helping them reach aha! moments during which they discover math insights and figure out how math works (see Figure 1.1). And these math insights are more easily retained, because they make sense to our students.

Consider the difference between telling students shortcuts versus helping them develop insights through investigations and discussions. You will surely recognize some of the following shortcuts and remember teachers telling you these things as their way of teaching you math. Without understanding, these rules become easily forgotten or confused. What if students explored the ideas rather than rushing to rules? Think about which approach might benefit students.

Figure 1.1 As students gather and explore data, they notice patterns in the numbers and gain insights about how math works.

Example: Adding 1

Young students often learn a shortcut for adding 1: When you add 1, it's the next number. But how might we guide students in an investigation to discover and make sense of this idea for themselves?

The teacher gives each student a ten frame and cubes or counters.

> *Show me 5 counters on your ten frame.*
> *Add 1 more. How many do you have now?*

Students count all of the counters to find the total. The teacher records $5 + 1 = 6$ on the board.

Show me 7 counters.
Add 1 more. How many do you have now?

Students count all of the counters to find the total (see Figure 1.2). The teacher records $7 + 1 = 8$ on the board.

The teacher continues with a few more examples. Students are then asked to talk with partners about what they notice about the equations they see recorded on the board.

$5 + 1 = 6$

$7 + 1 = 8$

$3 + 1 = 4$

$6 + 1 = 7$

What do you notice?
What is happening when we add 1?

Students observe the equations and look for patterns. As students share that 5 and 6 or 7 and 8 are just one number after the other when they count, the teacher might ask them to clarify what they are thinking.

When you add 1 it's just the next number? Why?
Can you predict the sum of 9+1? Explain.

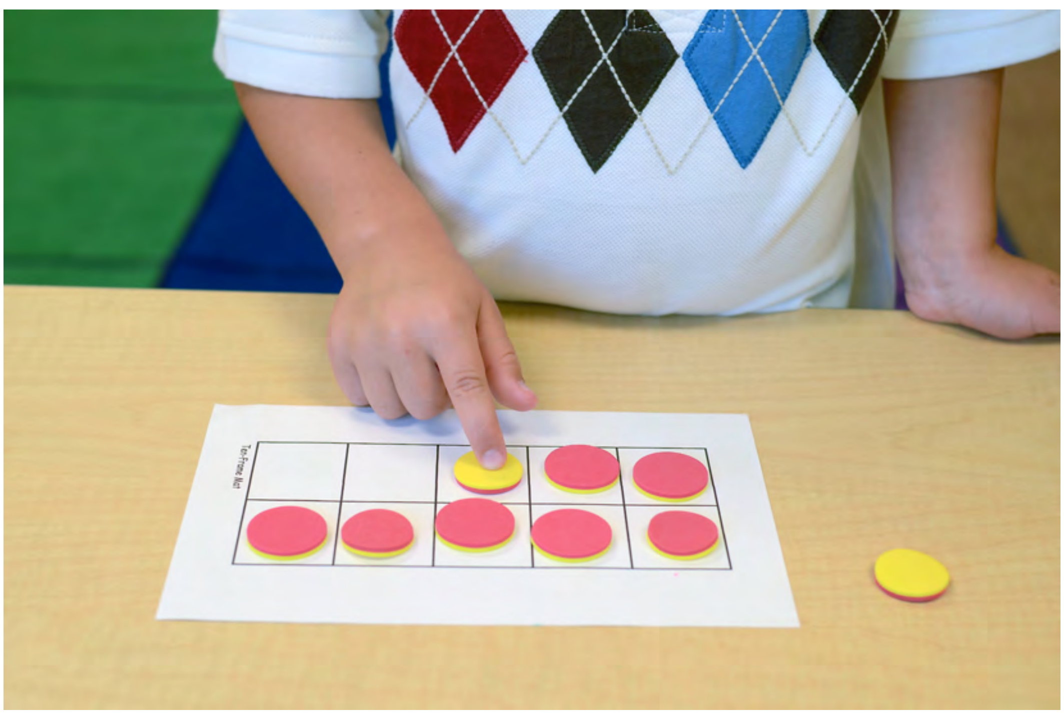


Figure 1.2 Students gather data as they show 7 + 1 with counters on the ten frame and find the sum. Once data are gathered, students observe to figure out what is happening when they add 1.

Try it and see if you are right.

What would you tell someone who can't remember how to find the sum of 6 + 1?

The understanding that has been built during these discussions has eliminated the need for students to memorize +1 facts. With these insights, students can tell you 14 + 1 or 24 + 1 or 36 + 1 without any memorization (see Figure 1.3). They have discovered what +1 means.

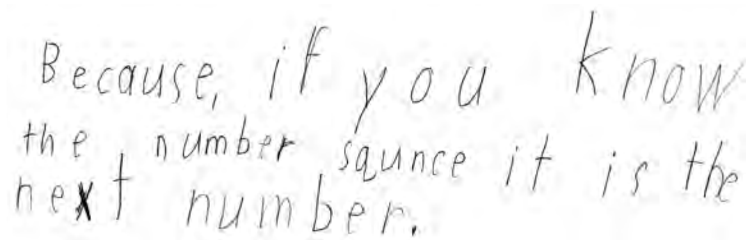


Figure 1.3 This student explains that "if you know the number sequence it is the next number," showing his understanding of what happens when you add 1 to a number.

Example: The Distributive Property

When introducing the distributive property, a teacher might simply ask students to remember the shortcut; that the distributive property says that 8 × 7 = (5 × 7) + (3 × 7). One way to investigate it instead:

The teacher gives each pair of students a set of square tiles and asks them to create an area model to show 8 × 7, using their experience modeling multiplication in this way.

How many tiles did you use to show 8 × 7? (56.)

How do you know? Did you have to count them? (No, we could multiply 8 × 7 because it is made with 8 rows and there are 7 tiles in each row so it is like 8 groups of 7.)

The teacher records *8 × 7 = 56* on the board, then asks students to split their 8 × 7 rectangle into two rectangles (see Figure 1.4).

How many square tiles are in each?

Talk with your partner about how you can use multiplication to tell how many squares are in your rectangles. (Multiply the number of rows by the number of columns.)

The teacher records some of students' data on the board.

8 × 7 = 56

2 × 7 = 14 and 6 × 7 = 42

3 × 7 = 21 and 5 × 7 = 35

8 × 4 = 32 and 8 × 3 = 24

The teacher then asks students to put their two rectangles back together to form the original rectangle.

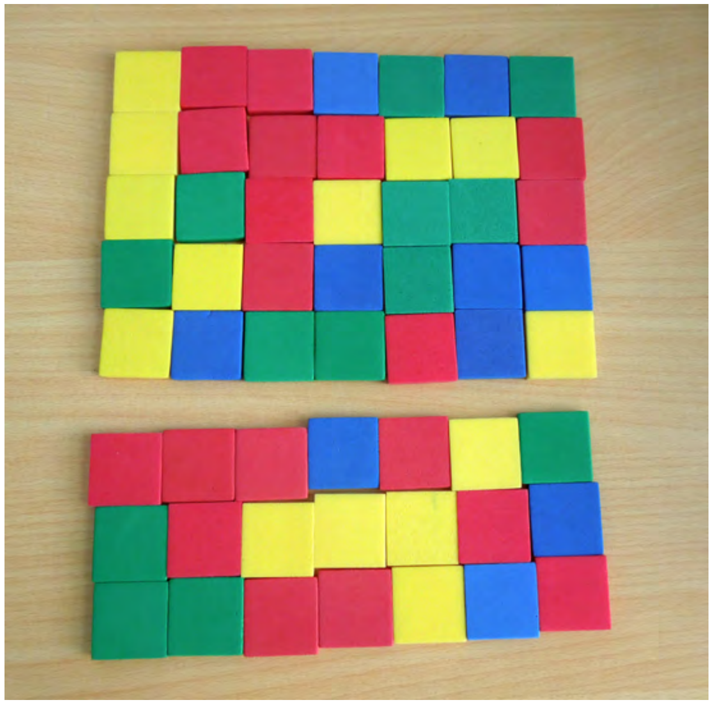


Figure 1.4 Students break apart their area model of 8 × 7 to explore the area of the two rectangles they create, discovering that 8 × 7 = (5 × 7) + (3 × 7).

Partners are asked to split the rectangle into two rectangles in a different way and record the equation for the original rectangle and the two equations to show the number of squares in the two new rectangles. Partners then observe the data on the board and their own data to discuss:

> *What do you notice? Is there a connection between the original rectangle and the two split rectangles?* (They are just parts of the big rectangle.)
>
> *Is there a connection between the original equation and the two new equations?* (One of the factors is the same in all three equations, but the other factor is split in the two new equations—like 8 becomes 2 and 6, but together 2 and 6 make 8.)
>
> *What do you notice about the products of the rectangles?* (The products of the two new rectangles are the same as the product of the big rectangle.)
>
> *Why is that?* (It's the same number of square tiles, we just moved them apart to create the two new rectangles.)
>
> *How could we use math notation to show someone that 8 × 7 is the same as 2 × 7 and 6 × 7? Talk to your partner to find a way to say this using numbers and math symbols.* (8 × 7 = 2 × 7 + 6 × 7.)

Although students may not have seen the use of parentheses before this, the teacher records the following and asks them why she may have written it in this way:

$$8 \times 7 = (2 \times 7) + (6 \times 7)$$

> *Why do you think I used parentheses?* (To show the two different rectangles we made.)

Students are challenged to gather more data to test their conjectures.

> *Do you think this will always be true? Will the product always be the same when we split one of the factors, when we split the rectangle?* (Yes, because you don't take any away or add any to it.)

Students work with partners to build a rectangle with different dimensions and then split it into two new rectangles in several ways. They are challenged to try splitting it vertically and horizontally and to write the equations to show what they found. They discuss their findings so they are ready to present them to the class.

> *How could you use this idea to find the product of 9 × 8 if you forgot it?* (You could break it apart like do 5 × 8 and 4 × 8.)
>
> *Does it matter which factor you break apart?* (No, you just have to remember to add them back together.)

The Benefits of Discovery

In both examples above, the students end up knowing the shortcut, but in the second approach, rather than being told it, they discover it. This approach has a number of benefits.

- Students are engaged in the lesson as they gather and observe data. They are doing math.
- Students use what they know about math to build new understandings.
- Students are learning the importance of observing data. They observe patterns and notice regularity, which leads them to insights.
- Students see how and when models can help them visualize math concepts and simplify math problems.
- Students develop their reasoning skills as they explore, look at evidence, make connections between math ideas, and struggle to figure out what is happening in each situation.
- Students develop communication skills as they attempt to explain what they see in the data, articulate generalizations, and justify their conjectures. They learn to listen to each other's ideas and evaluate whether the reasoning makes sense.
- Students gain confidence in their math abilities.
- Students view mathematics as an active endeavor as they solve problems and explore math ideas.

A key to helping our students arrive at the kinds of meaningful insights discovery can provide is the kind of questions we ask. Through deep questioning, we cultivate our students' reasoning skills. Let's now take a look at how our questions shape our students' learning.

Tips for Teaching Through Discovery

Setting Up Investigations

When setting up investigations to promote discovery, consider the following steps:

1. Have students gather data through models or computations.
2. Record the data and have students observe and discuss the data with others.
3. Have students share their insights and predict based on their insights.
4. Have them test their conjecture with additional examples.
5. Have them summarize what they learned and verbalize any rules or generalizations they have uncovered.

Focus on Understanding

Initial experiences with computational procedures should not begin with standard algorithms, but with explorations, models, and discussions related to place value, properties, and an understanding of the operations. Standard algorithms are introduced after students understand the procedures.

Record Observations

Record students' observations as you explore math ideas. Use words, pictures, and numbers to show their thinking. Refer back to the ideas frequently as students build their understanding.

What's the Rule?

Encourage students to hypothesize and test rules or generalizations. Have them predict based on their rule. Was their prediction correct?

Prove It

Encourage students to justify their rules or generalizations, explaining how and why the rule works.

Discovery Journal

Following classroom investigations, have students record their insights in a discovery journal. What did they notice? Why did this happen? Encourage them to use words, pictures, numbers, and/or examples to explain what they learned.

Essential Questions to Spur Insights

1. What do you notice?
2. Why is that happening?
3. Does it make sense? Why or why not?
4. Predict (for another similar example) based on your observations.
5. Will it always work? Explain.
6. What is the rule?

DEVELOPING PROBLEM-SOLVING STRATEGIES

Strategies are ways that students get to solutions. Elementary students might add, subtract, multiply, or divide to solve problems. They might draw diagrams, make tables or organized lists, look for patterns, or reverse their thinking to work backward toward the solution. Students who understand varied ways to find solutions are able to use the strategies flexibly, as needed depending on the problem.

Understanding, Not Key Words

Problem solving is not a rote skill. To solve problems, students make a series of decisions based on their math understanding. In the past, textbooks often attempted to simplify this task by offering key words that, when memorized, would simply tell a student what operation to use or how to solve the problem. This technique has not helped our students for two major reasons.

1. Simply looking for a word leads to lots of errors and misunderstandings. Key words can appear in the problem but are not meant to indicate the operation, and key words may not appear at all.
2. It gets in the way of students really understanding how to choose appropriate operations. The decisions are based on rote memorization of a list of key words, rather than real understanding.

Consider the typical key words for addition. When a student is told that *altogether* or *in all* tells them to add, what do they do for this problem?

> There were 13 brown dogs and 14 black dogs. How many dogs were there?

Neither key word appears in the problem.

And when students spot a key word, they often disregard everything else in the problem.

> Juan and Olivia picked $2\frac{3}{4}$ cups of strawberries altogether. They ate $\frac{1}{2}$ cup on the drive home. How many cups of strawberries did they have when they got home?

Although the word *altogether* appears, it is not intended to be an indication of the operation, and yet many students quickly add the two quantities in the problem.

Or what about this problem that mentions what is left?

> Caroline made a liter of tea. She drank $\frac{1}{4}$ of it on Tuesday. On Wednesday, she drank $\frac{1}{2}$ of what was left. How much did she drink on Wednesday?

Students often mistakenly subtract to find the solution, basing their decision on the words *what was left*, and yet this is a multiplication problem ($\frac{1}{2} \times \frac{3}{4}$).

Rather than relying on key words, which can often be misleading or unhelpful, we focus on helping our students think, by exploring the situations (the actions or relationships) that indicate each operation. These situations, or problem structures, help our students identify the operation that makes sense for a given problem.

For more on using problem structures to identify operations, see Chapter 2.

Building a Repertoire of Strategies

Our goal is to build a wide repertoire of strategies for solving problems, which includes knowing when to add, subtract, multiply, and divide, but also includes strategies such as

- looking for patterns
- making tables or lists to organize data
- drawing pictures and diagrams
- guessing, testing, and revising
- working backward
- using logical reasoning.

These strategies give students options for simplifying and solving math problems. But we cannot just tell students how to solve problems; that does not work. Providing focused tasks, guided questions, and opportunities to explore, model, and talk about varied problems is the key. When students have knowledge of a wide array of possible strategies, they are able to select ones that work for the given problem.

Strategies are not learned all at once or at one grade level; they are learned over time as they are applied to different content and are extended and refined throughout the elementary years (NCTM 2000). A critical way to build students' repertoire of strategies is having students share and justify their methods. Tasks that have one solution but multiple ways to get to the solution (as in Figure 1.12) provide opportunities for students to discuss their varied approaches and make sense of the different strategies they use (Spangler et al. 2014).

Problem: Bailey made 1 gallon of lemonade for her lemonade stand. She sold $\frac{3}{4}$ gallon of the lemonade on Monday. On Tuesday, she sold $\frac{1}{2}$ of what was left. How much did she sell on Tuesday?

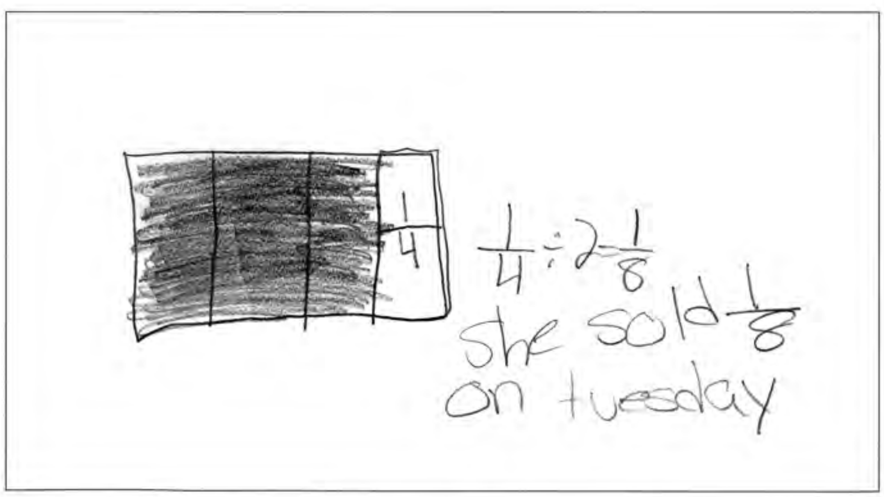


Figure 1.12

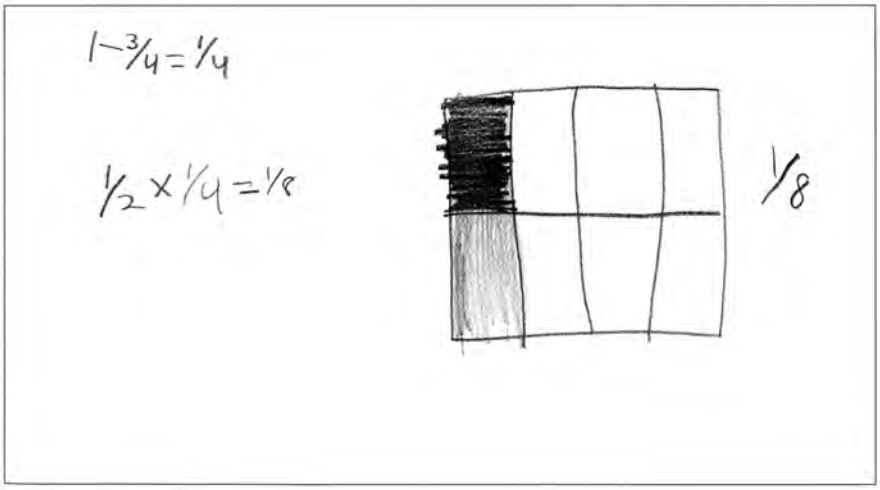


Figure 1.12 Both students use diagrams to visualize the problem. For the second step, one chooses to divide the remaining $\frac{1}{4}$ gallon by 2 and the other multiplies it by $\frac{1}{2}$, stimulating productive classroom discussions about their problem-solving decisions, the operations they use, and the connections between division and fractions.

Tips for Helping Students Choose a Strategy

Pinch Cards

Provide repeated practice selecting operations with quick but thoughtful pinch card discussions. Give each student a pinch card (see Figure 1.13) and pose a math word problem to the class. Students then:

1. silently pinch the sign that shows the operation they would use to solve the problem, showing the teacher
2. turn to a partner and share the operation they would use to solve the problem, justifying why they chose that operation
3. share their ideas with the class about which operation makes sense for the problem and, together with the teacher, build the equation to match the problem situation.

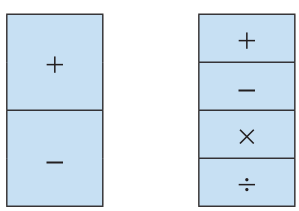


Figure 1.13 Pinch cards for primary grades show addition and subtraction, and those for intermediate grades show all four operations. All students are actively involved in thinking about operations as they select and pinch the operation they hear reflected in the problem context.

continues

Pinch cards offer a quick check for understanding as the teacher is able to see the operations selected by each student. Having students justify why they chose the operation is an integral part of the activity, allowing them opportunities to share their thinking with both partners and the class. It engages all students in thinking about the operations and prompts classroom discussions focusing on why specific operations make sense for specific problems.

Problem-Solving Strategies List

As strategies are introduced and discussed in class, record them on a class chart, as in Figure 1.14. This list of possible problem-solving strategies becomes a reference for students during problem-solving tasks. Following tasks, have students share the strategy they selected and justify why it made sense for that problem.

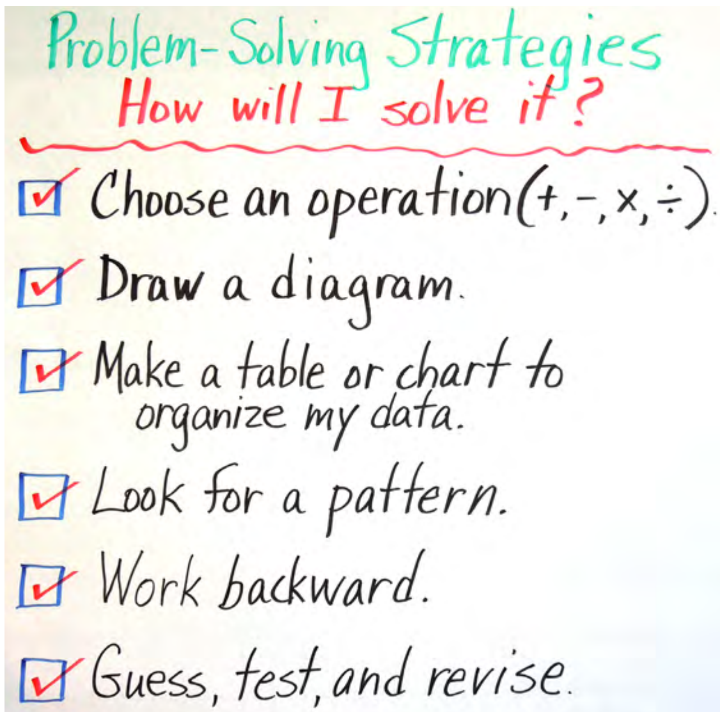


Figure 1.14 Each class' strategies list will look a bit different because the lists are compiled based on the strategies the class has discussed during their problem-solving investigations.

DEVELOPING A PROBLEM-SOLVING DISPOSITION

As important as it is for our students to know and apply a variety of problem-solving strategies, knowing strategies is not enough. Believing you can solve a problem, being willing to take a risk and try a strategy, and having the perseverance to stick with the task are critical qualities of effective problem solvers.

Effective problem solvers:

- believe they can solve problems; they are confident
- take risks; they act on hunches and are sometime wrong, but learn from their mistakes
- keep trying, even when the method or solution is not readily apparent
- reflect on their actions, make adjustments, and keep working toward solutions
- are patient, knowing that some problems take time to solve.

Problem solvers recognize that confusion is a part of solving problems and discover that persistence pays off.

Helping students develop a problem-solving disposition is as important as helping them acquire computational and procedural skills. We play a critical role in developing these dispositions by setting up K–5 classrooms that offer opportunities for students to explore cognitively demanding tasks in safe environments in which risk taking and patience are valued. By establishing a classroom climate that nurtures reflection and risk taking, accepts that students get stuck, encourages them to find ways to get unstuck, allows questioning of each other's methods, and acknowledges that learning arises from failures as well as successes (NCTM 2000), we are creating an environment in which problem solvers thrive.

The ability to persevere is vital to developing a problem-solving disposition, but there are many reasons students may not persevere as much as we would like them to. Students may give up for any of the following reasons:

- They don't have the skills they need to succeed at a particular task.
- They *think* they can't succeed.
- They are faced with a complex problem of a type they do not have experience with.
- They lack experience persevering, or don't know what to do to get unstuck.
- They don't believe that perseverance is valued.

Let's look at each of these situations and some ways we can help students deal with them.

Students give up when they don't have the skills to succeed.

Many of our students are right in feeling that they can't solve a problem because they lack the skills to do it. As we help them find ways to comprehend, visualize, and solve problems we are both arming them with the skills they need and building their confidence with problem-solving tasks.

But some students may be at different levels. A problem may be just right for one team, but at a frustration level for another. Consider differentiating problem tasks to allow all students to find success (see more in Chapter 5). Although all of the following problems require students to use their understanding of money, the data allow for different levels of skill.

- Colleen had 2 dimes, 5 nickels, and 15 pennies. How much money did she have?
- Colleen had 3 quarters, 2 dimes, 5 nickels, and 15 pennies. How much money did she have?
- Colleen had just the right amount of money to buy a granola bar for $1.25. What coins could she have used?

Students give up when they think they can't succeed.

A focus on the right answer causes many students to become immediately anxious when a problem-solving task is posed. Whether they have the skills or not, many of them believe they can't succeed.

- Do students feel safe to try, even if they can't find the answer?
- Does their anxiety stop them from moving through the task?
- Will building their confidence help them take risks?

As we create an environment in which process and effort are valued, students begin to relax and gain confidence. Consider the following to alleviate anxiety during problem-solving tasks:

- Focus on the process, not the answer. Spend more time discussing their thinking strategies than answers. Focus discussions on their methods and reasoning. Show students that you value the how and why of their problem-solving experiences more than the answer.
- Have students turn and share strategies with partners to allow them to discuss and process their ideas. Quick turn-and-share activities can jump-start reluctant students.
- Acknowledge and verbalize the complexity of some tasks. Let students know that everyone gets confused with some problems, including you! Acknowledge that confusion is part of being a problem solver, but that good problem solvers find ways to move through the confusion (e.g., visualizing the problem, talking with a partner, thinking about related problems).
- Have students solve problems with partners or teams. Not only does this allow them to process their ideas and gain new perspectives from others, but it offers the solidarity of finding a solution together, relieving the anxiety of being on one's own. And in the process, students are hearing others' thinking and seeing what it looks like to think like a problem solver.
- Have a class time-out as needed to refocus students on the task and decrease stress. As some students share tips and suggestions, others gain new direction or insights to be able to persevere through the task.
- Debrief after problem tasks to highlight methods. As students share varied methods, they recognize that problems do not have to be solved in one way; there are options for finding solutions. As students hear alternate methods, they build a greater repertoire of strategies and gain confidence for future tasks.

Students give up when faced with more complex problems if they lack experience with those types of problems.

Students need experience exercising perseverance. If all of the tasks they are posed are simple and brief, they aren't challenged to develop the skill to hang in with more complex tasks. Classroom experiences in which students explore and discuss two-step or multistep problems are vital. Discussing varied ways to find solutions for open middle problems (problems with one answer but multiple ways to get there) helps students see there is not just one path to the solution. And posing rich problems that have more than one answer, like the following, provides practice persevering to find those multiple solutions:

- If 10 children were at the party, how many were boys and how many were girls?
- How could Aidan pay for a cupcake that costs 35 cents, using exact change? Prove that each combination of coins equals 35 cents.
- What are some different rectangles that can be formed with 36 square tiles? What are their dimensions?
- Charlie had a string of licorice that was 26 inches long. He cut it into 3 pieces. How long could each of the pieces have been?
- 2 children shared a pizza. When they were done eating, $\frac{1}{6}$ of the pizza was left. What fraction of the pizza could each child have eaten?
- Mr. Short had 38 feet of fencing to build a pen for the dog. What could the area of the pen have been? Draw diagrams to justify your answers.

Students give up when they don't understand what it means to persevere or what to do when they get stuck.

Have a class discussion about perseverance. Create a class definition for the word (e.g., "We don't give up when it gets hard. We look for another way to find the answer."). Acknowledge that getting stuck is part of being a problem solver, but that good problem solvers find ways to get unstuck. Have students work in teams to come up with ideas for what they could do if they get stuck solving a problem. Create a chart to display their ideas and post it in the room for reference. The ideas might include:

- Use materials to model the problem.
- Draw a diagram of the problem.
- Reread the problem to remember what we were trying to solve.
- Think of another problem that is like that one and think about how we solved that problem.
- Make the data simpler.
- Think about the problem with no data at all.
- Ask a friend for an idea.

Frequently ask students if they got stuck during the problem-solving task, and if so, to share how they got unstuck.

Students give up when they don't understand that perseverance is valued.

Praise effort and perseverance, not just right answers. Be specific in your praise so students understand what you value. Through specific comments, they gain a better perspective of what it means to persevere (see Figure 1.15).

That wasn't what you were thinking in the beginning, was it? I like the way you changed your mind after you noticed . . .

Study Group Questions

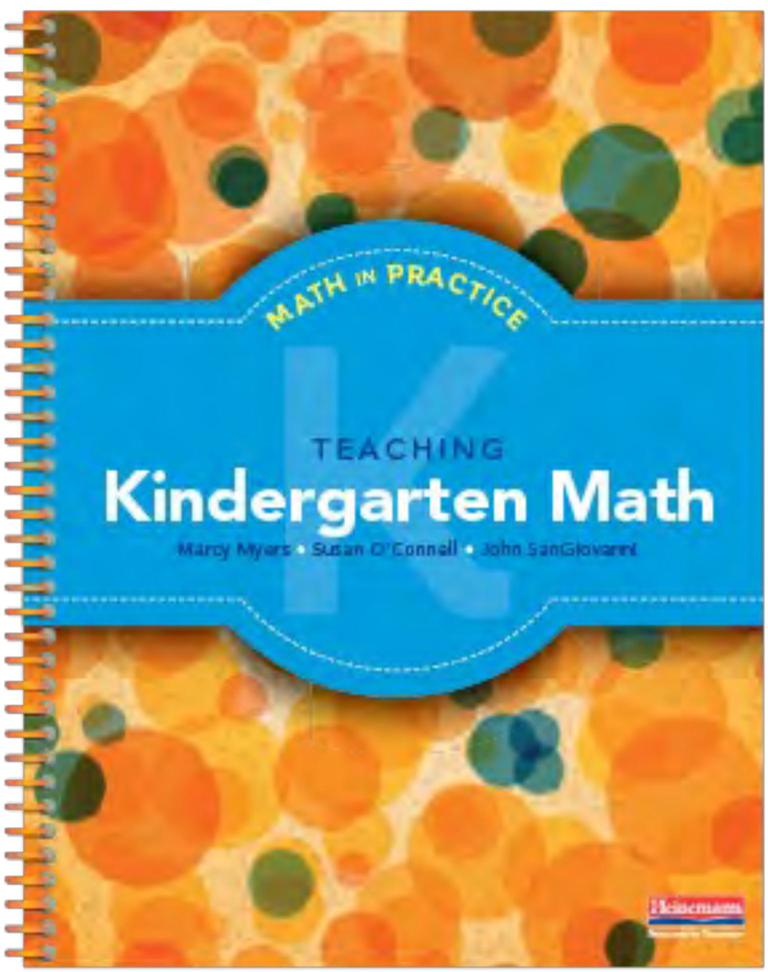
1. What math concepts lend themselves to discovery lessons?
2. How might you set up an investigation for students to discover a math idea? What data would students gather and observe? What questions might you ask?
3. How do your questions impact the level of thinking done by students?
4. In what ways might your comments to students' responses impact a class discussion?
5. How can you help students comprehend problems?
6. What does perseverance look like in a math classroom? Why is it important for your students to develop perseverance?
7. In what ways can you build your students' perseverance?

MATH IN PRACTICE

TEACHING

Kindergarten Math

Sample



MODULE

5

Comparing Numbers 1–10

About the Math

K.CC.6; K.CC.7

Students are beginning to understand that numbers represent a quantity. They have counted groups of objects and can name the quantity in a set (e.g., 5 stickers or 8 erasers). As students explore numbers, they create models of the numbers. They have made towers of connecting cubes or placed counters on ten frames to visualize quantities. They are noticing, with our help, that when they count, each successive number they say represents a quantity that is 1 more than the previous number said. Now, students use this information as they begin to compare quantities and numbers.

The key ideas focused on in this module include:

- comparing two quantities to decide which is more or less by using manipulatives or drawing pictures
- comparing two quantities to decide which is more or less by looking at the numbers.

When comparing groups of objects, students decide which group has more or fewer objects than the other. Counting and matching are two of the strategies students use. As students compare the heights of towers of cubes by placing them next to each other, compare rows of counters by matching them side by side, or count to compare the number of spaces filled by counters on a ten frame, they are making decisions about which number is greater or less (see Figure 5.1).

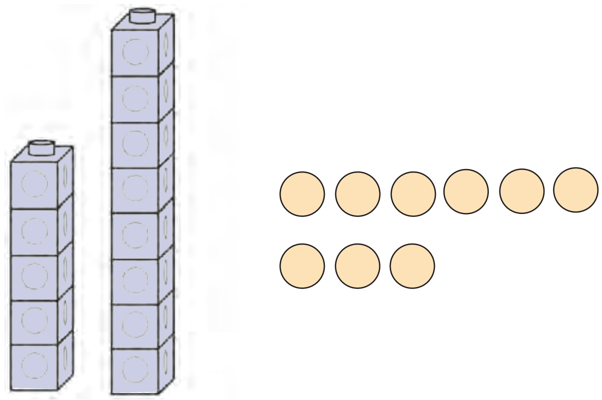


Figure 5.1 Students line up objects to compare quantities.

Students use matching and counting strategies to compare groups of objects, but after those experiences with objects, they are challenged to determine more or less when comparing numbers 1–10 without the objects.

Understanding the counting sequence, and the very important concept that each successive number is 1 more than the number before it, allows our students to make decisions based solely on the numbers. Looking at the order of numbers on number lines (see Figure 5.2) and hundred charts supports students as they internalize this concept.

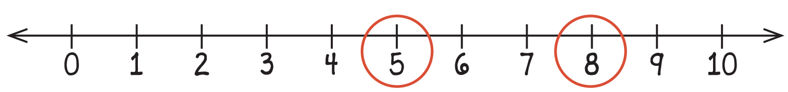


Figure 5.2 Students circle numbers on a number line to compare them.

Exploring the Progression

PREVIOUS		NOW		NEXT
Preschool Possible familiarity with the words *equal to*, *more* or *less*, though unable to express the meaning	»	**Grade K** Comparing two quantities, first by using manipulatives or drawing a picture and then by looking at the numbers	»	**Grade 1** Comparing two-digit numbers; learning the symbols for greater than (>) and less than (<)

The timing for this will vary depending on the student, but the goal is for students to be able to compare numbers 1–10 by the end of the kindergarten year.

Learning Goals

I can tell which set has more or fewer objects.

I can tell which number is greater or less.

Lessons in This Module

Comparing Groups of Objects

Comparing in the Classroom
Towers of Cubes
Matching Bears
Just Enough Carrots
Using a Balance to Compare
Building Numbers on a Number Strip

Comparing Written Numerals 1–10

Number Line Circles
Target Number

Ideas for Instruction and Assessment

COMPARING GROUPS OF OBJECTS

Students' initial experiences with comparing quantities help them make sense of the terms *more*, *less*, and *the same*. Students compare groups of objects to determine which group has more or less and draw pictures to compare quantities.

Comparing in the Classroom

It is grammatically incorrect to say there are less objects in a set. There are actually fewer objects in a set, but we say that one number is less than another number. With that said, young students often understand the idea of more or less and so using both less and fewer is acceptable at this level, as long as we make sure to make the connections between fewer/less than and more/greater than.

Consistently ask students questions to clarify their thinking. If a student responds that pencils has more, ask them why rather than quickly dismissing the answer. It may be that he was thinking about the length of all of the pencils combined. Through continued questioning, you may see the answer in a new light, and your questioning prompts students to be precise in their talk about math ideas.

This activity uses everyday classroom objects to help develop the meaning of the terms *more*, *less/fewer*, and *same*.

Show students 6 blue cubes and 4 red cubes.

Turn and ask your partner: Are there the same number of blue cubes as red cubes?

Is there more of one color? Which color?

What does it mean to have more?

How do you know there are more blue cubes?

Would it help if I line them up next to each other? Why?

Show students 3 pencils and 5 crayons.

Are there more pencils or crayons?

What could you do to help you decide which group has more?

Have 2 girls and 3 boys stand up.

Do we have more girls or more boys standing?

What could you do to help you decide which is more?

Record the word *more* on the word wall or Math Talk chart. Draw a row of 5 hearts and a row of 3 hearts. Ask students which is more and then circle the 5 hearts to show more.

Continue with examples of *more*.

If students are easily identifying *more*, continue with the following *less/fewer* and *same* examples. If not, continue working on *more* and address *less/fewer* and *same* at another time.

Differentiation

Many students arrive in kindergarten understanding the concepts of more, less, and same, but others may need repeated examples to develop the concept and language.

Show students 6 blue connecting cubes and 3 red connecting cubes.

Which group has less?

What could you do to help you decide which group has less?

What does it mean to have less?

We could also say we have fewer red cubes.

Record *less* and *fewer* on the word wall or Math Talk chart. Draw a row of 5 stars and a row of 3 stars. Ask students which is less/fewer and then circle the 3 stars to show fewer.

Show students two cups, each holding 3 pencils.

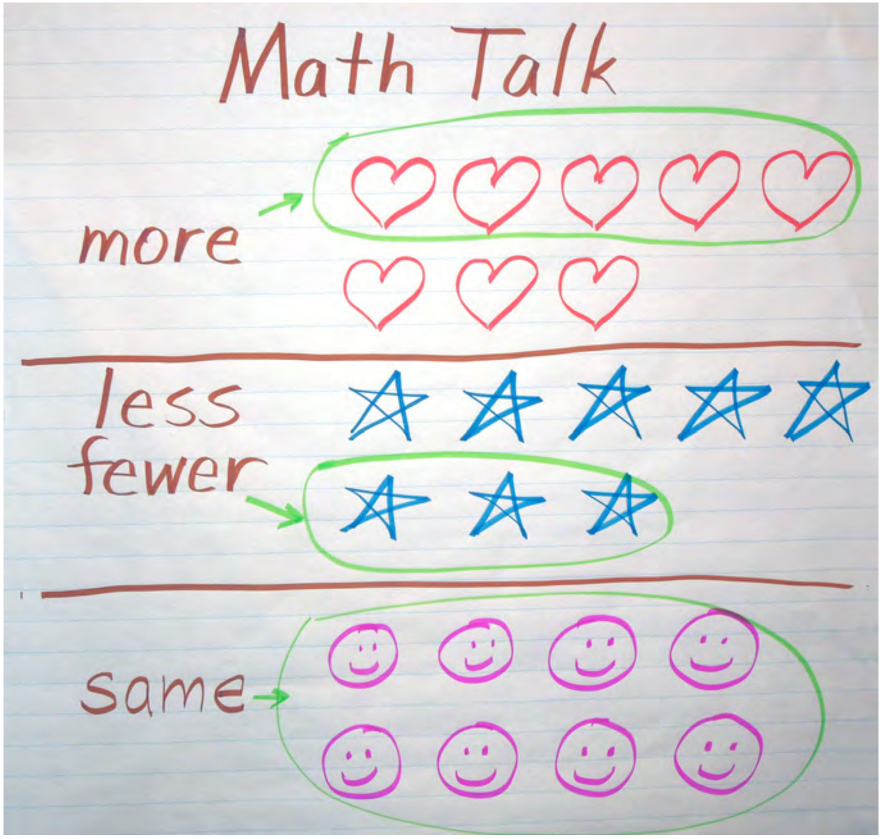
Which cup has more pencils?

Hold up one of the cups.

So this cup doesn't have more? And it doesn't have less?
What do we call that?
What does it mean if we say we have the same number of pencils?

Record the word *same* on your word wall or Math Talk chart. Draw a row of 4 happy faces and another row of 4 happy faces. Ask students which row has more and then circle both rows to show the same.

The formal terms of greater than, less than, *and* equal to *are introduced in a later lesson.*



Continue with examples as needed.

To conclude the lesson, provide a group of objects that all students can see (in the middle of a group seated on the floor, using a visualizer, among other ways).

Ask volunteers to divide the objects into two groups.

Which group is more? Explain.
Which group is less? Explain.
Are the groups the same? Explain.

Differentiation

For students who quickly understand more, less, and equal to, ask them to create their own groups of items to demonstrate each vocabulary term.

This math talk will provide students with more opportunities to gain understanding from other students' thinking.

Towers of Cubes SHOW IT!

Students build towers with more or less than a specified number of cubes.

Create a tower with 6 cubes. Have students count the cubes in the tower you make.

Can you build a tower that has the same number of cubes as mine?

Observe students as they build.

How do you know yours is the same as mine?

Can you make another tower, this time with more cubes?

Observe students as they build.

How do you know this tower has more cubes than your other tower?

Ask students to count the number of cubes they have in their new tower and share the number.

Record the numbers on the board. Talk about each one of them as you record it.

Luis used 7 cubes. Is 7 more than 6? How do you know?

Kellen used 8 cubes. Is 8 more than 6? How do you know?

This understanding has been developing as students explore counting, the recognition that each number in the counting sequence is 1 more than the number previously said.

Have students take apart their "more" tower so they just have the original tower with 6 cubes.

Look at your tower. How many cubes are in your tower again?

Can you make another tower, this time with less, or fewer, cubes?

Observe them as they build.

How do you know this tower has fewer cubes than your other tower?

Ask students to count the number of cubes they have in their new tower. Ask them to share how many they used. Record the numbers on the board. Talk about each as you record it.

Bailey used 4 cubes. Is 4 less than 6? How do you know?

Colin used 2 cubes. Is 2 less than 6? How do you know?

Have students take apart all of their towers.

Now, I'd like you to build me a tower with 5 cubes, a tower with less than 5 cubes, and a tower with more than 5 cubes. Be ready to show me your towers and tell me about more or less.

Have students share their towers and discuss how they knew the towers were more or less.

Matching Bears SHOW IT!

Students explore the strategy of lining up and matching materials to compare them. Colored bear counters are used in this activity, but any colored manipulative will work.

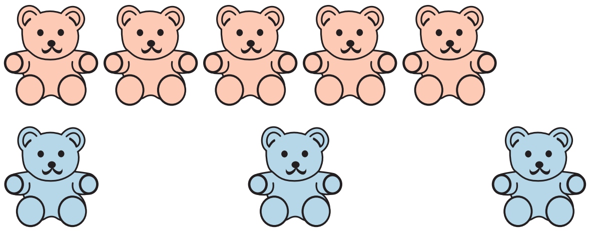
Make a pile of red bears and a pile of blue bears (no more than 10 in a pile). Do not line them up; just put them in a pile. It should be hard to determine the quantity simply from looking at the piles.

Which pile has more bears?

Talk to your partner. What could you do to make it easier to figure out which pile has more bears?

Have students share their ideas. Some may suggest counting them (a good suggestion). Others may suggest lining them up (another good suggestion).

Take the suggestion of lining them up and line them up without starting at a common starting point or with uneven intervals.

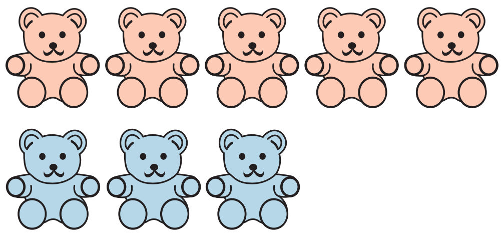


Does this help? Why or why not?

What could we do so we don't get confused?

Listen to students' suggestions. If no one suggests lining them up so they are next to each other (matching), suggest it through a think-aloud.

Maybe if I started here and lined them up so they are right next to each other, then I could see which line has more bears.



Which has more now? How can you tell?

Show with your fingers how you matched the first red and blue bears, then the second red and blue bears, then the third red and blue bears.

Now it is easy to see which has more.

Which has less? How can you tell?

Common Error

Students often line up counters with different starting points or gaps between objects so the comparison is not accurate. Specifically addressing ways to line up items is essential for many students.

Give students zipper bags with two colors of bears and have them line them up, using matching to figure out which color has more or fewer bears (see photo).

This student lines up the bears to see which quantity is greater. Although she wrote the numbers, she was actually using the line of bears rather than the numbers to determine which was more. The next step will be determining which is greater by looking only at the numbers.

Just Enough Carrots

Students listen to *Just Enough Carrots* by Stuart J. Murphy and use the story context to explore more, less, and same.

Before Reading:

Ask students to turn to partners and discuss:

If I gave you 2 yummy pieces of candy, would you want to have more or fewer? Why?

If I gave you 3 smelly skunks, would you want to have more or fewer? Why?

What would you like to have more of?

What would you like to have fewer of?

Tell students you are going to read a story about more and fewer of things.

During Reading:

Read *Just Enough Carrots* to students.

While they are listening, students should be thinking whether they would like to have more or fewer of the items in the story.

After Reading:

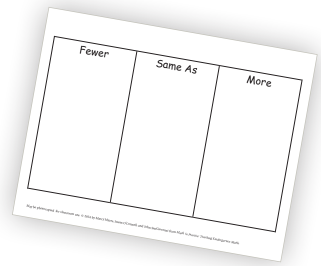
Talk about same, fewer, and more.

What does it mean to have more of something?

What does it mean to have fewer of something?

What does it mean to have the same amount of something?

Give students fewer/same/more mats.



Give students counters to place on their mat showing same, fewer, and more.

We have 5 carrots. Show me 5 in the same box.

Have students count to be sure they have 5 in the same box.

Show me fewer than 5 in the fewer box.

Have students share how many they have in the fewer box, checking to be sure that it is less than 5.

Show me more than 5 in the more box.

Have students tell how many they put in the more box.

How do you know it is more?

Do more examples with counters, discussing students' responses to each.

Then, have students draw 6 of something (anything they'd like) in the Same section of their mat. Ask them to draw fewer than and more than 6 of that item in the other sections.

Review their drawings to assess their understanding of fewer and more.

If students have drawn too many or too few items, ask them to count the items to check their work.

Using a Balance to Compare

Students use a balance to compare numbers using greater than, less than, or equal to.

Students in kindergarten learn to use the phrases greater than, less than, and equal to but do not use the symbols >, <, or = to compare numbers.

Each group of students will need a classroom balance and two sets of 10 connecting cubes in different colors for a total of 20 cubes.

Today we are going to use a balance to compare numbers.

Put 6 red cubes in one side and 3 blue cubes in the other side.

Watch as students place cubes.

What do you notice about the balance?

Why do you think the side with 6 cubes goes down?

Why do you think the side with 3 cubes goes up?

Look at the side with 6 red cubes. It is down because there are more cubes in that side.

When there are more cubes, we say the number 6 is greater than the number 3.

If students are not familiar with a balance, allow them time to play with it and the cubes so they can explore what happens when cubes are placed in each side.

Add *greater than* to the word wall or Math Talk chart.

It is important when comparing that students accurately count the cubes.

Connect to measurement, Module 10.

How can you use greater than *to tell about the numbers 6 and 3?*

There are more cubes on the side with 6 so 6 is greater than 3.

Look at the side with 3 blue cubes.

It is up because there are fewer cubes in that side.

When there are fewer cubes, we say the number 3 is less than the number 6.

Add *less than* to the word wall or Math Talk chart.

How can you use less than *to tell about the numbers 3 and 6?*

There are fewer cubes on the side with 3 so 3 is less than 6.

Have students remove the cubes from the balance.

Let's compare two other numbers.

Put 4 red cubes in one side and 9 blue cubes in the other side.

What happened to the balance?

Why did the side with 9 cubes go down?

Point to the words *greater than* on the word wall.

Use greater than *to tell your partner about the numbers 9 and 4.*

Listen as students talk with their partner.

How did you use greater than *to compare 9 and 4?*

There are more blue cubes so 9 is greater than 4.

Point to *less than* on the word wall.

Use less than *to tell your partner about the numbers 4 and 9.*

Listen as students talk with their partner.

How did you use less than *to compare 4 and 9?*

There are fewer red cubes so 4 is less than 9.

Have students remove the cubes from the balance.

Let's compare two other numbers.

Put 7 red cubes in one side and 7 blue cubes in the other side.

What happened to the balance?

Why didn't the balance go up or down?

Add *equal to* on the word wall or Math Talk chart.

How can you use equal to *to tell about the numbers 7 and 7?*

Yes, 7 is equal to 7.

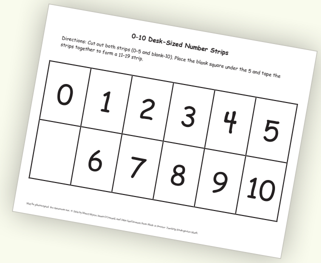
When there are the same number of cubes, we say the number 7 is equal to the number 7.

Students will need ongoing practice to use greater than, less than, and equal to compare numbers.

Students continue to show other numbers in the balance and use greater than, less than, or equal to as they compare the numbers.

Thinking Through a Lesson

Building Numbers on a Number Strip



Students build towers of cubes for each number on a 0–10 number strip to visualize that each counting number is 1 more than the number before it. They connect this idea to the concept of *more* as they transition to comparing quantities using numbers only.

Give pairs a large number strip (0–10) and connecting cubes.

SMP4

We are going to make towers to show each of the numbers on your number strip.

SMP2

How many cubes should we put on the 0? Why? (None because 0 means nothing.)

How many cubes should we put on the 1? Why? (1 cube to show 1.)

How many cubes should we put on the 2? Why? (2 means 2 cubes.)

Make towers to show all of the numbers on your number strip.

These initial questions allow us to see whether students understand the directions. After a couple examples done together, observe as students then work on representing the remaining numbers with towers.

When they have finished, ask them to talk with their partners about what they notice about the number strip towers.

Then, have them share with the group.

SMP8

What do you notice? (The towers get bigger from 0–10.)

How much bigger? (They get 1 cube bigger for every number.)

Does that make sense? Why or why not? (Yes, because you have more every time you count another number.)

Pose some comparison problems and have them look at the towers to solve them.

Listen for indications that students understand that each number is 1 more than the prior number. If they are simply saying they get bigger, ask for more precision with questions like, "What do you notice about the way they are getting bigger? What can you say that tells me exactly how much bigger the next number is?"

Which is more, 6 or 3? Explain. (6 is bigger.)

Which is more, 2 or 5? Explain. (5 is bigger.)

Which is more, 1 or 4? Explain. (4 is bigger.)

What do you notice about the numbers that are more than the others? (They are farther down the number strip.)

Focus on the term more. Ask students to tell you how they know which is more. They may use words ("It's taller") or refer to the towers placed side by side as they explain.

Do you need the towers to figure out which has more? (No, if it is after a number when you count then it is more.)

Focus students on the order of the numbers on the number strip. If needed, go back to the previous examples that students compared using their towers and look at the number strip for each. Have students place a transparent counter on each number on the strip and discuss the location of each number on the strip (Which is farther from 0, 6 or 3?)

Remove the towers, but leave the number strip.

Which is more, 7 or 3? How do you know? (7 because it is farther down the strip.)

We say that 7 is greater than 3.

For students who need additional support, allow them to rebuild towers to visualize each number pair, then locate the numbers on the number strip. Have them place counters on the numbers on the strip and talk about which is more (or greater than) as they look at the placement of their counters.

This lesson bridges comparing sets of objects to comparing with numbers only. Students who are still having difficulty comparing sets of objects may need the support of counters or cubes as they move to the following lessons. Continue to connect the quantities in each set to a number line, or a hundred chart, to help students gain insight into how to compare numbers using their understanding of the counting sequence rather than counting the objects.

In previous activities, we compared how many cubes or bears, but now we are beginning to compare numbers. We say we have more cubes when we have 6 rather than 3, but when we talk about numbers, we say that 6 is greater than 3. We are comparing the values of the numbers. Use both words frequently as you compare: "There are more cubes in our tower of 6 than in our tower of 3. 6 is greater than 3."

SMP6

Record *greater than* on your word wall or Math Talk chart next to the word *more*.

Which is more, 3 or 5? How do you know? (5 because it is farther down the number strip.)

We say 5 is greater than 3.

Which is more, 1 or 6? How do you know? (6 because it is farther from 1 when we count.)

We say 6 is greater than 1.

The symbols for greater than and less than are not introduced until first grade. Students in kindergarten are getting comfortable with the language of more and greater than, fewer and less than.

In closing, have students turn and talk as you listen to their comments. Then, have some students share with the group.

How does using a number strip help you compare numbers? (You know that every number you say is 1 more, so if you look at the strip you can see which are more.)

How does knowing how to count help you compare numbers? (Because every time you say another number, it is 1 more.)

Variation: This lesson can be repeated with an emphasis on identifying towers with *fewer* cubes or the number that is *less than* the other.

These questions help determine which students have begun to see that the counting sequence helps them compare numbers. Many students require additional exposure to gain this insight.

FORMATIVE ASSESSMENT

Option 1

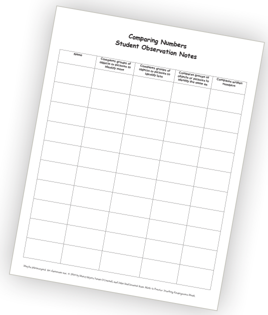
Show students a group of counters in random configurations, such as a group of 7 and a group of 5 counters. Ask:

> *Which group has more?*

They may touch and count each or line up the counters to compare.

Observe as the student works to count or arrange the counters.

Record observational notes of the student's work and responses.



When the student responds that 7 is more, ask:

> *How do you know?* (Students might explain that 7 is more because when the cubes are lined up, 7 is longer to show more. Students might also have developed the insight that 7 is more because it is farther along in the counting sequence.)

Then ask:

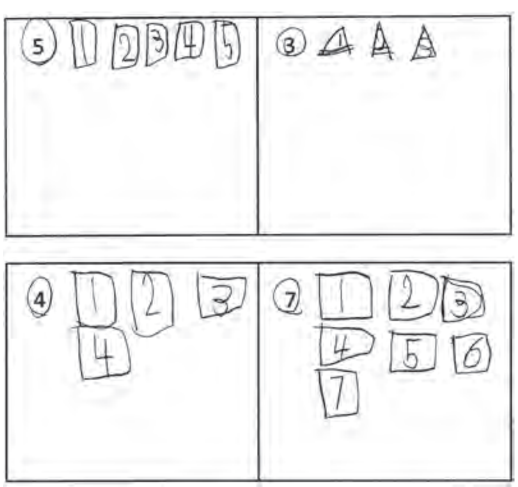
> *Which group has less and why?* (Students might explain that 5 is less because when the cubes are lined up, 5 is shorter to show less. Students might also have developed the insight that 5 is less because it is earlier in the counting sequence.)

If students are not able to identify which group is more or less and they do not line up the cubes to compare, suggest lining them up.

Option 2

Provide students with pairs of numbers and the following task:

> Which number is greater?
>
> Draw a picture to show each number. Circle the number that is greater.



This student correctly drew pictures to show the numbers but circled all of the numbers on the page rather than circling the number that is greater in each set.

It's not clear if the student misunderstood the directions or could not identify the number that is greater. To reassess the student, have him count aloud the pictures he drew for each set of numbers. Tell the student to use a crayon to circle the number for the set that is greater. If needed, rephrase the question asking him to circle the group that has more items in it. If the student is not able to identify the set that is greater using the pictures, have the student use cubes to count and compare numbers.

Additional Ideas for Support and Practice

The following ideas extend students' understanding of comparing sets of objects and provide meaningful practice.

Cups of Counters

Students practice comparing quantities with counters in cups.

Put a different number of counters in each of two cups (e.g., 3 in one cup and 5 in the other cup). Have students dump the cups and decide which cup has more. Have them explain how they know.

Did anyone count to help them decide? How did counting help?
Did anyone match counters to help them decide? How did matching help?

Have students draw the counters in each cup and circle the drawing that shows more.

Repeat with two different cups.

Students work to compare numbers by lining up the counters. She has 5 counters and he has 2 counters. They will be able to see that 5 is greater than 2.

Scoop and Compare

Students practice comparing with an interactive task of scooping pom-pom balls.

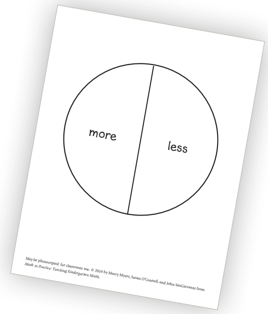
Prepare bowls (or bags) of large pom-pom balls with a scoop or ladle for each pair of students. Students can take a handful if scoops are not available.

Player A scoops out pom-poms and counts them. Player A will say, "I have ___ pom-poms."

Player B scoops out pom-poms and counts them. Player B will say, "I have ___ pom-poms."

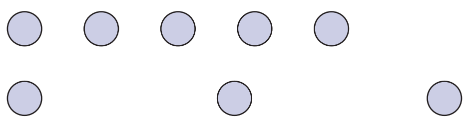
Players talk and decide who has more or less pom-poms, then spin the more/less spinner to see if the player with more or less wins!

Students place those pom-poms aside and scoop again. Continue play until all of the pom-poms are used.



Talk About It/Write About It

Jenny lined up counters to see which group had more. Was this a good way to line them up? Explain.



Ali had 4 crayons. Jamie had more than that. Draw a picture to show how many Jamie had. Label your picture with the number she had. Be ready to share. Is there more than one answer?

Draw a picture to show 8 is greater than 5.

Draw a picture to show 3 is less than 4.

Draw 6 balloons. Draw fewer than 6 balloons. Label the number of balloons you drew.

Draw 7 beach balls. Draw more than 7 beach balls. Label the number of beach balls you drew.

Count and Compare Center

This center provides counting practice and practice writing numbers, as well as practice comparing numbers.

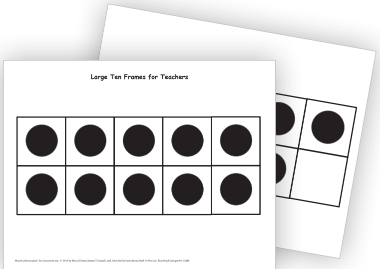
Place counters, tiles, bears, cubes, and so on of two different colors into bags.

Students dump out the materials in each bag, compare them by matching and counting. Students then record the numbers using that color crayon (red for red counters or blue for blue counters), and circle the number that is more.

Students then replace the materials in the bags and pick another pair of bags.

Having the students record their work in colors allows you to assess the accuracy of their counting because you know there were 5 green bears and 4 red bears or 6 purple cubes and 3 red cubes.

One More Fast Flash



Show students dots on a ten frame, and have them say the number that is 1 more.

When students have had ample experience with Fast Flash (see page 46), display a ten frame with 8 dots.

What number is 1 more than the number shown? (9.)

Add 1 more dot to the ten frame using another color to show 9.

Talk about how 9 is 1 more than 8, helping students understand that the next number in the counting sequence is 1 more.

Continue with other numbers showing 1 more.

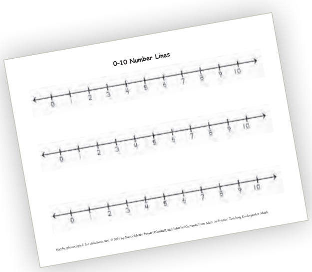
Eventually, students should be able to say the number that is 1 more when the ten frame is shown.

In their initial experiences, students compared sets of objects. Then they compared objects with number cards visible to show the quantity in each set. Numbers were recorded, but the objects were still present. Finally, the objects are removed and students compared quantities with numbers.

COMPARING WRITTEN NUMERALS 1–10

Students have investigated comparing quantities with objects and drawings and are now transitioning to comparing numbers looking only at the numerals.

Number Line Circles SHOW IT!



Students circle numbers on a number line to see which is closer to or farther from 0.

Give students two bags of counters with 1–10 counters in each bag.

Have them count the number of counters in each bag and circle the numbers on a number line.

Which bag has more counters? How do you know?

Were you able to tell which had more by looking at the number line? How?

How do you know that the number of counters that is farther to the right is more?

Use a laminated number line or place the number line in a transparent pocket so students can circle the numbers, make their decisions, and then wipe off the circles to start again. Record some of their ideas on the board.

Add some ideas using a think-aloud method if no one suggests it.

I was thinking the numbers get smaller the closer I get to 0.

I was thinking the numbers get larger the closer I get to 10.

I was thinking that the bigger number is farther from 0. What do you think about that? Talk with your partner about it.

To bring the lesson to a close, use the laminated number lines and dry erase markers.

Your first 2 numbers are 5 and 2.

Draw a circle around the number that is more and underline the number that is less. Explain.

Repeat with additional sets of numbers.

Differentiation

Some students may need to compare the actual objects longer than others. Allow them to use materials, but also have them select number cards or write numbers to show each comparison.

Target Number

Students use a target number and their understanding of more and less to compare quantities. This activity works well with a whole class.

Shuffle a set of large 0–10 cards (or use number cards displayed on an interactive whiteboard).

We have compared numbers using pictures and objects. Today we are going to compare numbers using number cards.

What words or phrases do we use when we compare numbers?

We are going to use greater than and less than to compare numbers to a Target Number.

Write *Target Number* on the board. To the left, write *Less Than*. To the right, write *Greater Than*.

Draw a card from the deck.

What number do I have? (6.)

6 is our Target Number.

Place 6 on the board under *Target Number*.

Have a student come up and select the next card.

Bobby picked the number 9. Our Target Number is 6. Is 9 greater than or less than 6? Where does 9 go on the board? Turn and talk to your partner and be ready to tell me where to place it and why.

Listen as students share with partners, and then have a student tell where it should go and why.

Do you agree? Does anyone have a different idea?

Allow students to question and share their ideas about whether 9 is greater than or less than 6.

Refer to the class number line or hundred chart to help students see that 9 comes after 6 when counting so it is greater than 6.

Let's pick another number card.

Have a student come up and select the next card.

Sarah picked the number 3. Is 3 greater than or less than our target number 6? Where does 3 go on the board? Talk to your partner and be ready to tell me where to place it and why.

Allow students to question and share their ideas.

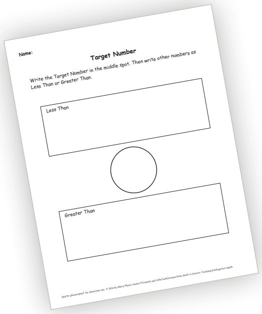
Why is 3 less than 6? How do you know?

Continue having students draw cards from the deck and sorting as greater than or less than the Target Number.

This activity can continue over several days with different Target Numbers.

When 10 is the Target Number, ask students why all numbers in the deck are less than 10. What numbers could you write that are greater than 10?

This activity can become a center using smaller number cards and the Target Number recording sheet.



Students draw a card and record the number in the Target Number Spot. As they pick numbers, they write the numbers in the Less Than box or Greater Than box.

> **FORMATIVE ASSESSMENT**
>
> Students are given pairs of numbers cards.
>
> They hold the greater number up in the air and the number that is less is held lower. (For example, give students cards with 8 and 3 on them. Students will hold up the number 8 and the number 3 will be lower.)
>
> Record student responses on the Observational Notes page.
>
> If students are not able to compare the quantities by looking at the numbers, put out counters to show the numbers 8 and 3.
>
> Then ask which number is greater and which number is less.
>
> Have students tell you why 8 is greater and 3 is less to assess whether they understand what *greater* and *less* mean.

Additional Ideas for Support and Practice

The following ideas extend students' understanding of comparing numbers to 10 and provide meaningful practice.

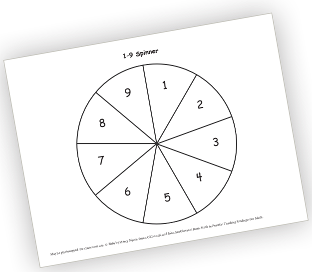
Talk About It/Write About It

Use the following as discussion starters:

Jay has 6 apples. Does he have more than 5? How do you know?

Maddie says 9 stickers is more than 7. Is she right? How do you know?

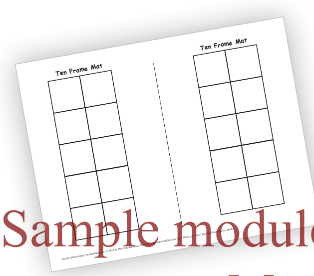
Students can draw or write to show that one number is more than or less than another.



Spin and Show 1 More

Partners practice showing 1 more.

Materials: 1–9 spinner for the pair; a ten frame and two colors of counters for each student



Player A spins the spinner and uses one color of counters to show the number he spun on the ten frame. He says, "I have 6; 1 more is 7," and places 1 counter of a different color on his ten frame.

Player B takes a turn following the same process.

After both players have shown their number, they compare frames. The player with the higher number wins that round.

Continue play.

Compare the Cards

This small-group activity provides practice in comparing numbers.

Have a stack of 1–10 cards.

Pull two cards.

Which is more/greater? How do you know?
Which is less? How do you know?

Allow students to use a number line, if needed.

Ask students to draw a model of each number to justify their answer.

Roll and Compare

This center or partner practice activity involves rolling dice and comparing numbers.

Students work with partners and take turns rolling a 1–6 number cube or a 0–9 ten-sided die (dependent on students' level of experience).

Partners record the numbers and a model of each die or cube in their math journals or on paper, then circle the number that is less (or more, whatever you designate).

Then, players roll and start again.

Which Number Is Greater?

This two-player interactive game can be used in centers or for partner practice. It is similar to the traditional card game War.

Materials: two sets of number cards 0–10 (22 cards in the deck)

Shuffle the deck and place it facedown between the students.

Player A draws a card and turns it over.

Player B draws a card and turns it over.

The player with the greater number says, "____ is greater than ___," and takes the two cards.

If the cards turned over are equal, players can turn another set of numbers to determine who gets the cards.

Play continues until all cards are drawn from the deck.

The player with the most cards is the winner. Then cards are shuffled and the game starts again.

Vocabulary

balance	greater than
compare	less than
count	match
equal to	more than
fewer than	same as

Online Resources

General Resources

Additional Problems

Comparing Numbers Student Observation Notes

Large Ten Frames for Teachers

Number Cards 0–10

Number Lines 0–10

Number Strip

Ten Frame Mats

Resources for Specific Activities

1–9 Spinner

Fewer/Same/More Mat Template

More/Less Spinner

Target Number Recording Sheet

Which Number Is Greater? Sheet

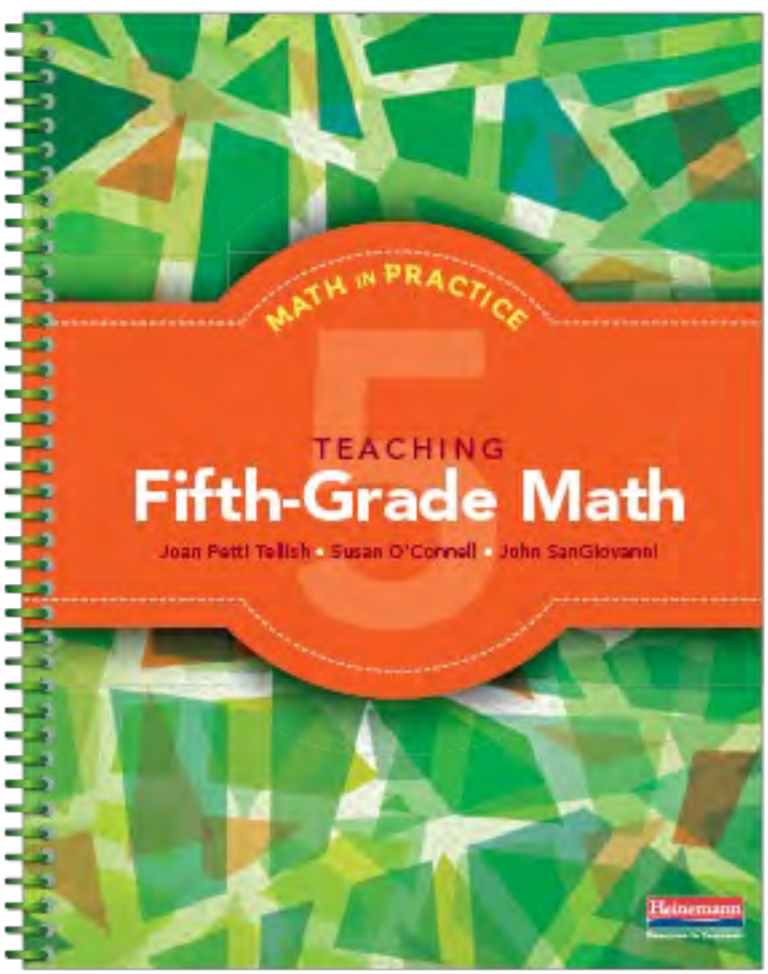
These resources are available at [redacted].

MATH IN PRACTICE

TEACHING

Grade 5 Math

Sample



MODULE

13

Exploring Volume

About the Math

5.MD.3.A; 5.MD.3.B; 5.MD.4; 5.MD.5.A; 5.MD.5.B; 5.MD.5.C

Although students measured liquid volume, or capacity, in third and fourth grades, the study of volume as an attribute of solid figures begins in fifth grade. At this level, students develop an understanding of volume, as well as explore ways to measure it.

The key ideas focused on in this module include:

- understanding the concept of volume, and measuring volume by counting the number of cubes it takes to fill a figure
- understanding that volume is measured in cubic units
- exploring the volume of rectangular prisms and making connections between volume and area
- discovering the formula for determining the volume of a rectangular prism
- solving problems about volume
- recognizing volume as additive and finding the volumes of complex figures.

Volume is a measure of the space inside a figure, typically measured in cubic units. At this grade level, volume is measured in whole number units. Initial explorations have students filling shapes with cubes and counting the number of cubes to find the volume. As they explore the concept further, they select appropriate units (e.g., cubic inches or cubic centimeters) and decide on appropriate strategies and tools (e.g., counting cubes or using addition or multiplication methods for finding the total volume). They use their experiences to estimate volume and solve problems related to volume.

Figure 13.1 Students pack boxes with cubes and then count cubes to find the volume.

Although students can count to find the total number of cubic units, they discover that there are other ways to find the volume without counting each cube. Rather than telling students the formula for finding the volume of a right rectangular prism, providing opportunities for students to gather data, observe those data, and gain insights of their own allows them to make sense of the formulas they discover (multiplying length × width × height or multiplying base × height).

As students explore volume, they revisit other math concepts. Students recognize that when calculating the volume, the order of the three measurements (length, width, height) is irrelevant because of their understanding of the associative property. Whether they calculate length × width × height or height × width × length, the volume will be the same. Students see connections with the formula for calculating area as they figure out why, for a right rectangular prism, multiplying the area of the base by the height would be the same as multiplying the length by the width by the height.

Decomposition is a common theme in mathematics. Students decompose numbers by place value in expanded form. They decompose monetary amounts into various coin denominations. And they decompose three-dimensional figures into other figures in order to determine the volume. Through this work, students recognize that volume is additive. They discover that they can split some shapes into separate rectangular prisms, find the volume of each one, and put those volumes together to find the volume of the original figure.

As important as understanding *how* to calculate volume is the understanding of *when* and *why* we calculate volume. Posing problems related to volume and having students explore, discuss, and solve the problems deepens their understanding of volume in real-world situations. These problems include situations where the volume is unknown, as well as situations in which one of the dimensions (length, width, or height) might be unknown.

Learning Goals

I can explain what it means to find the volume of a solid figure and how volume is measured.

I can find the volume of a rectangular prism by using a formula.

I can find the volume of complex figures.

I can solve math problems using my understanding of volume.

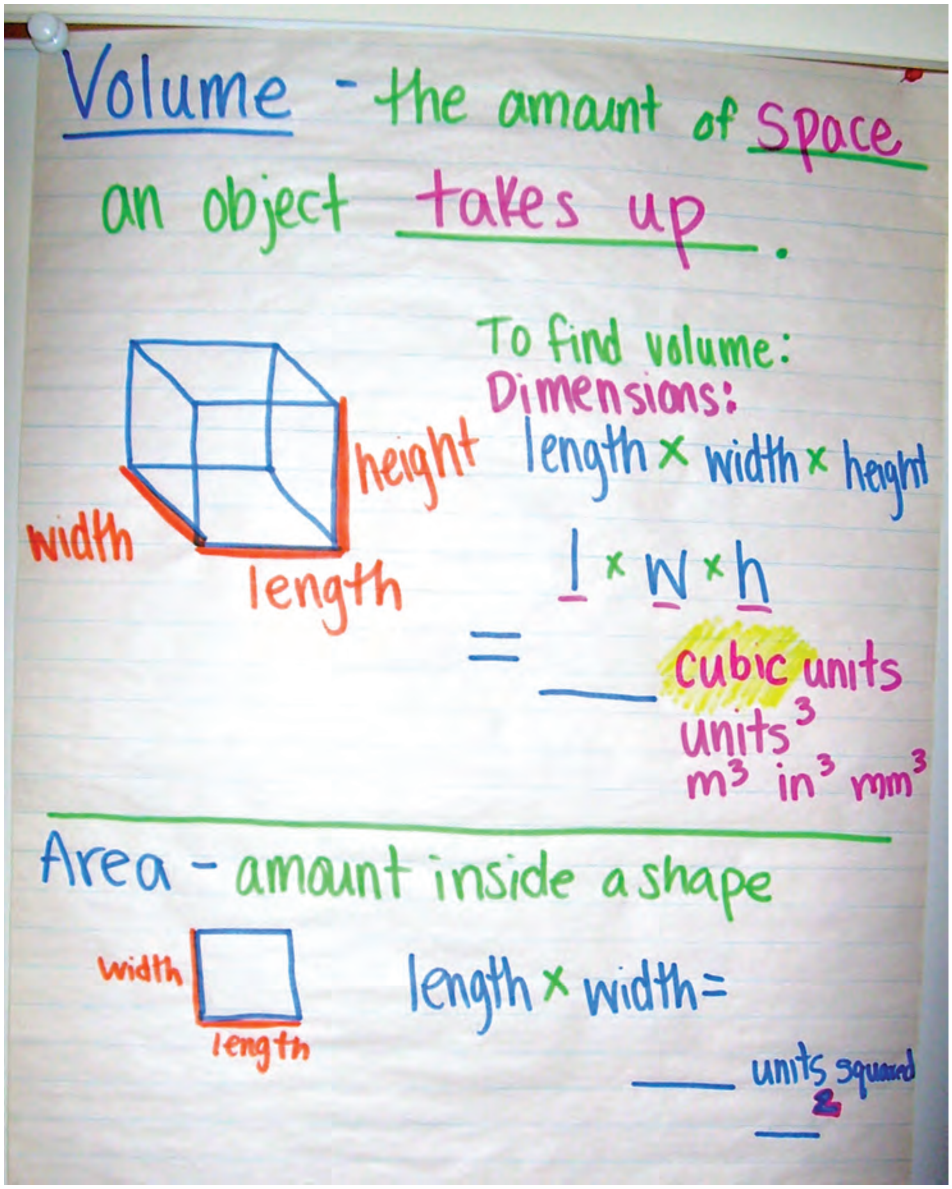
Lessons in This Module

Exploring the Concept of Volume

- Introduction to Volume: Counting Cubes
- Build It!
- Base × Height
- What's the Rule?
- Problem Solving

Understanding Volume as Additive

- Volume Is Additive
- The Birthday Cake



This anchor chart shows the labeling of area in square units for two-dimensional shapes and the labeling of volume in cubic units for three-dimensional shapes.

Exploring the Progression

PREVIOUS		NOW		NEXT
Grade 4 Exploring area; calculating area of rectangles	»	**Grade 5** Exploring volume; calculating volume of rectangular prisms	»	**Grade 6** Calculating volume of rectangular prisms with fractional measurements; calculating surface area

Ideas for Instruction and Assessment

EXPLORING THE CONCEPT OF VOLUME

Students explore the concept of volume by first filling solid containers with cubes and then observing and discovering the mathematical formula.

Thinking Through a Lesson

SMP4, SMP5, SMP6

Introduction to Volume: Counting Cubes

Students fill boxes with cubes and count them to explore volume.

Ask students about situations when it would be important to know how much something could hold, or how much it would take to fill an object.

Can you think of times when you might need to know how much could fit in a container?

When might you need to know how much it would take to fill something?

Common Error or Misconception

Students sometimes confuse the concepts of area and volume and the units used to measure them (square or cubic units respectively). Introductory investigations with materials (squares or cubes) help them develop deeper understandings of the differences between the concepts.

Have partners brainstorm ideas of real-world situations related to volume.

Make a list on a chart.

Have students work in groups of four.

Connecting to the real world makes the concept more meaningful for students. Real-world situations may include amount of cereal in a box, sand in a sandbox, or potting soil in a planter.

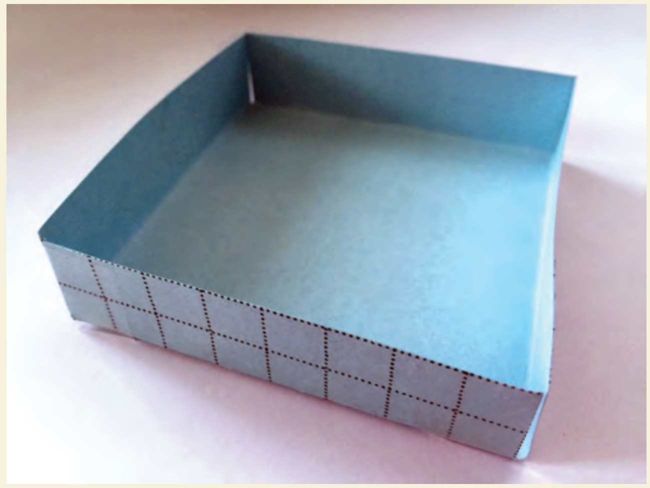
It is a good practice to have students work in groups so they can share their thinking and strategies.

Prepare centimeter grids copied on card stock paper for each student.

Have each student cut out a 12 × 12 square.

Have each of the four students cut out a different square section from each corner of his square (one cuts out a 2 × 2 section of each corner, another cuts a 3 × 3, another cuts a 4 × 4, and another cuts a 5 × 5).

Then students fold and tape the sides (grid side facing out) to make open boxes of various sizes.



This student makes a box, making sure the grid lines are facing out.

Compare your boxes with the other boxes in your group.

Which box is biggest?

Talk with your team members and be ready to share and justify your answer.

Have students share their ideas. Some may talk about the tallest or widest.

Is the tallest box the biggest? Why or why not? (It's the tallest, but it is not very wide.)

Is the widest box the biggest, why or why not? (It's the widest, but it's not very tall.)

Is there another way we might measure the boxes? (We could see how much they hold inside.)

Provide each team with a set of centimeter cubes.

Have students fill each box with cubes and count the number of cubes that fill the box.

How many cubes filled each box?

Which box has the most cubes?

Which box has the fewest cubes?

Explain that they just measured the volume of these boxes.

Hold up a flat rectangle.

What did we call the amount of space that covered the rectangle? (The area.)

How did we label the units for area? (Units squared, square units.) *Why?* (It was like counting squares.)

How might we tell someone the volume of our box?

What unit are we using to measure? (Cubes.)

Turn and talk to your partner about how we might label volume. (We might use cubic units.) *Why?* (We are using cubes. *Or,* It is a three-dimensional figure. *Or,* The cube is 1 unit on each side.)

So the tall box has a volume of 20 cubic units since 20 cubes filled it.

> *Ask students for their ideas before sharing the units used. They are more likely to remember it if they make sense of it.*

Add the phrase *cubic units* to the word wall or Math Talk chart.

Can we be more precise with the cubic unit? Turn and talk to your team.

Did anyone come up with a more precise label? (Yes. We said cubic centimeters since we used centimeter cubes.)

Turn and talk to your team about the volume of each box.

Write the word *volume* on the word wall or Math Talk chart.

Turn to your partner. How would you describe volume? What is it? (It is the amount something can hold. *Or,* It's the amount of space something takes up. *Or,* It's how many cubes fit in the box.)

How is it like area? How is it different? (It measures how much space something takes; area measures how much flat space something covers, while volume measures how much three-dimensional space something takes up.)

Write the word volume *in your journal.*

In your own words, tell what volume is.

Tell how you measure volume.

Build It!

Students compare volume by using cubes to build rectangular prisms.

Give each team of four students a set of multilink cubes.

What is a rectangular prism?

Use your cubes to make rectangular prisms. Each team member should make one that is a different size.

Allow time for students to build their rectangular prisms.

Talk to your team and predict which one will have the greatest volume.

Students may predict the tallest or widest, or one with the most cubes.

Predict which one will have the least volume.

Students may predict the shortest, or one with the fewest cubes.

Once students have made their predictions, have them find the volume of each one by counting the cubes. They may have to disassemble parts of the prism to do this, but can reassemble after they count the cubes.

Write the volume of each rectangular prism on a sticky note and place it by the prism.

How should you label your volume measurement?

Were your predictions correct?

Which has the greatest volume?

Which has the least volume?

Have students do a walk around the room to see the rectangular prisms made by each team.

As you look at each team's rectangular prisms, think about the ones with the greatest and least volumes.

What do you notice about them? Are they always the tallest? Are they always the widest?

Ask students to share their ideas about volume. (They might notice that the tallest doesn't always mean the volume is greater. Or that some of the shorter ones had a large volume because they were wide.)

Order the class' rectangular prisms from least to greatest volume, using the measurements on the sticky notes.

To end the lesson, have students discuss the following:

Do any of our rectangular prisms have the same volume but look different? Is that possible? Why or why not?

Differentiation

Challenge a group of students to work together to make prisms that look as different as possible but have the same volume.

Students use cubes to build rectangular prisms of various sizes.

Base × Height

Students discover that the volume of a right rectangular prism can be calculated by multiplying the area of its base by its height.

Have the students work in groups of four.

Provide each member of the group with centimeter grid paper on cardstock and centimeter cubes.

Have each student cut a 10 × 10 square out of the grid paper.

Then have them cut a 4 × 4 square out of each corner.

Have them fold and tape the sides (grid side facing out) to make an open box.

Have each student cover the bottom of their box with one layer of cubes.

How many cubes do you have?

Student 1 will keep his box as it is.

Student 2 will add 1 more layer.

Student 3 will add 2 more layers.

Student 4 will add 3 more layers.

Find the volume of the part of your box that is filled with cubes.

Record the data on the board.

Student 1: 4 cubic units

Student 2: 8 cubic units

Student 3: 12 cubic units

Student 4: 16 cubic units

What do you notice?

Does that make sense?

Have students flip over the box.

What would I be measuring if I measured just the flat bottom, the base, of the box?

How could you find the area of the base?

What is the area of the base? Think about the units you use for this measurement.

Turn and talk: How does the area relate to your volume measurements?

Have students repeat the layering activity with a box that is made from a 12 × 12 square with 4 × 4 cutouts in each corner.

Then, have them share their data and record it on the board on a chart like the following:

Area of Base	Number of Layers Tall	Volume
16 square units	1	16 cubic units
16 square units	2	32 cubic units
16 square units	3	48 cubic units
16 square units	4	64 cubic units

What do you notice about the area and the volume?

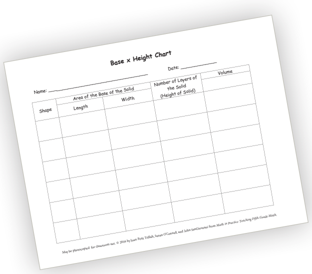
Does this make sense? Explain.

Have students make one more box with a 14 × 14 square with 4 × 4 cutouts in each corner.

Have students fill in the chart.

Turn and talk: Is there a way you could find the volume of all 4 layers of the box without filling it with cubes and counting them? Explain.

Have partners share their ideas.



What's the Rule?

Students discover that the volume of a right rectangular prism can be found by multiplying its length, width, and height.

Give each pair of students some multilink cubes. Pose the question:

How many different rectangular prisms might you build with a volume of 18 cubic units?

Build as many as you can and be ready to tell me the volume, the length of the base, the width of the base, and the height.

Make a class chart and have partners share the dimensions of their prisms.

Record the dimensions of each one.

Length of Base	Width of Base	Height	Volume
			18 units^3
			18 units^3
			18 units^3

What do you notice?

Are there any connections between the numbers in each row on our chart?

Is there a way to find the volume without counting every cube?

Students may suggest that multiplying the three measurements gives the same result as counting all of the cubes.

Do you think that will always work?

Build as many different rectangular prisms as you can with a volume of 24 units^3.

Record the dimensions for their rectangular prisms to test the conjecture.

What do you notice?

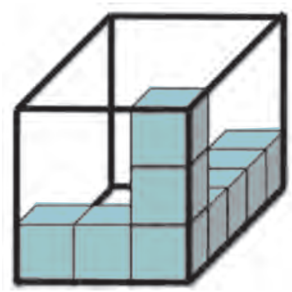
Does multiplying the dimensions give us the volume?

Have students practice finding volume by making a few more rectangular prisms of any size, recording the dimensions, finding the volume by multiplication, and then counting the cubes to check the volume.

FORMATIVE ASSESSMENT

Pose the following:

This prism is partly filled with cubes. What is the volume of the whole prism?



Look for students who understand that the dimensions are $3 \times 3 \times 5 = 45$ cubic units. If students are having difficulty, give them cubes to build the figure and count the cubes.

Problem Solving

Students solve problems using volume concepts.

Be sure that students are clear when presenting their findings. How did they arrive at the solution? What are the measurements of the boxes? How do they know if the boxes will fit?

Pose this problem for students:

Mr. Sines wants to take 3 boxes of materials home from school. He needs to know if they will all fit in his truck, or if he needs to make two trips to get all the boxes home. Here is some information you will need:

- 2 of the boxes are the same size. (2 ft. long, 3 ft. wide, and 2 ft. high)
- 1 box is larger than the others. (3 ft. long, 3 ft. wide, and 3 ft. high)
- Mr. Sines' truck has 60 cubic feet of space.

Can Mr. Sines fit all 3 boxes in his truck?

You may want to have students prepare posters with their solutions and then do a gallery walk in which students can read and comment on each other's work. This is also an excellent opportunity to have students ask each other questions to develop their skills in mathematical discourse.

Have students work in pairs to solve.

Have them present their findings to another partner pair.

Does the other pair agree or disagree? Why?

Pose another problem and have students work with a new partner or group to solve and present their findings.

Mrs. Petti is planning to ship a package. She needs a box that will hold at least 30 cubic feet.

Which box will be large enough? Prepare an argument to convince the class which box will work.

Box A		Box B	
Length	7′	Length	6′
Width	2′	Width	2′
Height	2′	Height	3′

Additional Ideas for Support and Practice

The following ideas extend students' understanding of volume and provide meaningful practice.

Check the Formula

Students fill open boxes (made with centimeter grid paper) with centimeter cubes.

They use the grids on the outside of the box to determine the length, width, and height.

Students then find volume by multiplying the dimensions and by counting cubes.

They check to see if the two volumes are the same.

They record the actual data for $V = l \times w \times h$ in their journals.

Pass-a-Problem

Tape one problem on the front of a manila envelope.

Prepare enough envelopes so that each group of four students will have an envelope with a different problem.

Students will solve a problem, place their solutions and work in the envelope, and pass it to the next group.

After the last rotation of problems, students will take out all of the solutions for the problem they are holding.

Each group will present that problem, solution, and strategies to the class.

Problems may include:

> Tom built a rectangular prism using 40 centimeter cubes. The length was 4 cm, the height was 5 cm, and the width was 2 cm. He took it apart and used the same 40 cubes to build another rectangular prism with different dimensions. What could the dimensions be?
>
> Heidi's new shoes came in a box that has a volume of 240 cubic inches. The length is 12 inches. What could the height and width be?
>
> A cereal box has a volume of 128 cubic centimeters. What could the dimensions be?

For additonal problems, see the online resources.

Talk About It/Write About It

> You have been asked to design a toy box for a child's bedroom. The toy box needs to hold 30 cubic feet. What could the dimensions be? Justify your answer.
>
> How is volume different from area? Explain.

What is the difference between cm^2 and cm^3? Explain.

How can you find the volume of a rectangular prism without filling it with cubes and counting them?

A rectangular prism has a volume of 48 cubic units and one of its dimensions is 4. What might the other dimensions be? Justify your answer.

The Same, But Different

Have students work with a partner to build two rectangular prisms with the same volume, but different dimensions.

Have them record the volume and the different dimensions in their journals.

Students may choose the number of cubes to use. Do they recognize that they cannot always build more than one rectangular prism with the same volume? This is a good connection to finding the factors of a number.

Pack It Up

Students construct a box to hold a small item.

Place small items such as a glue stick, stapler, small stuffed animal, or small toy in a basket on the table.

Provide grid paper, ruler, and scissors.

Have students build an open-top box to hold the item.

Have them record the dimensions in their journals and calculate the volume.

Have them explain how they determined what dimensions to make the box and if the box was too small, too large, or the perfect size.

Roll a Rectangular Prism

Students play with a partner to calculate the volume of rectangular prisms.

Each player rolls a 10-sided die three times to find the dimensions of each side of a rectangular prism.

Each student calculates the volume of the rectangular prism.

The student with the greater volume gets one point.

Repeat ten times. The student with the most points is the winner.

Provide calculators for students who need them, to keep the focus on the measurement concepts rather than on the calculations.

UNDERSTANDING VOLUME AS ADDITIVE

Students recognize that volume is additive. They split a shape into two or more rectangular prisms, find the volume of each one, and put those volumes together to find the volume of the original figure.

Volume Is Additive

Students find the volume of a complex shape by decomposing it into two rectangular prisms.

Have students work in pairs and each make a rectangular prism out of connecting cubes.

What is the volume of your prism?

Put your prisms together to make a new figure.

What do you think the volume of the new figure is? Why?

How would we find the volume of the figure?

Have students take their original figure back.

Combine your figure with a different partner's figure.

What is the volume of the new figure?

Have partners swap their composite figures with another team.

Find the volume of the figure you just received.

What did you have to do to find the volume of the complex figure made up of rectangular prisms?

Turn and talk to your partner about how you would find the volume of a complex figure.

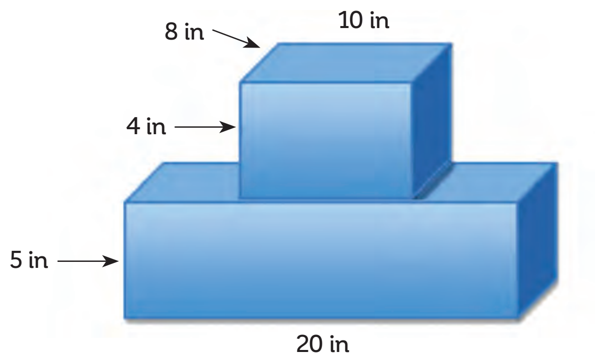
Review the definition of a composite or complex figure. It is a figure made from two or more geometric figures.

The Birthday Cake

Students solve problems about complex figures.

Post this visual and problem on the board:

Daniele is baking a cake for Grandpa Bill's ninetieth birthday party. She needs the cake to have a volume of 1,000 cubic inches so that there will be enough for all the guests. Here is an image of the cake. Will there be enough cake?



Work with a partner to solve the problem.

Observe as students solve the problem.

What strategies did you use to solve the problem?

What was the top layer's volume?

What was the bottom layer's volume?

Common Error or Misconception

Some students may multiply all the numbers together (5 × 4 × 10 × 8 × 20) instead of finding the volume of each rectangular prism. Ask students to explain why they made the calculations they did.

How did you find the volume of the whole cake?

Did Daniele bake a large enough cake?

Turn and talk to your partner about how you find the volume of a complex shape made up of rectangular prisms.

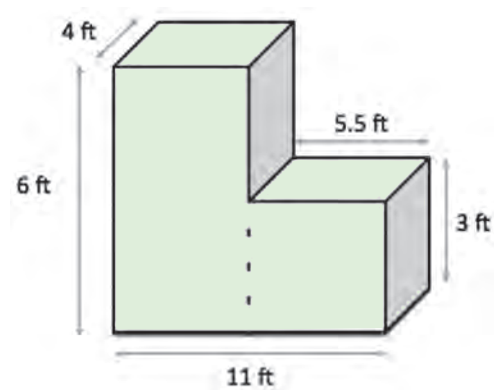
Differentiation

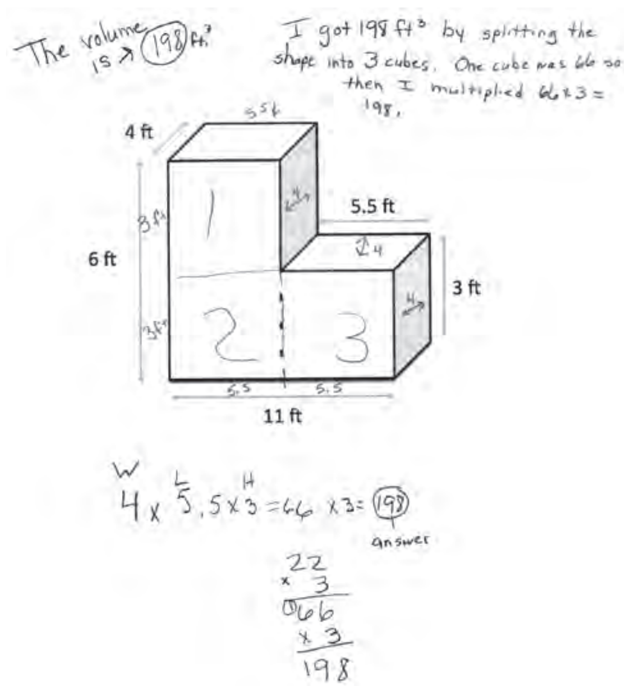
For a more challenging problem, use a figure involving decimals.

FORMATIVE ASSESSMENT

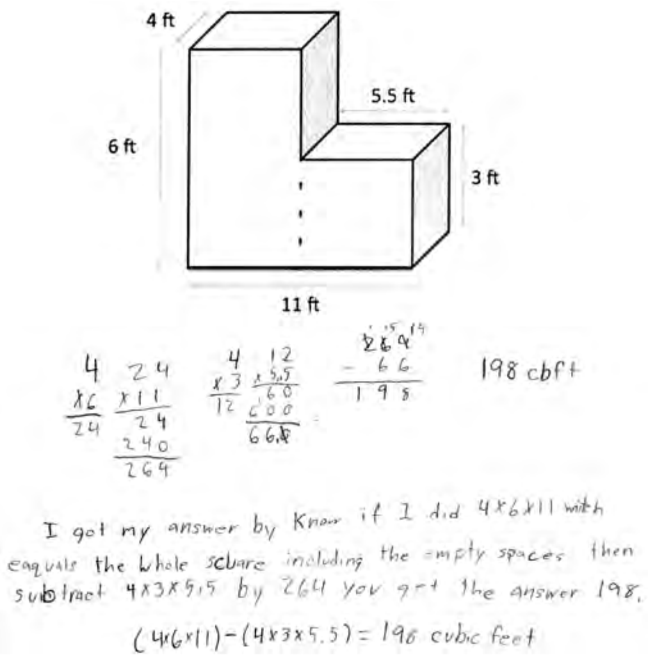
Pose:

Find the volume of this figure. Explain how you figured it out.





This student splits the figure into 3 equivalent figures. The student calculates the volume of one of the rectangular prisms and then multiplies it by 3 but does not explain her work or label her answer.



This student finds the volume of the entire figure, including the empty space. The student then finds the volume of the empty space and subtracts it from the total.

For additional formative assessment techniques, see Math in Practice: A Guide for Teachers, Chapter 5.

Additional Ideas for Support and Practice

The following ideas extend students' understanding of volume of complex figures and provide meaningful practice.

Talk About It/Write About It

Does it matter which way you break apart solid figures to determine volume? Explain.

Identify some real-world figures that can be decomposed into rectangular prisms to calculate volume.

Building Skyscrapers

Students find the volume of complex figures, using premade paper grid boxes to build a skyscraper.

Students may also build prisms from connecting cubes and use them to build the skyscrapers.

As a class, have each student make one box from grid paper to place in the center.

Vocabulary

base
complex figure
cube
cubic units³
height
length
unit
volume
width

Remind students that using a numbered list to explain each step might help them organize their ideas.

ONLINE RESOURCES

General Resources

Additional Problems

Grid Paper

Resources for Specific Activities

Base × Height Chart

These resources are available at [redacted]

Have each student cut a square out of the grid paper. Make sure sizes vary (7×7, 8×8, 9×9 . . .)

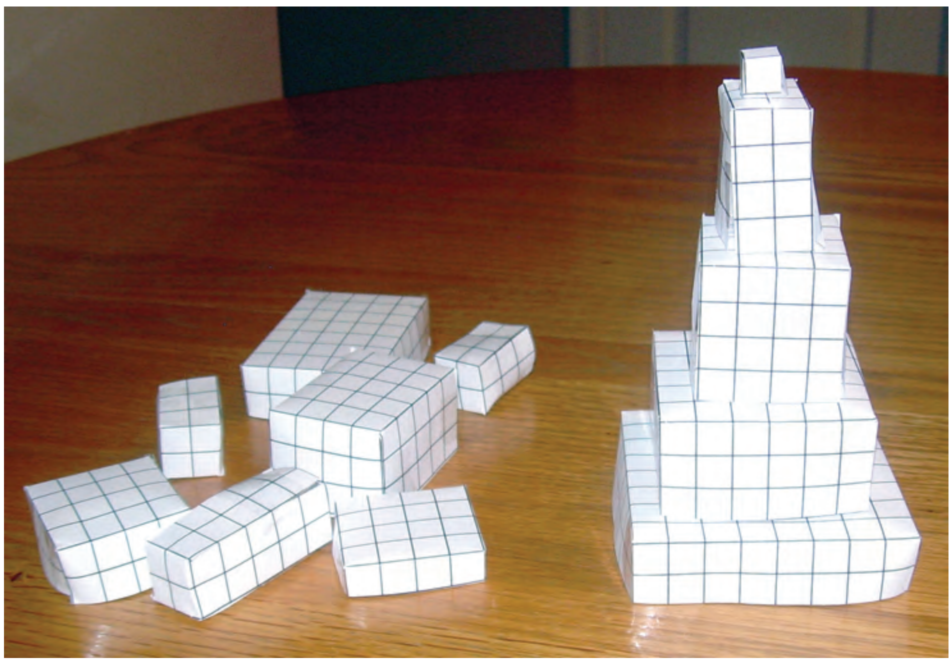
Then have them cut out the same size square from each corner on their paper. Students may choose the size (2×2, 3×3, 4×4).

They fold and tape the sides (grid side facing out) to make an open box.

Students work with partners to build a skyscraper with three or more boxes.

Partners figure out the volume of each level and then find the volume of their skyscraper.

Partners then write how they found the volume.



This is a sample of a figure that a student built using a variety of paper boxes.

Meet the *Math in Practice* team

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Sue O'Connell has decades of experience supporting teachers in making sense of mathematics and effectively shifting how they teach. A former elementary teacher, reading specialist, and math coach, she currently directs Quality Teacher Development, an organization that provides professional development for schools and districts across the country. Sue is also the coauthor of the bestselling *Putting the Practices Into Action* and the *Mastering the Basic Math Facts* series, and the author of *Now I Get It*.

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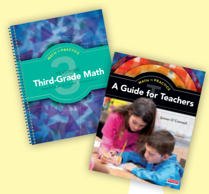

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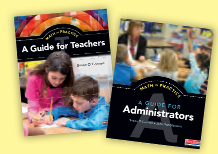

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