# No More Math Fact Frenzy 

LINDA RUIZ DAVENPORT<br>CONNIE S. HENRY<br>DOUGLAS H. CLEMENTS

JULIE SARAMA

## Dear Readers,

Much like the diet phenomenon Eat This, Not That, this series aims to replace some existing practices with approaches that are more effective-healthier, if you will-for our students. We hope to draw attention to practices that have little support in research or professional wisdom, and offer alternatives that have greater support. Each text is collaboratively written by authors representing research and practice. Section 1 offers practitioners' perspectives on a practice in need of replacing, and helps us understand the challenges, temptations, and misunderstandings that have led us to this ineffective approach. Section 2 provides a researcher's perspective on the lack of research to support the ineffective practice(s), and reviews research supporting better approaches. In Section 3, the authors representing practitioners' perspectives give detailed descriptions of how to implement these better practices. By the end of each book, you will understand both what not to do, and what to do, to improve student learning.

It takes courage to question one's own practice-to shift away from what you may have seen throughout your years in education and toward something new that you may have seen few if any colleagues use. We applaud you for demonstrating that courage and wish you the very best in your journey from this to that.

Best wishes,
—Nell K. Duke and M. Colleen Cruz, series editors

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守古需雱守

This book，the fourteenth in the Not This，But That series，marks the first with coeditor M．Colleen Cruz．Coeditor Ellin Oliver Keene，who has been with the series since its inception，is moving on to other proj－ ects．It has been a privilege to work with Ellin．Professionally，I will be ever in awe of her thought leadership．Personally，I am so grateful to call her a dear friend．

I also want to take the opportunity to welcome M．Colleen Cruz as a coeditor．How lucky I am that Colleen Cruz－the Colleen Cruz—has agreed to take on this role．It has been a pleasure already to engage with her on this series，and I look forward to our continued collaboration．
－Nell K．Duke，series coeditor

## INTRODUCTION <br> Nell K. Duke

I completely understand how we've ended up in a math fact frenzy. It seems as simple as $1+1$ that if we want kids to be able to retrieve math facts automatically, we should have them practice them over and over and over again as rapidly as possible. Plus, we've learned lots of ways to make that process seem fun, for example, in Around the World, Slam 10, and Math Facts Race.

The problem is that just being able to quickly spout out math facts does little for children if they don't have important underlying understandings. Without these understandings, many children will find it difficult to learn math facts in the first place. And what about those fun math fact competitions? When we look closely, we often find that those are fun for children who already know their math facts and far less fun for those who don't.

You want children to have more than math facts; you want them to be mathematical thinkers, to be aware of patterns, and to have tools to explore quantity, structure, and space. You want out of the frenzy and in to the calm of well-planned, well-executed-and, yes, fun-math fact education. To support you in that transition, the author team before you has expertise to the power of four. Linda Ruiz Davenport and Connie S. Henry are highly experienced practitioners who have helped countless teachers improve their practice. Douglas H. Clements and Julie Sarama are among the most respected mathematics education researchers in the United States, and their insights have been reshaping early math teaching.

Section 1 and the first portions of Section 2 identify the misconceptions and limitations of traditional approaches to developing math fact fluency. In the remainder of Section 2, you'll find lots of information pertaining to how children develop math fact fluency and how we can
best support them in that process. In Section 3, you'll find models that help you envision your own better practice regarding math fact fluency along with nitty-gritty details about teaching math facts, including how to ask children specific questions to assess and prompt their thinking, how to make effective use of timelines, and what to do before, during, and after particular math games to make them educative and, yes, fun.

Like many of our Not This, But That titles, this book is likely to lead you to do things quite differently than the teacher down the hallway, the school down the street, or the district down the highway. Be confident that these authors don't take the call to diverge from traditional practice lightly. They write with a large body of research and experience behind them. I encourage you to subtract from your existing practices those that the authors do not recommend, add to your practices those they suggest, and watch the effectiveness of your math fact instruction multiply.


## Worksheets and Drill $\neq$ Math Fact Fluency

## LINDA RUIZ DAVENPORT AND CONNIE S. HENRY

Remember back when we learned basic addition, subtraction, multiplication, and division facts in elementary school? We would have our pencils sharpened for our worksheets or our weekly timed test. There would be a page of fill-in-the-blank answers set up in neat rows and columns. We were told we would have three minutes to write down the answer to each math fact, and then the teacher would say "Begin!" and start the timer. We might have felt a sense of panic when our brains froze and we could not bring up the answers that only a few minutes earlier we knew.

Most of us really wanted to learn our math facts. Many of us practiced at home with flash cards, perhaps making our own in our neatest handwriting and grouping them by each number $(2 \times 1,2 \times 2,2 \times 3)$. We recited them out loud to ourselves or with a family member, trying to
memorize each one, only to be frustrated by facts that included larger digits, especially those 7 s ! We wanted to succeed on those timed tests, but somehow we never managed to do our best, as hard as we worked to prepare for them. How many students in math classrooms today have those very same experiences and those very same feelings?

What does it really take to help students learn their math facts in ways that allow them to access and use these facts fluently and flexibly to solve rich and challenging math problems? Are there strategies we could be using to help students learn their math facts more successfully, and with less stress and anxiety? In Section 2 we will hear more about what the research tells us about how students build their knowledge of math facts, and in Section 3 we will explore the kinds of classroom activities that help build that knowledge base, but for now, let's take a look at what we often see in elementary classrooms.

## What We Often Encounter

Below, we describe a few scenarios that capture what we encounter as we visit classrooms where teachers, with the best of intentions, are attempting to address number fact automaticity. As you read through these scenarios, we invite you to reflect on why these practices, and others like them, might be problematic but at the same time seem to endure in so many classrooms.

## Counting by Ones-Quickly!

This kindergarten classroom is colorful, welcoming, and full of energy. Students are working in centers and appear to be very focused on the tasks at hand. At one center, each student has a worksheet consisting of one hundred single-digit addition problems, all lined up vertically. At another center, students have a worksheet with one hundred single-digit subtraction problems, also lined up vertically. In both centers, students are quickly grabbing connecting cubes from a bin, quickly counting out the number of cubes needed to start the problem, then quickly adding more or taking some
away, and then quickly counting the total-all out loud. For example, one student solves $9+3$ by quickly counting out 9 cubes, then quickly counting out 3 more cubes, and then quickly counting all 12 cubes in front of her. She then very proudly and quickly writes the total on her worksheet.

My visit to this kindergarten classroom took place in the spring, and I was quite impressed with how quickly students, by this time of the year, were able to count collections of up to 20 objects and even beyond. I was also impressed with how they all seemed to understand that addition meant putting collections together and subtraction meant taking collections away.

But I couldn't help wondering how all this counting by ones was helping to build a foundation for learning number facts. True, students were very quickly getting right answers to these addition and subtraction problems. But what were they learning? Were they seeing any connections as they went from one addition problem to another or one subtraction problem to another? Did working so quickly get in the way of exploring these relationships? I also found myself wondering why they were doing all this work with addition and subtraction on separate worksheets, losing an opportunity to think about connections between addition and subtraction facts, or even to use addition facts to figure out subtraction facts.

Another question I asked myself was why they were doing number facts all the way up to totals of 20. It's true that you can use counting by ones to find those larger sums, just as you can use counting to find sums within 5 or 10 . But why move into those larger numbers? These students did not seem to need to practice their counting, as they all seemed to know their counting sequences perfectly, so what were they learning about these larger combinations?

Finally, I also wondered if the vertical notation, in conjunction with the speed, was also preventing the students from making connections. As we learn in Section 2, the format of arithmetic fact practice matters. Horizontal number sentences that include an equal sign help students notice number relationships and build number fact fluency. Students
who have opportunities to work with math facts expressed as number sentences (or equations, such as $3+4=7$ ) can begin to see relationships between each side of the equation, almost as if it were a balance scale. Adding something to one side of the equal sign has implications for what needs to change on the other side of the equal sign so everything can stay balanced. Seeing $2+3$ as "the same as" 5 can help us think about what might happen if we have 5 and then we take 2 or 3 away. It also invites a certain kind of playfulness: for instance, if $2+3$ is one way to make 5 , what are other ways I could make 5 ?

But here, with so many problems on a single worksheet that were to be solved quickly, and with a vertical notation that reinforced the idea that each math fact was its own independent problem, students quickly counted by ones with cubes to solve each and every math fact as quickly as possible, with little conversation about what they were doing or what they were noticing. Had I asked any student to give me the answer to a math fact, I am confident they would

> An observer in this classroom might think that students were well on their way to learning their addition and subtraction facts. But I left the classroom with lots of questions to consider about what might come next for these eager learners in terms of math fact fluency. have reached into the bowl for counters, counted by ones, and given me a correct answer. It looked like the goal was simply to finish the task using their cubes.

I could see that this lesson might look like a successful one in the moment, given the level of student engagement and the consistency with which students were getting correct answers. An observer in this classroom might think that students were well on their way to learning their addition and subtraction facts. But I left the classroom with lots of questions to consider about what might come next for these eager learners in terms of math fact fluency. For instance, did these students really need to count by ones with cubes to find each and every one of those math facts? Might some have been ready for slightly
more efficient strategies, such as using their fingers to find facts like $2+$ 3 or $3+2$, or adding on to find $5+2$ by starting with 5 and then adding on to get 6 and 7? Were students being held back by how the task had been presented?

How do students begin to move from counting by ones with counters or cubes to building the kind of number sense they need to learn their math facts, not just for addition and subtraction, but eventually for multiplication and division as well? How far can quickly counting by ones to find every math fact take you? (Section 2 aims to answer these questions, as well as others we raise in this section.)

## Worksheets Once a Week

In a first-grade classroom, a teacher asks all the students to be silent as she passes out a traditional worksheet with thirty or more addition and subtraction number fact problems, all set up with vertical notation. The students know the drill and get to work right away, perhaps hiding their fingers under their desks as they try to figure out the different sums or differences. Their faces are mostly earnest, though several seem to have a worried look. After some time, the teacher collects the worksheets and students go off to lunch and recess, looking as if this break was perhaps a welcome respite from math. The teacher puts away the worksheets to score later. She plans to pass back the scored worksheets tomorrow so students can practice any facts they did not answer correctly using their flash cards.

There are indeed lots of addition and subtraction facts: 110 addition facts for numbers 0 to 10 and another 110 subtraction facts. As I watched, it was clear that most students knew some facts from memory but used their fingers to help them solve those they did not yet know (for instance, $4+1=5,5+1=6$, or $5+5=10$ ). But I found myself wondering how these students would come to know all these facts for both addition and subtraction. Were these first-grade students able to make any connections from one math fact to another? Could they use
the facts they knew to figure out the facts they were still learning? Or did they think their goal was to memorize each fact independently? Had anyone ever suggested that they use what they knew to figure out what they didn't yet know?

I also wondered what would happen if students felt they could use their fingers more openly as they worked on these math facts. For instance, using fingers to find that $3+4$ is the same as 7 can also help you see that $4+3$ is also the same as 7 , just by switching hands. And by changing the 3 into a 4 , you can see that $4+4$ is the same as 8 . As we learn in Section 2, fingers can serve as manipulatives that support student thinking-they are tools that you always carry with you. There is also something playful about using fingers, and I couldn't help wondering if that playfulness could also contribute to the spontaneous generation of several related math facts, just for fun! We also know that using fingers can help students learn to work with numbers and operations at a more symbolic level.

Finally, I wondered about the goal of these weekly worksheets that provided practice with math facts. Was the teacher hoping that by using these worksheets on a weekly basis, students would become more motivated to learn and remember their facts? Perhaps they would be more likely to practice with flash cards at home or during extra time between classroom activities? But how well would they remem-

> It was hard to think about students facing failure at this young age and what this might mean for how they saw themselves as mathematics learners.
ber all these facts over the long term? And what about students who just weren't making progress? Would they give up? Would they think that mathematics is a boring subject that requires lots of memorization? It was hard to think about students facing failure at this young age and what this might mean for how they saw themselves as mathematics learners.

## Timed Tests: Knowing Math Facts Quickly

This third-grade teacher is starting math time with a timed test of multiplication number facts. The teacher distributes a problem sheet to each student, placing it face down so they cannot see the problems. She sets a timer for three minutes. "Ready, set, go!" she says as she starts the timer. Students quickly turn over their sheets and begin working, looking like they are concentrating hard as they write their answers to each problem, working as quickly as they can. Some students are further along than others. A few stop to erase an answer and replace it with what they think is a better choice. The alarm goes off. The teacher tells them to put their pencils down. There are a few sighs from students who didn't quite make it through the entire sheet. The teacher collects the sheets and lets them know she will be updating the wall chart showing how well each student was doing, adding a gold star for students getting all the problems correct. I could see that a few students had already earned their stars but some were still struggling to get there.

It looked like these timed tests were a classroom routine that students knew well. I could tell that some students had learned many of their multiplication facts and seemed to look forward to this opportunity to show off what they knew. But I could also see that many students looked uncomfortably stressed; they left some problems unanswered and some students completed only the first couple of rows, perhaps because they thought it best to solve each problem in the order given and had gotten stuck. It was clear that students could tell who knew their facts and who had not yet learned them, particularly with the wall chart showing everybody's progress.

I asked the teacher about these timed tests, and she responded that they were well worth the three minutes they took out of math class, given the importance of number fact fluency. She believed these timed tests communicated the importance of knowing math facts
quickly, and because she gave them often, students had opportunities to improve so they could all eventually earn gold stars. I could see how this made sense, but how did improvement happen? I remembered back to my own experience rotely memorizing number facts, going through the stack of flash cards with my father, and finally getting all the multiplication facts correct. I also remember

> But is rotely memorizing facts really the key if what we've memorized seems to disappear from memory so quickly? how quickly I forgot them! What was $7 \times 8$ ? $6 \times 9$ ? If I didn't keep practicing, some of these quickly faded from memory, coming back to me only after more work with those flash cards. But what about the students who just couldn't remember those facts from day to day, not to mention week to week?

I could see how these timed tests, along with the wall chart showing progress, might create incentives for students to know their facts with automaticity. But is rotely memorizing facts really the key if what we've memorized seems to disappear from memory so quickly? What helps us remember these facts? For some reason, some facts seemed easier to hold on to, like the 5 times table, where I could skip count by fives, or the 4 times table, where I could use what I knew about the 2 times table to help me think about the 4 times facts. In Section 2, we have opportunities to think about the important role of reasoning strategies in learning all our math facts, but unfortunately many of these reasoning strategies remain invisible to students.

I also kept thinking about the stressed looks on so many of these students' faces. As we'll see in Section 2, research suggests that students whose teachers used timed tests knew fewer math facts than students whose teachers did not use timed tests. Does speed prevent students from taking the time to use their reasoning strategies? Does speed create a stressful situation that makes students tend to forget what they thought they knew? How many of us perform at our best when we are being timed, especially if we are still just learning? It seems like there is lots to think about with regard to timed tests of number facts.

## Writing Down Our Math Facts to Help Us Remember Them

 In this fifth-grade classroom, all the students are working independently on their multiplication facts. The teacher has given them a template with five columns on the front and five columns on the back, and their assignment is to list every fact, in order, from 1 to 10 in each column. As I watch, I can see students listing their facts $-3 \times 1=3,3 \times 2=6,3 \times 3=9$-working down each column. The strategy most students seem to be using is skip counting, and I can see that sometimes they use their fingers to help them skip count as they get to the 6s, 7s, 8s, and 9s. Along the way I also see a few mistakes which throw off the multiplication facts that follow, but students don't seem to notice. They seem more focused on completing the task than on getting each fact correct. I ask a student if she can tell me what 5 times 7 equals. She smiles as she consults the " 5 "column to tell me it equals 35. I get the impression that the teacher asks students to create these tables of facts on a regular basis.I wondered, did this teacher believe that by listing number facts in order, over and over again, students would eventually come to know each fact? Or did this teacher think that by making these lists, students would begin to see some patterns in their times tables that could help them learn their facts? This might have been true for some students, but others seemed to be counting by ones as they skip counted, especially once they got into higher numbers, sometimes with the help of busy fingers. The fact that students were using so much skip counting to complete their tables suggested to me that they were likely re-creating their number facts each time they made their tables. I wondered if a conversation about the patterns they were noticing in their tables might have been helpful.

As I thought about what I was seeing, I felt encouraged by the fact that students seemed to understand the idea of multiplication-they understood that to get from $3 \times 6$ to $4 \times 6$, you needed 6 more, or another group of 6 . They also could easily find a fact they needed by looking in the right row in the right column, which showed they knew something about how times tables work. But this wasn't the same as knowing any
single multiplication fact as it was needed. In Section 3, we'll see some learning experiences that can help students really know their number facts without having to re-create the table.

I was also concerned that students were often counting on by ones to get to the next multiple as they skip counted. This suggested that there was still some work for many of them to do with their addition facts. How can we help students learn their multiplication facts if they are still working on their addition facts? How far back do we need to go to help these students? Relationships among operations turn out to be key as students learn their math facts, and this includes relationships between addition and multiplication, subtraction and division, addition and subtraction, and multiplication and division. In Section 2, we'll learn more about the importance of these rich connections as students learn and are able to remember their math facts.

## Number Games and Math Facts

In one third-grade classroom, students are sitting with partners in various parts of the room, playing number games. I know that such games are often considered a way to build number fact fluency, so I join a pair of students to see what they are doing and what number facts they are learning. One student rolls two number dice and gets a " 3 " and a "4," meaning he can move his piece 7 spaces along the path to the rocket ship. I hope to see this student add the 3 and 4 to get 7 and then move his piece 7 spaces, but this is not what happens. Instead, he moves his piece 3 spaces and then 4 more. What is this student having an opportunity to practice with this number game? I join another pair of students playing the game, this time with dotted dice. One student rolls a " 5 " and a " 3 ." He then proceeds to count each dot—"1, 2, 3, 4, 5, 6, 7, 8"-and moves his piece 8 spaces. "Did you really need to do all that counting?" I asked. "No," he admitted. "I know 5 plus 3 is 8 ."

Many elementary math curriculum programs include number games that are intended to be a tool for helping students learn their math facts,
and most elementary classrooms seem to have a collection of math number games on their shelves. These are intended to provide students with practice learning their math facts, often with a focus on addition or multiplication, depending on the game. Teachers typically introduce these games to students in the "rug area" of the classroom and then send them off to work in pairs, making sure they all understand the rules and know the expectations for working together as partners.

I knew from watching this teacher introduce the game that students were to get their game board pieces from home base to the rocket ship by rolling two dice, adding the two numbers together, and then moving their pieces that many places along the designated path. While one student added the numbers on the dice, the other was responsible for checking the sum and making sure the piece was moved exactly that many places. Some dice were dotted and some had numbers, and in some cases students had one of each, depending on which dice they drew out of the bin. I had the sense that this game was similar to many others these students had learned to play, and their focus seemed to be on getting their pieces to the final destination, rather than on practicing their math facts.

Students were certainly engaged and seemed to be having a good time together. But I also noticed that students were using quite a range of strategies to add their two numbers together. Some students using the dotted dice were counting by ones to get their total. Some students using a combination of a number die and a dotted die seemed to be adding on, sometimes using their fingers. Some students using number dice used their fingers, but more often they just moved the number of spaces indicated by the first die and then moved the number of spaces indicated by the second die. It was interesting for me to see that the strategies they used to add their two numbers seemed to depend on the kind of dice they had. This raised questions for me about how the tools students have inform the strategies they use when practicing their math facts. How do teachers decide which are the best tools to support students as they learn their math facts? Or
do they think of dice as tools that are pretty much the same regardless of how numbers are represented on them? Some of these questions about choosing tools to support fluency in math facts are discussed in Sections 2 and 3, where you'll also find some advice about what different tools offer and how they might be used during classroom activities.

As the teacher circulated, there seemed to be little conversation about the strategies students were using and the extent to which they were drawing on their known number facts. But what incentives were there to draw on any known number facts, especially if their known math facts were not yet solidly held in memory? All of this made me wonder about exactly what students were practicing while they played this game and the extent to which it helped students learn their math facts with greater fluency. We will hear more about how to use games effectively to support math fact fluency in Sections 2 and 3.

## Using "Fun" Worksheets and Puzzles

During one of my school visits, I stop by the copy room, since here I often get some insight into what resources teachers are using to teach math apart from what comes with the district's adopted math curriculum materials. I notice stacks of worksheets that did not copy correctly in the recycling bin. This worksheet is titled "Secret Multiplication Word Puzzle Valentine's Day-Sixes" and contains twenty multiplication problems, written vertically, all including a factor of 6. After students complete the twenty problems, they use a key at the bottom of the worksheet to convert products to letters that then spell out a holiday message.

Math fact worksheets seem to be everywhere, in one form or another. I often find them in copy rooms but I also see plenty of them in teachers' classrooms, organized into stacks at the side or back of the room. When I ask teachers about these worksheets, they often reassure me that these do not replace the math that is part of their lesson, but are designed to provide students with "extra practice" once they finish their work. An added bonus is the fact that many of these worksheets do not even need to be corrected, since checking the decoded message (or
whatever else is built into the worksheet as the "reward") can tell both students and teachers whether success has been achieved.

I've noticed that students often complete these worksheets easily, partly because they might know some facts like $1 \times 6$ and $2 \times 6$ and partly because they can figure out the rest by counting, especially if they skip around the worksheet to do the facts in order, or if they do some skip counting on the side. I've also noticed that students often complete these somewhat leisurely; they are in no big hurry to get them finished, even with a message or some other "reward" for getting them all correct.

But what are students learning by completing these worksheets? Teachers tell me the extra practice provided by these worksheets helps support automaticity with math facts. I imagine that administrators and parents would also agree. But I would guess that if I asked students, "What's $4 \times 6$ ?" they would need to look at their worksheet to find the answer. Just writing down the answers to these multiplication problems doesn't seem to affect what they know or how quickly they know it, especially since they can work backwards from the message to figure out the fact they need. What might be a more meaningful alternative to these holiday worksheets?

## Some Final Questions

In these visits, I can see that teachers value fluency with number facts, and I can see they are committed to helping students learn them, though I wonder how teachers are making sense of the standards indicating that students should learn these facts "from memory." But I also find myself thinking of all the middle school and high school math classrooms that I visit where I see so many students who still, as young adults, struggle with their math fact knowledge. I see some teachers using worksheets even with these middle school and high school students, but more often I see teachers who have given up on helping students learn their number facts, giving them multiplication tables to tape to their desk that they can consult as needed, or handing them
calculators. But why is achieving number fact fluency so difficult for so many of our students? Why is it surrounded with so much stress? Why do so many students come to think of themselves as not good in math because they cannot get these number facts memorized?

It feels like something about the way we approach number fact knowledge is just not working for our students. Writing our facts over and over again, whether this work is a timed test or "dressed up" to seem like fun, doesn't really seem to help. Activities with "cute" contexts for finding number facts are often just an opportunity for students to keep using what they already know, even if it's counting by ones, because there is little incentive to move into less comfortable strategies that are new to them. So how do students learn their number facts? Why don't these kinds of approaches work? What does work? What does the research tell us?

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