first steps®

First Steps in Mathematics

Data Management and Probability

Understand Probability; Collect, Process, and Interpret Data

Improving the mathematics outcomes of students

PEARSON

Government of Western Australia
Department of Education and Training

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First Steps in Mathematics: Data Management and Probability

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Diagnostic Map: Probability—Measuring Chance

Number Sense Phase: *Matching*

Enter: 3–5 years Exit: 5–6 years

- Students use one-to one matching to judge quantity.

Measurement Phase: *Emergent*

Enter: 2–3 years Exit: 5–7 years

Students initially attend to the overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

Number Sense Phase: *Quantifying*

Enter: 5–6 years Exit: 6–9 years

- Students trust the count to describe quantity without variance.

Measurement Phase: *Matching and Comparing*

Enter: 5–7 years Exit: 7–9 years

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to *use counting to directly compare* things so as to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Number Sense Phase: *Partitioning*

Enter: 6–9 years Exit: 9–11 years

- Students use additive thinking to deal with many-to-one relations.

Measurement Phase: *Quantifying*

Enter: 7–9 years Exit: 9–11 years

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Number Sense Phase: *Factoring*

Enter: 9–11 years Exit: 11–13 years

- Students think both additively and multiplicatively about numerical quantities.

Measurement Phase: *Measuring*

Enter: 9–11 years Exit: 11–13 years

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Number Sense Phase: *Operating*

Enter: 11–13 years Exit: —

- Students can think of multiplication and division in terms of operators, and reason proportionately.

Measurement Phase: *Relating*

Enter: 11–13 years Exit: —

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students work with measurement ***information*** and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

Diagnostic Map: Probability

In Probability, students extend what they know about measuring *perceptual* attributes to measuring the *chance* something is likely to happen.

In this way, it follows the same development as Measurement: students become aware of the attribute (Could it happen? Won't it happen? Is it likely?); they compare and order from more to less of that attribute (Which is more likely? Equally likely?); they measure the attribute by comparison with a unit (being able to say how likely), and then to the standard unit (placing events on a scale from 0 to 1); they understand the relationships that enable us to indirectly work out the likelihood.

However, in another way, Probability does not follow the same development as Measurement. The fact that measuring chance is not a perceptual attribute makes it more abstract. As a result, when it comes to this new attribute, there is a lag when students move through the phases of development for Measurement.

Probability: Measuring Chance *Emergent*

Enter: 2–3 years Exit: 7–9 years

Students develop awareness that some things are more and less likely to happen and begin to use some of the comparative language of their communities to describe likelihood.

As a result, they use this type of language themselves and describe familiar, easily understood events as being more or less likely, e.g., Mom said we *might* go to grandma's after school; we are *more likely* to go home than to grandma's; we *usually* go home after school.

By the end of the Emergent phase in Probability, students typically

- are beginning to show that they recognize an element of chance in many things that are a part of their lives
- understand expressions such as "will happen," "won't happen," and "might happen"
- are able to distinguish impossible events from events that are possible but unlikely

But, as they enter the Matching and Comparing phase in Probability, they

- distinguish between certain and uncertain events, but may not realize that certainty must also include events that are certain not to occur
- may be unable to distinguish equally likely events, e.g., may assume all colours are equally likely to appear when given a four-colour spinner with unequal sectors
- may understand that some things are more likely than others, but not be able to provide relevant reasons why events might be more or less likely to occur (e.g., believe they will spin a 6 because 6 is their favourite number)

Probability: Measuring Chance *Matching and Comparing*

Enter: 7–9 years Exit: 9–11 years

Students draw on their experience to describe familiar things as more or less likely. They use expressions such as "very likely," "less likely," "equally likely," and "quite unlikely."

As a result, they are able to directly compare and order events from more to less likely and are able to justify their decision with relevant reasons.

By the end of the Matching and Comparing phase in Probability, students typically

- construct simple experiments and use counting to determine which event is more likely
- understand that that certainty includes those events that must happen and those that cannot happen
- understand what it means for simple events to be equally likely, e.g., can see why a spinner with four equal sectors is equally likely to stop on any colour, and that one divided into unequal sectors will not
- can list all possibilities for straightforward situations when prompted

But, as they enter the Quantifying phase in Probability, they

- may be uncritically influenced by other dominant features when ordering objects by likelihood, e.g., may be influenced by personal preference or personal experience and so say, "It is less likely to rain tomorrow because it never rains on my birthday" or "I'm more likely to roll a 6 because I always roll a 6"
- may construct an experiment to determine likelihood, but be casual about ensuring fairness, even to the point of altering or fixing outcomes to produce the predicted results

Probability: Measuring Chance *Quantifying*

Enter: 9–11 years Exit: 11–13 years

Students connect the idea of likeliness to the frequency of an event. They come to understand that repeated trials provide a count that can predict future likeliness. This count enables two things to be compared without directly comparing them.

As a result, they trust information gained from repeated trials as an indicator of probability and are prepared to use this to order events and determine how likely they are.

By the end of the Quantifying phase in Probability, students typically

- draw on personal experience to compare and order a variety of chance-related events and order them along a continuum that acts as an informal scale
- draw on numerical information alone to decide whether two simple events are or are not equally likely to occur
- are careful to ensure that an experiment is fair, e.g., that the sections of a spinner are equal or that the coin toss is consistent
- systematically list all possibilities, unprompted, to work out numerical probabilities for one-stage actions
- use experimental results to determine a range of possible outcomes and informally use relative frequency to estimate probabilities

But, as they enter the Measuring phase in Probability, they

- may be unable to create devices such as spinners or bags of coloured balls to produce specified orders of probability, e.g., make a spinner which is most likely to come up red and equally likely to come up blue or green
- may trust numerical information but may not be able to accurately order events where the total number of trials is not the same, e.g., might say that an event that occurs 8 out of 12 times is more likely to occur than an event that occurs 5 out of 7 because 8 is greater than 5

Probability: Measuring Chance *Measuring*

Enter: 11–13 years Exit: 15+ years

Students begin to quantify the chance of events occurring using probability as a measure for "how likely" or "how much more likely." They realize they can produce ratios by comparing the total number of occurrences to the total number of trials (experimental probability) or by comparing the number of desired outcomes to the number of possible of outcomes (theoretical possibility).

As a result, students understand that probability is the way we measure chance and that probability statements give a measure of how likely something is to happen.

By the end of the Measuring phase in Probability, students typically

- use a range of information sources to put things in order from least likely to most likely, e.g., use research data or experimental data to form conclusions
- understand that the greater the number of trials, the greater its reliability as an indicator of likelihood
- use their understanding of equivalent fractions to judge equally likely events
- interpret the 0 to 1 scale in general usage and understand why the probability that a toss of a fair die will produce 5 is one-sixth
- identify all the outcomes for two- or three-stage situations, e.g., rolling a die and tossing a coin

But, as they enter the Relating phase in Probability, they

- may not be able to overcome deep personal instincts about the likeliness an event should occur in spite of experimental or theoretical data
- may not recognize or trust calculations that would determine all possible outcomes for multiple-stage situations
- are able to simulate a situation where it would be difficult, costly, or inappropriate to generate real data, by designing simple experiments that replicate a significant aspect of the situation; they use their understanding of ratios and numerical probabilities

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Diagnostic Map: Data Management

Number Sense Phase: *Matching*

Enter: 3–5 years Exit: 5–6 years

- Students use one-to one matching to judge quantity.

Measurement Phase: *Emergent*

Enter: 2–3 years Exit: 5–7 years

Students initially attend to the *overall appearance* of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than as describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

Implications for Data Management

By the end of this phase, students are in a position to

- look at physical displays of familiar data and say which is most or has more
- sort and arrange data they have collected into familiar groupings
- count when asked to say how many in each group in a data display

But, students

- may count when asked to say how many in pre-arranged data, but focus on overall physical size rather than the numerical size of the group e.g., count 3 bananas and 5 strawberries, then say there are more bananas
- may focus on overall physical size rather than the numerical size of the group
- may say one group has more than another, but cannot say how much more
- may lay out objects, but lack the intention to compare

In Data Management, students use their understanding of Measurement to gather, compare, represent, and interpret data. In this way, students' understanding of Data Management is dependent upon their understanding of Measurement, which is dependent upon their Number sense.

As a result, the phases outlined in the Measurement Diagnostic Map should be considered when interpreting students' responses to Data Management activities. Doing so will help in understanding why some students may struggle to achieve certain outcomes while others do not.

The accompanying chart shows what students will be in a position to do and understand as they move through an appropriate program in Data Management, phase by phase.

Ultimately, Data Management becomes a quest to determine the likelihood or chance of something happening; it is intricately connected to Probability.

Number Sense Phase: *Quantifying*

Enter: 5–6 years Exit: 6–9 years

- Students trust the count to describe quantity without variance.

Measurement *Matching and Comparing*

Enter: 5–7 years Exit: 7–9 years

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to *use counting to directly compare* things: to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Implications for Data Management

By the end of this phase, students are in a position to

- suggest counting as a way of answering data questions that focus on comparing collections, e.g., will suggest counting our pets to answer the question "Which pets are more popular?"
- use skip counting to say how many in a tally
- suggest direct comparison when prompted to record growth data, e.g., we can cut a streamer to match the sunflower plant each week to see how much it grows
- use counting to help construct their data display, e.g., construct a block graph by counting how many in each group, then counting how many squares to colour in
- understand the need for a baseline and space blocks regularly to allow comparisons to be made
- place direct measurement data in sensible sequences using a baseline, e.g., cut paper strips to fit around their heads and make a bar (column) graph by lining up the bottom of strips
- choose to count to compare the sizes of groups, without prompting
- look at a bar graph and say which bar has more based on its length

But, students

- may not attend to equal units when grid lines are not provided, e.g., they may create the correct number of pictures for each group, but not use the same size for each picture
- cannot construct a scale on the vertical axis to represent frequencies or measurements (although they can use a common baseline and label the horizontal axis with the groups)
- may not realize that the relative lengths of the bars relate to quantities in the collected data
- may not use a scale on the axis to tell how many, instead preferring to count

Number Sense Phase: *Partitioning*

Enter: 6–9 years Exit: 9–11 years

- Students use additive thinking to deal with many-to-one relations.

Measurement *Quantifying*

Enter: 7–9 years Exit: 9–11 years

Students connect the two ideas of directly comparing the sizes of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Implications for Data Management

By the end of this phase, students are in a position to

- see that, when organizing data, the categories can be reorganized without changing the overall total, e.g., to combine two brands of hamburgers, they need to add the two brands, or if they separate boys' and girls' responses for favourite take-out foods, the total must match the combined result
- see that they need to ensure that the data they collect is consistent
- produce and read pictographs or block graphs where each unit represents more than one piece of data
- produce simple two-way tables and Venn diagrams, partitioning totals between the cells or sections for straightforward data
- recognize that the length of bars in a bar graph can represent any numbers or measurements of data
- represent whole-number data in different ways, e.g., after measuring everyone's height, can produce a measurement graph that shows each child's height, or a frequency graph that shows the number of people of each height
- recognize that the length of bars and simple whole-number scales on the axis refer to quantities in the data collected, e.g., given a frequency graph about people's favourite take-out food, they know that the lengths of the bars represent the number of people naming each food in the survey
- use column and row headings to interpret what the numbers in simple two-way tables represent

But, students

- may be able to represent only whole numbers of units
- may not be able to work out how to represent the data when it is not a multiple of the unit
- may not realize that measurement data can be grouped
- may be unable to represent or read data using a continuous scale
- may not be able to convert to proportional measures to make comparisons
- may be unable to interpret the meaning between marked intervals on scales of frequencies or measures
- reading frequency data in two-way tables may not realize that to make sensible comparisons, the total frequencies need to be taken into account, e.g., students may say that more girls than boys like orange juice because 15 girls and 12 boys say they like it, but not realize that this information is misleading if there are 30 girls and 16 boys in the sample

Number Sense Phase: *Factoring*

Enter: 9–11 years Exit: 11–13 years

- Students think both additively and multiplicatively about numerical quantities.

Measurement *Measuring*

Enter: 9–11 years Exit: 11–13 years

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Implications for Data Management

By the end of this phase, students are in a position to

- think carefully about the accuracy of their data and recognize that data collection is about measuring different aspects of a situation
- understand that they can group measurement data in their display
- create axes that show discrete or continuous quantities, including time scales
- use simple proportional comparisons when interpreting data in tables and graphs, e.g., half as many people prefer pizzas to hamburgers; the height of the wheat is three times higher than it was at the end of week 2

But, students

- may not recognize when they need to convert their data to fractions or percentages to make sensible comparisons

Number Sense Phase: *Operating*

Enter: 11–13 years Exit: —

- Students can think of multiplication and division in terms of operators, and reason proportionately.

Measurement *Relating*

Enter: 11–13 years

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students *work with measurement information* and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

Implications for Data Management

By the end of this phase, students are in a position to

- choose from a wide range of measurement options when planning data investigations, using indirect measurements and creating measurement scales for non-standard attributes, e.g., they may create a scale from 1 to 5 to measure people's concern for environmental issues
- plan complex scales on axes to produce a wide range of graphs, including using class intervals, fractions, and percentages
- represent growth or change data over time by using a time scale, and approximate value within the intervals by joining the points when appropriate
- see that both axes can be made into number or measurement scales and used to show relationships between data, e.g., they could sketch a line graph to show how the amount of food on a plate varies over time at a buffet lunch as the plate is repeatedly filled with food, which is then eaten
- interpret displays showing relational information between measurements or frequencies
- interpret complex scales on graphs where not all scale markings are labelled

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INTRODUCTION

The *First Steps in Mathematics* resource books and professional learning program are designed to help teachers plan, implement, and evaluate the mathematics program they provide for students. The series describes the key mathematical ideas students need to understand in order to achieve the principal learning goals of mathematics curricula across Canada and around the world.

Unlike many resources that present mathematical concepts that have been logically ordered and prioritized by mathematicians or educators, *First Steps in Mathematics* follows a sequence derived from the mathematical development of real children. Each resource book is based on five years of research by a team of teachers from the Western Australia Department of Education and Training, and tertiary consultants led by Prof. Sue Willis at Murdoch University.

The *First Steps in Mathematics* project team conducted an extensive review of international research literature, which revealed gaps in the field of knowledge about students' learning in mathematics. Many of these findings are detailed in the Background Notes that supplement the Key Understandings described in the *First Steps in Mathematics* resource book for Data Management and Probability.

Using tasks designed to replicate those in the research literature, team members interviewed hundreds of elementary school children in diverse locations. Analysis of the data obtained from these interviews identified characteristic phases in the development of students' thinking about mathematical concepts.

The Diagnostic Maps—which appear in the resource books for Number, Measurement, Geometry and Space, and Data Management and Probability—describe these phases of development, exposing specific markers where students often lose, or never develop, the connection between mathematics and meaning. Thus, *First Steps in Mathematics* helps teachers systematically observe not only what mathematics individual children do, but how the children do the mathematics, and how to advance the children's learning.

It has never been more important to teach mathematics well. Globalization and the increasing use of technology have created changing demands for the application of mathematics in all aspects of our lives. Teaching mathematics well to all students requires a high level of understanding of teaching and learning in mathematics and of mathematics itself. The *First Steps in Mathematics* series and professional learning program help teachers provide meaningful learning experiences and enhance their capacity to decide how best to help all students achieve the learning goals of mathematics.

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Chapter 1

An Overview of *First Steps in Mathematics*

First Steps in Mathematics is a professional learning program and series of teacher resource books that are organized around mathematics curricula for Number, Measurement, Geometry and Space, and Data Management and Probability.

The aim of *First Steps in Mathematics* is to improve students' learning of mathematics.

First Steps in Mathematics examines mathematics within a developmental framework to deepen teachers' understanding of teaching and learning mathematics. The developmental framework outlines the characteristic phases of thinking that students move through as they learn key mathematical concepts. As teachers internalize this framework, they make more intuitive and informed decisions around instruction and assessment to advance student learning.

First Steps in Mathematics helps teachers to

- build or extend their own knowledge of the mathematics underpinning the curriculum
- understand how students learn mathematics so they can make sound professional decisions
- plan learning experiences that are likely to develop the mathematics outcomes for all students
- recognize opportunities for incidental teaching during conversations and routines that occur in the classroom

This chapter details the beliefs about effective teaching and learning that *First Steps in Mathematics* is based on and shows how the elements of the teacher resource books facilitate planning and instruction.

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Beliefs about Teaching and Learning

Focus Improves by Explicitly Clarifying Outcomes for Mathematics

Learning is improved if the whole-school community has a shared understanding of the mathematics curriculum goals, and an implementation plan and commitment to achieving them. A common understanding of these long-term aims helps individuals and groups of teachers decide how best to support and nurture students' learning, and how to tell when this has happened.

All Students Can Learn Mathematics to the Best of Their Ability

A commitment to common goals signals a belief that **all** students can be successful learners of mathematics. A situation where less is expected of and achieved by certain groups of students is not acceptable. School systems, schools, and teachers are all responsible for ensuring that **each** student has access to the learning conditions he or she requires to achieve the curricular goals to the best of his or her ability.

Learning Mathematics Is an Active and Productive Process

Learning is not simply about the transfer of knowledge from one person to another. Rather, students need to construct their own mathematical knowledge in their own way and at a pace that enables them to make sense of the mathematical situations and ideas they encounter. A developmental learning approach is based on this notion of learning. It recognizes that not all students learn in the same way, through the same processes, or at the same rate.

Common Curricular Goals Do Not Imply Common Instruction

The explicit statement of the curricular goals expected for all students helps teachers to make decisions about the classroom program. However, the list of content and process goals for mathematics is not a curriculum. If all students are to succeed to the best of their ability on commonly agreed concepts, different program implementations will not only be possible but also be necessary. Teachers must decide what type of instructional activities are needed for their students to achieve the learning goals.

> *A curriculum that enables all students to learn must allow for different starting points and pathways to learning so that students are not left out or behind.*
>
> ***—Darling-Hammond, National standards and assessments, p. 480***

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Professional Decision Making Is Central in Teaching

It is the responsibility of teachers to provide all students with the conditions necessary for them to achieve the curricular goals of mathematics. This responsibility requires teachers to make many professional decisions simultaneously, such as what to teach, to whom, and how, and making these professional decisions requires a synthesis of knowledge, experience, and evidence.

> *Professionalism has one essential feature;…[it] requires the exercise of complex, high level professional judgments…[which] involve various mixes of specialised knowledge; high level cognitive skills; sensitive and sophisticated personal skills; broad and relevant background and tacit knowledge.*
>
> ***—Preston, Teacher professionalism, pp. 2, 20***

The personal nature of each student's learning journey means that the decisions teachers make are often "non-routine," and the reasoning processes involved can be complex. These processes cannot be reduced to a set of instructions about what to do in any given situation. Teachers must have the freedom and encouragement to adapt existing curricula flexibly to best meet their students' needs and to move them forward. The improvement of students' learning is most likely to take place when teachers have good information about tasks, response range, and intervention techniques on which to base their professional decisions.

"Risk" Relates to Future Mathematics Learning

Risk cannot always be linked directly to students' current achievement. Rather, it refers to the likelihood that their future mathematical progress is "at risk."

Some students who can answer questions correctly might not have the depth of understanding needed for ongoing progress. Others might have misconceptions that could put their future learning "at risk." A number of students may make errors that are common when they try to make sense of new mathematical ideas and, therefore, show progress. For example, students may erroneously expect equally likely outcomes to appear equally frequently with small numbers of spins or tosses. However, this expectation signals progress because their idea is closely related to the notion of fairness.

Students who are learning slowly, or whose previous experiences are atypical, might nevertheless progress steadily if their stage of learning is accommodated with appropriate, but challenging, learning experiences.

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Successful Mathematics Learning Is Robust Learning

Robust learning, which focuses on students developing mathematics concepts fully and deeply, is essential if learning is to be sustained over the long term.

A focus on short-term performance or procedural knowledge at the expense of robust knowledge places students "at risk" of not continuing to progress throughout the years of schooling.

Learning Mathematics: Implications for the Classroom

Learning mathematics is an active and productive process on the part of the learner. The following section illustrates how this approach influences the ways in which mathematics is taught in the classroom.

Learning Is Built on Existing Knowledge

Learners' interpretations of mathematical experiences depend on what they already know and understand. For example, when collecting data, young students may focus on overall size rather than the numerical size of the group. Other students may say one group has more than another, but cannot say how much more.

In each case, students' existing knowledge should be recognized and used as the basis for further learning. Their learning should be developed to include the complementary knowledge with the new knowledge being linked to and building on students' existing ideas.

Learning Requires That Existing Ideas Be Challenged

Learning requires that students extend or alter what they know as a result of their knowledge being challenged or stretched in some way. For example, a student may believe that two entirely different events could not have the same chance of happening. Students need both reasoning and practical experiences to develop the understanding that two different events can be equally likely. Or, it may happen when the student finds that peers think about a problem in a different way. The student must find some way of dealing with the challenge or conflict provided by the new information in order to learn.

Learning Occurs when the Learner Makes Sense of the New Ideas

Teaching is important—but learning is done *by* the learner rather than *to* the learner. This means the learner acts on and makes sense of new information. Students almost always try to do this. However, in trying to make sense of their mathematical experiences, some students will draw conclusions that are not quite what their teachers expect.

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Also, when students face mathematical situations that are not meaningful, or are well beyond their current experience and reach, they often conclude that the mathematics does not make sense or that they are incapable of making sense of it. This may encourage students to resort to learning strategies based on the rote imitation of procedures. The result is likely to be short-term rather than effective long-term learning. Teachers have to provide learning experiences that are meaningful and challenging, but within the reach of their students.

Learning Involves Taking Risks and Making Errors

In order to learn, students have to be willing to try a new or different way of doing things, and stretch a bit further than they think they can. At times, mistakes can be a sign of progress. For example, when comparing chance events, students may begin to use numerical information to decide whether events are equally likely, but make mistakes because they do not take into account the total number of trials. Such errors can be a positive sign that students are beginning to recognize that probability can be quantified and are trying to make sense of the process.

Errors can provide a useful source of feedback, challenging students to adjust their conceptions before trying again. Errors may also suggest that learners are prepared to work on new or difficult problems where increased error is likely. Or, learners may try improved ways of doing things that mean giving up old and safe, but limited, strategies. For example, a student who can successfully list the number of possible outcomes for an event takes a risk when using multiplication to find the answer, since multiplying may result in increased mistakes in the short term.

Learners Get Better with Practice

Students should get adequate opportunities to practise mathematics, but this involves much more than the rote or routine repetition of facts and procedures. For example, students may not understand that each toss of the number cube is independent of what happens before or after. They need to compare the variability in small numbers of tosses with the relative predictability of large numbers of tosses.

If students are to develop a rich understanding of data management, they will need spaced and varied opportunities to notice and reject unrealistic results, including things they have not actually seen or experienced. Repetitious procedures of routine questions are unlikely to provide this rich understanding. In fact, they are more likely to interfere with it.

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Learning Is Helped by Clarity of Purpose for Students as well as Teachers

Learning is likely to be enhanced if students understand what kind of learning activity they should be engaged in at any particular time. This means helping students to distinguish between tasks that provide practice of an already learned procedure and tasks that are intended to develop understanding of mathematical concepts and processes. In the former case, little that is new is involved, and tasks are repetitive, so they become habitual and almost unthinking. Students should expect to be able to start almost immediately and, if they cannot, realize that they may need to know more and seek help.

With tasks that are intended to develop understanding, non-routine tasks and new ideas may be involved. Students should not expect to know what to do or to be able to get started immediately.

Students may spend considerable time on a single task, and they will often be expected to work out for themselves what to do. They should recognize that, for such activities, persistence, thoughtfulness, struggle, and reflection are expected.

Teaching Mathematics

Teachers assume considerable responsibility for creating the best possible conditions for learning. The kind of learning tasks and environment teachers provide depend on their own view of how learning is best supported. The perspective that learning is an active and productive process has two significant implications for teaching.

First, teachers cannot predict or control exactly what and when students learn. They need to plan programs that provide students with a wider and more complex range of information and experiences than they would be expected to understand fully at any given time. For example, considerable life experience is involved in distinguishing those things that are subject to chance variation from those that are not. Events that appear unpredictable may simply be those we do not know enough about, and those that we regard as quite predictable can surprise us. By providing these types of experiences, teachers can stimulate their students' curiosity about connections within mathematics, helping students develop notions about probability at many different levels preceding the prescribed teaching of these connections.

> This represents a significant change in program planning. It is a movement away from an approach that exposes students only to content and ideas that they should be able to understand or do at a particular time.

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Second, for students to become effective learners of mathematics, they must be engaged fully and actively. Students will want, and be able, to take on the challenge, persistent effort, and risks involved. Equal opportunities to learn mathematics means teachers will

- provide an environment for learning that is equally supportive of all students
- offer each student appropriate mathematical challenges
- foster in all students processes that enhance learning and contribute to successful achievment of goals

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Understanding the Elements of *First Steps in Mathematics*

The elements of *First Steps in Mathematics* embody the foregoing beliefs about teaching and learning and work together to address three main questions:

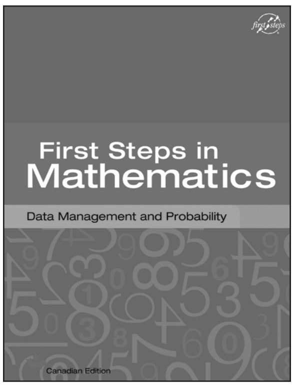
- What are students expected to learn?
- How does this learning develop?
- How do teachers advance this learning?

Learning Outcomes for the Data Management and Probability Strand

The Data Management and Probability strand in *First Steps in Mathematics* focuses on chance events and data-handling processes. In Probability, the focus is on developing students' ability to make predictions about how likely an event is in situations where there is uncertainty. In Data Management, the focus is on collecting, organizing, analyzing, and presenting data. As a result of their learning, students should be able to use their understanding of likelihood to compare and order everyday chance events, and be able to collect, organize, interpret, and present their data.

During the elementary years, students should

- learn to recognize unpredictability in familiar daily activities and refine their use of the language of probability
- describe and order events from least to most likely
- begin to quantify how likely it is that something will happen



Students should also begin to develop the understanding and skills needed to clarify the questions they want answered, and collect and handle data to help answer those questions. They should also consider such questions as "What kind of data?" and "How much do we need so we can feel reasonably confident in our conclusions?" The uncertainty involved in drawing conclusions from data is what connects probability and data. Thus, learning experiences that will enable students to understand chance, collect and organize data, summarize and represent data, and interpret data should be provided.

As a result of their learning experiences, students at all levels should be able to achieve the following outcomes.

Understand Probability

Understand and use the everyday language of probability and make statements about how likely it is that an event will occur based on experience, experiments, and analysis.

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Collect and Process Data

Plan and undertake data collections and organize, summarize, and represent data for effective and valid interpretation and communication. There are two parts to this outcome—Part A: Collect and Organize Data and Part B: Summarize and Represent Data—with a separate chapter for each.

Interpret Data

Locate, interpret, analyze, and draw conclusions from data, taking into account data collection techniques and probability processes involved.

Integrating the Outcomes

The outcomes for Data Management and Probability are each dealt with in separate chapters of this book. This is to emphasize both the importance of each and the difference between them. For example, students need to learn to represent their data in graphs, tables, or diagrams for others to read (Summarize and Represent Data), and to read, analyze, and draw conclusions from data (Interpret Data).

This does not mean, however, that the ideas and skills underpinning each of the outcomes should be taught separately or that they will be learned separately. The links between the outcomes are significant and inevitable. Consequently, many of the activities will provide opportunities for students to develop their ideas about more than one of the outcomes.This will help teachers to ensure that the significant mathematical ideas are drawn from the learning activities so that students achieve each of the outcomes for Data Management and Probability.

How Does This Learning Develop?

First Steps in Mathematics: Data Management and Probability describes characteristic phases in students' thinking about the major mathematical concepts of the Data Management and Probability strand. These developmental phases are organized in Diagnostic Maps.

Diagnostic Maps

These maps help teachers to

- understand why students seem to be able to do some things and not others
- realize why some students may be experiencing difficulty while others are not
- indicate the challenges students need to move their thinking forward, to refine their preconceptions, overcome any misconceptions, and so develop deep reflective understanding about concepts
- interpret students' responses to activities

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Number Sense Phase: *Partitioning*

Enter: 6–9 years Exit: 9–11 years

- Students use additive thinking to deal with many-to-one relations.

Measurement Phase: *Quantifying*

Enter: 7–9 years Exit: 9–11 years

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Number Sense Phase: *Factoring*

Enter: 9–11 years Exit: 11–13 years

- Students think both additively and multiplicatively about numerical quantities.

Measurement Phase: *Measuring*

Enter: 9–11 years Exit: 11–13 years

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Probability: Measuring Chance
Matching and Comparing

Enter: 7–9 years Exit: 9–11 years

Students draw on their experience to describe familiar things as more or less likely. They use expressions such as "very likely," "less likely," "equally likely," and "quite unlikely."

As a result, they are able to directly compare and order events from more to less likely and are able to justify their decision with relevant reasons.

By the end of the Matching and Comparing phase in Probability, students typically

- construct simple experiments and use counting to determine which event is more likely
- understand that that certainty includes those events that must happen and those that cannot happen

Probability: Measuring Chance *Quantifying*

Enter: 9–11 years Exit: 11–13 years

Students connect the idea of likeliness to the frequency of an event. They come to understand that repeated trials provide a count that can predict future likeliness. This count enables two things to be compared without directly comparing them.

As a result, they trust information gained from repeated trials as an indicator of probability and are prepared to use this to order events and determine how likely they are.

By the end of the Quantifying phase in Probability, students typically

- draw on personal experience to compare and order a variety of chance-related events and order them along a continuum that acts as an informal scale
- draw on numerical information alone to decide whether two simpl

The Diagnostic Maps include key indications and consequences of students' understanding and growth. This information is crucial for teachers making decisions about their students' level of understanding of mathematics. It enhances teachers' decisions about what to teach, to whom, and when to teach it.

Each developmental phase of the Diagnostic Maps has three aspects. The first aspect describes the learning focus during the phase. The second aspect details typical thinking and behaviours of students by the end of the phase. The third outlines preconceptions, partial conceptions, or misconceptions that may still exist for students at the end of the phase. This aspect provides the learning challenges and teaching emphases as students move to the next phase.

Diagnostic Tasks

First Steps in Mathematics encompasses series of short, focused Diagnostic Tasks that have been validated through extensive research with students. A number of the Sample Learning Activities in the *Data Management and Probability Resource Book* are based on these tasks and thus can be used by teachers to help them locate individual students on the Diagnostic Maps.

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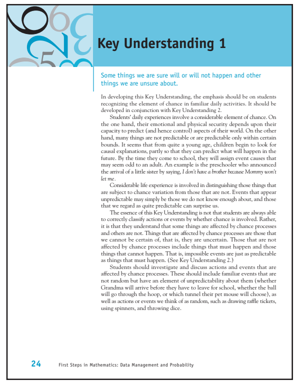
How Do Teachers Advance This Learning?

To advance student learning, teachers identify the big mathematical ideas, or key understandings, of the outcomes, or curricular goals. Teachers plan learning activities to develop these key understandings. As learning activities provide students with opportunities and support to develop new insights, students begin to move to the next developmental phase of mathematical thinking.

Key Understandings

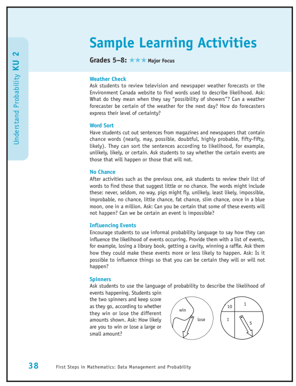

Key Understanding 1

The Key Understandings are the cornerstone of *First Steps in Mathematics*. The Key Understandings

- describe the mathematical ideas, or concepts, which students need to know in order to achieve curricular goals
- explain how these mathematical ideas form the underpinnings of the mathematics curriculum statements
- suggest what experiences teachers should plan for students so that they move forward in a developmentally appropriate way
- provide a basis for the recognition and assessment of what students already know and still need to know in order to progress along the developmental continuum and deepen their knowledge
- indicate the emphasis of the curriculum at particular stages
- provide content and pedagogic advice to assist with planning the curriculum at the classroom and whole-school levels

The number of Key Understandings for each mathematics curricular goal varies according to the number of "big mathematical ideas" students need to achieve the goal.

Sample Learning Activities


Sample Learning Activities

For each Key Understanding, there are Sample Learning Activities that teachers can use to develop the mathematical ideas of the Key Understanding. The activities are organized into three broad groups:

- activities suitable for students in Kindergarten to Grade 3
- activities for students in Grades 3 to 5
- activities for students in Grades 5 to 8

If students in Grades 3 to 5 or Grades 5 to 8 have not had enough prior experience, then teachers may need to select and adapt activities from earlier groups.

Case Studies

The Case Studies illustrate some of the ways in which students have responded to Sample Learning Activities. The emphasis is on how teachers can focus students' attention on the mathematics during the learning activities.

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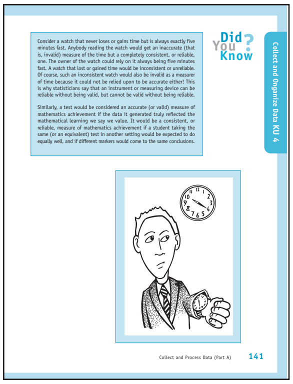
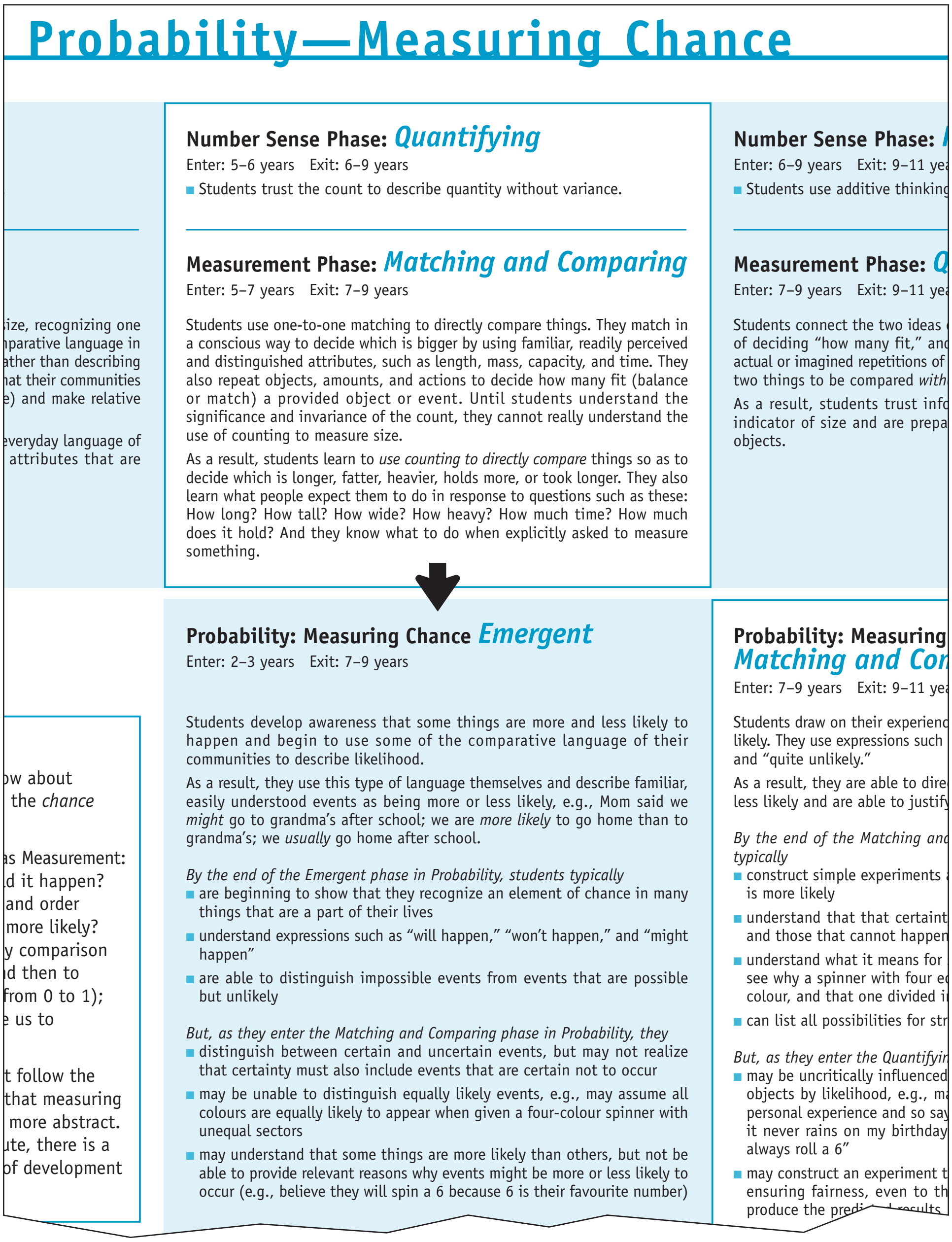


"Did You Know?" Sections

For some of the Key Understandings, there are "Did You Know?" sections. These sections highlight common understandings and misunderstandings that students have. Some "Did You Know?" sections also suggest diagnostic activities that teachers may wish to try with their students.

How to Read the Diagnostic Maps

The Diagnostic Maps for Data Management and Probability have five phases: Emergent, Matching and Comparing, Quantifying, Measuring, and Relating. The diagram on this page shows the second phase, the Matching and Comparing phase.



Probability—Measuring Chance

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hat their communities
e) and make relative

everyday language of
attributes that are

Number Sense Phase: *Quantifying*
Enter: 5–6 years Exit: 6–9 years
- Students trust the count to describe quantity without variance.

Measurement Phase: *Matching and Comparing*
Enter: 5–7 years Exit: 7–9 years

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to *use counting to directly compare* things so as to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Number Sense Phase: /
Enter: 6–9 years Exit: 9–11 ye
- Students use additive thinking

Measurement Phase: Q
Enter: 7–9 years Exit: 9–11 ye

Students connect the two ideas
of deciding "how many fit," an
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two things to be compared *with*

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Probability: Measuring Chance *Emergent*
Enter: 2–3 years Exit: 7–9 years

Students develop awareness that some things are more and less likely to happen and begin to use some of the comparative language of their communities to describe likelihood.

As a result, they use this type of language themselves and describe familiar, easily understood events as being more or less likely, e.g., Mom said we *might* go to grandma's after school; we are *more likely* to go home than to grandma's; we *usually* go home after school.

By the end of the Emergent phase in Probability, students typically
- are beginning to show that they recognize an element of chance in many things that are a part of their lives
- understand expressions such as "will happen," "won't happen," and "might happen"
- are able to distinguish impossible events from events that are possible but unlikely

But, as they enter the Matching and Comparing phase in Probability, they
- distinguish between certain and uncertain events, but may not realize that certainty must also include events that are certain not to occur
- may be unable to distinguish equally likely events, e.g., may assume all colours are equally likely to appear when given a four-colour spinner with unequal sectors
- may understand that some things are more likely than others, but not be able to provide relevant reasons why events might be more or less likely to occur (e.g., believe they will spin a 6 because 6 is their favourite number)

Probability: Measuring
Matching and Co
Enter: 7–9 years Exit: 9–11 ye

Students draw on their experienc
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and "quite unlikely."

As a result, they are able to dire
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By the end of the Matching and
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it never rains on my birthday
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produce the predi... results

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The text in the "During" section of each phase describes students' major preoccupations, or focuses, during that phase of thinking about Data Management or Probability.

The "By the end" section of each phase provides examples of what students typically think and are able to do as a result of having worked through the phase.

The achievements described in the "By the end" section should be read in conjunction with the "But as they enter" section. Together, these two sections illustrate that although students might have developed a range of important understandings as they passed through the phase, they might also have developed some unconventional or unhelpful ideas. Both of these sections are intended as a useful guide only. Teachers will recognize more examples of similar thinking in the classroom.

How Do Students Progress through the Phases?

Students who have passed through one phase of the Diagnostic Maps are entering the next phase. They bring behaviours and understandings from one phase to the next. For example, the text in the "But as they enter the Matching and Comparing phase in Probability" section describes the behaviours students bring to the Matching and Comparing phase. This section includes the preconceptions, partial conceptions, and misconceptions that students may have developed along the way. These provide the learning challenges for the next phase.

Linking the Diagnostic Maps and Learning Goals

Students are unlikely to achieve full conceptual understanding unless they have moved through certain phases of the Diagnostic Maps. However, passing through the phase does not guarantee that the concept has been mastered. Students might have the conceptual development necessary for deepening their understanding, but without access to a classroom program that enables them to learn the necessary foundation concepts described in a particular phase, they will be unable to do so.

The developmental phases help teachers interpret students' responses in terms of pre- and partial conceptions. For example, a student may assume that more girls than boys play volleyball because 10 girls and 6 boys in their survey play volleyball, but the student may not consider that there were 30 girls but only 12 boys in the survey. In this case, the students may not be through the Measuring phase.

How Will Teachers Use the Diagnostic Maps?

The Diagnostic Maps are intended to assist teachers as they plan for mathematics teaching and learning. The descriptions of the phases help teachers make informed decisions about students' understandings of the mathematical concepts. The maps will help teachers understand why students can do some things and not others, and why some students may be having difficulty achieving certain outcomes.

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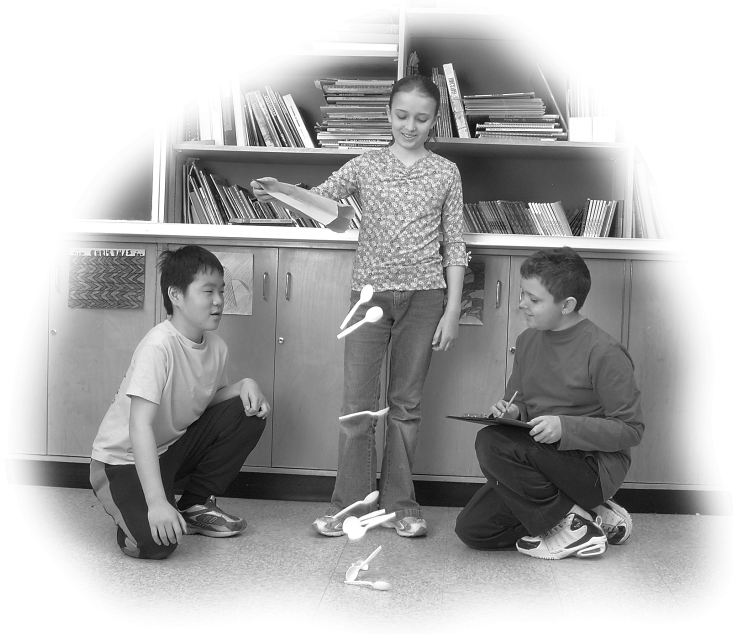
Initially, teachers may use the Diagnostic Maps to extend their own knowledge about how students typically learn mathematics. Knowing about the major conceptual shifts in each phase and their links to mathematical learning goals will help teachers decide which Key Understandings should be the major focus for classroom planning.

Familiarity with the behaviours described in the phases will enhance the informed decisions teachers make about what they observe students doing and saying during lessons. The information obtained over time about the major preoccupations of students informs ongoing planning. As teachers begin to understand the typical behaviours of each phase, this planning process will become more efficient.



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Planning with *First Steps in Mathematics*

Using Professional Decision-Making to Plan

The *First Steps in Mathematics* resource books and professional development support the belief that teachers are in the best position to make informed decisions about how to help their students achieve conceptual understanding in mathematics. Teachers will base these decisions on knowledge, experience, and evidence.

The process of using professional decision-making to plan classroom experiences for students is fluid, is dependent on the situation and context, and varies from teacher to teacher. The selection of learning activities and appropriate focus questions will be driven by each teacher's knowledge of his or her students and their learning needs, the mathematics, and mathematics-related pedagogy. The *First Steps in Mathematics* resource books and professional development focus on developing this pedagogical content knowledge.

The diagram on the next page illustrates how these components combine to inform professional decision-making. There is no correct place to start or finish, or process to go through. Circumstances and experience will determine both the starting point and which component takes precedence at any given time.

Different teachers working with different students may make different decisions about what to teach, to whom, when, and how.

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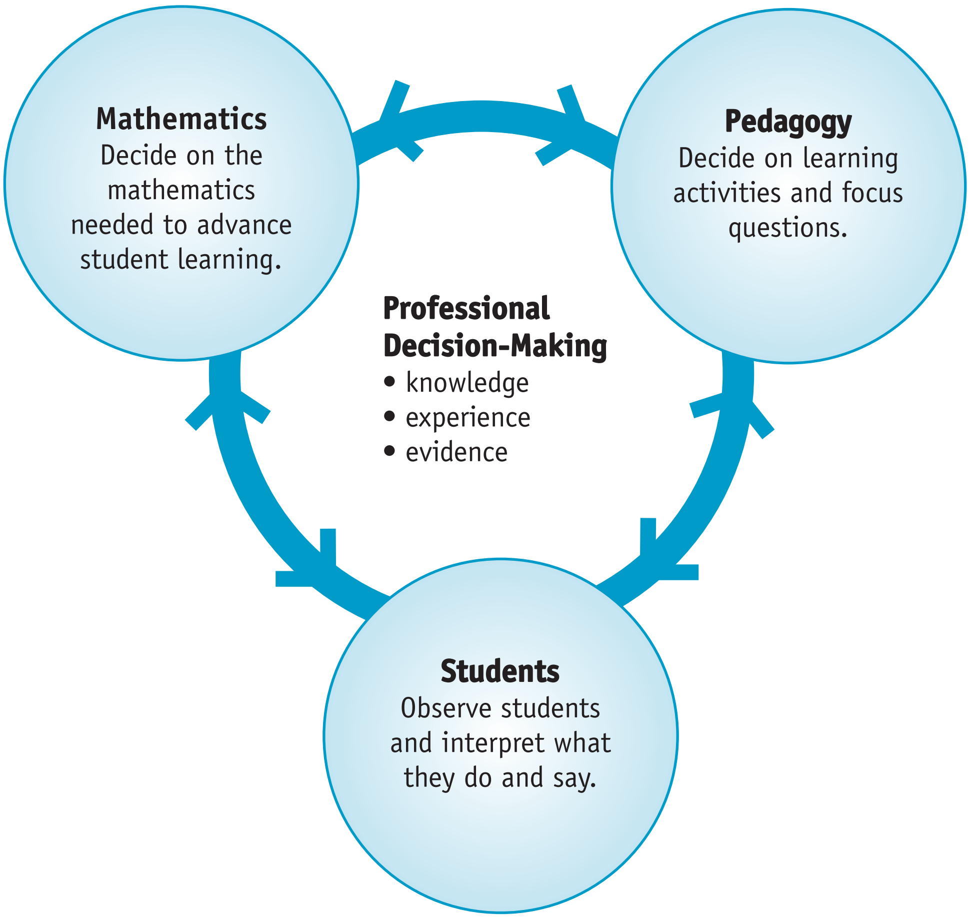
The process is about selecting activities that enable all students to learn the mathematics described in curriculum focus statements. More often than not, teachers' choice of activities and focus questions will be driven by their knowledge of their students and the mathematics. At other times, teachers might select an activity to help them assess students' existing knowledge or because of the specific mathematics in the task. Whatever the starting point, the *First Steps in Mathematics* resource books and professional development will help teachers to ensure that their mathematics pedagogy is well informed.

The examples on the opposite page show some of the different ways teachers can begin planning using *First Steps in Mathematics*.



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Focusing on the Mathematics

Teachers may choose to focus on the mathematics, deciding on the mathematics they think they need to move students on.

What mathematics do my students need to know?

Mathematics
Decide on the mathematics needed to advance student learning.

What sections of ***First Steps in Mathematics*** do I look at?

- Key Understandings and Key Understandings descriptions

Understanding What Students Already Know

Teachers may choose to start by finding out what mathematics their students already know.

What do my students know about these mathematics concepts?

Students
Observe students and interpret what they do and say.

What sections of ***First Steps in Mathematics*** do I look at?

- Key Understandings and Key Understandings descriptions
- "Did You Know?" sections
- Diagnostic Maps
- Diagnostic Tasks

Developing Students' Knowledge

Teachers may begin by planning and implementing some activities to develop their knowledge of students' learning.

What activities will help my students develop these ideas? How will I draw out the mathematical ideas from the learning activity?

Pedagogy
Decide on learning activities and focus questions.

What sections of ***First Steps in Mathematics*** do I look at?

- Sample Learning Activities
- Case Studies
- Key Understandings and Key Understandings descriptions

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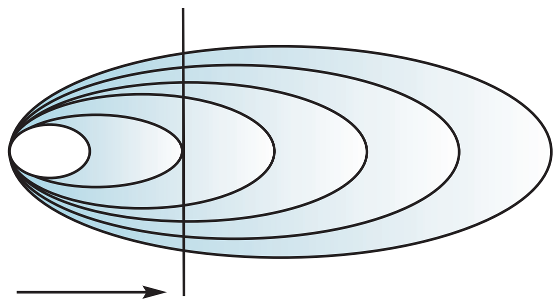
Planning

The mathematics curriculum goals and developmental phases described in the Diagnostic Maps help teachers to know where students have come from and where they are heading. This has implications for planning. While day-to-day planning may focus on the mathematics of particular Key Understandings, teachers must keep in mind the learning required for progression through the developmental phases.

If a student has reached the end of the Matching phase, for example, then the majority of experiences the teacher provides will relate to reaching the end of the phase. However, some activities will also be needed that, although unnecessary for reaching the Matching and Comparing phase, will lay important groundwork for reaching the Quantifying phase and even the Measuring phase.

For example, students do not need to know that when they organize data, the categories can be reorganized without changing the overall total, in order to reach the end of the Matching and Comparing phase, but they do need to know this to reach the end of the Quantifying phase.

Developmental Phases



There are a number of reasons for this approach. First, it is expected that a considerable number of students will enter the middle years of elementary school having reached the end of the Quantifying phase. Second, if teachers are to wait until this time to start teaching about the consistency of the overall total in a graph, then it is unlikely that those students would develop all the necessary concepts and skills in a timely fashion. Third, work in the middle years of elementary school should not only focus on the Quantifying phase, but also provide the groundwork for students to reach the Measuring phase in the next year or two, and the Relating phase some time later.

Teachers who plan on the basis of deepening the understanding of the concepts would think about the expected long-term learning in the early years of schooling. They would provide experiences that lead to the learning goals at the Quantifying and Measuring phases. This means students may be challenged about the significance of the consistency of the total in a graph despite the rearrangement of data. They may not yet be ready to use this knowledge to solve particular problems. It will take several years of

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learning experiences in a variety of contexts to culminate in a full understanding

Monitoring Students over Time

By describing progressive conceptual development that spans the elementary school years, teachers can monitor students' individual long-term mathematical growth as well as their long-term progress against an external standard. This long-term monitoring is one of the reasons why a whole-school approach is essential. For example, Sarah has reached the end of the Measuring phase for each of the Data Management and Probability concepts while another student, Maria, has only just reached the Matching and Comparing phase.

By comparing the girls' development against the standard, their teacher is able to conclude that Sarah is progressing as expected, but Maria is not. This prompts Maria's teacher to investigate Maria's thinking about Data Management and Probability and to plan specific support.

However, if two years later, Sarah has not reached the end of the Relating phase while Maria has reached the end of the Quantifying phase and is progressing well towards reaching the Measuring phase, they would both now be considered "on track" against an external standard. Sarah's achievement is more advanced than Maria's, but in terms of individual mathematical growth, Sarah appears to have stalled. Her progress may now be of greater concern than Maria's.

Reflecting on the Effectiveness of Planned Lessons

The fact that activities were chosen with particular mathematical learning goals in mind does not mean that they will have the desired result. Sometimes, students deal with an activity successfully, but they use different mathematics than teachers anticipated. Different activities related to the mathematics not learned may need to be provided in the future.

On other occasions, what students actually learn may not be what teachers intended them to learn. Students may surprise teachers and cause them to rethink the activity.

In some instances, activities that teachers think will help students develop particular mathematical ideas do not generate those ideas. This can occur even when students complete the activity as designed.

The evidence about what students are thinking and doing during their learning experiences is the most important source of professional learning and decision-making. At the end of every activity, teachers need to ask themselves: *Have the students learned what was intended for this lesson? If not, why not?* These questions are at the heart of improving teaching and learning.

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Teachers make constant professional, informed evaluations about whether the implemented curriculum is resulting in the intended learning goals for students. If it is not, then teachers need to change the experiences provided.

Teachers' decisions, when planning and adjusting learning activities as they teach, are supported by a clear understanding of

- the desired mathematics conceptual goal of the selected activities
- what progress in mathematics looks like
- what to look for as evidence of students' deepening understanding

When planning day-to-day lessons, it is important for teachers to appreciate that many of the same activities will be appropriate for students who are working within a range of developmental phases. Teachers can accommodate the differences in understanding and development among students by

- asking different questions of individual students and groups of students
- providing extension activities for selected students
- giving particular students opportunities to do different things with the activities

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Chapter 2

Understand Probability

Understand and use the everyday language of chance and make statements about how likely it is that an event will occur based on experience, experiments, and analysis.

Overall Description

Students recognize that many situations are somewhat unpredictable: these include scoring a goal, getting caught in rain on the way home from school, and getting a good hand in a game of cards. They make appropriate use of the everyday language of chance, such as "might," "could," "likely and unlikely," "certain and uncertain," "possible and impossible," "probable," "odds," and "fifty-fifty." They realize that situations with uncertain individual outcomes may show long-term patterns in their behaviour and that we use these patterns to help interpret data and make predictions in order to address questions such as these: "How many mice will we have by next month? What will the weather be like for our celebration in June? How long will the battery last?"

Students compare events using numerical and other information to order the events from those least likely to happen to those most likely to happen. They know that probability is the way we quantify how likely it is that something will happen, and they can interpret the probability scale from 0 to 1. They estimate probabilities from experiments and simulations using the long-run relative frequency. They also use systematic lists, tables, and tree diagrams to help them analyze and explain possible outcomes of simple experiments, and to calculate probabilities by analysis of equally likely events.

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Understand Probability:
Key Understandings Overview

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU). These Key Understandings underpin achievement of this family of concepts. The learning experiences should connect to students' current knowledge and understandings rather than to their grade level.

Key Understanding		Description
KU1	Some things we are sure will or will not happen and other things we are unsure about.	page 24
KU2	There are special words and phrases we use to describe how likely we think things are to happen.	page 32
KU3	We can compare and order things by whether they are more or less likely to happen.	page 42
KU4	We say things have an equal chance of happening when we think they will happen equally often in the long run.	page 56
KU5	We can use numbers to describe how likely something is to happen.	page 64
KU6	Sometimes we list and compare all the possible things that could happen to predict how likely something is to happen.	page 72
KU7	Sometimes we use data about how often an event has happened to predict how likely it is to happen in the future.	page 80

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Grade Levels— Degree of Emphasis			Sample Learning Activities
K-3	3-5	5-8	
★★★	★★★	★★	K-Grade 3, page 26 Grades 3-5, page 28 Grades 5-8, page 30
★★★	★★★	★★★	K-Grade 3, page 34 Grades 3-5, page 36 Grades 5-8, page 38
★★	★★★	★★★	K-Grade 3, page 44 Grades 3-5, page 46 Grades 5-8, page 49
★★	★★★	★★★	K-Grade 3, page 58 Grades 3-5, page 60 Grades 5-8, page 62
★	★★	★★★	K-Grade 3, page 66 Grades 3-5, page 67 Grades 5-8, page 69
★	★★	★★★	K-Grade 3, page 74 Grades 3-5, page 75 Grades 5-8, page 77
★	★★	★★★	K-Grade 3, page 82 Grades 3-5, page 84 Grades 5-8, page 86

Key

★★★ **Major Focus**
The development of this Key Understanding is a major focus of planned activities.

★★ **Important Focus**
The development of this Key Understanding is an important focus of planned activities.

★ **Introduction, Consolidation, or Extension**
Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

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Key Understanding 1

Some things we are sure will or will not happen and other things we are unsure about.

In developing this Key Understanding, the emphasis should be on students recognizing the element of chance in familiar daily activities. It should be developed in conjunction with Key Understanding 2.

Students' daily experiences involve a considerable element of chance. On the one hand, their emotional and physical security depends upon their capacity to predict (and hence control) aspects of their world. On the other hand, many things are not predictable or are predictable only within certain bounds. It seems that from quite a young age, children begin to look for causal explanations, partly so that they can predict what will happen in the future. By the time they come to school, they will assign event causes that may seem odd to an adult. An example is the preschooler who announced the arrival of a little sister by saying, *I don't have a brother because Mommy won't let me*.

Considerable life experience is involved in distinguishing those things that are subject to chance variation from those that are not. Events that appear unpredictable may simply be those we do not know enough about, and those that we regard as quite predictable can surprise us.

The essence of this Key Understanding is not that students are always able to correctly classify actions or events by whether chance is involved. Rather, it is that they understand that some things are affected by chance processes and others are not. Things that are affected by chance processes are those that we cannot be certain of, that is, they are uncertain. Those that are not affected by chance processes include things that must happen and those things that cannot happen. That is, impossible events are just as predictable as things that must happen. (See Key Understanding 2.)

Students should investigate and discuss actions and events that are affected by chance processes. These should include familiar events that are not random but have an element of unpredictability about them (whether Grandma will arrive before they have to leave for school, whether the ball will go through the hoop, or which tunnel their pet mouse will choose), as well as actions or events we think of as random, such as drawing raffle tickets, using spinners, and throwing dice.

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Links to the Phases

Phase	Students who are through this phase in Probability...
Emergent (Early)	■ are beginning to show that they recognize an element of chance in many things that are a part of their lives ■ understand expressions such as "will happen," "won't happen," and "might happen" ■ are beginning to show some understanding that repetitions of chance actions are likely to produce different results
Emergent (Late)	■ can list things that might happen in relation to daily events or actions
Matching and Comparing	■ have a reasonable sense of the difference between certainty and uncertainty
Quantifying	■ understand that the essential nature of chance processes means things that are very unlikely are still possible and things that are very likely may not happen

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Sample Learning Activities

K–Grade 3: ★★★ Major Focus

Guess What

Help students to distinguish between situations they feel sure about, and those they are unsure about. For example, when playing Guess Who or Guess What games, let students see you put a toy car under the cover. Ask: Can you be sure what is under the cover? Could it be anything else? Prompt students to close their eyes while you put something under the cover and ask them to guess what it is. Ask: Can you be sure what is under the cover this time? Draw out the difference between being certain and making a guess.

Is It Possible?

Extend the Guess What game above by asking students to think about what could be under the cover. For example, place a pillow under a blanket and ask: Could there be an elephant under the cover? Why not? What could possibly be under there? Have students take turns to suggest what it could be.

Certain or Uncertain?

Use everyday classroom events to talk about uncertainty, contrasting these with things we can be sure about. For example: Who is going to be the messenger next week? We know the answer because it is written on the bulletin board. Or: We cannot be sure who is going to be the sports monitor because we have not yet decided.

Could, Will, Will Not

Give students activity cards showing different events, such as riding a bike, watching television, and playing in the rain. Ask them to sort the cards into piles depending on whether the event could happen, will happen, or will not happen tomorrow. Ask: Why do you think that?

Challenges

When students tell news, challenge their causal explanations, whether or not you think they are valid, to provoke them to explain. For example, a student may say *Our new baby will be a boy because that is what Dad wants.* Ask: Does that mean it has to happen? Can Dad really decide whether the baby is going to be a boy or a girl? Challenge the student in cases where the explanation may be reasonable (e.g., *The doctor told Mommy and Daddy the baby will be a girl.*).

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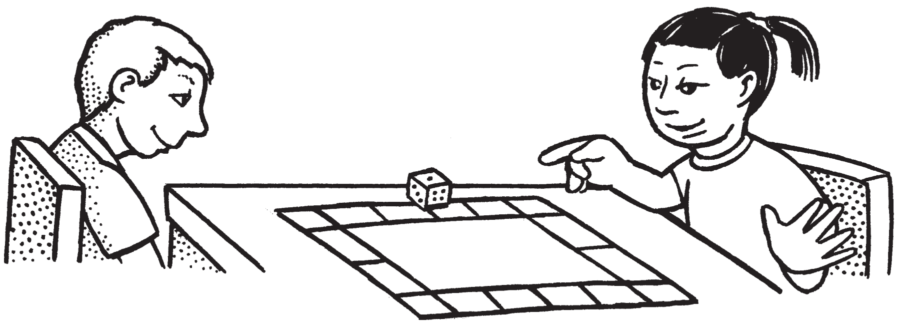
Board Games

Invite students to play board games that require a particular number to come up on a number cube. Afterwards, ask: Can you make sure that the number you want will come up? Does the position of the number cube before you throw it make a difference? Does wishing for your number help? Have students test each suggestion several times to see if it helps the number to appear.



Tosses

Have students toss several red/blue counters and try to get all counters to land with one colour face up. Ask: Can you do anything to make sure that you get one colour every time you toss the counters? Why can you not be sure that you will get exactly what you want to come up?

Who Will It Be?

When choosing students for tasks, place some names into a hat and ask: Which name do you think will be picked? Could we arrange it so that name does come out first?

Random Events

Invite students to predict the outcome of random events, then test their predictions. For example, give students a box of buttons and ask them to predict which colour they will get if they pick one without looking. Ask: What colour do you hope you will get? What colour could you get? Will Anne get a red one because that's her favourite colour?

What Colour?

After activities like the previous one, ask students to think about what might happen if the event is repeated. For example, ask: Liam got a red button that time. It is Steven's turn next, so will he get a red one, too? Could he get a different colour? Test their predictions and ask: Why did Steven not get the same colour as Liam?

Story Predictions

While reading a story to the students, ask them what might happen next and what cannot possibly happen next, for example, after Goldilocks breaks Baby Bear's chair. Ask: Do you think the bears will come back? Do you think Goldilocks will talk to the bears if they come back or will she run away? Could the bears talk?

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Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Who Will It Be?

Extend *Who Will It Be*?, page 27, by asking students to think about the possibilities for the events that are certain. For example, you could say: We know it is not Paulo. He did it this week and his name does not go in the hat. We know it is not Inez because she is not in our class.

Is It Possible?

Extend *Guess What,* page 26, by showing students a range of objects (pencil case, apple, ruler). Give them clues that allow them to eliminate certain possibilities that might be under the cover. For example, if the clue is "You cannot eat it," they can be certain that the object is not the apple. Draw out that while they cannot be certain what is under the cover, there are some things it certainly is not.

Tricky Clues

Extend the previous activity by introducing clues that are more ambiguous, such as "I have one of these at home." Include items that may or may not fit this criterion. Ask, for example: Can you really be certain it is not a skateboard? Is it possible I have one at home? Draw out the difference between unlikely and impossible events.

Could, Will, Will Not

Extend *Could, Will, Will Not*, page 26, by asking students to list other activities that could, will, or will not happen after school today, and explain why. Draw out that while some students can be certain some things will happen, others can be equally certain they will not happen.

Are You Sure?

After the previous activity, ask students to consider the events they said will happen and decide whether they can really be certain about them happening. For example, you could ask: You said you will watch television this afternoon, but could something happen to make that change? Can you really be certain?

Possible or Impossible?

Ask students to use the words "possible" and "impossible" to explain their choices when classifying books as fiction or non-fiction. Draw out that non-fiction books are always about possible events, but fiction can include impossible events, some of which we can be certain will not happen.

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Hangman

Invite students to play Hangman. Give them the first letter of the word and ask them to guess each letter in order. Ask: If our first letter is "g," which letters would it be possible to have next? Which would be impossible? How do you know?

More Hangman

Extend the previous activity by allowing students to use a list of letter frequencies they have created from samples of text, or obtained from a website, to help them guess which letters are in the word. Ask: Does knowing which letters are used most often help you predict the letters in this word? Draw out that letter frequency helps us decide which letters are more likely, but it does not help us determine the correct letters for sure.

Will It Happen?

Ask students to determine whether they can be certain that familiar events will happen. Mention events that are certain to happen (the sun will rise tomorrow) and those that are certain not to happen (if I drop this glass, it will float in the air). Mention a range of uncertain events, too. Ask students to explain their decisions.

Tug of War

Conduct a class tug of war contest. After students have predicted which teams they think will win, compare predictions with results. Ask: Did the team you thought was most likely to win actually win? Why do you think this happened? Draw out that some things that may seem certain always have an element of probability affecting the outcome.

Who's Next?

Invite students to predict who will be the next person to walk through the door. If they predict correctly, ask: How did you know that (person X) was going to come into our room? If they predict incorrectly, ask: Why can you not be sure who will come into our room next? Draw out that some things happen by chance and are not predictable.

Weather Forecasting

Have students create a list of different weather types, including fog, hail, snow, sunshine, cloud, and drizzle. Ask them to choose one as their weather prediction for the next day. After a couple of weeks of weather predictions, ask students to consider how predictable the weather was. Ask: Why were we sometimes wrong? Is it possible to predict the weather, or is it unpredictable?

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Sample Learning Activities

Grades 5–8: ★★ Introduction, Consolidation, or Extension

Timetables

Ask students to write a timetable showing what they expect to happen in their next school day. Ask: Are you certain about any of these events? How can you be certain? Which events are you uncertain about? What do you need to know to be certain that these events will happen?

What Might Happen?

While reading a shared fiction story, ask students to predict what might happen next and to create a list of suggestions. Have students review the list to decide which events are possible and which are impossible in real life. They can review their list of possible events and say which are likely and which are highly unlikely. Then they can review the list of impossible events and say how they can be certain the events are impossible.

Predict and Test

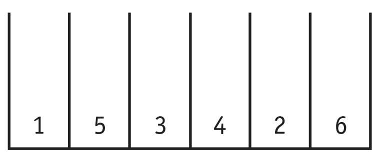
Ask students to predict and test the outcome of chance events. For example: Draw names out of a hat, replacing the name and shaking the hat each time, and have students list the names as they are pulled out. Before each name is pulled out, have them predict if it will be the name of a boy or a girl. Ask: Can we be certain? What if we drew out the names of four boys in a row? Would that change the likelihood of a boy's or a girl's name being drawn out next? *(No, because the number of boys' and girls' names in the hat hasn't changed.)* Under what circumstances could we be certain that the next name will be a girl's? *(If the names were not replaced each time and all the boys' names had been pulled out.)*

Luck of the Draw

Have students investigate the number of tickets that prize-winners have bought in a raffle. Ask them whether increasing the number of tickets bought increases the chances of winning. Ask: Is there any way to be certain you will win?

Possible or Impossible?

Ask students to decide whether events (including some that are very unlikely) are possible or impossible. For example, consider tossing balls at a target marked as below. Ask: Using five balls, is it possible to get a score of 50? of 5? How do you know? Is it really impossible to get a score of 50, or just very unlikely?



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Certain or Uncertain?

Invite students to classify the following situations as certain or uncertain:

- A porcelain plate will break if dropped.
- You roll a number cube and get a 8.
- You draw a green marble from a bag of only red marbles.
- You get either a head or a tail when you toss a coin.
- It will rain tomorrow.
- Tomorrow you will be a day older.
- You will have homework tonight.

Ask: What factors do you have to think about when you make your decision? Will the plate always break, or does it depend on the type of floor you drop it on? Ask students to re-sort the certain situations into "certain to happen" and "certain not to happen."

Coloured Tiles

Have students analyze various lucky draw situations, with and without replacing the selected object each time. In the situation where the object is not replaced, ask students to estimate how many times they would have to draw a coloured tile to be certain they would get, say, a red tile (from a selection of five yellow tiles, two blue tiles, and one red tile). Ask: Why can we not say the same thing for the situation where we replace the tile each time? Draw out that it is possible to not draw a red tile.

Chance Cards (1)

Invite students to give examples of situations that match different likelihoods. For example, students take turns to select a chance card from a collection labelled "certain," "uncertain," "very likely," "likely," and "very unlikely." On a blank card, they write an example of a situation that matches their chance card. Ask them to share and justify the situations they wrote about. Draw out that very unlikely situations are still possible and very likely situations may not happen.

Chance Cards (2)

Extend the previous activity. Ask students to exchange their situation cards with another group to sort in their own way. Have students compare their categories. Ask: Did you sort into certain/uncertain or possible/impossible? Encourage students to question the categories selected by others. Ask: Why did you choose certain, possible, and impossible? Draw out that impossible situations are also certain: they are certain not to happen.

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Key Understanding 2

There are special words and phrases we use to describe how likely we think things are to happen.

The language of chance is widely used in a colloquial way, as in these examples: "Tomorrow will probably be sunny" or "It is a sure thing." In developing this Key Understanding, the emphasis should be on clarifying, refining, and extending students' use of this everyday language of chance in conjunction with development of Key Understandings 1 and 3.

The students' discussions of chance aspects of daily life and of their experiments should be based on natural language use, and ideas should be expressed in their own terms. Students need to hear terms such as "*possible*," "*impossible*," "*unlikely*," "*likely*," "*certain*," "*probable*," and "*improbable*" modelled, and have opportunities to practise using them appropriately in context. From time to time, explicit attention should be paid to the use of expressions such as "*unlikely*," "*it might happen*," "*being lucky*," "*that's not fair*," "*always*," and "*tomorrow it will probably rain*." For example, students in the middle and upper elementary years might be asked to justify and explain their use of certain expressions based on past experience and the range of possible outcomes (*It is unlikely that we will go to Sam's house after school today because we usually only see him on the weekend*), rather than on idiosyncratic associations based on single occurrences or feelings (*I bet it's going to rain today because I really want to play soccer*).

Students might try to place chance expressions in order from "certain not to happen" to "certain to happen." (There is unlikely to be one right order; the intention is to consider the ways we use such expressions.)

The binary pairs "possible-impossible" and "certain-uncertain" are important, but they are different (see diagram, page 33) and will each require specific attention. Students who appear to distinguish things that must happen from things that might happen may have difficulty with the "possible-impossible" distinction. Some may stretch the meaning of "possible" to include almost anything and hence describe what adults regard as impossible events as "might happen but do not." Conversely, others may believe that things they have never personally experienced or do not wish to happen, cannot happen, and so stretch the meaning of impossible to include things that are possible.

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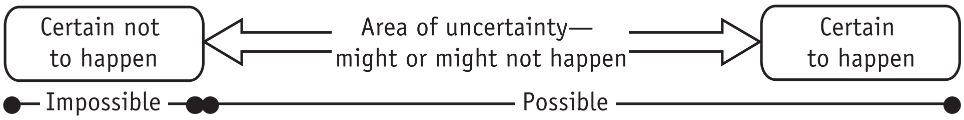
Many students also find the notion that impossible events are "certain" quite difficult. They should learn that, in English, we use the word "certain" to describe events about which we believe there is no doubt, including those that must happen and those that cannot happen (*I am certain that I cannot get a black button because there are none in the bag*).



Links to the Phases

Phase	Students who are through this phase in Probability...
Emergent (Early)	■ show that they understand expressions such as "will happen," "won't happen," and "might happen" by how they respond and use them
Emergent (Late)	■ are beginning to understand the idea of "impossible" and to distinguish impossible things from those that are possible even if unlikely *For example:* A student can tell you that it is impossible to get a white button from a box that does not contain any white buttons.
Matching and Comparing	■ distinguish certain from uncertain things and know that certain events include those that must happen and those that cannot
Quantifying	■ can sensibly sequence chance-related expressions along a continuum
Measuring	■ understand that probability is the way we measure chance; that is, probability statements give a numerical measure of how likely something is to happen

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Sample Learning Activities

K–Grade 3: ★★★ Major Focus

Language of Certainty

During conversations and news, respond to students' recounts, requests, and descriptions of situations with comments such as: Did that really happen? I was not certain that you would be able to do that. Who else thought he could not do that? Use language such as this: Are you certain you left it? If you are uncertain about how much you will need, go and check, then come back when you know, that is, when you are certain.

Please Explain

During conversations and news, respond to students' use of certainty language by encouraging them to explain their meaning. For example: You said you will definitely go for a swim after school today. What did you mean?

Likelihoods

Repeat the previous activity, but focus on language describing the likelihood of events. If a student says, *I am probably going to the zoo this weekend,* say: I understand it is quite likely you are going to the zoo. How do you know it is quite likely?

Picture Sort

Have students sort cards that depict related events, some of which are obviously impossible and others possible, for example, an elephant sitting in a bus, a child sitting in a bus. Ask students to explain why they think the event is impossible or possible. (See Case Study 1, page 40.)

Pickles to Pittsburgh

Read *Pickles to Pittsburgh* by Judi Barrett or a similar book to students and ask them to identify the impossible events. Ask: How do you know it is impossible to snow popcorn? Why?

What Will Happen? (1)

Provide students with cards showing pictures of classroom activities such as using a calculator, reading a book, and playing with play dough. Prompt them to suggest words that describe the likelihood of each event. Ask: Will these things happen? Do we really know they will happen, or are we better to say they might happen?

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What Will Happen? (2)
Repeat the previous activity for events that have been ordered. For example, you said you were more likely to do math tomorrow at school than watch a video. Could we say we will probably do math? Is it possible that we will not do math?

Lucky Draw
Invite students to wrap a number of items for a lucky draw game and then ask others to guess what might be in the packages. The students who made the game then use chance words to say whether each suggestion is possible or impossible, certain, certain not to be, or probable. Record each example and then sort them to say what has or has not been used in making the game. Then ask the students playing the game to say what they can possibly get in the lucky draw.

Uncertainty Words
Collect uncertainty-related words students use in their everyday speech, such as "maybe," "could be," "might," "perhaps." Use the words to stimulate storytelling, drawing attention to the difference between events students have some control over and events that just happen.

Is It Safe?
Ask students to help design a circuit of play equipment using tires, boards, ladders, climbing frames, and so on. Help them check the safety of their suggestions by asking: Is it likely that children will fall off if the board is steeper? How do you know? Where could the board go to make it less likely that children will fall? If we use Clare's suggestion and put the tires in a zigzag, are more children likely to want to play on it?

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Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Correct Language
Ask students to consider whether everyday use of chance language is correct. For example, say: Jamila said there is no way she could win the raffle—it is impossible. Do you agree with her? Can she be certain she will not win the raffle? How do you know? Draw out that events that have not happened yet or have not happened in students' experience are not necessarily impossible.

Possible Outcomes
During conversations and news, ask students to justify their use of chance language by referring to past experience or possible outcomes. For example, you said you were certain that your sports team would win this week. What made you so sure? Ask: Is that the only thing that could happen? Can you really be certain? Model appropriate justifications that refer to possible outcomes or to past experience.

Story Predictions
When reading stories, ask students to list events that could occur next. Encourage them to consider events that might happen, but do not happen often. Draw out that these events are still possible.

Possible or Impossible?
Present students with scenarios they will not expect to happen and discuss whether they are possible or impossible. You might say: Sam said that it is impossible that an eight-year-old boy would drive a car. Was he right? Draw out that events might be possible even if you have never seen them happen or do not expect them to happen.

It Could Happen
Ask students to list events that they are certain will happen, that might happen, and that they are certain will not happen. Before a field trip, for example, have students list various events and ask them: How do we know these events will happen? Can we really be certain? At the end of the field trip, compare these lists with what actually happened. Were there some events that could have happened, but did not?

Weather Forecasting
Extend *Weather Forecasting,* page 29, so that students choose a weather type for the following day and then say how likely they think it is to happen. Record their responses and then consider why they all indicate a high likelihood of the event.

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Lucky Tickets

Have students investigate the number of tickets prize-winners have bought in a raffle. Ask: Does increasing the number of tickets you buy increase your chances of winning? Is there any way you can be certain you will win?

Likely Events

Ask students to collect magazine cuttings and sort them into those depicting events that cannot happen (impossible) and those events that can happen (possible). Focus on their explanations and draw out the difference between impossible and highly unlikely. Ask: Can this ever happen anywhere? Could someone else ever see such a thing happening?

Sporting Events

Have students review newspaper articles on sports and find words used to describe likelihood. Make a class collection of words and then place these in groups showing examples of words that seem to indicate the same likelihood.

Science Certainties

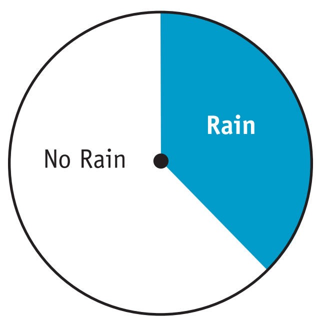
When watching science documentary videos, such as about space exploration, ask students to make a list of the words used to indicate that some events are not certain. Ask: Why would these words be used?

Did You Know?

We sometimes use the word "random" as though it means haphazard, but randomness involves a kind of order that emerges in the long run. To a statistician, a phenomenon is random if individual outcomes are uncertain, but the long-term pattern of many individual outcomes is predictable.

We often deliberately plan randomness to produce a particular long-term pattern. For example, we may know from local meteorological records that it rains on one in three days in April. We could produce a spinner with a two-thirds sector shaded yellow to represent no rain and a one-third sector shaded grey to represent rain. We could use it to simulate a series of days and to study possible weather patterns for April in our location. The spinner represents the two possible unequal outcomes (rainy and not rainy). The result of any particular spin will be uncertain but, in the long term, one in three spins will result in grey representing a rainy day.

Another example involves number cubes that must be perfectly balanced. In this case, the intention is to ensure that each of the six faces is equally likely to come up in the long run. Again, while the individual outcomes are unpredictable, the number cubes are carefully made to ensure predictability in the long-term pattern of outcomes.



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Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Weather Check

Ask students to review television and newspaper weather forecasts or the Environment Canada website to find words used to describe likelihood. Ask: What do they mean when they say "possibility of showers"? Can a weather forecaster be certain of the weather for the next day? How do forecasters express their level of certainty?

Word Sort

Have students cut out sentences from magazines and newspapers that contain chance words (nearly, may, possible, doubtful, highly probable, fifty-fifty, likely). They can sort the sentences according to likelihood, for example, unlikely, likely, or certain. Ask students to say whether the certain events are those that will happen or those that will not.

No Chance

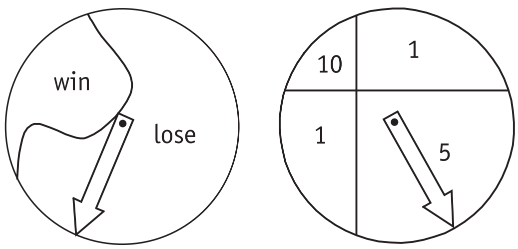
After activities such as the previous one, ask students to review their list of words to find those that suggest little or no chance. The words might include these: never, seldom, no way, pigs might fly, unlikely, least likely, impossible, improbable, no chance, little chance, fat chance, slim chance, once in a blue moon, one in a million. Ask: Can you be certain that some of these events will not happen? Can we be certain an event is impossible?

Influencing Events

Encourage students to use informal probability language to say how they can influence the likelihood of events occurring. Provide them with a list of events, for example, losing a library book, getting a cavity, winning a raffle. Ask them how they could make these events more or less likely to happen. Ask: Is it possible to influence things so that you can be certain they will or will not happen?

Spinners

Ask students to use the language of probability to describe the likelihood of events happening. Students spin the two spinners and keep score as they go, according to whether they win or lose the different amounts shown. Ask: How likely are you to win or lose a large or small amount?



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Fitting Expressions

Have students suggest events they think fit some of the expressions below, and discuss with their peers about why they think it so:

- a small chance
- quite certain
- more than likely
- a good chance
- almost certain
- not much chance
- highly improbable
- extremely likely
- a very good chance
- very likely
- fairly likely
- in all probability
- highly unlikely
- likely
- a 50-50 chance
- pigs might fly
- even chance
- probably
- not in a month of Sundays

Chance of Rain

Invite students to use words and phrases such as those above to give meaning to numerical statements about chance. For example, you could say: There is a 10 percent chance of rain in the next week. Does this mean it is highly likely it will rain in the next week? Does it mean it certainly will not rain? What do you think it might mean? Extend to include other numerical statements. For example:

- The chance of getting a red tile is 1 chance out of a possible 10.
- The chance of one person drawing a heart from a hand of cards is 2 in 7.
- The chance of getting a blue button is 3 out of 10.

Ask: What do these statements mean? Are they likely or unlikely, or is there an even chance that they might or might not happen?

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CASE STUDY 1

Case Study Learning Activity: K–Grade-3—Picture Sort, page 34

Key Understanding 2: There are special words and phrases we use to describe how likely we think things are to happen.

Working Towards: Matching and Comparing Phase

TEACHER'S PURPOSE

I noticed that many of the students in my Grade 1/2 class used words like "impossible" and "no chance" in their everyday talk, but I wondered what distinctions they made between possible and impossible events. Children often hear adults say things like "I do not have a chance of winning the lottery," even when they have a ticket. I realized that students may not often be exposed to the real meaning of impossibility as referring to things that cannot happen. I decided to provide an activity that could stimulate discussion about what is possible and what is not.

I planned to later ask the students to re-sort the collection of possible events into those that must happen and those that might happen. They could then begin to understand certainty in chance as a general concept that included all the things that cannot happen as well as all the things that must happen. Knowing that these ideas are difficult to grasp, I initially focused only on the cannot happen/can happen categories.

ACTION AND REFLECTION

The students and I spent some time cutting out pictures from magazines. I made sure the collection depicted a wide range of events, including some that seemed to me to be obviously impossible.

I suggested that we create a collage on the bulletin board under two headings: Possible (can happen) and Impossible (cannot happen). I held up several pictures and asked students to think about whether what they saw in the picture could really happen in real life. "Is it really possible for a cow to jump over the moon like that, or is it impossible? Why do you think that?"

Pairs of students took piles of cuttings and sorted them into the two categories. They were to tell each other what they could see happening in each picture, then decide if it should go in the cannot happen pile or in the can happen pile.

Erin had an advertisement for milk that showed a drawing of a cow in a busy city street. She insisted it went in the cannot happen pile, while her partner Jason argued that it should go with the can happen pictures. I asked Erin to explain why she thought it impossible.

"Because cows have to live on farms, not in the city. There is no grass to eat," said Erin. Then I asked Jason to explain his reasoning. "It might have fallen off a truck or something," he said. "A truck crashed and some cows ran off, I saw it on television." Erin seemed to be focusing on what should happen, rather than what could happen, while Jason seemed to have a more conventional way of reasoning about what is possible.

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DRAWING OUT THE MATHEMATICAL IDEA

I asked a "what if" question to draw out the distinction further. What if it was a tiger? Erin was sure that would also belong in the cannot happen pile, because tigers live in a zoo and they can kill people. Interestingly, Jason switched his decision to cannot happen.

"I have only seen a tiger in a zoo, or it could be in a circus but they would not let a tiger be there," he explained, pointing at the picture. I realized Jason's *judgements* were based more on direct personal experience than on what might be possible, with the result that his reasoning seemed to be inconsistent.

The personalized ways Erin and Jason made their judgements were what I expected, but I decided to help them extend their ideas about impossibility. I asked Rachid, who was sitting at the next desk listening to the conversation, what he thought. (Because of an earlier conversation with Rachid, I guessed he would be able to give a more conventional explanation.)

"No, it has to be in the can happen pile. It is just standing there in the picture. It is not driving a car or anything impossible. Cows and tigers could be standing like that anywhere, any animals could be," explained Rachid.

Erin was not really convinced; her ideas were still focused on what normally happens—cows and tigers do not belong in the city. Jason, though, was ready to be swayed by Rachid's explanation, and expanded on the idea. "And lions and elephants and pigs—they would be 'can happen' as well. They can get there, maybe on a truck or something. It is only silly things in 'cannot happen'—animals cannot drive cars, they cannot talk, they cannot play baseball." He had taken on a more general notion of impossibility.

I helped Jason notice and resolve an anomalous or conflicting idea without correcting him or simply telling him how to do it. Jason resolved his own cognitive dissonance, thereby constructing his own correct knowledge.

CHALLENGING CURRENT IDEAS

I then asked another question to further challenge their understanding: "What about if it was a dinosaur just standing there, not doing anything silly?" Jason hesitated, looking at Rachid for guidance. Rachid, though, was quite sure about this situation as well: "Dinosaurs got extinct and they are only bones now so it has got to be a cannot happen thing." Jason was happy to agree.

I asked Rachid to tell the whole class his reasoning about Erin's and Jason's picture, and I went on to reinforce this more conventional way of thinking about impossibility. There was opportunity to revisit the language when the collage was in place, and we began to try to separate the Possible group into those things that might happen and those things that must happen.

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Key Understanding 3

We can compare and order things by whether they are more or less likely to happen.

Just as we can compare and order objects and spaces according to size, or events by how long they take, we can compare and order events by how likely they are to happen.

In developing this Key Understanding, students should be assisted to draw on their experience to describe familiar things as more or less likely. For example, they could say that where they live, it is more likely to snow in December than in June. Having compared two events in this way, they should be assisted to put several events in order from those they think least likely to those they think most likely. For example, they could use expressions such as "very likely," "quite likely," "equally likely," "quite unlikely," "very unlikely" (see Key Understanding 2) to describe and order unrelated events:

- We will do some mathematics in school today.
- The egg will crack if I drop it.
- It will rain today.

They could also order related everyday events such as the likelihood of four possible destinations after school (home, the mall, the pool, a friend's house), explaining their reasoning.

Just as we would help students first develop the idea of area by comparing regions of obviously different areas, so too the idea of "how likely" will be best developed if initially the events being ordered are obviously different in likelihood. As suggested in Key Understanding 1, students' lack of experience may result in their suggesting orders that seem odd to an adult. The criteria for evaluating the order of events that students produce should relate to whether their explanations show an understanding of the idea of "more likely," rather than whether they understand the events themselves or have sufficient experience or knowledge to make an accurate assessment of likelihood.

As they gain experience, students should be asked to order events that are closer in likelihood and, during the middle and upper elementary years, begin to place events in order based on numerical or measurement information provided to them, or on frequency data collected from their experiments. They should also develop the understandings necessary to make probability devices such as number cubes, spinners, or bags of coloured tiles to produce specified orders of probability (e.g., make a spinner that is most likely to come up red and equally likely to come up blue or green).

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Links to the Phases

Probability Phase	Students who are through this phase in Probability....
Emergent (Late)	■ understand some things are more likely than others
Matching and Comparing	■ draw on their personal experience to describe familiar things as more or less likely, and provide relevant reasons (1)
Quantifying	■ are able to use a range of sources of information to put things in order from least likely to most likely (2)

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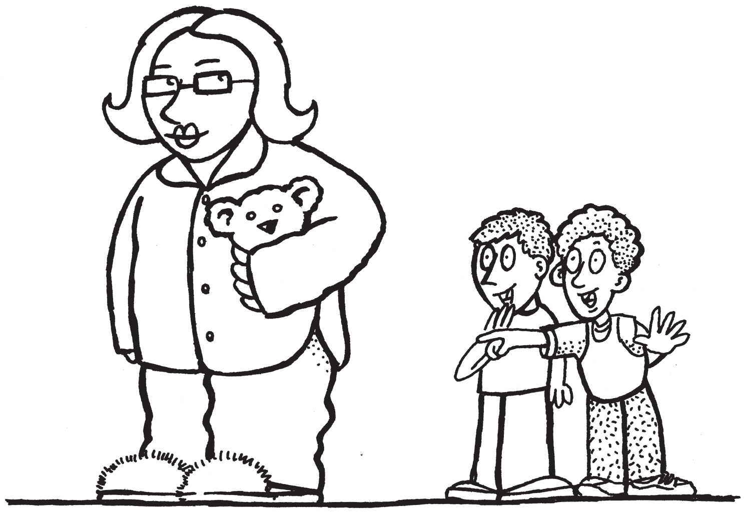
Sample Learning Activities

K–Grade 3: ★★ Important Focus

More or Less Likely? (1)
Ask students which of two unrelated events is more likely. For example: Is it more likely that we will start school on time tomorrow or that the principal will come to school in her pyjamas? Choose events that are obviously more or less likely, since the intention is to focus on the meaning of the expression "more likely."



More or Less Likely? (2)
Repeat the previous activity for two related events. For example: Are you more likely to go straight home after school today or to drive to your cottage? How do you know you will not go to your cottage?

What Will Happen?
Provide students with cards showing pictures of classroom activities such as using a calculator, reading a book, and playing with play dough. Ask them to order the cards from most likely to least likely to happen at school tomorrow. Ask: What makes you think that we are very likely to read a book tomorrow? (See Case Study 2, page 52.)

What Is for Lunch?
Before lunchtime, ask students to predict what they are most likely to have for lunch. Record their predictions and the actual contents of their lunch for a week. Ask: How did you know what you were likely to get for lunch? What are you most likely to get tomorrow?

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More Lunch
Following the previous activity, ask students to list what they are most likely and least likely to have for lunch for the next week, and the reasons for each decision. Ask: How often do you think you will get the items on your "most likely" list? How often will you get the items on your "least likely" list? Why do you think that?

Story Predictions
When reading a story to students, stop periodically and ask them to predict what might happen next. Write each suggestion on a card and have students order the cards to say which is most likely and least likely to happen next. Ask: What is in the story that helps you to decide which is most or least likely?

After School
Ask students to record their after-school activities for a week or so, and then predict what they are likely to be doing over the next few afternoons. Ask: What are you most likely to be doing? Why do you think it is likely? Why do you think [name an activity] is more likely than [name another activity]?

Two Steps Forward
Have students toss an uneven object, such as bottle cap, button, or chalkboard eraser. When it lands face up, the thrower takes two steps forward and when it lands face down, the thrower takes two steps backward. Have students play with a partner and see who can be first to a given line. Ask: Is the bottle cap likely to land face up or face down? Which side is more likely? Would it be better to take forward steps when it lands face down instead?

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Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Will It Rain?

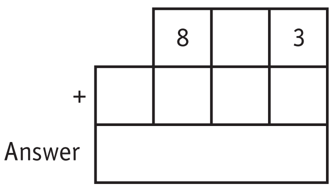
Ask students whether they think it is likely to rain today/tomorrow. Ask: What helps you make your prediction? (Talk about seasons and weather predictions.) Have students record their predictions and compare with the actual weather.

Tug of War

Hold a class tug of war competition, with students predicting which team will win each week. Ask: How did you decide which team is more likely to win? Discuss which predictions were based on evidence and which were based on such things as loyalty. Ask students which strategy is likely to give the best predictions.

How Can It Change?

Have students brainstorm factors that might change the probability of an event happening. For example: What could you do to make it less likely that you will have an accident in the playground, or will be late for school?

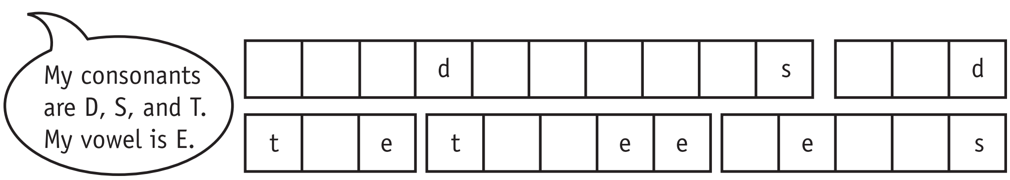


Number Cube Games

Have students toss a 10-sided number cube and put the digits into a grid (see left) to build a three-digit number and then a four-digit number that will add to the largest amount. Ask: If the first number is 3, where would you put it? If the next number is 8, where would you put it? Would you be likely or unlikely to get a number larger than 8? For each toss of the number cube, consider the likelihood of other numbers coming up that are larger or smaller.

Wheel of Fortune

Play a "Wheel of Fortune"–type word-guessing game on the whiteboard. Allow students to choose, for example, three consonants and one vowel. Place these letters in the appropriate slots and ask children to guess the word. Ask: How did you know which letters to choose? What would happen if you pulled letters out of a hat instead of choosing them? Draw out the fact that some letters are more common and thus more likely to be needed. (Link to *Hangman*, page 29.)



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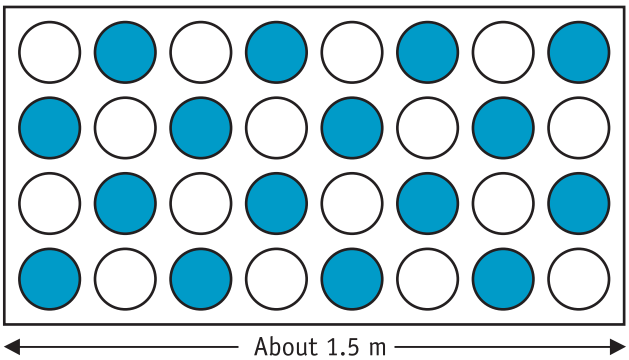
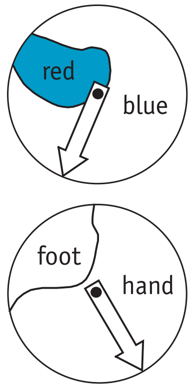
Weather Forecasting

Ask students to list different types of weather conditions and then place them in order from the least to the most likely to occur tomorrow. Ask them to justify their decisions.

Round the Twist

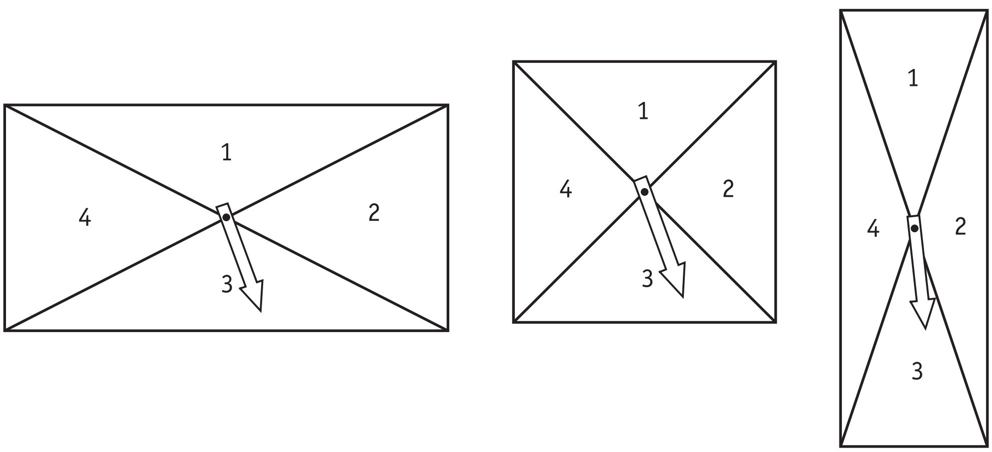
Construct a modified version of Twister™ using a 4 x 8 array of red and blue circles. Provide students with two spinners like those below. To play, the teacher spins both spinners and a student follows the instruction, that is, puts a foot or hand on a red or blue spot. This is repeated for several more students, then turns are rotated through the players, each trying to follow the instructions on the spinners without falling or knocking other players over. After the game, ask: Are you more likely to get red or blue? Are you more likely to get a hand or a foot? Why?





Making Spinners

Invite students to make their own spinners and use them for games such as Snakes and Ladders. For example: Rectangular spinners can be made using a lid from a shoebox by marking the centre point of the rectangle and connecting this with the corners (see below). Have students compare a spinner with unequal sections with one that has equal sections and say why some numbers are more likely to come up than others. Ask: If you wanted to get a 4, which spinner would you choose to use?



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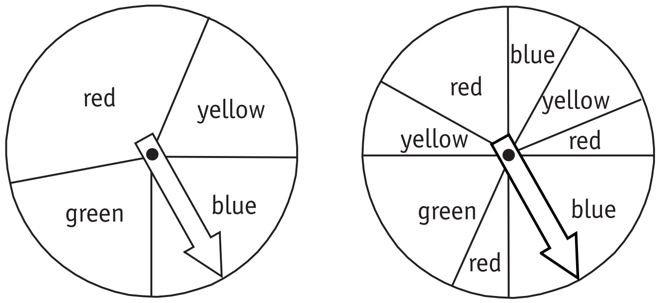
Grades 3–5: ★★★ Major Focus

Comparing Spinners

Show students a range of spinners, some with four equal sectors, others with unequal sectors. Ask: If you need red to win a game, which spinner would you choose? Why?



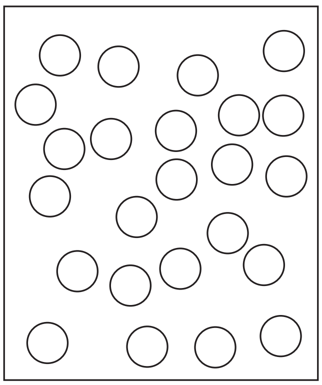
Pattern Block Pictures

Ask students to make a picture using pattern blocks and a cube that has its faces marked with three triangles, two hexagons, and one rhombus. They roll the cube a set number of times, collecting the shapes that appear. They then create their picture or design with those shapes. Ask: What shapes are impossible to get? Which shape are you most likely to get? Why can you not be certain you will have a hexagon in your picture?

Coloured Candies

Give students diagrams of a number of scattered circles to represent a collection of coloured candies. Say: Pretend the candies will be put in a bag and you will be selecting one without being able to see which one you are selecting. Colour them to produce these results:

- Colour the candies so you would be certain to get red.
- Colour the candies so that it would be impossible to get green.
- Colour the candies so you would be more likely to get red than green, and more likely to get green than blue.
- Colour the candies so you would be equally likely to get red, green, or blue, but could not get any other colour.



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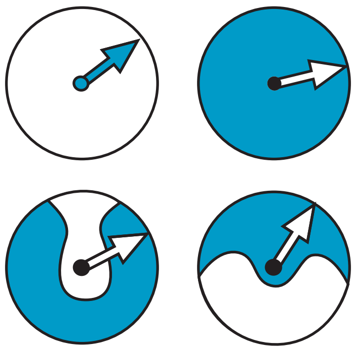
Sample Learning Activities

Grades 5–8: ★★★ Major Focus

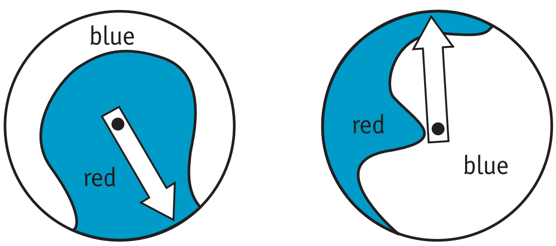
Ordering Spinners

Have students order spinners (see below) according to how likely they are to result in particular outcomes. For example: You need to spin white to win. Put the spinners in order according to which one you would rather have. Ask: Why did you put them in that order? If you needed blue to win, how would the order change? Why?



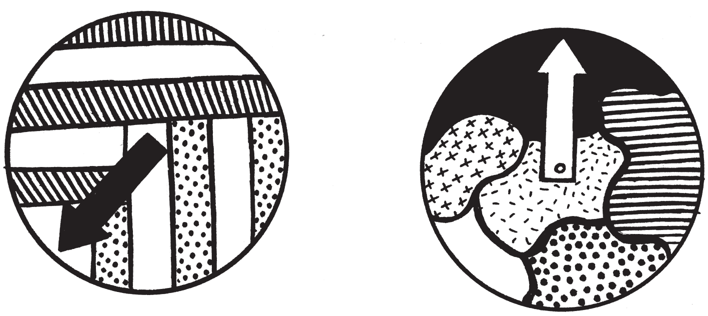
Which One?

Present students with spinners like the ones below. Ask: If you had to choose a colour to spin before you could start a game, which colour would you choose from the first spinner? Which would you choose from the other spinner? Why?



Wrapping Paper Spinners

Ask students to use patterned wrapping paper to make spinners. Without looking at the pattern, they can cut out an indiscriminate circle and place a spinner in the centre. Ask: Which colour is most likely/least likely to come up? Why?



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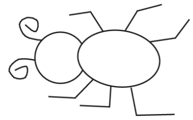
Grades 5–8: ★★★ Major Focus

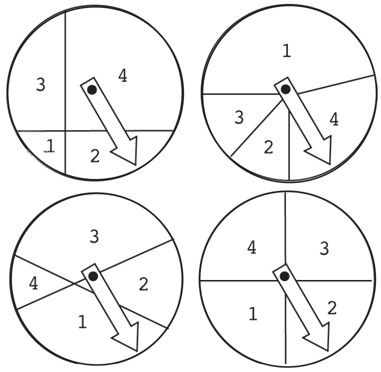
Different Spinners

Present students with a variety of spinners designed to point to numbers from 1 to 4. Ask students to choose one of the spinners to use in a game such as Make a Bug (see below). To start the game, they need to throw a 1 for the body and then roll a 2 to draw a head, 3 for feelers, and 4 for legs. Ask: Which spinner would you choose if you want to draw a head? Which would give you the best chance of being able to draw a feeler? Which spinner would you choose if you could use only one throughout the game? Would you choose a different spinner if you could draw the parts in any order? Why?

To make a bug, you need

One body - spin 1 for a body
One head - spin 2 for a head
Two feelers - spin 3 for each feeler
Six legs - spin 4 for each leg





Cards

Ask students to compare the likelihood of drawing particular cards in different situations. For example: When selecting one card from a hand of five spades, three hearts, and one club, which suit is most likely to be drawn next? Which suit is least likely to be drawn next? Why do you think that?

Influencing Events

Invite students to discuss how they can influence the likelihood of events occurring. Provide them with a list of events, such as losing a library book, getting a cavity, and winning a raffle. Ask students how they could change the circumstances surrounding these events to make them more likely and then less likely to happen.

Make a Spinner

Have students make probability devices, such as number cubes, spinners, or bags of coloured tiles, so that some events will have more chance of occurring than others. For example: Make and test a spinner that you think is most likely to stop on red, least likely to stop on green, and have the same chance of stopping on yellow and blue. Ask students to compare the different ways they constructed their spinners.

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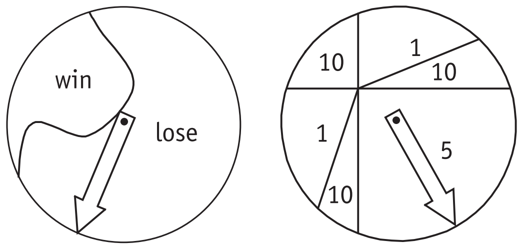
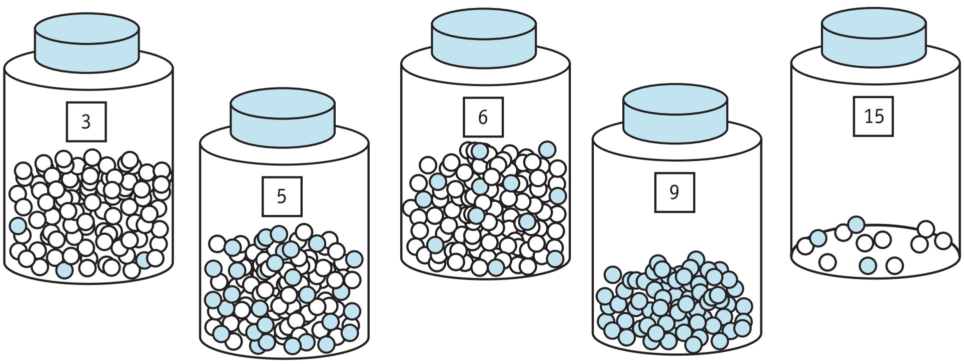
Spinners

Prompt students to examine the spinners below and determine the most likely outcome. Ask: Are you more likely to win an amount or lose an amount? Are you more likely to spin a 10, a 5, or a 1, or are they equally likely to come up? Explain your thinking.



Chance Situations

Invite students to examine and compare different chance situations and predict which one is more likely to generate a specific event. For example: Provide a range of large, sealed plastic jars that contain different quantities and proportions of red and white beads. Have students work with two jars at a time, handling them and turning them around to examine the beads inside. Ask: If you put your hand into a jar and pulled out a bead without looking, from which jar would you be most likely to draw a red bead? How did you decide? What makes it difficult to choose? Have them exchange jars and repeat the process. (See Case Study 3, page 54)



Pigs Might Fly

Ask students to order the likelihood of unlikely events in their daily life or from a book. For example, they could list some of the unlikely events that occur in the book *Pigs Might Fly* (Rodda, 1997) or some other book that discusses the chance of something happening. They could then rate the events by likelihood of their happening.

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CASE STUDY 2

Sample Learning Activity: K–Grade 3—What Will Happen?, page 44

Key Understanding 3: We can compare and order things by whether they are more or less likely to happen.

Working Towards: Matching and Comparing phase

TEACHER'S PURPOSE

When listening to my Grade 3 students talking to each other, I noticed that many referred to more and less likely everyday events using appropriate language. For example, I heard Kenny say to Gavin, "Yes, I will probably be allowed to play at your house today." And when Gavin asked if he would also be allowed to sleep over, Kenny said: "Not much chance of that. It's school tomorrow." I decided to give them an activity that would help them think about how we can directly compare and order a range of events according to how likely they are to happen.

ACTION AND REFLECTION

I thought of 10 different but familiar events, most of which obviously varied in likelihood, and recorded them on cards. Some were about everyone in the class while others were personal and likely to cause disagreement. To give students a clear focus, I asked them to think about how likely these events were to happen tomorrow, rather than in general. The events were as follows:

- We will drink some water.
- We will do some reading in class.
- A dog will bark.
- Someone in our class will go to the washroom.
- We will all stay home from school.
- One of us will travel on a plane.
- I will ride on a bus.
- My teacher will wake up on the Moon.
- It will rain.
- I will not watch television.

I divided the class into four groups and gave each a full set of cards. First they worked in pairs, each pair taking two of the cards and deciding whether one was more likely to happen than the other, and why. They then joined with another pair in their group and tried to order the four cards. Finally each group together ordered all 10 from least to most likely.

Students find it difficult to work out what we are talking about when we use terms like* on average, normally, *and* in general *to refer to events. For example, asking a young child to think about the likelihood of going shopping after school in general requires a more complex set of judgements than asking about the likelihood of going shopping after school tomorrow.

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Most in Abbie's group thought drinking water and using the washroom had to go together, saying, "We all have to do both things every day so one cannot be more likely." Abbie disagreed, saying that going to the washroom had to be more likely. I tried to get her to justify this assertion by asking: "Why do you think it is more likely?"

She replied: "We could drink juice or milk, we do not have to drink water, but everyone has to go to the washroom." This was surprisingly advanced thinking for this level.

DRAWING OUT THE MATHEMATICAL IDEA

Kathryn and Ellie were arguing about whether riding on a bus was more likely than rain.

"So tell me why you think riding on a bus is more likely, Kathryn," I said.

"Well, I go to after-school care every day on the bus, and Mom and I go to the movies at the mall most Saturdays, and it does not rain as often as that," she said.

"What about you, Ellie? What do you think?" I asked.

"I went on a bus once last spring when we went to Grandma's, and it has rained lots since then," claimed Ellie, "so rain's got to be more."

"So what is the same and what is different here?" I asked. "You ride on the bus a lot, Kathryn; Ellie hardly ever rides on the bus. Ellie said it has rained a lot since spring; Kathryn said it does not rain very often."

"But the rain cannot be different," said Ellie. "The rain *must* be the same for us both. If it rains, it rains. We both have to have the same chance of raining."

"I get it," said Kathryn. "The rain is the same, but riding on buses is different."

"So what can you say about the chances of these things happening tomorrow?" I asked.

"It can be both things," said Kathryn. "I have got more chance of going on a bus, because Ellie's mom picks her up in a car. It is more chance of rain for Ellie, because she hardly ever goes on a bus."

I invited Kathryn and Ellie to explain what they had found out to the class.

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CASE STUDY 3

Sample Learning Activity: Grades 5–8—Chance Situations, page 51

Key Understanding 3: We can compare and order things by whether they are more or less likely to happen.

Working Towards: Measuring phase

I chose to partly fill large plastic jars so that children could more easily examine their contents and see the relative proportions of red and white beads, without opening the jars. The 16 jars were made up as follows:

Jars 1 to 7: half full of beads

Jar 1—all white
Jar 2—all red
Jar 3—white with 3 red
Jar 4—red with 3 white
Jar 5—half red and half white
Jar 6—one-quarter red and three-quarters white
Jar 7—one-quarter white and three-quarters red

Jars 8 to 14: quarter full of beads

Jar 8: all white
Jar 9: all red
J***ar 10:*** white with 3 red
Jar 11: red with 3 white
Jar 12: half red and half white
Jar 13: one-quarter red and three-quarters white
Jar 14: one-quarter white and three-quarters red

Jars 15 and 16: 12 beads each

Jar 15: 9 white and 3 red
Jar 16: 9 red and 3 white

TEACHER'S PURPOSE

My Grade 6 students had already had many experiences using informal reasoning to order everyday chance events according to how likely they are to occur. I decided they were ready for a more structured experience that required them to think about number when making their judgements.

ACTION

I made up 16 large plastic jars containing varying numbers of red and white wooden beads. In small groups, they compared pairs of jars, deciding which gave the best chance of drawing a red bead. We had a whole-class discussion, with pairs telling the rest of the class how they made their judgements. This gave students opportunity to use words such as "very likely," "impossible," "100 percent," "more chance," and "less chance."

After they had compared a few pairs of jars, I asked them to make a poster to record their judgements for any jars they found particularly interesting or tricky to compare. This gave me opportunity to move around and interact with the groups—manipulating some of the comparisons to draw out particular ideas. For example, Roselyn's group was comparing Jar 3 (half full of beads) and Jar 10 (quarter full of beads). Each jar contained three red beads with the rest white. By turning and shaking the jars the group decided there were just three red in each. "So, that means they have to have the same chance," said Roselyn. Others in the group agreed.

CHALLENGING CURRENT IDEAS

I did not question Roselyn's judgement, but arranged for the group to swap Jar 10 with another group's Jar 15 (9 white, 3 red beads), which they then compared with Jar 3 (half full of white with three red). Initially, Roselyn said, "Oh, it is three again so that is just the same," and others started to agree.

I asked: "Are you sure it is the same chance of getting a red from these two? If the reds were candies and you had one chance to put your hand in and take one ball without looking, which jar would you choose?"

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Ivan immediately grabbed the jar with the smaller number of beads. "I choose this one," he said. "You have got much more chance of getting a candy."

"But why?" I asked. "There are only three red in each, so how can it make a difference which jar you draw it from?"

"But you can see, just by looking at it?" said Ivan.

"But what are you looking at?" I persisted, "What makes it different?"

Roselyn, who had previously focused only on the number of red beads, recognized that the quantity of white beads was also important. "You have to look at how many white ones too—not just red ones. You would be more likely to get a white in that one, even if the red is the same chance."

I then retrieved Jar 10 and asked them to re-think their previous decision. Roselyn immediately questioned her earlier judgement. "I think it cannot be the same; look, there are more white ones in that one."

Ivan was not so sure. "But there are a lot of white. I think there are too many whites to make a difference. It would still be hard to get a red."

Many students, even in Grades 7 and 8, do not fully understand how we can use numbers to describe proportional relationships. Nevertheless, they may use an intuitive sense of proportion to compare situations such as these involving mixtures in different ratios.

I was not concerned that there were still inconsistencies in their thinking. I knew these were very complex ideas and they would need more time and many more such experiences to develop these understandings.

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Key Understanding 4

We say things have an equal chance of happening when we think they will happen equally often in the long run.

The analysis of "equally likely events" is generally the basis for the calculation of probabilities, and so it is important that students develop sound foundational ideas about what we mean when we say that events are equally likely. In the same way that two quite different looking regions could have the same area, two entirely different events could have the same chance of happening. Students should develop this understanding both through reasoning and practical experiences.

Being able to say that two events are equally likely does not require students to have any idea what the numerical probability is. Focusing too early on numerical probability is likely to be unhelpful and to obscure, rather than help, the development of this concept. Students can be helped to understand what it means to say that two events are equally likely, by thinking about events that are more and less likely. For example, if given a list of events to order which includes both "a head will come up on the toss of a coin" and "a tail will come up on the toss of a coin," students will be confronted with deciding which is more likely. In the absence of any convincing reason to believe one or the other is more likely, the notion of being "equally likely" should arise. Similarly, you might present students with a cube where each of the faces is a different colour, and ask them to try to put the colours in order from the most to least likely to appear. They should realize that there is no reason to believe that one is more likely than another.

Students should experiment with situations about which they have made predictions of equal likelihood. Such experiments require careful handling, however, since it is in the nature of chance events that the outcomes are unpredictable! The difficulty for students beginning to make sense of chance is that equal chance does not mean that we expect the same empirical results in small trials.

However, students may expect equally likely outcomes to appear equally frequently, even with small numbers of spins or tosses, and may lose confidence when they do not. This expectation may also lead them to expect the numbers to "even out," as if chance is a self-correcting mechanism that promptly takes care to restore the balance whenever it is disrupted.

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Typically, students do not understand that each toss of the number cube is independent of what happens before or after. They need to systematically record the outcome of large numbers of number cube rolls, for example, to be convinced that each number will come up equally often *in the long run*. They also need to compare the variability in small numbers of tosses with the relative predictability of large numbers of tosses. (Computer simulations are very valuable here.)

The idea of equally likely outcomes is also closely related to notions of fairness. Thus, because we expect and rely on each face on a coin or number cube "coming up" equally often, we would say that a coin or number cube is "unfair," or biased, if the faces were unevenly weighted so that in the long run they would not come up equally often.

Students should investigate random devices for fairness. In some cases, deciding whether two events are equally likely will involve ideas about ratio. For example, recognizing there is an equal chance of getting a red ball from two jars, one containing 20 red and 40 blue balls and one containing 40 red and 80 blue, requires a basic understanding of proportion.

Links to the Phases

Phase	Students who are through this phase in Probability...
Emergent (Late)	■ may not distinguish situations that involve equally likely events from those that do not
Matching and Comparing	■ understand what it means for events to be equally likely, e.g., a student will say that a spinner coloured in four equal sectors is equally likely to stop on any of the colours, but that another spinner coloured in four unequal sectors will not.
Quantifying	■ will use provided numerical information such as the average number of rainy days each month, or the number of each colour in a jar of balls, to decide whether two events are or are not equally likely to occur
Measuring	■ can use their understanding of equivalent fractions to judge equally likely events

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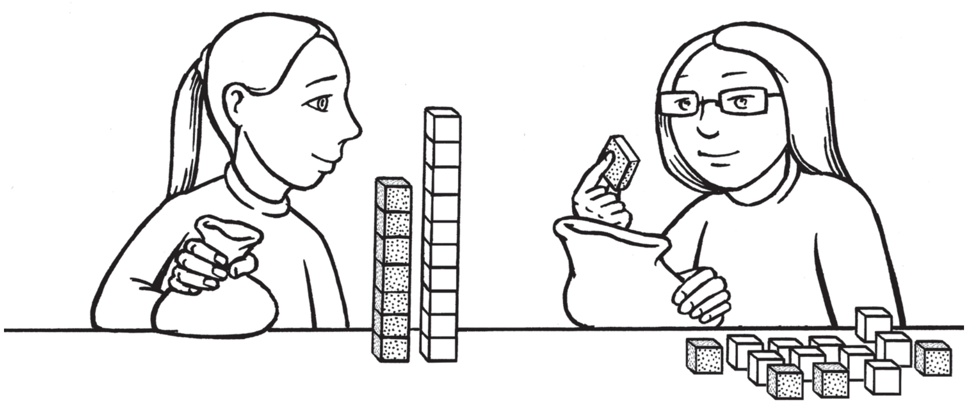
Sample Learning Activities

K–Grade 3: ★★Important Focus

Fruit Time
Prepare fruit to be shared on plates. Ask students to predict what might be on the plates. Ask: Will we have more pieces of apple or pear? Why? (Responses might include *There were more apples than pears in the fruit box* and *We always have more apples than pears.*) Will there be more apples on this plate than that? (*They should be the same because we try to share them fairly.)*

Towers
Ask pairs of students to cooperate to build, for example, a blue tower and a yellow tower. Give each pair of students a bag that contains two square tiles (one yellow and one blue) and a pile of yellow and blue blocks. They take turns to draw out one square and add a block of that colour to the correct tower. The square is replaced after each turn. Before the game begins, ask students to say which tower will be taller. Focus on the suggestions that they could be the same and that either could be higher. As the towers grow, interrupt the game and ask the students to observe all of the towers built and say why they are different.



Same Chance
Invite students to use playing cards to play Snap. When they have finished, ask how they should play so that each player has the same chance of being able to snap the cards. Ask: What do we need to think about to make it fair? Does it matter if you look at a card before you put it down? Does where you sit and how you sit make a difference? What rules could we make to give everyone the same chance of saying Snap?

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Fair Choice
Ask students to decide whether random methods (e.g., picking names out of a hat) for assigning jobs or picking groups are fair. Ask: Could anyone in the class be picked? Does everyone have the same chance? Can we make it fairer?

What Will Happen?
Provide students with cards showing various classroom activities such as using a calculator, reading a book, and playing with play dough. Ask them to place the cards in a line according to which activities are most likely to happen tomorrow. Ask: Were there any you had trouble deciding between? Why were these two so difficult to order?

Two Steps Forwards
After playing *Two Steps Forward*, page 45, have students play it again using a red/blue counter. After the game, ask whether they prefer to play it with a counter or a bottle cap. Ask: Would you expect to have red and blue come up equally often? Why is it better to play the game with something that does not have an equal chance of coming up?

Building Spinners
Ask students to construct their own spinners and use them for games such as Snakes and Ladders. Invite them to compare a spinner with unequal sections with one that has equal sections and say why some numbers do not have an equal chance of coming up. Ask: Should we try to make all the sections the same?

Jam Sandwich
Read *The Giant Jam Sandwich* (Lord 2007) or a similar book about probability from your classroom library to the students. Then ask them whether they think a slice of buttered bread that is dropped is likely to land with its buttered side up, or whether either side has an equal chance of facing upwards. Have students test out their predictions individually and then combine their results to get a large sample. Ask: Which side landed up the most? Do you think there was a reason for this happening? If you did the testing again, would you get the same results? Why or why not?

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Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Tosses
Have students toss counters that have a different colour on each side (e.g., red/blue counters) and score points depending on whether the counters land with red or blue facing up. Before they begin, ask them to predict whether more blue or red will be face up. Ask: Has one colour more chance than the other? How do you know? Would it make any difference if we used more counters? What if we just used one counter?

Two Steps Forwards
Play *Two Steps Forwards*, pages 45 and 59, using a coin. Ask: Would you expect heads or tails to come up more often? Play the game again with a number cube with four blue and two red faces. Ask: Where might you end up after six throws? Why might one side come up more often than the other?

Changing the Numbers
Ask students to consider the effect of changing numbers in number cube games that require particular numbers to be rolled. For example, in a Make a Bug game (see page 50), you need 6 to start by drawing a body, then rolling 1 allows you to draw a head, 2 a feeler, 3 a front leg, 4 a back leg, and 5 a middle leg. Suggest that students change which numbers are required for which parts. Ask: Would this change your chances of making a beetle or starting the game? Are any of the numbers harder to get than others?

Roll of the Number Cube
Invite students to decide whether numbers on a number cube have an equal chance of coming up. Get them to play a game involving number cubes, such as Snakes and Ladders. Give each student a different number to roll before they can start the game. Ask: Are any of the numbers more difficult to throw? Is a 6 really the most difficult? Why not?

Choosing Teams
Have students think of ways to decide which of two teams will go first. Ask: Could we toss a bottlecap or thumbtack instead of the usual coin or bat? How could we test to see if the alternatives were fair? What would we need to do?

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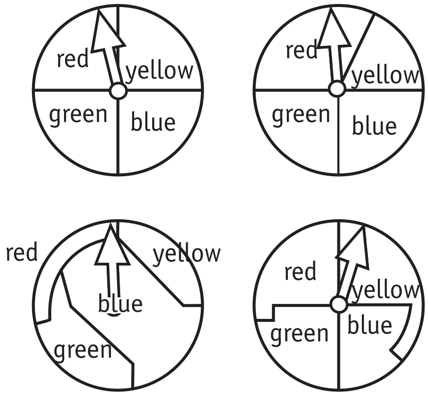
Spinners (1)

Show students a range of spinners with four colours, some with equal sectors, others with unequal sectors. Ask: Which are fair? Why do you think that? Include some with different areas, but equal distances around the perimeter.



Spinners (2)

Extend the previous activity by asking students to make and design a range of spinners that look different, but have an equal chance of scoring, say, red or blue. Ask: How many different spinners can you design with equal chances of the two colours coming up? What about with three colours? What do you need to think about when you make a spinner so that each colour comes up equally often?

Round the Twist

After playing the modified version of *Twister*™, page 47, ask: How could we make the spinners so that red is as likely as blue to come up? Have pairs of students design a spinner and then justify why their spinner is fair. Draw out how spinners can look different but still provide an equal chance for the two colours to come up.

Equal Chances

Ask students what their chances are of pulling out a white or black block from a bag when one of each colour is in the bag. Ask: Could we put more blocks in the bag but still have equal chances of getting a white or a black? What numbers would not give us an equal chance? If we put in other, different coloured blocks, would black and white still have equal chance?

Fifty-Fifty

Give students a range of situations to sort into those which are a 50-50 chance and those which are not. Ask them to explain why some are not a 50-50 chance. For example, picking a time at random and seeing if it is daylight or not in Thunder Bay in January is not a 50-50 chance, as Thunder Bay has fewer daylight hours in January than dark hours. Draw out that we say things have a 50-50 chance if there is no reason to expect one to occur more often than the other. Link 50-50 to the idea of half of 100 percent.

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Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Outcomes

Challenge students to order the likelihood of events where some or all of the events are equally likely to occur. Have them list and order the outcomes from most to least likely to occur for each of these situations:

- rolling a number cube
- flipping a coin
- spinning an equally divided spinner

Ask: Why is it difficult to order the different outcomes in each situation? Draw out that when there is no reason to think that one outcome is more likely than another, then the outcomes are equally likely.

Weather Conditions

Ask students to list possible weather conditions and place them in order from the least to most likely to occur tomorrow. Ask: Which conditions were easy to order? Which conditions are more difficult to put in order? Why is it difficult to say which one should come before the other? Could these conditions be equally likely? Why?

Choosing Socks

Have students examine different situations and say why two events are or are not equally likely. For example, they can identify drawers from which they are equally likely to take a black or a white sock in the dark:

- a drawer with 22 whites and 11 blacks
- a drawer with 8 whites and 8 blacks
- a drawer with 21 whites and 21 blacks

Collections

Have students predict the likelihood of drawing out a blue or a red counter from the following collections:

- 10 reds and 15 blues
- 34 reds and 20 blues
- 18 reds and 20 blues
- 10 reds and 10 blues
- 40 reds and 40 blues

Ask: Which ones are easy to decide? Which ones are not? Which collections are equally likely to come up red or blue? Why? Would it make any difference if we doubled all the numbers? Why? Why not?

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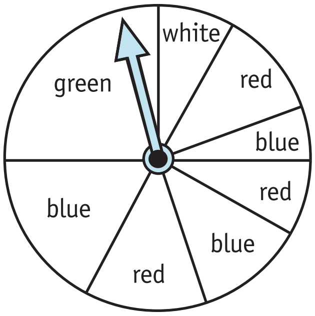
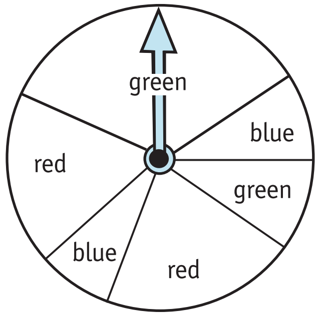
Chance Spin

Have students examine two spinners (see below). Ask: Which one would you use if you wanted to get red? Why is it difficult to choose? Ask them to fix the spinners so that both spinners have the same chance of coming up red.





Polyhedra Number Cube Game

Invite students to consider events as less likely, more likely, and equally likely to describe the fairness of games. For example: Give each partner in a pair of students a number cube. The pair plays a simple track game where one partner moves a space if he or she rolls a 1, 2, or 3, and the other moves a space if a 4, 5, or 6 comes up. After a few games, exchange one partner's number cube with a regular polyhedra number cube (tetrahedron, octahedron, dodecahedron, or icosahedron). Repeat the game using the same rules. Ask: Is the new game fair? Why? Why not? Which shape are you more/less likely to win with? Why? How can you make the game fair? Draw out that fair games require that each winning combination (or winning set of numbers) should have an equal chance of coming up.

Why Is It Fair?

Have students discuss why some games are fair and others are not. Ask pairs of students to draw up a simple track game to resemble a running race. They take turns to roll two number cubes. Runner 1 moves a square if the difference between the two cubes is 0, 1, or 2. Runner 2 moves a square if the difference is 3, 4, or 5. Ask: Is this game fair? Draw out that unfair games are where one player's chance of winning is not equal to the chances of the other players.

Make a Spinner

Have students examine the probability devices (number cubes, spinners, or bags of coloured tiles) they made (see page 50) and say which events are equally likely. Have them explain how they constructed their devices so that two events were equally likely to occur. Ask: How did you plan your spinner so that it had the same chance of stopping on yellow and blue? Encourage students to compare the different ways they constructed their spinners to ensure that yellow and blue were equally likely to come up.

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Key Understanding 5

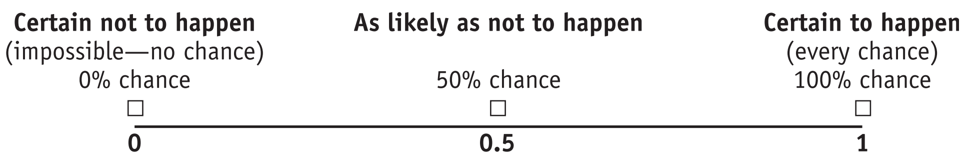
We can use numbers to describe how likely something is to happen.

Just as area is a measure of how big a region is and time is a measure of how long something takes, probability is a measure of chance, or of how likely something is to happen.

We can compare and order objects, spaces and events without reference to numbers, but when we want to say, for example, how big or how much bigger an area is, we use numbers. Similarly, we use numbers when we want to say how likely or how much more likely an event is. In each case, we use a unit as the basis for quantifying our comparisons.

Events that *cannot* happen all have the same chance of happening, that is, *no* chance, so it makes intuitive sense to say they have a probability of 0. Events that *must* happen are also all equally likely and so it makes sense that they all have the same probability. We have decided to give all events that *must* happen a probability of 1 (or 100%). We then use this certainty of happening as our unit and compare all other events to it in order to quantify how likely they are to occur.

Events that might happen are more likely than events that cannot happen and less likely than events that must happen, so we would expect events that might happen to have a probability somewhere between 0 (no chance) and 1 (every chance). It is also reasonably intuitive to think of events that are just as likely to happen as not, as having a probability halfway between cannot happen and must happen. In effect, we have developed a numerical scale like this:



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Initially, students should informally and intuitively place events, such as the chance of it getting dark tonight, swimming to Jupiter, doing math tomorrow, and getting a tail when a coin is tossed, on the scale from 0 (cannot possibly happen) to 1 (must happen). They could compare their placements and debate them, perhaps coming to compromise or average positions. Only after they understand the ideas of more, less, and equally likely should they begin to quantify chance (See Key Understanding 3 and 4).

Without actually working out probabilities for themselves, students in Grades 6 to 8 should interpret simple probability statements in everyday use. For example, they might say that a 90% chance of rain means rain is very likely and therefore decide to cancel the picnic, but a 10% chance of rain means there is little likelihood of rain and so they are prepared to take the small risk of rain and will not cancel. Given a 50% probability of rain, they might find it difficult to decide. Thus, they should come to understand that events are at their least predictable in the centre of this scale and become more predictable as you move towards either end.

Links to the Phases

Phase	Students who are through this phase in Probability…
Quantifying	■ can use the scale from 0 to 1 in an informal way, placing everyday expressions of chance such as "impossible," "poor chance," "even chance," "good chance," and "certain" on the scale ■ have an intuitive sense of the meaning of probability statements such as those associated with weather predictions
Measuring	■ understand that probability is the way we measure chance; that is, probability statements give a measure of how likely something is to happen ■ understand the 0 to 1 scale and can interpret expressions of probability in general usage, such as the probability of rain tomorrow is 30%, and there is a 50-50 chance

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Sample Learning Activities

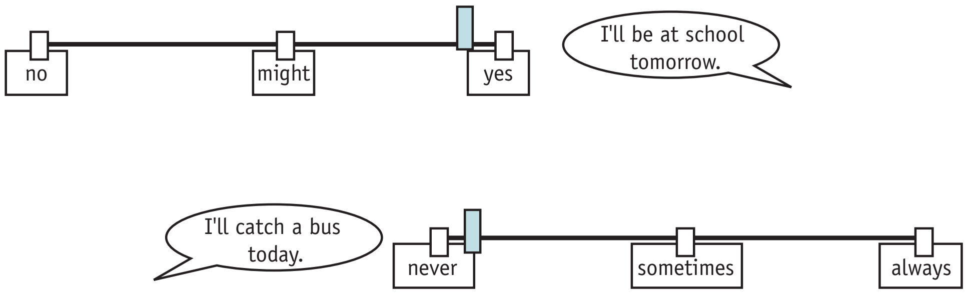
K–Grade 3: ★ Introduction, Consolidation, or Extension

What Will Happen?

Make two labels: "cannot happen" and "must happen." Place the labels far apart. Prepare a set of cards showing various classroom activities, such as using a calculator, reading a book, and playing with play dough. Invite students to order the cards from most to least likely to happen tomorrow, and place their cards somewhere between the labels. Ask them to say why they put the cards closer to one of the labels, or towards the middle.

String Line

Ask students to pin labels such as "no," "might," and "yes," or "never," "sometimes," or "always" (indicating the likelihood of things happening) in order along a string across the room. They then take turns choosing where events such as "I'll be at school tomorrow" or "I'll catch a bus today" should go, by moving a marker along the string line.



Fifty-Fifty

In incidental discussion of chance events, such as rain, use the term "50-50 chance" to describe events that have an equal chance of happening. Explain it as having an even chance of happening: maybe it will, but maybe it will not. Draw out that 50 is half of 100.

Paper Scale

Have students cut a long strip of paper and label one end "cannot happen" and the other end "must happen." Ask them to draw pictures of things that could happen after school that day, including unusual as well as routine events. They then place their pictures on the strip of paper where they think they would be on the scale, and say why.

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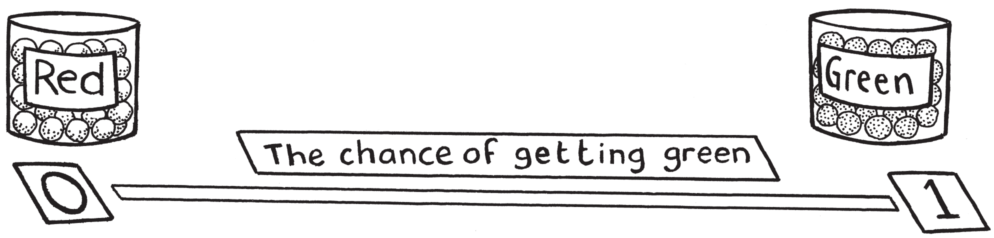
Sample Learning Activities

Grades 3–5: ★★ Important Focus

Coloured Balls

Place a tape across the front of the room, marked with 0 at one end and 1 at the other. Label it "The chance of getting green." Show students two jars, one containing only, say, red balls, the other containing only green. Point to the jar containing red and ask: If you drew one ball from this jar, what chance is there of getting green? (None) If we gave that a number, what should it be? Place the jar at the appropriate place (0) on the tape. Repeat for the jar containing green balls, asking: What is the chance of getting green? Place that jar at the other end of the tape (1). Make up another jar containing mostly red and one or two green balls. Ask students where on the tape would show how likely you are to get a green ball. Discuss why. Repeat for various combinations using an intuitive sense of whether you should be closer to the red (0) or green (1) end. Repeat with a jar containing equal numbers of red and green.



What Will Happen?

After students do *What Will Happen?*, page 66, introduce the idea of a numerical scale. Link this to measuring other things such as length and time, for example, if no time has passed, we would say there are zero seconds or zero minutes. Ask: If we used the number 0 to show that something is certain not to happen, what number could we use to show that an event is certain to happen? What kinds of numbers might we expect to see on the things that might happen?

Fifty-Fifty

After students identify a situation where there is a 50-50 chance, ask them to identify where this would be on the number scale. Find other situations that would be similarly placed in the centre, such as 50% chance of rain, one in two chance of a coin landing head side up, or odds or evens on a six-sided number cube.

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Grades 3–5: ★★ Important Focus

Combined Possibilities

Invite students to identify all the possibilities in a simple chance situation. Draw out that one of these must happen or is certain to happen. For example: Show students a jar containing red and blue balls and ask them to list the possible outcomes if you were to draw out one ball. Ask: How likely am I to get either a red or blue ball ? (*Certain to, a sure thing*) So, if we were to give the chance of getting a red or a blue ball a number, what would it be? (100%, 1) What is the chance of pulling out a yellow ball? (*No chance, impossible*) So if we were to give that a number, what would be the sensible choice? (0)

More Possibilities

Extend the previous activity to include halfway points. Ask: How could we fill this jar so that you had an equal chance of getting a red or a blue ball? (equal numbers of both) What chance would there be of getting a red ball if you drew out one ball? Would the chance be more than 0? more than 1? less than 1? Where between 0 and 1 would it be?

Rating the Chances

Have students use a 0 to 1 chance scale to estimate the chance of a range of events, such as "I will drink some water some time tomorrow" and "I am going shopping after school today." Ask: Why did you decide to put that one close to the 0 and that one close to the 1?

Coloured Balls

Encourage students to use numbers informally to rate their chances of events occurring. For example: Ask them what colour they hope to draw out from an enclosed box containing six blue balls, three yellow balls, and one red ball. Students informally predict their chance of getting what they want and then indicate where this sits on a scale, with 0 being "impossible" and 1 being "certain." Those who choose the same colour can compare and justify the positions they chose.

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Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Rating the Chances

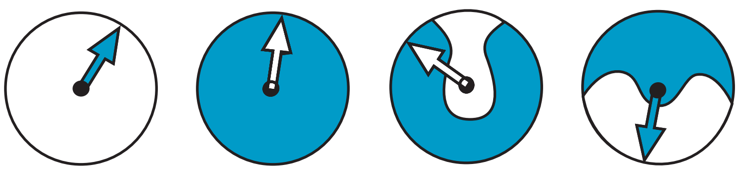
Ask students to use newspapers to help generate a list of national or international events that might happen. Using a number line between 0 and 1, have students informally rate the chances of the events occurring, and explain their reasoning.

Word Sort

Have students sort chance words such as "nearly," "may," "possible," "highly probable," and "fifty-fifty," and place them on a 0 to 1 probability scale according to their description of whether an event will, might, or will not happen. Ask them to justify their positioning of words.

Ordering Spinners

Show students the spinners below. Ask them to put the spinners in order according to which they would rather have, if they needed to spin white to win. Ask: Why did you put them in that order? Have students use a number line between 0 and 1 to informally rate the chances of spinning white, and explain their reasoning for ordering each of the four spinners.



Pigs Might Fly

Extend *Pigs Might Fly*, page 51, to have students place the unlikely events they have already ordered, on a scale from 0 to 1. Ask them to justify their order and say why all the events are close to 0.

Rulak

Have students interpret probability statements as an aid to making decisions. For example: Present monthly probability information about the weather in their hometown and an imaginary location (Rulak) in the southern hemisphere for each month in the year. Ask students to imagine a friend from Rulak will be coming to visit. Have them write to their friend, suggesting suitable clothes to bring and comparing the local weather to their friend's home, justifying their decisions.

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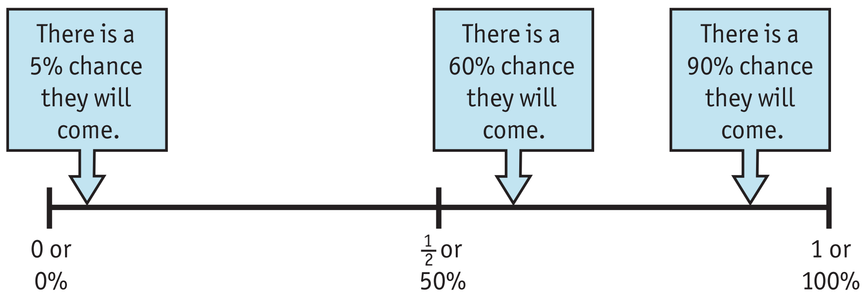
Grades 5–8: ★★★ Major Focus

Percentages

After working with percentages as a part of a whole, have students place percentage estimates of probability on the 0 to 1 scale. Reinterpret percentage estimates as approximate fractions. For example: Collect percentage estimates from newspapers, magazines, or news programs. Ask: Where on the number line would you place the statement "The chance of rain tomorrow is 30%"? What does that mean? If the percentages are closer to 0 or to 1, how does that affect our prediction? *(The closer it is to 0 or 1, the surer we are that it will either not rain, or rain; the closer it is to 50%, the less sure we are about rain or no rain.)*

Sorting Percentages

Extend the previous activity and have students sort everyday probability statements into those that make it easy to make a decision and those that do not. For example: Consider the following probability statements: there is a 90% chance of rain; there is a 10% chance of rain; there is a 50% chance of rain. Which probabilities help you make a decision about whether or not to cancel tomorrow's sports day? Which statement does not help you make a decision? Or: The school cafeteria manager is deciding how many more drinks to order based on whether visitors from a neighbouring school are coming. Consider the following probability statements: there is a 90% chance they will come; there is a 60% chance they will come; there is a 5% chance they will come. Which probabilities help you make a decision? Which statement does not help you make a decision? Draw out that events are more predictable at either end of the scale and less predictable in the centre of the scale.



Fruit Box (1)

Have students use numbers informally to rate the chances of events occurring. Invite them to examine a fruit box containing six apples, three oranges, and one grapefruit. The fruits are drawn from the covered box so students cannot see what they are choosing. Have them informally predict their chance of drawing out each of the fruits and then indicate where this chance sits on a scale with 0 being certain not to happen and 1 being certain to happen. Students compare and justify the positions they chose for each type of fruit.

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Fruit Box (2)

Extend the previous activity by asking students to remove one fruit from the box. Invite students to again predict their chance of drawing out each of the fruits and then indicate where this chance sits on a new scale. Ask: Are your predictions the same as before or have they changed after one fruit has been removed? How have your predictions changed? Remove a second fruit and then invite students to use another scale to again predict the chance of drawing out each of the fruits. Continue drawing out fruits and ask: When might you get a 0% chance of drawing out a particular fruit? When might you get a 100% chance of drawing out a particular fruit? Would the chances change if we replaced the drawn fruit after each turn? Why? Why not?

Predict and Test

Ask students to predict and test the outcome of chance events with and without replacement. For example: Put all the students' names in a hat and have them consider their chances of having their names drawn out to win prizes. After establishing that they have one chance out of the number of names in the hat, invite them to approximate this on a 0 to 1 chance scale. Ask: What is your chance of having your name drawn out? Where would that chance appear on the scale? Draw out a name for first prize, and then ask if and how students' chances have changed for winning second prize. Repeat for third and fourth prize. Ask: What is the chance of the first-prize winner also winning second prize? What affects whether their chance is 0% or the same chance as everyone else? *(It is whether or not the name is replaced after it is drawn out.)* If we continue drawing prize winners without putting the names back and you end up the very last to be drawn out, what happens to your position on the 0 to 1 chance scale each time? *(The chance moves from near the 0 end at the first draw to 1 before the last draw, because the name is certain to be drawn out next.)*

Chance of Red

Have students examine the contents of a box of buttons. Ask: If you were to shake them and take one out without looking, what would your chances be of getting a red one? Ask students to estimate this by plotting a point on a number line between 0 and 1. Take one button out. Ask: What is your chance of getting a red now? Has it gone up, stayed the same, or gone down? Where do you think the new point is now? How do you know? Continue until all the buttons have been removed, focusing students' attention on the fact that the whole (what 1 represents) changes with each button removed, because the whole chance is the total number of buttons left in the box before each draw. The chance of getting a red each time will depend on the relationship between the number of red buttons not yet removed and the total number of buttons still left in the box.

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Key Understanding 6

Sometimes we list and compare all the possible things that could happen to predict how likely something is to happen.

One of the ways we can make predictions about chance events is to analyze the situation. We study the event, identify what could happen, make some assumptions, and then use reasoning to work out how likely it is that a particular outcome will occur.

The dual notions that some events are equally likely and that an event certain to happen has a probability of 1 are together needed to analyze situations and make numerical statements about how likely a particular outcome is. Consider tossing a number cube that has six different-coloured faces. We know (or assume) that all of the possibilities (red, tan, blue, green, white, and black) are equally likely. We are also certain that one of those colours must happen. It therefore makes intuitive sense to think that if each colour has the same probability of showing face up and the six probabilities add to 1, then each colour has a probability of one-sixth.

To be able to undertake such analyses, students need to be able to list all the possible outcomes for an event and make decisions about whether or not the possible outcomes are equally likely. Neither is straightforward for children. Developing the capacity to identify all possible outcomes and to think about whether they are equally likely should be the major focus of this Key Understanding for primary students, rather than the actual computation of probabilities.

Early experiences should emphasize "What are all the possibilities?" rather than "What is the chance of one possibility happening?" Often students in the early elementary years focus on a particular outcome of interest to themselves or that they believe most likely. Through Grades 4 and 5, they should develop their capacity to systematically list all of the relevant and possible outcomes for one-stage events (such as throwing one coin or number cube). It may be considerably later before they can identify all the outcomes where there are two and three stages or components (e.g., for throwing two coins or two number cubes).

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Initally students may assume that each outcome always has an equal chance of happening. For example, given a four-coloured spinner with unequal sectors, they may assume all colours are equally likely to appear, either not noticing that the sectors are of different sizes or not seeing the relevance. Similarly, having identified the possible results of tossing two coins as two heads, two tails, or one of each, they may assume the three possibilities are equally likely. Students will need many experiences analyzing possible outcomes, as well as experimenting with materials, before they will be convinced that all possibilities need not be equally likely.

Some younger students may learn to describe chance numerically in simple situations involving equally likely events by counting the number of ways an event can happen out of the total possible events. However, it is likely to be more helpful for children to take the time needed to develop sound basic concepts about chance processes through experimentation and ordering events non-numerically than to rush to numerical probability.

Links to the Phases

Phase	Students who are through this phase in Probability...
Emergent (Late)	■ can identify possible outcomes for events that are familiar or that they have observed, but may not systematically list all possibilities
Matching and Comparing	■ will list, when prompted, all the possibilities for straightforward situations
Quantifying	■ will, unprompted, systematically list all the possible outcomes for a "one-step" action and use this information to work out numerical probabilities
Measuring	■ are able to see why the probability that a toss of a fair number cube will produce 5 is one-sixth and, if one-quarter of a spinner is red and the rest is yellow, why the probability of getting red on a spin is one-quarter and yellow is three-quarters

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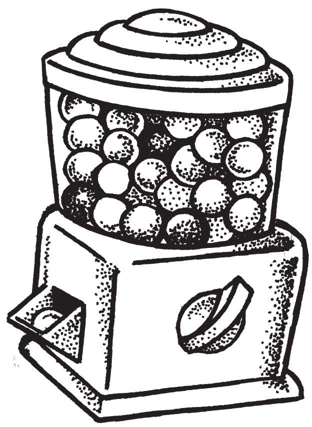
Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

What Colour? (1)
Invite students to say what colour button they might get if they closed their eyes and chose one from small box. Ask: Is it possible that you might get another colour? Which colours might you get? Have you listed them all? How many different colours could you get? If there are no orange buttons, ask: Can you get an orange button? Why not?

What Colour? (2)
Vary the previous activity to other simple familiar contexts where the possible selections are unambiguous, such as gumball machines, small sets of cards, Pick-up Sticks™ or Lego™. Ask similar questions to draw out the need to list all the possibilities and to also think about what is not a possibility.



Number Cube Variations
Ask students to systematically list all possibilities to make checking easier. For example: Give students number cubes, some with each face a different colour, some with each face a different number, for example, one numbered conventionally 1 to 6; another labelled 10, 20, 30, 40, 50, 60. Ask: What could you get if you toss this number cube? Is it possible to get a red/blue? Is it possible to get an 8 on the 1 to 6 number cube? Draw out that with the number cube labelled 10 to 60, it is easy to say we cannot get 55; however, with the coloured number cube, we would need to check.

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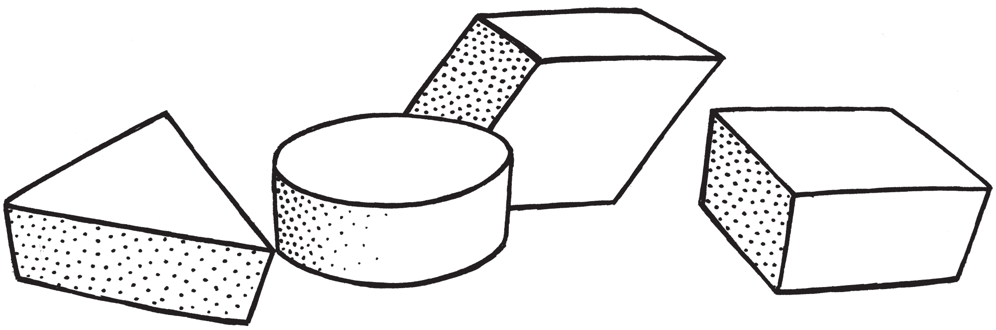
Sample Learning Activities

Grades 3–5: ★★ Important Focus

Blocks

Place four different-shaped blocks (e.g., sphere, square, rhombus, triangular prism) in a box. Prompt students to list the possible outcomes if they drew one out without looking. Ask: Are you more likely to get a triangle, a square, a circle, or a rhombus? Why? If you drew out a shape, wrote it down, put it back, and then did it many times, what do you think would come up most often—a triangle, a square, a circle, or a rhombus? Why? What chance do you have of selecting a triangle? Draw out that there are four possibilities and no reason to think one would be chosen over others.



Selections

Extend the previous activity to other situations where there is only one of each item, and then informally describe the chance of a particular item being selected. For example: names of 10 children in a hat, to be drawn for a job; prizes on an equi-spaced wheel; and weekday on which you could be born.

Pathways

Provide students with a simple map of a few streets in your community. On an overhead copy of the map, trace along a route and, as you reach an intersection, ask students to list all the choices a driver would have at that intersection. Make a choice and move on. Have students repeat this activity with a partner, starting at a different spot.

What Colour? (1)

Extend *What Colour? (1)*, page 74, by providing each group with a small box of buttons. Ensure some boxes contain the same range of colours. Ask students to list the colours in the box and record how many of each colour. Ask them to order the colours from those they would be least likely to get if they closed their eyes and selected one, to the colour they are most likely to get, and to explain their order. They then predict how likely they are to get each colour. Ask: Can you use numbers to say how likely you are to get a red? What numbers do you need to know? (*I need the number of the colour and the total number.*)

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Grades 3–5: ★★ Important Focus

What Colour? (2)

Extend the previous activity by asking students to compare their responses with those of the neighbouring group. Ask: Are both groups equally likely to get red? green? Explain why. Display each group's results for the whole class to see and discuss. Draw out that, because some boxes contain the same range of colours, there might be the same possibilities, but the possibilities may not be equally likely.

Tosses

Invite students to select a fixed number of counters with a different colour on each side (e.g., red/blue counters), say eight, and list all the possible outcomes when the counters are tossed together (e.g., eight blue, seven red, and one blue).

Which Area?

Identify separate areas of the classroom and number them 1, 2, 3, 4, 5, and 6. Invite students to choose one of the areas to stand in. Roll a number cube. Students standing in the area whose number was rolled must leave the game. Show students a range of six-sided number cubes, for example, one with faces marked 1, 2, 3, 4, 5, 6 and one marked 1, 3, 6, 6, 6, 6. Ask: Which number cube would you rather play this game with? Draw out that you have more chance of winning with the weighted number cube.

Card Hands

Use one pack of playing cards to provide groups of students with a "hand" of seven cards. Ask them to list all possible outcomes when drawing a card from their hand. (Seven possibilities.) Ask: Could a person get [name a number and suit] if he or she drew from your hand without looking? (Only one group will say yes.) What chance is there to get that card from your hand? (One group should say 1 in 7, the other groups 0.) Which suit (or type of card) is someone drawing from your hand most likely to get? (Groups will have different responses.*)* Ask: Why do you think that?

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Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Making Selections

Repeat one-stage selection activities such as *Blocks, What Colour?*, and *Card Hands,* pages 75 and 76. Ask students to list all possible outcomes in a table, with frequencies if appropriate, and use the list to make statements about the likelihood of various possibilities. As they gain confidence, help them to express chance statements numerically, including with fractions. For example:

- **What Colour?** Extend and ask students to predict how likely they are to get each colour. Ask: What numbers do you need to know? (How many of each colour and how many in all.) Have students express their probabilities in statements such as *There are 3 chances in 10 that I will get red* or *The chance of getting red is 3/10.*
- **Card Hands** Extend and have students record the frequency of each suit for their hand (e.g., one group might have 4 spades, 2 hearts, 1 club, and 0 diamonds). Ask: Can you use that data to quickly say which suit a person choosing one card from your hand is more likely to get? What chance is there of a person getting a heart? (*2 in 7*) a diamond? (*None.*)

Spades	4
Hearts	2
Clubs	1
Diamonds	0

- **Fruit Box** The prize box for the class contains 6 apples, 3 oranges, and 1 grapefruit. Ask students to use various ways to describe the chance of drawing each fruit. For example: *The chance of getting a grapefruit is 1 chance out of a possible 10, 1 in 10, 1/10, or 10%.*
- **Names in a Hat** Put the name of each student in a hat. Ask students to record how many boys' and how many girls' names are in the hat. Ask: If I draw a name without looking, am I more likely to draw a girl's name or a boy's name? What chance is there that I will draw a boy's name?

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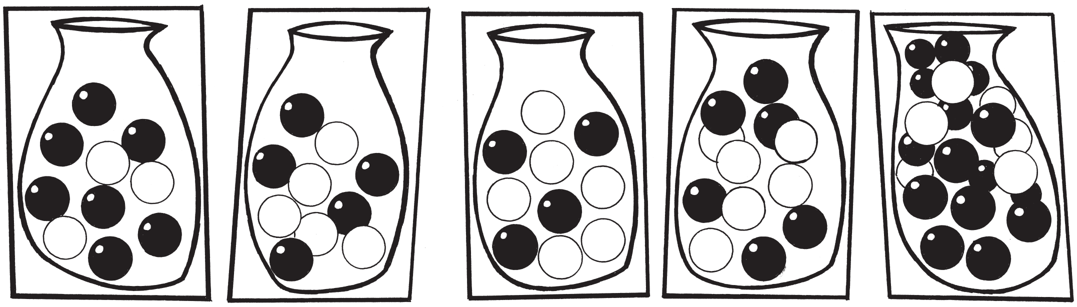
Grades 5–8: ★★★ Major Focus

Frequencies of Balls

Provide students with pictures of bags containing coloured balls in differing proportions (e.g., 3 red and 7 blue, 5 red and 5 blue, 6 red and 4 blue, 6 red and 6 blue, 6 red and 14 blue), with variations structured to allow various comparisons to be made as described below. Ask students to imagine drawing a ball from a bag without looking.



- Students choose a bag from which they are equally likely to select a red or blue. Ask: What is the chance (probability) of getting a red from that bag? What is the chance of getting a blue from that bag? Are there any other bags for which there is an equal chance of getting a red or a blue? What is the chance (probability) of getting a red from that bag? a blue? Is it the same chance as for the other bag? Draw out that 5 chances in 10 and 6 chances in 12 are both one-half. Red and blue are equally likely and nothing else can happen, so each has a half chance of being selected.
- Have students say whether they are more likely to get red or blue from the first bag (three red and seven blue). Repeat for other bags. Sort bags into those where red is equally likely as blue, those where red is less likely than blue, and those where red is more likely than blue. Remind students that when red and blue are equally likely, each has a chance of a half. Draw out that when red is less likely than blue, we would expect its chance to be less than a half, and when red is more likely than blue we would expect its chance to be more than a half.
- Ask students what chance there is of getting red from the first bag (3 chances in 10, or $\frac{3}{10}$). Is this less than a half?
- Reinforce, asking students which of the last three bags they are more likely to get red from (six red and four blue, six red and six blue, six red and 14 blue). Why? Ask: What is the chance of getting red in each case? Do the fractions follow the order of size they expected?

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Make a Spinner

Have students design a spinner so that the chance of it stopping on red is $\frac{1}{4}$, on green is $\frac{1}{2}$, and on blue is $\frac{1}{4}$; OR stopping on blue is $\frac{3}{8}$, on red is $\frac{1}{4}$, on green is $\frac{1}{4}$, and on yellow is the rest; OR stopping on red is 0.5, on green is 0.1, and on blue is 0.4.

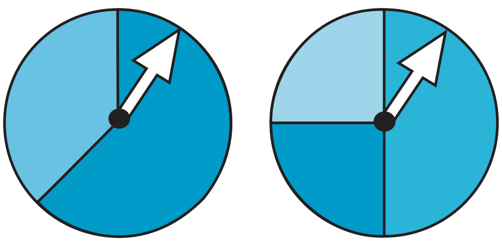
What Colour?

Extend *What Colour?*, page 75. Have students colour and draw diagrams to produce specified chances of getting specific colours:

- Provide pictures of bags with 5, 10 and 15 balls. Ask students to colour each bag so that the chance of getting red is $\frac{2}{5}$.
- Ask students to draw three different bags with a probability of one-quarter of getting red.
- Provide a picture of a bag with 10 balls. Ask students to colour it so that the chance of getting green is $\frac{3}{10}$ and of getting blue is $\frac{5}{10}$. Ask: If the rest of the balls are yellow, what is the chance of getting yellow? Can you draw another bag with more balls, which has the same chances of getting blue, green, and yellow?

Spinners

Provide different spinners with differently proportioned segments, but with easily perceived fractions, for example, one-quarter, one-quarter, one-half; or one-third, two-thirds (see below). For each spinner, ask students to list the possible outcomes, discuss whether each outcome is equally likely, and say why some outcomes should occur more than others. They then can use numerical statements to describe the chance of each outcome occurring.



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Key Understanding 7

Sometimes we use data about how often an event has happened to predict how likely it is to happen in the future.

One of the ways we make predictions about chance events is to use data about what has happened in the past, or to carry out experiments (simulations) that we think are sufficiently like the situation we are interested in to act as a kind of proxy for it.

Students should carry out experiments that involve chance processes (spin a spinner, toss a bottle cap) and examine the outcomes. Initially, discussions and predictions should refer to the range of possibilities—what could happen. As suggested for Key Understanding 6, students should learn to make complete lists of possible outcomes from simple experiments (e.g., tossing a number cube), demonstrate how each outcome may occur, and argue why they think they have the complete list. Having worked out by analysis (as described in Key Understanding 6), what they expect the chances are of, say, a 6 coming up when they toss a number cube, students should experiment to test their predictions. They will need many experiences before they realize that their prediction of 1 in 6 refers to the long term, not to the short term. Only if we make a very large number of throws, would we expect the results to be consistently close to one-sixth.

Students' experimental work should also include many situations that are difficult or impossible to analyze, but are experimentally simple. For example, it may be difficult to work out the probability of a thumbtack falling point up or down by thinking about it. However, it is quite straightforward to throw a thumbtack a large number of times and use the relative frequency of appearance of the two outcomes to estimate how likely each is to occur in future. Working out the chance of future events is often based on the idea that if phenomena, or people, behaved in a certain way in the past, they are likely to behave similarly in the future. For example, if in the past 10 years, it rained in April on 250 days, we can use the ratio 250:300 (rainy days/total days) or about 83% as an estimate of the chance of rain on future April days.

This exercise does not tell us if it will rain on a particular day in April, or even how many rainy days there will be this April, but it will tell us there are likely to be more rainy days in April than August, when the probability of rain is, for example, 8%.

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With help, students in Grades 6 to 8 can use local data to similarly estimate chance. For example, they can use ratios like the number of accidents occurring at school compared to the number of school days over a period to estimate the likelihood of an accident occurring on any one school day. They can examine factors that could affect the data, such as changes in student numbers or playground equipment. We expect variation from year to year but, if there have been no changes in population or in the school environment, we would expect any fluctuations to be within a range.

Where it is impossible, difficult, costly, or inappropriate to use past information or generate real data about a situation, a simulation or model that replicates the important aspects of a situation can be created. For example, tossing a coin can replicate a situation that has two equal outcomes occurring randomly, like girl or boy births, so families can be modelled using a number of coin tosses to model a family of that size.

Links to the Phases

Phase	Students who are through this phase in Probability...
Quantifying	■ will, with assistance, be able to use experimental results to determine a range of possible outcomes and informally use relative frequencies to estimate probabilities
Measuring	■ will be able to plan simple experiments and derive the ratios from their data to generate numerical probability statements

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Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

What Is for Lunch?

Have students record the contents of their lunches for a week and then use this to say what they are likely to get each day for the next week. Ask: How did you know what you were likely to get for lunch? Did you ever find something that you did not expect? How did this happen?

After School

Ask students to record their after-school activities for a week, and then predict what they are likely to be doing over the next few afternoons. Ask: Does the list of what you did last week help you to predict what you might be doing this week?

Weather Watch

During spring or winter, ask: Do you think it is likely to rain today? tomorrow? Ask students to record their predictions and then record the weather over a couple of weeks. They then review the data to see what the weather has been like and use this information to predict the weather each day for the next week. Ask: How did the data help us to know what the weather might be?

Tosses

Ask students to toss several red/blue counters and try to get all counters to land with one colour face up. Have them record the results of each toss. After a few tosses, ask students to predict whether they are likely to get two blues, two reds or similar, on their next toss. They then toss and record a few more times. Ask again: Are you likely to get two reds on your next toss?

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Team Sports

Invite students to look at the teams that have won the school sports competitions over the past few years and predict which team will win this year. Ask: What makes you think this team might win? Have the teams changed since last year? Can we be sure which team will win?

Games

Have students play board games that require a particular number to come up on a number cube, for example, Make a Bug, page 50. Afterwards, ask: Can you make sure that the number you want will come up? Does the position of the number cube before you throw it make a difference? Does wishing for your number help? Have them test out each suggestion and record their throws to see if it helps the number to appear.

Safety First (1)

After students have played on climbing equipment, ask them if they felt unsafe on any parts of the equipment and why. Have them reflect on how the equipment was set out, what they were doing, and how many others were using it at the same time. Invite students to suggest ways to alter the design of the equipment and try it the next day.

Safety First (2)

Extend the previous idea to other activity areas, for example, painting corner, home corner, blocks, cooking, water play, sand table. Use this information for students to contribute to the classroom rules for each area. If they are ever playing in an unsafe way, ask them to recall the rules, and say what might happen if they continue.

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Sample Learning Activities

Grades 3–5: ★★ Important Focus

Which Is Harder?

After students have played a number cube game requiring a 6 to start, ask: Are sixes really harder to get than other numbers? Would changing the starting number to a 2 be better? Have students work in pairs to find out, by throwing the number cube 20 times and producing a tally against the list of six possible outcomes. Invite them to describe what they found. Ask: Who found 6 came up less often than 2? Who found 2 came up less often than 6? Who found they were the same? Who found they were close to the same? Have each pair combine their results with two other pairs to produce results for 60 throws. Write the results on the board. Discuss differences and similarities in data. (*More variation in 20 throws; reduced variation in 60 throws.*) What does the data suggest? Are all numbers equally likely or are some more likely than others? Total the scores for the whole class so they now have 200 to 300 throws. Ask: What does this suggest to you? Is 6 less likely to come up in the long run? Why does it feel that way? (*Partly that there are five times as many chances that 6 will not come up; partly because they want to get started!*)

Triangular Spinner

Repeat the previous activity for several other simple random devices, such as an equilateral triangular spinner showing three different colours.

Number Cube Patterns

During games involving a number cube, ask students to record each number they throw. Afterwards, look at the sequence of numbers and see if there are any patterns. Ask: Can writing down this sequence help us to say what the next number will be when we roll a number cube? How often did each number come up? Will each number come up the same amount of times if we play the game again?

Two Steps Forwards

Have students toss a bottle cap, thumbtack, chalkboard eraser, or button. When it lands face up, they take two steps forwards; when it lands face down, they take two steps backwards. They can play with a partner and see who is first to reach a given line. Have students record the outcome of each toss and use this information to decide if it would it be better to take forward steps when it lands face down instead. Ask: Which side comes up more often? Is it likely to continue to come up more often?

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Testing Predictions

Play the previous game again, with students carrying out an experiment as in *Which Is Harder?*, page 84, to test their predictions about one of the devices they used (e.g., bottle cap, thumbtack, chalkboard eraser).

Fair Number Cubes

While playing number cube games, invite students to compare a regular six-sided number cube with one made from a non-regular rectangular prism. Present this scenario: Rebecca says we should not use the rectangular prism because some numbers will not have as much of a chance of coming up. Ari says it does not matter which number cube we use: the numbers are all still equally likely to come up. Ask: How can we find out? Students carry out an experiment, as in *Which Is Harder?*, page 84, to test their predictions.

Tug of War

Conduct a class tug of war competition. Have students look at each team's previous wins and losses when making their predictions for the next round. Ask: Does knowing how often this team has won before help you? How?

Heads and Tails

Invite students to predict the outcome of tossing two coins by standing up and placing two hands on their head (two heads), two hands on their bottom (two tails), or one hand on each (one head and one tail). Toss two coins. Students who predicted correctly continue to play. Keep a record of what was thrown each time and ask: What do you think you should pick to give you the best chance of staying in the game? Ask them to justify their decision.

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Sample Learning Activities

Grades 5–8: ★★★ Major Focus

Making Selections

After completing the *Making Selections* activities on page 77, have students test their predictions. Build up data sets starting with pairs, then groups, then the whole class, as for *Which Is Harder?*, page 84, and ask similar questions. For some of these examples, the possible outcomes may not be equally likely. Draw out that for small amounts of data such as the 20 collected by pairs, results are quite variable and one set of 20 would not be enough for you to be confident. For 60 bits of data, there is less variability and results may be more convincing. Several hundred bits of data collected by the whole class give a long-term pattern more consistent with expectation. Draw out that if the short-term pattern does not seem to fit our conclusions from analysis, we might not be concerned, but if the long-term pattern does not fit, we might go back and check the thinking we used that led us to our prediction. (See Case Study 4, page 90.)

More Testing

Repeat the previous testing for other predictions made in relation to Key Understanding 6.

Irregular Spinner

Provide students with a spinner that has three obviously unequal sectors, differently coloured. Ask them to list the possible outcomes of a spin, and order from least likely to most likely (without numbers). Have students hypothesize about the frequency with which each colour will appear. They experiment to test their prediction, building up from pairs to groups to the whole class, as for the previous activity. Ask students to describe their data, as in *About half the time the green came up; the red came up about twice as often as the blue.*

Families

Have students hypothesize about the likelihood of a family with two children having two boys, two girls, or one of each. Ask them to suggest different ways they could represent (simulate) a birth with an equal chance of being a boy or a girl, for example, toss a coin and make it heads for a girl and tails for a boy, or use a spinner with half marked B for a boy and half marked G for a girl. Toss or spin once for the first birth, and then again for the second birth, or use the coin for one birth and the spinner for the next birth. Invite students to experiment to test their prediction, building up from pairs to groups to the whole class, as for the previous activity. Have students record their experimental results in a two-way table and use it to describe their data: *About half the*

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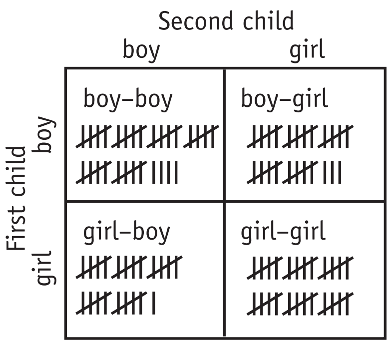
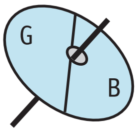
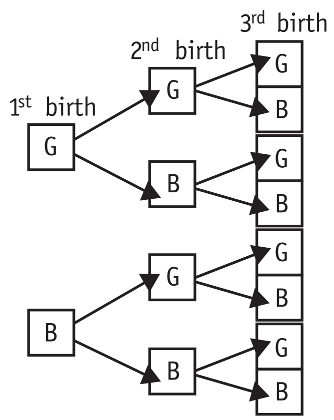
time there was one of each; about a quarter of the time there were two girls; and about a quarter of the time there were two boys.





Bigger Families

Have students extend the previous activity to families with three children, using the spinner or coin three times to simulate the gender of the three births. Ask: How will you record the results? Why will a two-way table not work? What fraction of three-child families do you think might have all boys or all girls? What fraction do you think will have two boys and a girl, or two girls and a boy? Have students use their spinners or coins to represent a large number of families and then help them use the data to predict the fraction of three-child families that are likely to have same gender children and the fraction that are likely to have either two boys and a girl, or two girls and a boy. (The diagrams below may help students understand the variation in probability and record the results of their experimentation more systematically.)



GGG
GGB
GBG
GBB
BGG
BGB
BBG
BBB

Testing Bigger Families

Students could test the result of their predictions in the previous activity by collecting real data in the school. Negotiate with other classes to find all the families of three children and record how many boys and girls they have, and the order of the births. Ask: How well do the fractions of three-child family combinations in the school match your simulated families?

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Grades 5–8: ★★★ Major Focus

Race to 50

Have students play a number cube game called Race to 50, by tossing two number cubes and adding the total, then using their calculator to keep a cumulative total. After the game, ask: What numbers can you get from each toss of the two number cubes? Have students list all of the possible totals and toss the number cube to see how often they get each number. Ask: Did you get one total more often than another? Why do you think that happened?

Weather Watch

Ask students to get weather data for several years from Environment Canada about various conditions such as sunny days, rain, and snow late in the year. Use the data to compute probabilities for the different months in the year. Keep corresponding data throughout the year and compare the statistics from each month with the probability predictions.

Counters (1)

Have students take four cardboard counters, each of which is red on one side and blue on the other. Have them list the possible results after the counters are tossed in the air (that is, four red; three red and one blue; two red and two blue; one red and three blue; four blue). Invite them to predict whether these results are equally likely and if not, to say which results they think are more likely than others. Have them experiment by throwing and recording the results each time, building up the quantity by combining results in pairs, groups, then the whole class, as in *Which Is Harder?*, page 84. Ask: How do the results match your predictions? Students should find that the five different outcomes are not equally likely.

4 B 卌 ||
3 B, 1 R 卌 卌 ||||
2 B, 2 R 卌 卌 ||
1 B, 3 R 卌 卌 |||
4 R 卌 |

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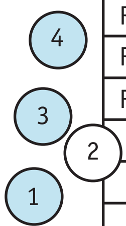
Counters (2)

Extend the previous activity by having students record exactly how each of the counters falls on each toss. Invite them to number the counters 1 to 4 on each side, and then use a grid to record the colour of each counter after each throw. As before, have students combine the data to obtain several hundred results. Invite them to use their recordings to work out why an all-blue result or an all-red result is less likely than a two-blue and two-red result. Ask: How many different ways did the counters show two red and two blue? How many different ways did the counters show all red? Draw out that there is only one way the four counters can fall to be all red, but any of the following six different arrangements results in two reds and two blues: RRBB, BBRR, BRRB, RBBR, RBRB, or BRBR.

1	2	3	4
R	B	B	R
B	R	R	B
B	B	B	R
B	B	B	B
R	B	R	B
B	R	R	B
R	R	R	R
R	R	B	B
R	B	R	R



March Break

Have students gather local weather data about the average number of wet days in March. Help them to formulate questions, such as these: What is the likelihood of no rain or snow over the five weekdays of March Break? Ask students to make a spinner to represent the proportion of rainy/snowy and dry days in March. They experiment by spinning five times and recording whether all days were dry or not. Repeat 10 times with a partner recording. Exchange and combine results, giving 20 trials. What fraction of simulated five-day periods had no rain/snow? Combine results for three pairs. Compare group results. Combine results for whole class. Discuss this as an estimate of the chance of five particular days in March being dry.

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CASE STUDY 4

Sample Learning Activity: Grades 5–8—Making Selections, page 86

Key Understanding 7: Sometimes we use data about how often an event has happened to predict how likely it is to happen in the future.

Working Towards: Factoring/Measuring phase

To make the process efficient, we put the coloured number cubes into large foam cups and taped a sheet of clear plastic firmly over the opening. The students tipped it upside down, shook it several times, then turned it right way up and stood it on the desk. They could then look through the clear cover to check which face was showing. With this easily constructed device, a large number of results could be generated in a short time without the noise or confusion of throwing number cubes.

TEACHER'S PURPOSE

I had been helping my class of 11 to 12 year-olds extend their ideas about equal likelihood, using number cubes that had different-coloured faces: two red, two blue, one yellow, and one green. They could list the four possible outcomes, and most agreed that red and blue should have the same chance because they each had two faces. Some also recognized that yellow had the same chance as green, but each had less chance than red or blue. However, they could not agree about what this meant or predict the results of actually throwing the number cubes.

CHALLENGING THEIR IDEAS

I decided to organize a way of recording the results of their number cube throws that would challenge their ideas about what it is we are estimating or predicting when we say that, for example, red has a 2 in 6 chance.

I provided some strips of grid paper six squares wide and asked students to label the columns according to the colour of each face. In pairs, students threw their number cubes, colouring the squares to match as each result came up. (They had one strip between two, and took turns throwing and colouring.)

I initially asked them to predict what their page might look like after 12 throws. Some expected the results to show about four red, four blue, two yellow, and two green. Others thought they would be about even for each colour, while others insisted anything could happen: "It is just the luck of it, anything could come up so you just cannot say."

Each pair threw 12 times and compared their results with what they thought would happen. Many were surprised at the results, which seemed to confirm the opinions of those who believed it was just luck and you could not predict the outcomes. The results of the pairs were quite variable. The students drew a thick line outlining the totals to date (see diagram on page 91) to keep a record of their result after 12 throws.

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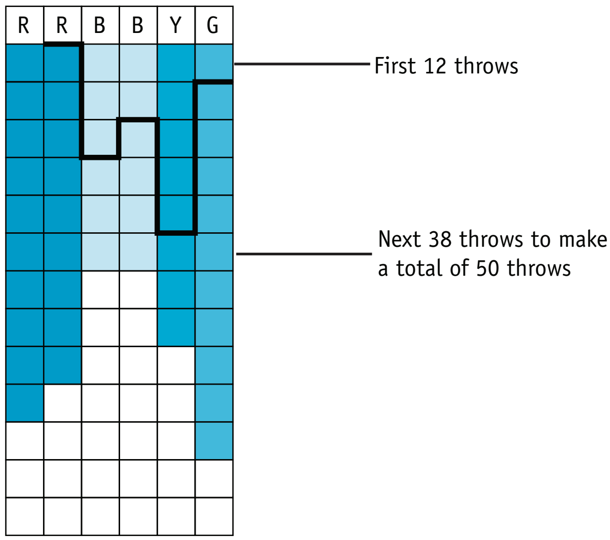
I then asked them to predict what might be the result of 50 throws. They were less keen to predict balanced results this time and most thought we could end up with anything. However, some thought that the columns might even up. After each pair had obtained 50 results, they again outlined the shape and we pinned their strips to a board for the class to look at. The variation in the results seemed to further confirm for many that there was no way to predict the outcomes. "They are all different. You just cannot tell."

"So what might we mean when we say red has a 2 in 6 chance?" I asked.

"I think it might be that you should get two reds when you do six throws," said Sandy, "but it is not going to happen like that really."

"What do you mean?" I asked.

"Well, I thought in six throws it is supposed to be two red because there are two red faces, but really it does not happen like that. They are all different."



I asked the students to look across everyone's results and see if they could see any patterns. A variety of observations were made, but Christalla noticed something important. "There are two reds in every six throws for a lot of rows in the 50 results, but hardly any in the 12 results," she said. This comment opened up an opportunity to build further on this idea. "I wonder what would happen if we threw some more? Would we expect to get more finished rows?"

Over the next day, students added to their columns, being careful to record every result. Each pair recorded about 200 results and students saw that they all had a lot more completed rows of two red, two blue, one yellow and one green, as well as a lot of "left-over" results. I then suggested we put all of our results together to see how they looked overall.

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DRAWING OUT THE MATHEMATICS

The pairs cut off their left-overs, then all pasted their completed rows on the display board, one under each other in columns. They then cut up the left-overs into strips of the different coloured columns and added them to the display, matching them up, somewhat like a jigsaw, to make as many completed rows as possible. When all had been combined, we found we ran out of red first, leaving extra blue, yellow, and green.

As a class, we looked at the display and talked about what it told us about the idea of a 2 in 6 chance of red. In discussion, we drew out the following observations:

Although few 11 to 12 year-olds understand ratio numbers, the visual effect of combining the recorded outcomes in this way enables them to see what is meant by, for example, a two out of every six chance. They can see concretely how much variation there is after a few results, but also the result of collecting a very large number of outcomes.

- There were not many people who got even one complete row in the first 12; there were some more who got complete rows when we did 50, and a lot more in 200.
- After 200 throws, there was still a lot of variation, but everyone ended up with a lot more reds and blues than yellows and greens.
- When we put all the results together, we ended up with more than 3000 throws, but we did not get exactly two reds for every six throws—there were still a lot of left-overs. We would have needed to get about 100 more reds to make it even.
- We could say that for the 3000 results there were, on average, about two reds for every six throws.

I was satisfied that my students now had a better intuitive feeling about what we mean when we express a probability as a ratio. They were beginning to understand that this was an indication of the pattern we expected in the long run, not what we would expect in the short run.

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Chapter 3

Collect and Process Data

(Part A) Collect and Organize Data

Plan and undertake data collection and organize data for effective interpretation.

Overall Description

Students systematically collect, organize, and record data to answer their own questions and those of others. Examples of questions include these: Which school lunch is liked best? Which animal is most scary? What shapes and proportions do people like best? How much water is used in the school each year and for what? Does having a part-time job affect school results? Students clarify and refine questions and plan surveys, experiments, and simulations to help answer them in unbiased ways, considering both the data collection instruments and the size and nature of samples.

Students understand the following:

1. Classification underlies the organization of data.
2. How we classify depends upon the questions we want to answer.
3. The way the data is organized can illuminate or mask certain data features.
4. This influences how the data is interpreted and used.

For example, one classification of sports preferences might suggest that students prefer ball games; another might suggest that balls are not relevant, rather that students prefer team to individual sports. They, therefore, realize that data can be distorted accidentally or deliberately to reach inappropriate conclusions. They consider the impact of technological change on the collection and handling of data and the issues this raises about matters of privacy and social monitoring. They also consider ethical issues in the collection and organization of data and act responsibly in this regard.

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Collect and Process Data (Part A):
Key Understandings Overview

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of this family of concepts. The learning experiences should connect to students' current knowledge and understandings rather than to their grade level.

Key Understanding		Description
KU1	We can answer some questions (and test some predictions) by using data.	page 96
KU2	We can produce data by counting or measuring things, asking groups of people, watching what happens, or reworking existing data.	page 106
KU3	Organizing data in different ways may tell us different things.	page 114
KU4	We should make our data as accurate and consistent as possible.	page 130
KU5	Sometimes we collect data from a subset of a group to find out things about the whole group. There are benefits and risks in this.	page 142

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Grade Levels—Degree of Emphasis			Sample Learning Activities
K-3	3-5	5-8	
★ ★ ★	★ ★ ★	★ ★	K-Grade 3, page 86 Grades 3-5, page 88 Grades 5-8, page 90
★ ★ ★	★ ★ ★	★ ★ ★	K-Grade 3, page 108 Grades 3-5, page 110 Grades 5-8, page 112
★ ★	★ ★ ★	★ ★ ★	K-Grade 3, page 116 Grades 3-5, page 118 Grades 5-8, page 121
★	★ ★ ★	★ ★ ★	K-Grade 3, page 120 Grades 3-5, page 122 Grades 5-8, page 124
★	★ ★ ★	★ ★	K-Grade 3, page 144 Grades 3-5, page 145 Grades 5-8, page 146

Key

★ ★ ★ **Major Focus**
The development of this Key Understanding is a major focus of planned activities.

★ ★ **Important Focus**
The development of this Key Understanding is an important focus of planned activities.

★ **Introduction, Consolidation, or Extension**
Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

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Key Understanding 1

We can answer some questions (and test some predictions) by using data.

There are many ways that we answer the questions we have about the world. One way is to ask someone we believe to be an authority *(Mommy, why does it get dark at night?)*, another is to refer to a textbook (perhaps an astronomy text). However, many questions can be answered by the production of data. Using data to answer questions is the essence of "the scientific method."

In developing this Key Understanding, students should learn that

- we (and others) can answer many of the questions we have about the world by referring to data
- not all questions can be answered by referring to data
- some questions cannot be directly answered by data, but can be reframed into questions that can be
- some questions cannot be completely answered by data, but data may contribute to the answer
- questions often have to be made more clear or precise before we can decide what data is needed
- we must make sure that the revised questions still get at what we wanted to know in the first place
- sometimes our questions can be answered by data we produce ourselves (primary data collection)
- sometimes our questions can be answered by data produced by others and/or already available (secondary data collection)
- sometimes data we (and others) have already collected suggests new questions

Students should also learn that

- predictions are not simply guesses; they are best guesses, informed guesses, or judgement calls based on our previous experience and knowledge, and our theories and analysis
- when we test a prediction, we formulate a question or hypothesis and produce data to answer this
- when we test a prediction, we answer the specific question "Will what I predict happen?"

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Although young children ask many questions, often they do not consciously think of producing data as a way of answering these for themselves. It is often possible to build on the questions students spontaneously ask so that they learn to think of data collection as an appropriate question-answering strategy. However, many of their questions are not simple enough to be readily answered by the kind of data they can produce. Thus, teachers may need to model the posing of simple questions about collected objects or pictures, directly prompt students, or help them to ask more searching questions.

Sometimes the question itself makes it clear what type of information is needed (see Key Understanding 2), but many questions and predictions are framed in rather general terms. For example, "How safe is the playground?" cannot be answered directly by data, since students are unlikely to have an obvious or immediate measure of safety (although health and safety organizations may). In this case, we have to ask ourselves whether we can reframe the question into one that can be answered by data, or alternatively we need to produce data on something that we think would be an indicator of safety. Students should experience the processes involved in posing questions for themselves and refining and reframing them to make them accessible to data that they produce afresh for this purpose or which is already available.

Links to the Phases

Phase	Students who are through this phase in Data Management...
Emergent	■ pose simple questions about things they can observe and participate in discussions about how to find out, e.g., a student can ask, "What type of fruit is there most of on the table?"
Matching and Comparing	■ realize that they can answer some questions by collecting data ■ are able to offer some appropriate data-oriented questions and make predictions, e.g., a student may predict, "I think the most popular fast food will be pizza."
Quantifying	■ will, when prompted, attempt to clarify and refine their questions to decide what data to produce, e.g., a student could explain what is meant by "the most popular pet."
Measuring	■ can use these skills on practical problems that may not be obviously mathematical ■ will collaborate to develop sub-questions that contribute to addressing a general concern

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Sample Learning Activities

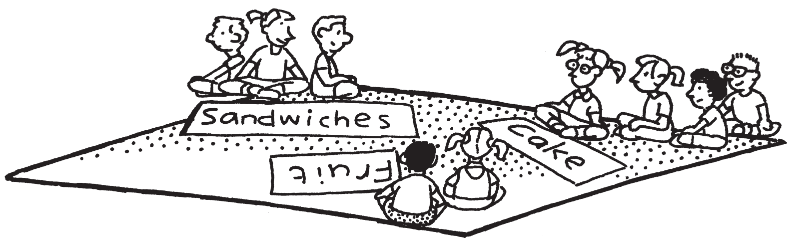
K–Grade 3: ★ ★ ★ Major Focus

Yes/No

Hang a card with "yes" on one side and "no" on the other next to individual name cards. Display a question above the names, for example, Have you had fruit? or Do you need to change your book? Have students turn their card to the appropriate side to show the answer relevant to them.

Picnic Plan

Pose the following question: What food should we take on our picnic? Write each suggestion on a card and position them around the mat. Ask students to sit next to their choice and then count and record the number at each card. Remove the least popular and continue until there is an appropriate result. Repeat this activity using other questions, including those that arise spontaneously in the classroom.



Finding Answers

Respond to questions students spontaneously ask by suggesting data collection. For example: Are there going to be enough brushes for everyone? Ask: Do we have to rely on guessing? How could we find out? Draw out that collecting data enables us to answer questions.

Questions, Questions

Model the posing of simple questions. For example, after lunch, ask: Do you think most of us had sandwiches for lunch? What type of fruit do most people like? How many different sorts of biscuits do we eat? Next, ask: How could we find out if most of us had sandwiches for lunch? Do you think that is the same for the class next door? How could we find out? Draw out that these are questions that could be answered by collecting information. Ask students to volunteer some more questions. Discuss with the class whether the questions can be answered by collecting information. Revise the questions if necessary. Ask each student to make up a question that a partner could answer by collecting information.

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Prompting Questions

Prompt students to find questions that they can answer by using data. When working on a theme such as wolves, ask: What do you wonder about wolves? Which of these things could we write a question about? Which of these questions could be answered by counting things? Which could we read about to find out?

Searching Questions

Model the process of extending simple questions into more searching questions. For example: "Are the boys or girls in our class taller?" could be extended to "What if we were in a Grade 6 class? What if we were talking about Grade 12 girls and boys? What if we were asking about adults? Does age make a difference?"

Clarify and Refine

After students have framed a general question, model questions they could ask themselves to clarify and refine their questions. For example: "Is soccer the most popular game for children?" could be refined by asking "By most popular, do we mean the game that is liked the best or the most commonly played?"

Grandpa's Breakfast

Encourage students to decide which questions can and cannot be answered by collecting data. After reading *Grandpa's Breakfast* by Josephine Croser or a similar book from your classroom library, ask students to decide what they would like to know about breakfasts. Ask: Which of these questions could we answer by collecting information? Can we rewrite some of our questions so that we could answer them by collecting information?

Making Predictions

Have students make predictions based on experience when growing plants in science class. Ask them to predict which plant will grow the fastest and decide what data they need to collect to test their prediction. Ask: Why did you predict that plant would grow the fastest? What question does the data answer?

Story Time

Before reading a book to the students, ask them what they would like to know from the book. List the questions, then have students work through the list and say which could be answered by counting something, and which by recording other information from the book.

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Sample Learning Activities

Grades 3–5: ★ ★ ★ Major Focus

Question Box

Have students replace general questions with more specific ones that can be answered using data. For example, ask students to decide which questions from a class question box they would like to answer. Ask: Could we answer this question by collecting data to answer a different question? To find out why dogs bark, we cannot survey dogs, but we could ask dog owners what is happening at the same time as their dog barks.

Muliplying Jar

After reading *Anno's Mysterious Multiplying Jar* by Mitsumasa Anno or a similar book from your classroom library, ask students to think of ways to keep track of the number of kingdoms, islands, and rooms. Have students then pose their own questions that can be answered in a similar way.

Refining Questions

Have students refine a question so they can use data to answer it. During technology lessons, invite students to decide which questions to ask in order to evaluate their designs. For example, after they have designed paper planes from different materials, they might first ask: Which are the best materials to use? Help them to rework the question along these lines: Which plane flies the farthest? Ask: Does the second question really answer the first question? What else might be considered as a measure of the best design?

New Questions

After collecting some data, have students pose new questions. For example, after reading one book about glaciers, ask: Do glaciers really move at 1 m per year? What information would we need to collect to answer that question? After students produce data from reference material, ask: What else could we find out from the data? What other questions could we ask? Does this data help us or do we need to collect some more data to answer the other questions?

Using Data

Encourage students to decide whether they could answer questions by collecting data. For example, ask students to brainstorm things they would like to find out about pets—Which pets are most popular? What should you do if your pet is sick? Ask: Could we answer any of these questions using data? What questions might you have to change to collect data to answer them? For example, what do we mean by “most popular”? How could we check this?

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Recycling

After collecting data to answer a question about recycling in their community, ask students whether the data answers their original question. Ask: What did you find out by collecting the data? Has it helped us answer our question? Has it answered a part of our question?

Model Cars

Ask students to use predictions to frame questions. For example, after making model cars in technology lessons, have students predict which car will travel the farthest down an incline and then decide what data they need to collect to test their prediction. Ask: What question is your prediction testing? The students may modify this question so that it directly suggests what data to collect.

Early Birds

Have students reframe questions so they can collect data to answer them. Students collect popular sayings and ask a question about one. For example: What does "the early bird catches the worm" mean? How could we rework the question so that we could understand it and find out if this is true? Draw out how the question could become "Do people who wake up early get more done during the day?"

Did You Know?

While some young students may not know the difference between a statement and a question, for some, the way a question is formulated may be culturally specific. In some cultures, it is considered more appropriate to hint at what you want to know rather than to ask directly. The hint may be in the form of a leading statement, such as "I think the best part of the book was when Kurt reached the top of the mountain." This has particular importance in Data Management and Probability when we ask students to think about answering questions by collecting data. For example, with activities from Key Understanding 1 that suggest we model the posing of simple questions, care must be taken, on the one hand, to acknowledge and respect the different forms of questions that cultures may find acceptable and, on the other hand, to help all students develop the ways of speaking used within mathematical discourse.

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Sample Learning Activities

Grades 5–8: ★ ★ Important Focus

Question Box

Invite students to ask questions and decide which can be answered with data. Have them sort questions from their class question box into categories according to whether they can answer them

- from their own experience
- from an information source such as a book or a website
- by producing some data

Students place the questions onto a Venn diagram showing the three options, and add to it over time. Can some questions be answered in more than one way? How do you show that on your diagram?

Why Not?

Ask students to say why they would not use data to answer certain questions. For example: Why would it not make sense for you to collect data to find out what day of the week March 1 fell on this year? Do you have to go to different calendars to find out the answer to this question?

Balloon Power

Encourage students to clarify their questions so they know what data is needed. For example, during a science lesson involving the construction of balloon-powered cars, students could examine a question such as "Which car works best?" Ask: Would you be able to use data to answer this question as it stands? Why? Why not? Have students rework the question and come to a consensus about a question that can be answered using data. For example: Which car travels the longest distance? They then compare the new question to the original. Ask: Is that what you really wanted to know or is it something else?

Predictions

Ask students to formulate questions based on their hypotheses or predictions. For example, in the science lesson mentioned in the previous activity, students could predict factors that may affect how far the cars travel and then frame questions about those factors. Do long cars go farther? Does the weight affect the car's performance? Do the cars that go farther have large wheels?

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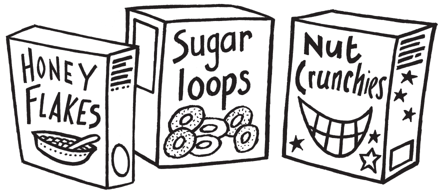
Is It True?



Have students reframe topical statements or hypotheses into questions, so that they can use data to find out if the statement is true. For example: Present a statement like "Supermarkets place cereals with high sugar content on shelves at the eye level of young children." Ask: How could we find out if the statement is true? What questions do we need to answer to help us find out if it is true? (See Case Study 1, page 104.)

Families

Have students define terms in a question so that data can be produced to answer the question. For example: Initiate a discussion about what is meant by "family," perhaps comparing what restaurants include in their "family meal deals." What do restaurants consider to be a typical family? Ask students to consider who counts as family in their minds, in their home culture, or in government statistics. You may need to take some children's family situations into consideration when talking about "our family." Talk about the concepts of the nuclear family and the extended family. Ask students to describe their own definition of family and then, through a process of building consensus, arrive at a definition of family to be used in their data collection.

Producing Data

Ask students to decide if their questions can be answered by existing data or whether they need to produce the data themselves. For example: Should you carry out a survey of family size or go to census data on the Internet? Which data helps you answer your original question? How?

Using Data

Invite students to frame questions from existing data or from data they have collected themselves. They might explore databases on the Internet as a stimulus to asking questions they find interesting. One idea is to find the last census data and ask which professions are the most popular for women under 30 years old. Ask: Can you answer your questions directly from the data? Do you need to find other data to answer your questions?

Mathematical Data

As the opportunity occurs, have students record their mathematical questions and decide which would require them to produce data. For example: What do we mean by polygon? Is 3 ÷ 4 the same as 4 ÷ 3? Are sixes harder to get on number cubes than other numbers?

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CASE STUDY 1

Sample Learning Activity: Grades 5-8—Is It True?, page 103

Key Understanding 1: We can answer some questions (and test some predictions) by using data.

Working Towards: Quantifying phase and Measuring phase

MOTIVATION AND PURPOSE

After considering some of the persuasive techniques used in advertising, my class of 11 and 12 year-olds was becoming more questioning of the media. So I wrote this statement on the board:

Supermarkets place cereals with high sugar content on shelves at the eye level of young children.

I then asked, "How could we find out whether this statement is true?"

CONNECTION AND CHALLENGE

The students were keen to share their opinions with the class.

"We could keep a record of overweight children."

"We could ask a dentist to ask the children with bad teeth what cereal they eat."

There were other similar responses. It seemed many did not think carefully about what data would be useful to answer the question I had asked. With this in mind, I said: "Let's say a dentist gave us a list of cereals eaten by children with decayed teeth. How could we use this list to help us decide whether supermarkets do put the cereals with high sugar content on the shelves at the eye level of young children?"

"Well, I do not think we could. We still would not know which shelves the cereals with high sugar content were on," said Akila.

"I know! We could catch them stacking the shelves on a surveillance camera," said Josh.

I responded, "But what would you actually be looking for?"

Elizabeth suggested, "We would be able to see what shelf they were putting the ones with high sugar on."

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Akila added, "But we would need to know which cereals have the high sugar content so we could tell. We need to look on the boxes."

The students were beginning to isolate which data could help, so I prompted them to refine the question. "So if you find the sugar content of all the cereals, how will you know which cereals have a high sugar content? A word like 'high' can be tricky. How much sugar is 'high'?"

"We could ask a dietitian about what is high sugar."

"Or we could look on the box and just pick the five highest ones."

Satisfied that the students were beginning to grapple with this idea, I challenged them to look carefully at the initial statement to see if they would need to make other decisions.

Mark commented, "We do not know which shelves are at children's eye level because some children are tall and some are short."

Li Ming added, "And we don't know what counts as young!"

DRAWING OUT THE MATHEMATICS

Through discussion, I drew from the students that to find out whether the initial statement was true, they would have to reframe it into questions that could be answered directly by data. I also drew out that more than one set of data might be needed. The class brainstormed a list of questions we needed to answer, these among them:

- Which cereals have high sugar content?
- Which shelves are at the eye level of young children?
- What will we call high?
- What will we call young?
- What is the eye level of young children?
- What shops will we call supermarkets?

The focus of the next lesson was for the students to consider how they would produce the data to answer each of these questions.

During this discussion and the following lesson, opportunities arose to talk to the students about whether it would be reasonable to generalize from their data. For example, if the students only produced data from one supermarket, would it be reasonable to generalize their findings to all supermarkets? Similarly, if they chose to measure the eye level of children and calculate an average, how many and which children would need to be measured?

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Key Understanding 2

We can produce data by counting or measuring things, asking groups of people, watching what happens, or reworking existing data.

In developing this Key Understanding, students should begin to learn that

- the information we record about an object, event, or experience, is our data
- although we say we collect data, we really produce it
- there is a range of ways to produce the data that will help us to answer our questions
- the data does not tell us everything about the original objects, events, or experiences
- we can answer questions about only the aspects of things that we have data on
- we should think ahead and try to imagine how we will use our data and how useful it will be

Data production often involves counting or measuring things in fairly straightforward ways, and in the early years most data will be of this kind. As they progress, students should learn that when we collect frequency data (How many children like each type of book?), we may lose information that is difficult to retrieve later (Which children like each type of book?). Students need to make conscious decisions about whether frequency information is sufficient or whether information about each case should be recorded. In addition, older students might begin to use simple rating scales.

Asking people to tell you what they have done, what they want to do, or what they think can be a useful way to get information. However, "asking people" is no simple matter. Students need considerable opportunity to explore the effect on responses of wording questions in different ways, and of the effect of the type of response required (oral or written, yes/no, forced choices, simple information, open-ended). As their experience increases, students should be encouraged to predict what sorts of responses people might give.

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Recording our observations of things as they are happening or of the effect of things that have happened is another way we produce data. These observations may involve naturally occurring events (what students choose at the cafeteria) or involve an experiment (rearrange the food in the cafeteria on successive days and record what students choose). Students should begin to ask themselves what exactly they will record.

Students should learn that we do not always have to produce fresh data to answer a new question. We may be able to use data others have produced. We may also be able to rework (reorganize) data we or others originally produced to answer other questions, and to use data on one thing to answer questions about another (using the number of mats laid out to find out how many students came to school today).

Links to the Phases

Phase	Students who are through this phase in Data Management...
Emergent	■ offer suggestions about what objects they could collect or make to answer simple questions posed by the teacher
Matching and Comparing	■ participate in group discussions about how to create data to answer specific questions
Quantifying	■ will try to refine questions to make it clearer what data to collect ■ will suggest for themselves what data to collect to answer questions that make sense to them
Measuring	■ will suggest indirect ways of getting data when direct data is not available ■ will attempt to reframe a simple survey question to make it less ambiguous and to make responses easier to interpret
Relating	■ are able to work in small groups to survey people, observe aspects of their environment, measure things, run experiments, and generate data mathematically ■ work autonomously but collaboratively in developing and testing short sets of questions

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Sample Learning Activities

K–Grade 3: ★ ★ ★ Major Focus

Playground Survey
Display a picture map of the playground showing the outdoor equipment. Have students attach a picture of themselves to the place or the piece of equipment they enjoy using the most. At circle time, ask students to count to see what activity most of them enjoy. Extend this by asking them to look for different information. Ask: What else can we find out? (For example: There are more boys than girls on the slide.)

Modelling
Model the process of deciding what data to collect by making suggestions about how to collect information to answer students' own questions. For example: Perhaps we could count how many children like different sorts of foods or perhaps we should watch what everyone eats for lunch. Which method would best tell us which is our favourite food?

Shoelaces
Ask students to decide what information they need to collect so that they can find someone to tie their shoelaces when they come undone. Ask: Will knowing how many students can tie shoelaces help? Would a list of everyone's names help?

Scary Things
After reading a story such as *Scaredy Squirrel* by Melanie Watt, ask students what scares them the most. Ask: How could you find out what scares most students in this class, or in this school? Should everyone say, write, or draw what scares them the most? Be aware that some students may be shy about discussing what scares them.

Birthdays
Have students write their birth month on a card and then find other students with the same birth month. Ask: Which month has the most birthdays? How can we find out? Should we count the groups, or would matching children one to one in the groups for different months help?

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Popular Toys

Invite students to decide how to collect information to answer questions such as, What is the most popular toy in Grade 1? Draw out that asking everyone in the class is a sensible way, but not the only way. Ask: Would asking everyone give us the best idea? How else could you find out?

Counting

Ask students to write a list of things they can count, for example, how many of each coloured pencil they have. Then ask them to think of questions that could be answered by counting the things they have listed. Ask: Do you need to count the things in different ways to answer different questions? Compare the questions "How many pencils do I have?" and "How many pencils of each colour does our group have?"

Grandpa's Breakfast

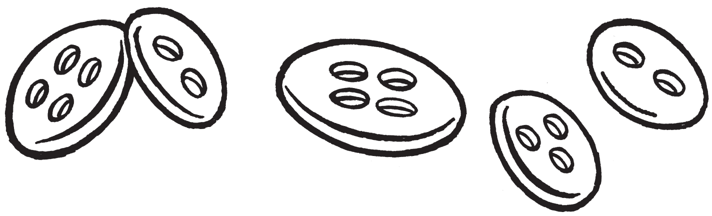
Extend *Grandpa's Breakfast*, page 99, by asking: Where could you find the information to answer a question like "What breakfast cereal has nuts in it?" (*By looking on the supermarket shelves, asking people in our class, looking on breakfast cereal boxes.*)

Out at Play

Ask students how they could decide how many children there are in another class when that classroom is empty. For example, they could count chairs, desks, bags, or hats. Ask: How can we be sure that what we have counted will tell us how many students in that class are at school today?

Buttons

Have students decide how they could find out whether most people wear buttons with two, three, or four holes in them. Ask: Would asking everyone in the school help? Students could interview their parents and others in their community instead.



Getting to School

When considering how students get to school in the mornings, ask: Would we get better information by observing others or by asking how they get to school?

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Sample Learning Activities

Grades 3–5: ★ ★ ★ Major Focus

Tosses
Have students toss a fixed number of counters that have a different colour on each side, for example, 12 red/blue counters. Ask them to decide how to record what is happening with each toss in order to keep track of the colour combinations. Ask: What is it that we need to record?

Which Method?
Have students compare methods of collecting data. For example: Which method should be used to decide whether there are more girls or boys in the school? Data could be collected by tallying students as they arrive at school, using the class roll, counting everyone at an assembly, or having all students pair up. Ask: Which method would be the simplest? the most helpful?

Finding Information
Ask students to prepare one or two questions to collect data about their classmates, such as their after-school activities and interests. Have students swap questions and complete them. Students then read the responses of others and reflect on the usefulness of their questions. Ask: Did your question give you the information you wanted? Was there any information you were not expecting to get? Could you change your question to make sure you get the information you want?

Popular Foods
Have students collect data in different ways and compare results. For example: To find out which are the most popular foods sold in the cafeteria, one group might check sales for a week and another group might survey students. Ask: Did you find the same information? Which method best helped you answer your question?

Paper Chase
Ask students to decide how they can find out how much paper is used in the school for photocopying. Ask: Could you find out by using information that is already in the school?

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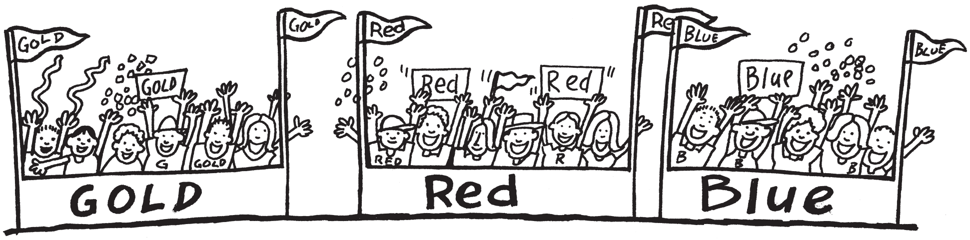
Sports Day

Invite students to decide how to collect data during a sports day. Have them brainstorm different types of data they could collect to write an article for the school newspaper. Ask: What would people like to know about our sports day? How could we collect this information? Students may observe the teams at various times in the day to decide which is cheered the most, or they may decide to use the information recorded for the long jump to compare the jumps of different age groups. Ask: Did your method of collecting data help you plan your article? Should you have used a different method to collect your information? Why?



Speeding Up

Have students rework data produced to answer one question in order to answer other questions. For example, students collect data to answer the question "What are the fastest times for the different track and field events in the last few Olympics?" They then use this data to answer other questions such as "Are the athletes really getting faster?"

Recording Data

Have students decide how to record information so that nothing is lost in the process. For example: Are students from the same family always on the same team? Are the students who play on the school volleyball team the same students who play on the basketball team? Ask: Can you record this so that you can find which students are on which team?

Question Box

Encourage students to consider a range of ways to produce data that will help them answer a question. Have pairs of students take a question of interest from a class question box. Ask: Is it possible to answer the question by counting or measuring something, asking groups of people, watching what happens, or reworking some existing data? Which of these ways would you choose to answer your question? Why?

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Sample Learning Activities

Grades 5–8: ★ ★ ★ Major Focus

If the World Were a Village
After reading *If the World Were a Village* by David Smith, ask students what question the boy in the story might have been asking himself for each calculation. Have them then decide what data needs to be collected to answer each of the questions.

Cereal Survey (1)
Have students each prepare a two- or three-question survey to collect data from their classmates about a particular new cereal. For example: How many times have you eaten the cereal? Did you enjoy it? What did you enjoy about it? When did you feel hungry again after eating it? Would you recommend your parents buy it again? Do you think it is good value for money?

Cereal Survey (2)
Ask students to test and revise questions. For example: Before using questions from the previous activity in a real survey, have them exchange questions, complete them, and return them to the owner. Students then read the responses and reflect on the usefulness of their questions. Ask: Did your question give you the information you wanted? Was there any information you were not expecting to get? Could you change your question to make sure you get the information you want?

Cereal Survey (3)
Have students decide whether the answers to the questions in the previous activity could be recorded using numbers or words. Ask: How does this affect how you will record your data?

Anticipating Answers
Ask students to anticipate what people might say in response to a question before deciding how to collect and record the data. For example, they can anticipate responses to an open question like "What did you enjoy about … [a certain book, movie, new cereal]?" Ask students to consider asking people to rate their enjoyment on a 1–10 scale. Ask: Which data gives the best indication of people's enjoyment? Why? When would it make sense to use a rating scale and when would it not?

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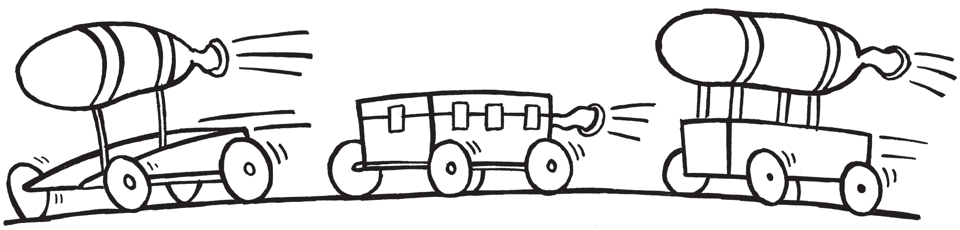
What Kind of Response?

Invite students to decide between fixed choices or open-ended responses when planning a short series of questions. They can consider whether the responses to questions should be in the form of a tick, a word, a sentence, or a paragraph, by discussing the benefits and risks of each. Draw out that while collecting data in categories may close some options, it may open up possibilities that students had not thought of before. The responses from open-ended questions may be difficult to sort but give broader information.

Balloon Power

Encourage students to think ahead to what data will answer their question. For example, have them brainstorm data that might help them find out which balloon-powered car works the best. Ask: Would frequency data of the number of cars travelling certain distances be enough to answer the question? What information would you record prior to or during the trial? Is there other information that you could record during the trial that might help answer the question?



Make a Spinner

Have students collect data to test predictions. Ask them to construct a spinner that they think is most likely to stop on red, least likely to stop on green and have the same chance of stopping on yellow and blue. After they have designed their spinners, ask: How do you know it works? Students decide what data they will need to collect to test the spinners they designed.

Allowance

Ask students to consider how the way a question is asked can bias responses. For example, have them write, test, and revise two or three questions to find out what parents think about the idea that children should get an allowance only if they do special jobs at home. Compare responses to an open-ended question, a multiple-choice question, a yes/no question, and a statement with a rating scale. Ask: How does each method affect the data you get?

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Key Understanding 3

Organizing data in different ways may tell us different things.

In developing this Key Understanding, students should learn that

- classification underlies the organization of data
- how we classify depends on the questions we want to answer
- data organized in different ways may tell us different things
- data organized in a particular way may expose some things and mask others
- new questions may require a reorganization of data
- reorganizing data often suggests new questions for investigation

A central part of designing data production is thinking carefully about how data is organized during the collection phase, and how it might ultimately be further organized and reorganized for analysis. The process of producing data is usually based on classifications (Do we just record "bird" or do we record different types of birds?), and it is important to think about this ahead of time. Also, data is often recorded in an organized way (e.g., in a tally sheet) and the form of organization used can determine how useful the data is. Planning ahead in this way is not an approach students commonly use. Possibly the best way for them to learn to do so is from seeing what goes wrong when they do not follow a logical, organized process. Teachers can also model the process, not by making all the decisions for the students, but by exposing their own thinking processes and providing scaffolding to enable the students to plan for themselves.

In the early grades, students will classify and sequence in straightforward ways to answer simple questions. Initially, they classify and sequence actual objects or pictures of objects rather than data. Later, their classifications broaden to include data represented in various forms such as paper strips, words, and numbers. Even relatively simple classifications can be ambiguous (Is it blue or green?), and students need experience with the kinds of dilemmas people face when making decisions about whether an item belongs to one category or another. If the purpose of classifying is clear and makes sense to them, students are more likely to recognize ambiguities in their classifications and try to clarify or improve on their descriptions of categories.

Students should be expected to make suggestions about how to classify and sequence their data in order to answer particular questions. Over time they should explore the effect of different classifications on what they can or cannot learn about the things they are investigating. Thus, in investigating their favourite sports, one way of organizing their data might suggest that

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students generally prefer ball games over non-ball games. However, another way might suggest it is not ball games students like so much—rather that they prefer team to individual sports. Similarly, one way of classifying favourite books might enable students to decide whether children like books that have pictures better than those that do not. Another way of organizing the books might mask this distinction. Older students should develop strategies for improving their efficiency in sequencing and classifying larger sets of data and represent two-way classifications in diagrams and tables.

Links to the Phases

Phase	Students who are through this phase in Data Management…
Matching and Comparing	■ can apply familiar and unambiguous criteria to classify and sequence data consistently
Quantifying	■ make suggestions about how to classify their data to answer straightforward questions ■ are beginning to understand that the same data might be reorganized to answer different questions
Measuring	■ are consciously aware that different classifications may be necessary to answer different questions ■ can suggest how to improve a classification strategy to better suit the purpose
Relating	■ can plan class intervals as a way of organizing and classifying their measures, e.g., a student may find the mass of various rocks and group the data from under 250 g, 250 g to 500 g, and over 500 g.

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Sample Learning Activities

K-Grade 3: ★ ★ Important Focus

Sorting
Ask students to sort a collection of things, such as shells, leaves, seed pods, or toys, and make general statements about how they are the same/different.

Building
Have students sort objects into those that are useful for building and those that are not. Discuss their reasons for grouping them, then construct a house using the useful objects and see if they change their mind about the grouping.

Shoeboxes
Invite each student to keep 10 different things in a shoebox to sort. Ask students to sort them according to different criteria on different days. Use their suggestions for groupings and include a group of things that "are not."

Labelling Groups
Have students collect a container full of different items from inside/outside their classroom and sort them using their own groupings. Students make labels for groupings, but hide them. Other students then guess what labels they have used for their groups. When the label suggested is different, ask: Could both labels be used for the group?

Different Groups
After the previous activity, have students sort the same collection into different groups. Ask: Is it possible to put them into two groups instead of four (or four instead of two)? What would you call each group?

Shoelaces
Prompt students to decide what information to collect so they can find someone to tie their shoelaces. Ask: When you ask people, how can we keep track of who can and who cannot tie their shoelaces? Would writing down the names of the students help? Which names should we write down?

Transport Groups
Have students create a chart using pictures of different sorts of transport. Ask them to decide on their own categories and then compare their groupings with those of others. Ask: Why do the groups show different things? Is one way of grouping better than another?

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Once Upon a Time

Invite students to find more efficient ways to organize their data. For example, after reading *Once Upon a Time* by John Prater or a similar book, have them work out how many different characters are in the book. Ask: How could we organize the data so that we do not miss any of them, or count them twice? Encourage students to list the names of the different stories included, tally the characters for each, and then find the total.

Language Data

Have students organize their data into their own categories and then compare how others have organized their information. For example, after they have recorded the different languages spoken in the class, ask: How could we organize the information to make it easier to read? Draw out that lists and tables help us organize our data.

What Is Work?

Ask students to clarify categories to sort objects. When talking about working in the home, ask students to define what can be included in their categories. Ask: What would we include as work? Is cooking supper work? Is helping with cooking supper work?

Where Do You Fit?

Invite students to decide which of two categories they belong to. Place labels around the room so that all students fit into one category or the other (boys/girls, wearing sneakers/not wearing sneakers, brought their lunch with them/did not bring lunch). Have students decide where to stand. Ask: What made you decide to stand on that side of the room? What do we know about Claire if she is standing over here?

Favourite Vegetable

Ask students to draw and colour a picture of their favourite vegetable on a small square of paper and then arrange all the pictures into groups according to the type of vegetable. Have students count the groups to work out which is the most popular. Ask: What would happen if we placed our vegetables into groups according to their colour? Students then count the new groups and say which colour of vegetable is the most popular. Ask: Why do we get different results from the way we group our pictures?

Scary Things

Extend *Scary Things*, page 108. Ask students to draw a picture of the thing that scares them the most and sort the pictures into categories that they decide on. Students need to work out how to deal with pictures that do not seem to go into their existing categories or belong to two categories. Be aware that some students may feel shy about drawing or discussing what scares them the most. (See Case Study 2, page 124.)

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Sample Learning Activities

Grades 3–5: ★ ★ ★ Major Focus

Where Do You Fit?

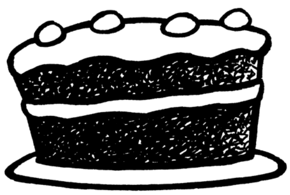
Ask one student to decide on two categories (e.g., blue eyes, not blue eyes) and start to put other students into these two groups. Invite the others to work out what the categories are by looking at what the students in the groups have in common. Extend this to include attributes that are more ambiguous (e.g., light brown and dark brown hair).

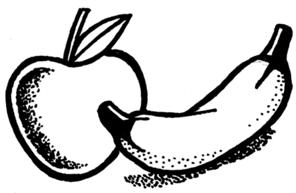
Recording Information

After the previous activity, help students to record the information in a table. Ask: What were our two categories? Write this on the board, then ask: Which students belong in each category? Record the names under each heading. Draw out that organizing information in a table helps us to see at a glance who belongs in each group after they have physically moved away.

Food Groups

When creating a table of different food types, ask students to clarify the categories that have been used. Have them add items to the table to show what foods they have eaten. Ask: What sorts of foods would we add to the fruit and vegetable group? What foods would we add to the dairy group? Where would we put things that are made up of foods from different groups, such as chocolate cake?





Classifying Objects

Have students classify objects using ambiguous classifications. For example, a collection of leaves or rocks could be sorted according to size and/or colour. Ask: Are there more large ones or small ones? Are there more brown ones than red ones? When students begin to struggle with placement, ask: What is the problem? How could we decide where to put that item? How can we clarify what we mean by large and small?

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Categories

Have students decide on categories before asking their questions or observing the situation, and then decide how useful their categories were. For example, prior to a class trip to the zoo, students might have chosen the categories "hair" and "fur" to group animals. During the trip ask: Were there some animals you found difficult to put into these groups? How could you change the groups to make it easier? Do these groups help us to answer the questions that we now have?

Birthdays

After students have answered one question with data they have collected, ask another question that requires them to reorganize their information. For example, after they have worked out which months students have birthdays in, ask which season most birthdays are in. Ask: What do we have to do with the data?

Tosses

Invite students to choose a fixed number of counters with a different colour on each side, say eight red/blue, and list all the possible outcomes when the counters are tossed together (eight blue; seven red and one blue; and so on). Ask them to decide how they can record the information to show how many times each pair of colours come up. Students can record as they go, then compare the different recording methods and say which are easier to use to record and which are easier to read. Draw out that lists and tables are useful in recording information in an organized way. Help students to decide which table would be the easiest to use to record the information and repeat the activity using this table.

Getting to School

Have students decide how to create a table to record information on how students get to school. Ask: Would you list all of the modes of transport and tally under each category as you ask? Students can then say whether they were able to record all information on the table or whether it needs to be modified. Ask: How else could you have created the table?

Favourite Sport

Ask students to organize information and then try different categories to see if this changes the result. For example, ask them to write their favourite sport on a sticky label and then decide how to group the labels to answer the question "What is our favourite sport?" Ask: What does this organization of the information show? How else could we organize this information? Have them rearrange the sticky labels and then decide whether the new organization shows something different. Ask: Which organization best helps us to answer our question?

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Grades 3–5: ★ ★ ★ Major Focus

Venn Diagrams (1)

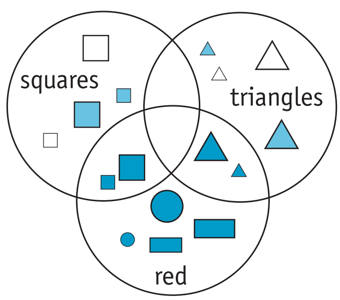
Have students decide which category they belong to and place themselves in a Venn diagram created with rope or tape. Start with two separate circles and ask students to stand in one circle if they play hockey and the other if they play soccer. Ask: How can we show that some people play both? Move the circles so that they overlap. Students then stand in the appropriate place. Ask: Where do people stand if they play neither? (*Outside both circles*)

Venn Diagrams (2)

Extend the previous activity to include a third group, such as baseball. Start with another separate circle. Ask: How could you show that some people play both baseball and hockey? How can we show that some people play three different sports? Move the circle so that it overlaps the first two and have students move into the appropriate place.

Venn Diagrams (3)

Invite students to identify discrete categories by using Venn diagrams. For example, ask them to sort attribute blocks into three circles showing squares, triangles, and red shapes. Ask: Why are there not any shapes in the overlap between squares and triangles?



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Sample Learning Activities

Grades 5–8: ★ ★ ★ **Major Focus**

Sorting Cars

Ask students to sort pictures of similar objects and describe the spatial likenesses and differences between them. For example, prompt them to consider what is the same/different about the shape of the following:

- minivan and pick-up truck
- kitchen chair and lounge chair
- running shoes and basketball shoes
- mountain bike and BMX bike
- bear and beaver

Have students give a partner some of the pictures to sort into their categories. Ask: Were they able to use your categories? Are some of the categories ambiguous? How can you clarify your categories?

Food Groups (1)

Ask students to decide on their own food groups and define what characteristics each group will have in common. Have them record everything they eat and drink over a day and sort their list into their groups as a way of categorizing what they ate. Ask students to define their categories and see if partners can use these to sort their own lists of food. Ask: Is it possible to sort all the foods into these groups, or do you need to define another group? How are these groups similar to or different from the standard food groups? Which way of grouping is the most helpful in designing a healthy eating plan?

Food Groups (2)

Extend the previous activity and have students compare how they and their partner presented their information. Ask: How did you show what your categories are? How did you show what is in each category? Students compare how they presented their information in a table with other ways of presentation.

Sorting Students

Extend *Venn Diagrams*, page 120, to topics that include more categories. For example: Students decide on three categories of pet ownership and show which category they belong to by standing in a Venn diagram created with rope or tape. They overlap circles to show that some people own more than one type of pet. Ask: Where do people stand if they do not own a pet or do not own a pet that is in one of the categories? Have students rename the categories to include more children.

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Grades 5–8: ★ ★ ★ Major Focus

Question Box

Have students sort questions from their class question box into categories according to whether they can answer them

- from their own experience
- from an information source such as a book or a website
- by producing some data

Prompt students to place the carded questions on a Venn diagram showing the three options and add to it over time. Ask: Can some questions be answered in more than one way? How do you show that on your diagram?

Favourite Sport

Invite students to suggest how to change a classification to answer different questions. Students can write the name of their favourite sport on a sticky label and together organize the labels to see which event is liked most by boys and which by girls. Ask: Does this tell you what event is most liked by all students? How would you reorganize the labels to find out? (See Case Study 3, page 126.)

Favourite Food

After using data to answer a particular question (such as "What is our favourite food?"), ask students to decide whether their data could be used to answer a different question. For example: Do we like to eat healthy foods? Do we prefer burgers or chicken? Is price an issue? Do students in our class like more expensive food? Ask: Would the same organization of our data answer this new question? How could we organize our information to answer our new question?

Allowance (1)

Show students the following table in electronic format.

Amount of allowance children receive each week		
Amount ($)	**Gender**	**Work/Do not work for it**
5	G	DW
7.50	G	W
5	G	DW
10	B	W
5	B	DW

Can you see how much money children receive each week from this table? Ask: Can you see any interesting trends?

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Allowance (2)

Extend the previous activity to see that sorting in different ways can expose some things and mask others. Students can use the "sort" option to sort by the different columns and then say which way of sorting the information best helps them to answer their question. Ask: Does one way of sorting hide the "interesting" information? When might you use this way of sorting?

Balloon Power

Have students structure and record data electronically (e.g., a table with a "sort" option or a spreadsheet), then sort to answer specific questions. For example, they could record data from the trials of their balloon-powered cars. They then sort the data in different ways to answer questions such as these: Is there a relationship between the distance travelled and the weight of the car? Do the cars with larger wheels travel farther than the cars with smaller wheels?

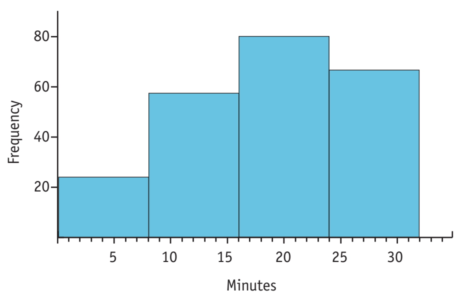
Two-way Tables

Invite students to decide how to set up a table to investigate relationships in simple data. For example: Which is most popular—white or whole-wheat bread with jam or peanut butter? Ask: How could you record the information in a table as you gather it? Could you use labels on both the columns and the rows? Does the table help us to see at a glance which is most popular?

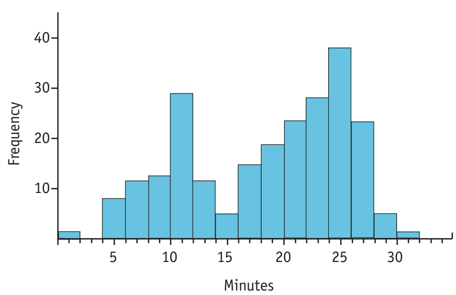
Graphing

Ask students to use graphing software to see how grouping measurement data into different-sized intervals may expose some things and mask others. For example: Examine graphs such as those below and say how the different intervals affect what you can see in the graph.

Length of time it takes to get to school



Length of time it takes to get to school



Collect and Organize Data KU 3

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CASE STUDY 2

Sample Learning Activity: K–Grade 3—Scary Things, page 117

Key Understanding 3: Organizing data in different ways may tell us different things.

Working Towards: Matching and Comparing phase

I noticed that many of the students' pictures depicted more than one idea, such as a monster in a dark room. Production of authentic data may often mean that students are dealing with "messy" data that does not fall easily into neat and obvious categories. However, students need experience in making decisions about organizing this type of data if they are to achieve the outcome.

MOTIVATION AND TEACHER'S PURPOSE

A group of Grade 2 students were in the classroom reading corner.

"I love this book," said Jasmine. "It is about monsters, and monsters are scary."

"Monsters are not scary," said Hashim.

"Yes they are!" added Laila.

I recognized an opportunity for the students to produce and organize some data!

ACTION

I read the class the first few pages of a story about a little dog that imagined all sorts of scary things. I then presented them with a question: Is there something that scares most of us?

The students each drew a picture of what scared them most and took turns to tell the group about their pictures as they stuck them haphazardly on the board. I led a discussion in which the students decided to sort their pictures and count the number in each group to answer the question. Soon they began to argue about whether monsters and ghosts belonged in the same group, eventually agreeing that ghosts were white and monsters were any other colour. They continued to sort, using their chosen categories of monsters, ghosts, sharks, spiders, bad dreams, lightning, and the dark.

DRAWING OUT THE MATHEMATICS

I drew my students' attention to a picture of ghosts looming in a dark room. "Where will we put this picture?"

"That one is ghosts, so it goes in the ghost group," said Hashim.

"But it should be in the dark group!" said Laila.

"That picture could go in two groups," I responded. "How can we decide where to put it?"

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"I know," said Jasmine. "We should look at what is most in the picture. There is only a little bit of dark and lots of ghosts, so it is a ghost picture."

The other students consented and continued to classify pictures until they came to a picture of a monster with the word "dream" written underneath.

"It is a dream picture because it is a dream!" said Jacob.

"No, it is a monster picture. It is a dream monster," argued Maggie.

I said, "We will need to choose which group to put this picture into."

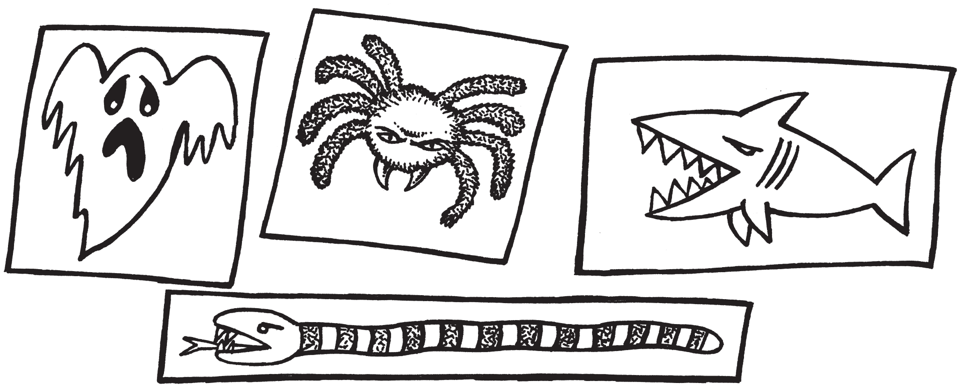
"But which is the right group?" asked Jacob.

"There is not a 'right' group," I said. "We have to make up our minds about what groups we want so we can sort the other pictures in the same way."

They sorted the remaining pictures and counted the numbers of pictures in each group. They concluded that bad dreams scared most of the children.

REFLECTION

I was satisfied that my students had participated in making decisions to organize some "messy" data. Next lesson I planned to draw their attention to the possible different answers they may have reached if they had made other decisions during the organization process.



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CASE STUDY 3

Sample Learning Activity: Grades 5–8—Favourite Sport, page 122

Key Understanding 3: Organizing data in different ways may tell us different things.

Working Towards: Quantifying phase

MOTIVATION AND TEACHER'S PURPOSE

Some Grade 5 students were discussing which events they had liked and not liked at the school's recent sports day. At my suggestion, they were keen to further investigate their preferences. I anticipated that the type of data they would collect would lend itself to a discussion about organizing data in different ways.

CONNECTION AND CHALLENGE

The students came to a consensus about the question they wanted to investigate: Did Grade 5 boys and girls both like the same event best? They decided that the students in their class and in the Grade 5 class next door should write their name and the event they liked best on a sticky label. The labels were then stuck on the board and sorted, first into boys and girls, and then into events. The number in each group was recorded. The students saw that hurdles was liked best by more boys than any other event and that ball toss was liked best by more girls than any other event. The Grade 5 boys and girls did not like the same event best.

DRAWING OUT THE MATHEMATICS

In this case, the data could have been first sorted according to event, and then according to gender.

	Long jump	High jump	Ball toss	Running	Hurdles
Boys	5	5	2	8	10
Girls	7	4	9	6	5

I said "So, if we were going to do just one of these activities during phys.ed. on Friday, how could we use this data to help us decide?"

"We would do hurdles because that has got the biggest number," said Flavio.

"But that is what the boys like," said Josie "The girls liked ball toss best and there is only five who voted for hurdles."

"We are trying to decide on just one activity for everybody to play," I said.

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"We might have to think about grouping our data in a different way."

Sam reasoned, "Well, it does not matter if they are boys or girls—we all have to play. You have to look at the big groups. Like you have to add the boys and girls up."

I invited Sam to draw on the board to help him explain his idea to the class.

I commented, "Sam has reorganized our data into new groups so we can see how many people, boys and girls together, like each event best. What event is liked best by most Grade 5 students?"

	Long Jump	High Jump	Ball Toss	Running	Hurdles
Boys	5	5	2	8	10
Girls	7	4	9	6	5
	12	9	11	14	15

George said, "Hurdles, because that has got the biggest number."

"But that is still not fair," Fatima added. "The girls like ball toss best."

I said: "When our data are all together, we cannot see the difference between boys and girls. When we think about just the girls, ball toss is liked best by the greatest number. When we think about just the boys, the hurdles is liked best by the greatest number. But when we think about all Grade 5 students, hurdles is liked best by the greatest number."

"But when we do sports on Fridays," commented Diedre, "it is just our class. The other class is not there."

I responded: "I wonder if this would make a difference to our answer? How could we group our data to help us find out?"

After some discussion, they decided to group the data into classes and then group the events within each class. They then re-sorted the data to find out if both Grade 5 classes liked the same event best. The students were quite surprised to see that this reorganization of data showed that neither ball toss nor hurdles was liked best by the greatest number of students in their class or in the other class. In fact, long jump was liked by the greatest number in their class and running races were liked best by the greatest number in the other class.

I took the opportunity for students to consider that different classifications may affect what they can and cannot find out about things they are investigating.

The data could have been first sorted according to event and then according to class.

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Using sticky labels on the whiteboard enabled the students to physically reorganize the same set of data into new categories.

	Long Jump	High Jump	Ball Toss	Running	Hurdles
Our class	9	3	7	4	7
Other class	3	6	4	10	8

If the data had not lent itself to demonstrating these ideas, I could have focused on recording this type of information in two-way tables and then used another data set on another occasion to explore the effects of organizing data in different ways.

I was satisfied that my students had seen that organizing data in different ways may tell them different things, and that how they organized their data depended on what they were trying to find out. They had also had an opportunity to see how organizing data in a particular way may expose or mask some things.

Quantifying students are able to make suggestions about how to classify data to answer straightforward questions, and are beginning to understand that the same data may be reorganized to answer different questions.

EXTENSION

I decided to work with my students to show how the same information could be recorded and sorted in a table or spreadsheet on a computer. They went on to collect information about the best-liked events from the rest of the students in the school and put all of their data into a spreadsheet program. They could see how using the computer to help organize their data made reorganizing categories much easier than trying to manipulate their original data sheets.

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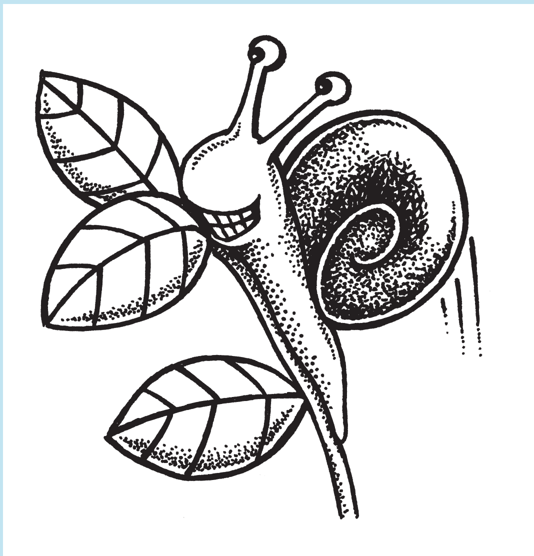
Did You Know?

To investigate how a snail travels across the garden, we can record different types of information about its journey:

- quantitative (its trail was 2.3 m long) or qualitative (its trail was transparent and gooey)
- subjective (it was travelling really slowly) or objective (it is under that table)

We can record observations of the snail itself as it moves (in real time or on film), reconstruct its path by observing the snail trail it left or the damage to plants along its route, or ask several people to remember and tell us about its path. Thus, we could produce many different kinds of data from and about the one phenomenon. The data we choose to record depends on what question we want to answer.



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Key Understanding 4

We should make our data as accurate and consistent as possible.

An important but subtle aspect of this Key Understanding is that we work with data itself rather than the original things and we cannot expect to be able to fill in the gaps by adding personal knowledge to our data or recreating the circumstances that produced it. Students need to understand that data has to stand alone and be good enough to represent features of objects, events, and experiences that are no longer there. They need opportunities to make decisions individually and collaboratively, and note what goes wrong when they do not plan their data collection well so that, with the help of their teacher, they can improve their techniques.

We usually judge whether data is "good enough" by two criteria—its accuracy and its consistency. Within statistics, these are referred to as *validity* and *reliability*, respectively, although students do not need to use these terms.

Being accurate (or valid) is about making sure that we are getting a true measure or indicator of the thing we are interested in. When determining pet popularity by counting pets owned, we need to decide whether an aquarium of fish counts as one or whether each fish counts as one. Both numbers are correct, but the issue is which is the better indicator of popularity of fish over dogs and other pets. When asking for people's opinion, we need to think carefully about how to ask the question so that we do not bias their responses. To test a prediction that the area of one's hand print is about one-fortieth of one's total body area, we need to know whether it is palm only or fingers as well. The production of accurate data links closely with Key Understanding 1, since we need to ensure that the way we reframe questions and define terms remains true to the original question.

Being consistent (or reliable) is about making sure that we do not introduce chance variation into our data through erratic data collection processes. We want to feel confident that the same information would be recorded on different occasions or by different observers. Inconsistencies in data occur when we

- are careless in making or recording our observations (not counting some of the people arriving, entering tally marks in the wrong column)

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- do not do it the same way each time (sometimes leaving our shoes on when measuring height and sometimes not, sometimes including aqua as blue and sometimes as green)
- do not make sure that we are all doing it the same way (some measuring arm length from armpit to wrist and others from shoulder to wrist)

Students should be helped to consciously plan approaches to data collection that minimize these sources of error and variation.

Links to the Phases

Phase	Students who are through this phase in Data Management...
Quantifying	■ will, when prompted, clarify what to record in each category, e.g., a student will clarify whether an aquarium of six fish should be counted as six or as one in the fish category ■ are beginning to be careful in their data collection, e.g., when making tallies, a student will try not to miss making any marks
Measuring	■ recognize the need to measure what they think they are measuring, e.g., a student will think about how she or he phrases a survey question ■ take care with their data collection but may not anticipate difficulties or plan ahead
Relating	■ will anticipate problems, plan ahead, and do test runs to ensure that their measurements or frequency counts are accurate and consistent

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Sample Learning Activities

K-Grade 3: ★ Introduction, Consolidation, or Extension

Sorting Toys
Have students sort a collection of toys into two groups: inside toys and outside toys. On a different day, ask them to sort the same set of toys again. Ask: How can we make sure that we always put the same toys into the same groups?

Once Upon a Time
Invite students to find ways to organize their counting so they do not recount, or miss anything. For example, after reading *Once Upon a Time* by John Prater or a similar book, have them count to say how many different characters are in the book. Ask: How could we organize our counting so that we do not miss any of them, or count them twice? Encourage students to list the names of the different stories included, tally the characters for each, and then find the total. Ask: Are there other ways we could check that we have counted correctly?

Long Jump
Ask students to use a piece of paper tape to record how far they can jump, then say whether their jump is possibly the longest in the class. Ask: How can we make sure that each piece of tape is measuring all of a jump and no more? Students share and compare where they placed the tape to mark the start and where they tore the tape off to mark the end of their jumps. Help them come to a consensus about how to get an accurate measure.

Favourite Colours
Encourage students to consider situations where data is collected inconsistently to influence the outcome. For example: Let the students know that your favourite colour is, for example, blue and ask them to vote on which colour to use to make a sign for the classroom door. Count several blue votes more than once, and do not count some of the other votes. When students complain the vote is not fair, ask: Why do you think it is not fair? What would make it more fair?

Favourite Vegetables
Have students collect data to determine their favourite vegetable. Before they start, clarify what data is going to be collected. For example, ask: Are we going to give one favourite vegetable each, or several?

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Bingo

As students play Bingo, have them record the letters or numbers that come up on their sheet. Ask: How can we make sure that the person who says "Bingo" first has correctly marked a card? Would it help to keep a record of all of the numbers or letters that come up as they happen?

Foot Size

Have students each measure the size of one of their feet, then compile the data and compare their results. Ask: Have we all measured the same foot part? Which part could we measure? If we were to compare the size of a Grade 2 student's feet with a Grade 7 student's feet, which part of the feet should we all measure?

Language Data

After students have decided on categories to use to collect their information, clarify what they might put into each group. For example, when collecting data on the languages spoken in the class, ask: Where will you put someone who can speak both French and Arabic?

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Sample Learning Activities

Grades 3–5: ★ ★ ★ Major Focus

Measuring Chicks

After students collect measurement data, have them check that they have measured the same thing. For example, when measuring the growth of the class chicks using blocks, prompt them to show how they measured a chick. Ask: Did you all measure in the same way? Do you think it matters? Why? Draw out that to find out how much the chicks have grown, we need to get a true measure of the chick's height and we need to all measure in the same way.

Making Hats

Have students collect measurement data and then consider why it matters whether they have measured correctly and all in the same way. For example: They measure the distance around the heads of Grade 1 students to make hats. After using this information to make the hats, ask them why some hats did not fit.

How Tall?

After students collect measurement data, have them review the numbers and see if they all seem reasonable. For example: Have students measure the heights of everyone in the class to find out if students born in the first half of the year are taller than those born in the second half. Ask: Is Jodie really taller than Zak? Does it matter if we have our shoes on or off? Does it matter how each person stands, or how we find the top of a person's head? Invite students to suggest how to make their measurements more accurate and then re-measure both groups and compare the results with those obtained the first time.

Car Colours

Have students make decisions about ambiguous categories. For example, ask students to collect data on car colours, then wait a few days and ask them to collect more data on car colours. When ambiguities arise, ask: Who can remember what we did last time? Did we call "silver" grey or white? How could you have described the categories so you all sort the colours in the same way? Would more categories help avoid having to make difficult decisions?

Different Methods

Invite students to compare different counting and recording methods across different situations. For example: They could use tallies, electronic counters, the constant function on a calculator, and counting to say how many students

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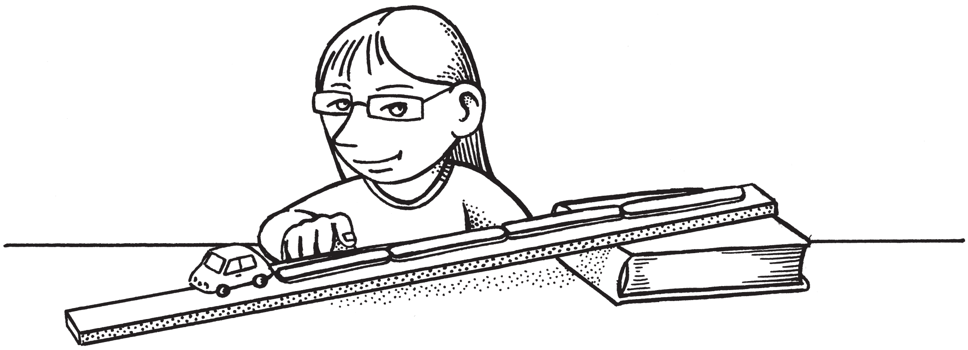
were wearing yellow, or to keep track of how many laps around the basketball court a student runs. Ask them which methods are most reliable (that is, consistent) for each situation.

Model Cars

Encourage students to refine their measuring technique of an object or event prior to collecting data to answer a specific question. For example, have individual students use craft sticks to measure and record the distance travelled by their model cars down an inclined plane. Ask: How is it that we can get different measurements for the same distance? How should we carry out our measurements so that we all get the same answer each time for the same distance?



Keeping Track

After students have recorded data to answer a question, ask them to reflect on how easy it was to keep track of their data. For example, have a group of students go around to each class to find out how many boys and girls are at school that day. Ask: What could you have done to make it easier to record your data and check that you had the correct amount? Would organizing it into a table have helped? How?

Checking for Consistency

Before collecting data, have students brainstorm ways of checking that their data will be consistent. Ideas include working in pairs and cross-checking their results. Have them try out their suggestions and reflect on their usefulness. Ask: Did you both collect the same data? If there were differences, why do you think they occurred?

More Scary Things

Have students collect and organize data about themselves in response to a question like "What scared you the most when you were younger?" Students then choose how to collect comparative data from others and reflect on the problems with accuracy that occur when they do not carefully plan and agree on the question and collection methods. (See Case Study 4, page 138.)

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Sample Learning Activities

Grades 5–8: ★ ★ ★ Major Focus

What Are You Measuring?

When students take measurements to use as data, ask them to consider how to make their measurements accurate. If they want to measure how long their pets are, do they include the tail? Do they measure in a straight line from nose to tail? Or do they allow the tape to follow the curves on the head and back? Ask: Which would be a better indicator of the length? Are you measuring what you think you are measuring? How do you know?

Growth Experiments

Invite students to consider how to ensure measurements are consistent when recording data. For example, during growth experiments of living things, like the height of wheat plants or the length and mass of baby guinea pigs, ask students to write instructions for taking the measurements so that different students at different times will take the measurements in the same way. Ask: How could our data be affected if we do not write clear instructions?

Growth Patterns

Arrange for students to visit the Kindergarten or Grade 1 classroom each month and take careful height measurements of each student to examine the growth pattern of young children. Explain that the overall growth will not be more than a few centimetres. Ask: So what do you need to think about to ensure that your measurements are accurate and consistent? (For example, a standard height measuring instrument, ensure not wearing shoes, stand the same way each time, careful reading of the scale.)

How Do We Measure?

When students are collecting measurement data, ask them to describe how they measured and to consider questions like these:

- Why does it matter if we all use the stopwatch in different ways to check how long it takes people to walk a distance?
- When weighing seeds collected, why does it matter that some people weighed the container as well?
- When measuring to see how much children have grown, why does it matter if you let them leave shoes on one time and take them off another time?

Encourage students to say why it is important to measure correctly and all measure in the same way, and to predict what effect this would have on the conclusions they can draw from their data.

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Inaccuracies

Show what can go wrong if students do not measure correctly and in the same way. For example, have students collect data about the diameter and circumference of lids to investigate relationship. Ask: Why would it be difficult to plot the results in a scatter plot if everyone has measured in a different way? How does inaccurate measuring affect how clearly you can see relationships in the data?

Balloon Power

Ask students to consider whether their data represents the situation well enough or whether they have to think back to the situation to answer their question. For example, if considering which balloon-powered car is the most reliable, ask students to say whether a record of the average distance travelled by all the cars is sufficient to answer their question. Why? Why not? What data would you have collected if you knew then what you know now?

Consistent Data

Have students decide how they could ensure consistent data when recording more difficult or complex frequency data. For example:

- Counting vehicles that pass the school could be shared so that one student counts cars, one counts trucks, and another counts bicycles and motorbikes.
- When counting weeds on a section of a lawn, the area could be marked into small squares and different students could count different squares.
- To count the number of ants in a particular place, four students could all count at the same time, and then average their totals.

Ask: How can you ensure that you are recording all that you can see?

Missing Answers

When students are organizing data obtained from written questions, ask them to discuss what they should do about missing answers, or answers that are obviously incorrect. Ask them to consider how including such information could affect their data and conclusions. For example: If they want to find out the mean allowance that students in their class receive, and one child writes $2000 per week, would you include that response or not? What effect would this have on conclusions?

Zero Values

Have students decide when it is or is not appropriate to include values or measures of zero in data. For example: When collecting information about the number of children in a family, would you need to include families with no children? Why? Why not? Draw out that the decision to include zero values depends on the original question.

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CASE STUDY 4

Sample Learning Activity: Grades 3–5—More Scary Things, page 135

Key Understanding 4: We should make our data as accurate and consistent as possible.

Working Towards: Quantifying phase

I wanted my students to experience for themselves that "asking people" is no simple matter and that problems may arise if data collection is not carefully planned.

TEACHER'S PURPOSE

My class of nine and ten year-olds had made many decisions in the organization of their data in response to an initial question, "What scared you the most when you were younger?" However, the way their responses were recorded had been planned by me. I wanted to give them some experience in making decisions so they could see that the accuracy and consistency of their data could affect their conclusions.

ACTION

The students decided that, although some of them were more scared of one thing than another, the list was fairly predictable—they had thought the same kinds of things were scary as did the Grade 2 students who had also worked on the project. We wondered if adults would have been scared of the same kinds of things when they were children.

With a little prompting, the students decided they could collect some data to find out for themselves. Angelina suggested they could ask some adults that night and bring the information to class the next day. I hoped their data would provide me with the opportunity to draw out the need for more careful planning.

DRAWING OUT THE MATHEMATICS

The following day, I asked my students to work in small groups and talk about what they had found out. As I walked around the room, I listened to what they were saying. I then asked students to share what they had found.

"I found out that my mom was scared of snakes just like I am, but Josiah's stepdad was scared of vampires," said Rhiannon.

"But vampires were not on our list," said Elizabeth.

"That does not matter; that is what my stepdad said he was scared of," said Josiah.

"But you were supposed to ask if it was the same things," responded Elizabeth. "I gave Grandma the list and she ticked what scared her."

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"Yes," added Rhiannon. "I thought we had to find out if they were scared of the same stuff—I just asked Mom if she was scared of snakes like me when I was little."

Others joined in, arguing about what they thought they were supposed to find out. They realized that, although they assumed they all knew what had been needed, they had interpreted the big question differently. They decided they should have all asked, "What scared you most when you were young?" so they could match their own responses to the same question.

Josiah explained, "But that is kind of what I asked my stepdad: 'When you were a kid, what was the scariest thing?' "

I asked, "And can you remember exactly what he said?"

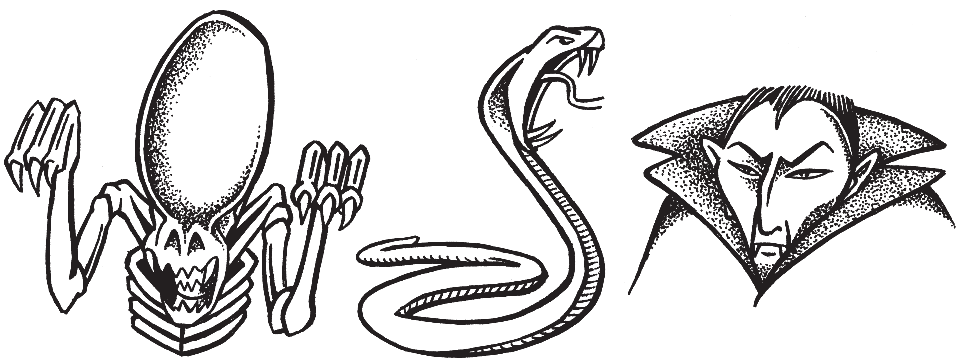
"He told me about when he was my age or a bit older, he saw a movie about vampires and he got really scared by it."

"But that means your dad was older than we are," said Hari. "I'm not scared of monsters now, only when I was in preschool."

This stimulated a lot of talk about just what was asked and how the adults responded. The students also realized the data they had gathered about adults had to stand alone. When they were categorizing data about themselves, they had the opportunity to add and clarify what they had initially written. They could not do this with the new data. For example: "My uncle was scared of aliens, but he did not say how old he was."

Hari pointed out, "Rhiannon's mom might have been *most* scared of something else, but Rhiannon only asked her if she had been scared of snakes when she was little, and we do not even know how old she was either."

Although in the initial class discussion we did not clarify what younger meant, students often prefaced what they said with "When I was in preschool" or "When I was in Grade 1," which tended to focus the rest on that age group when answering the question themselves. We cannot make the assumption that "we all know what we mean" when the data is collected from others.



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Through this conversation, we were able to focus on how accurate their data was for answering our initial question, and the students themselves decided that there were too many problems for it to be useful. They wanted to go back and ask again. With help, they formulated a question that they thought would elicit more accurate data: "When you were about five years old, what scared you the most?"

Once they were satisfied with the question, I drew attention to the idea of consistency. I asked: "Now that you have this question, do you think you will all remember it and not change it at all when you ask it tonight?" This stimulated more discussion, and they decided to type the question on the computer, print, and photocopy it so the adults could write their answers on the sheet. That way, we would be sure they would all answer exactly the same question.

REFLECTION

The next day, when students brought back their responses, they discovered another problem that we had not anticipated. Some adults had written down several different scary things. We did not know which scared them the most or if they were equally scared of several things.

This finding helped students to reflect on the need to carefully think through the way we produce our data, and, when asking people, to even anticipate possible responses, to make sure that the information we get back from our question is as accurate and reliable as it can possibly be.

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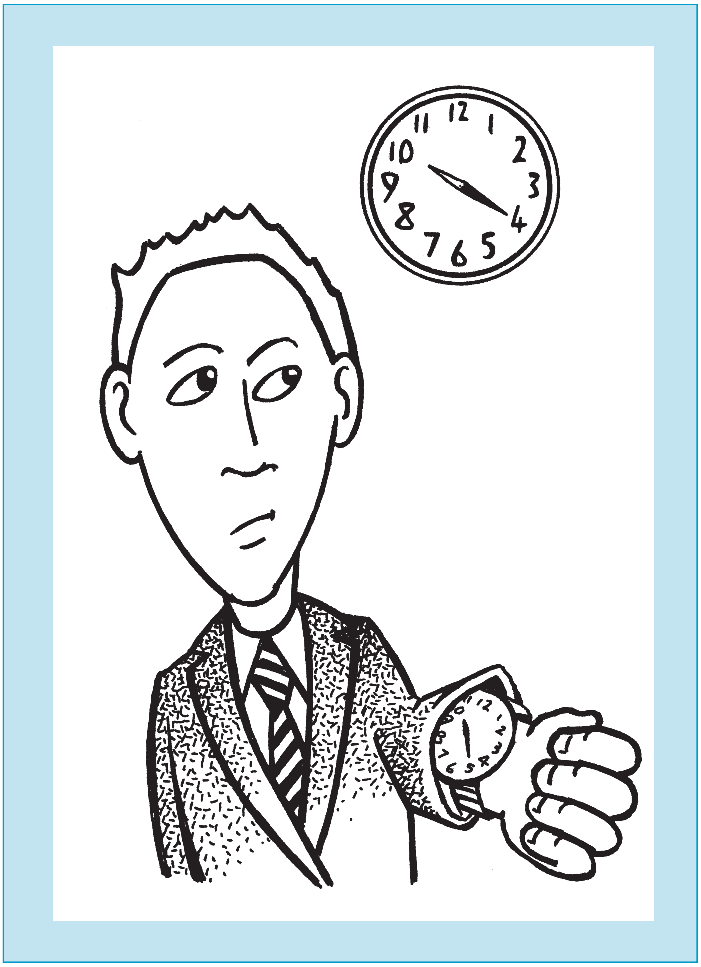
Did You Know?

Consider a watch that never loses or gains time but is always exactly five minutes fast. Anybody reading the watch would get an inaccurate (that is, invalid) measure of the time but a completely consistent, or reliable, one. The owner of the watch could rely on it always being five minutes fast. A watch that lost or gained time would be inconsistent or unreliable. Of course, such an inconsistent watch would also be invalid as a measurer of time because it could not be relied upon to be accurate either! This is why statisticians say that an instrument or measuring device can be reliable without being valid, but cannot be valid without being reliable.

Similarly, a test would be considered an accurate (or valid) measure of mathematics achievement if the data it generated truly reflected the mathematical learning we say we value. It would be a consistent, or reliable, measure of mathematics achievement if a student taking the same (or an equivalent) test in another setting would be expected to do equally well, and if different markers would come to the same conclusions.



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Key Understanding 5

Sometimes we collect data from a subset of a group to find out things about the whole group. There are benefits and risks in this.

We can sample some of the people, some of the things, some of the time, some of the results. … The essential idea in sampling is that we can make inferences about a whole group from data produced on a subset of that group. That is, we assume that the sample (subset) will behave in a roughly similar fashion to the population (whole group) and hence draw conclusions about the whole from the part. In doing so, we make statements about how confident we are in making those generalizations.

Considerable care needs to be taken in selecting samples from populations and in forming conclusions about populations from samples so that we can make clear statements about how confident we are of the conclusions drawn. However, the technical processes involved in selecting samples, determining sample size, and making inferences from samples are not straightforward, and developing these more technical skills is not a goal for the elementary years.

The essence of this Key Understanding for the K–8 programs is twofold. First, students should learn to distinguish between collecting data on a whole group (a census of the population) and collecting data on a subset of the group (a sample of the population), with the intention of drawing conclusions about the whole group. Second, they should consider in an informal way whether it makes sense to collect data from a subset of a whole group and, if so, how they should choose the smaller group.

Distinguishing a sample from a population is not as obvious as it first seems. Sometimes, the population is infinite, as in the number of times you could toss a coin. Sometimes it is changing constantly, as in all of the people in Canada. And sometimes it is hard to determine the population a sample represents. For example: What is the population to which we can generalize results from our class of Grade 6 students? Students eventually learn to distinguish samples from populations in reasonably straightforward situations. They compare situations where a census is needed and those where a sample may be sufficient or necessary. For example, in investigating the insect population, they may develop strategies for sampling parts of the garden. In investigating music tastes, they may decide that surveying the Grade 6 students in their school would give a pretty good indication of music taste among 11-year-olds, but not older or younger children.

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Students should also discuss things that might introduce bias into samples and consider ways of overcoming bias in the selection of samples. In doing so, they could informally consider the three main ways that we construct samples. First, we might construct stratified samples where parts of a sample are chosen from each of several groups such as gender, grade level, or geographic location. Second, we might select random samples, as in drawing names out of a hat where every member of the population has an equal chance of being selected in the sample. And third, we can select convenience samples in which we decide to work with the people or objects or times readily available to us.

The approach should be quite informal, with sampling activities ranging over these three approaches without students having to name them or distinguish them during the elementary years. Students should informally ask themselves this question: How confident are we that the sample fairly represents the population?

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Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

Autumn Leaves

Have students collect fallen leaves during autumn and then group them to show which ones come from the same tree. Ask: Which trees drop the most leaves? Do some trees keep their leaves and not drop them? Does our grouping of the leaves show which trees we have more of in the school yard?

Buttons

Invite students to group a collection of buttons according to how many holes in each. Ask: Does this mean that most people have two-holed buttons on their clothes? Draw out that looking at just a few buttons does not tell us much about all button wearers.

Eye Colour

Have students draw a picture of their eyes to show their eye colour and then group all the pictures to find out how many students have each colour. Ask them to consider the numbers in each group and to say whether the same number of students would have the same colour in different classes.

Did You Know?

Telephone polls where people give a response by phoning a particular number can give completely misleading results. A telephone poll in the United States once asked people to phone in to respond to the question: If you had the chance to live your life over again, would you have children? The overwhelming response was "no." The government was so concerned that it commissioned a survey that asked the same question of a carefully stratified sample of the population. This time there was an overwhelming "yes" response. One problem with self-selected samples, such as the telephone poll, is that there is no way to control for bias in the sample—and more than one call is often accepted from the same telephone. In this case, the phone-in was invited at about 6:30 p.m.—a particularly difficult time of day for most parents. It is not difficult to see how that particular phone-in sample may have been biased towards a "no" vote.

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Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Favourite Cereal

Present students with situations where we cannot collect information on everyone or everything of interest, and ask them to decide what to do. For example: Which breakfast cereal is sold the most? Ask: We cannot find out what every single person in the country has bought this week, but what else could we do to find out? Who could we ask? Draw out that sometimes we have to collect a little bit of data and use it to give us an idea about what might happen in general.

Hangman Extended

After playing Hangman, page 29, ask students how they could find out which letters are the most common. Ask: Can we check every single word ever written? What could we do instead? Where should the words come from? Should we look in a dictionary? Draw out that sometimes it is appropriate and sensible to collect a bit of data, but we need to be careful about where it comes from.

Extraordinary Deals

After reading *Ordinary Mary's Extraordinary Deed* by Emily Pearson or a similar book, ask students: What types of ordinary deeds that people do every day help other people? (Example: Helping someone carry groceries) How would these deeds become extraordinary? (*They'd become extraordinary, if they led to five other deeds*...)

Fair Sample

Invite students to decide if selecting a certain sample would be fair. For example: We want to find out what sort of sports equipment to buy but only have time to ask 30 students. Should we ask just the boys? Just the Grade 6s? What would be a fair sample?

Pen Pals

When writing to pen pals about what typical Canadian students do, ask: Is what our class does enough of a sample? How would you find out what other Canadian children do? Where would these children live?

Comparing Data

Have students compare class "where we were born" data with national figures. Ask: Is our class typical? Could we use our class as the representative for the country? Can you think of reasons for the difference? Draw out that often a sample does not represent the whole population.

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Sample Learning Activities

Grades 5–8: ★★ Important Focus

Sampling

Encourage students to research and brainstorm how samples and sampling are used in the world around us. Ask: When you hear the phrases "taking a sample" or "getting a sample," what do you think of? Draw out ideas of sampling through free samples, phone polls, tagging of members of a certain species of animal to estimate their population, and samples taken during an election.

Is It Fair?

Give students situations that would clearly be non-representative. For example, ask: If I wanted to find out what sports we should include on our sports day, would asking the Grade 6 boys what they like be a good way to get that information? Would that be fair? Why not? What part of the school population would that sample favour?

Choosing a Sample

Have students explore different ways of selecting a sample. For example: We want to find out if Grades 5, 6, and 7 students think they should or should not line up before entering a classroom. To avoid disrupting the classrooms, we decided to ask 30 students during lunchtime to sample opinion from the six classes. We could ask just our Grade 5 class, which would be convenient, or the first 30 people from any of those grades we come across. We could put everyone's name in a hat and draw out 30 names, or we could choose a stratified sample of five (drawn randomly) from each classroom. Discuss which option is preferable. Ask: Which is the easiest sample to choose? Which is the more representative sample? Which would give us the most confidence in our prediction? Have students carry out the survey in these different ways, and also survey all Grades 5, 6, and 7 students. Compare the results with their ideas about which sampling strategy was best.

School Representatives

Prompt students to discuss whether students in their class would be representative of the whole school for collecting data about a specific topic. Ask: Would it be fair to ask just our class? Would a better sample be a few students from each class—the same number in the sample but more representative of the whole school? Why would you think this kind of sample is better? Would we also try to choose equal numbers of boys and girls in a sample? Why? Why not?

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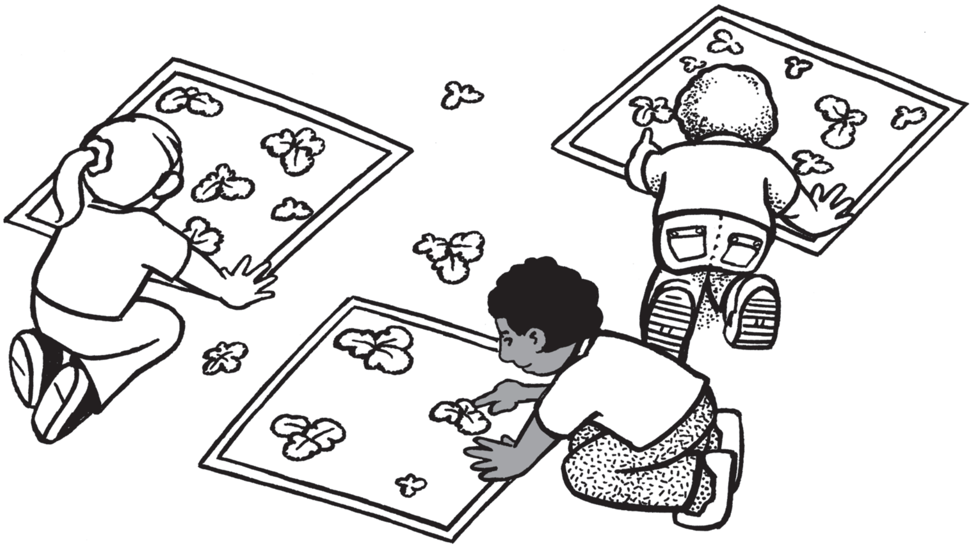
Generalizations

After students collect data from their own class, ask: Is it reasonable to generalize? Could we say this is the case for all Grade 5s? all students? Why not? What other data would need to be collected?

Weed Growth

Discuss ways of sampling weed growth on the playground. Ask: Would it be sensible to count all the weeds on the whole playground? What could we do? Draw out that we could, for example, mark out a square metre, and count those weeds, multiplying the number by the total square metres on the playground. Ask: Would several squares in different parts of the playground be more representative of the overall weed growth?



Making Assumptions

While carrying out chance activities, draw attention to the assumptions we make when we use data such as rainy days or accidents at school to make predictions about the future. Discuss how confident we are that our sample (the data we have collected) represents its population (all of what has already happened). Draw out that for this kind of population, we can only ever use a sample.

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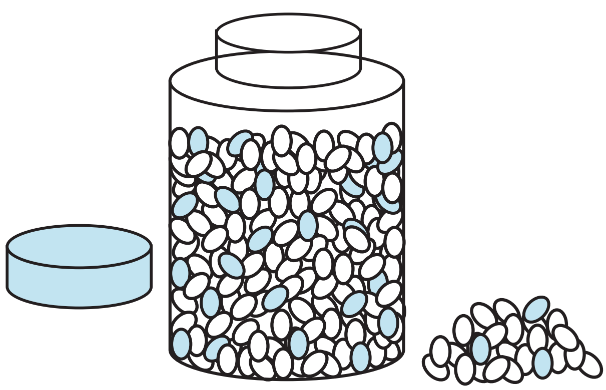
Grades 5–8: ★★ Important Focus

Counting Fish

Have students consider how it is possible for scientists to estimate the number of fish in a lake. Explain that they do this by catching, tagging, and then releasing a set number of fish. Later they catch some fish again and work out what fraction of the catch is tagged. Because they already know the total number of tagged fish in the lake, they can use this relationship to estimate the total fish population. For example, they release 100 tagged fish. They later catch 100 fish and find 10 of them are tagged. This is one-tenth of the caught fish, so they assume that the 100 tagged fish should be about one-tenth of the total fish population. This means the total number of fish in the lake should be about 10 times 100 (about 1000).

Have students simulate part of this process using beans in a jar to represent fish in a lake and consider how useful such a strategy might be. As a class, help students count out 800 white beans (total fish in the lake), then colour 200 of those (a quarter of the beans) with a felt pen (tagged fish). Put all the beans in a large jar and mix them well. Have students take turns to "catch" a small number of "fish" (that is, grab a small handful of beans without looking). They count how many fish they have caught and check how many of them are tagged fish. We know that the true fraction of tagged fish is one-quarter (200 out of 800), so we can check how close our different samples are to this. Ask: Are about a quarter of your fish tagged? What if you put your catch together with your partner's fish, or your group's fish? Are the tagged fish about a quarter of the total catch now? How many fish do you think we would need to catch to give an accurate estimate of our fish population?

Repeat the process using different sized samples to test their suggestions. Draw out that for small samples, the results vary a lot, but for larger samples the fraction of tagged fish will be closer to the true relationship. Note: This strategy works only if the tagged fish are well mixed in with the rest of the fish.



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Chapter 4

Collect and Process Data

(Part B) Summarize and Represent Data

Plan and undertake data collection and organize Data for effective interpretation.

Overall Description

Students summarize and represent data produced by themselves and others. They describe patterns in data and make concise but meaningful summaries using statistics to describe proportions, averages, and variability. They understand that none of these statistical tools are ends in themselves—they are useful only insofar as they assist interpretation and communication. They choose and use diagrams, tables, plots, and graphs that are suited to the kind of data and the purpose of the display.

They understand, on the one hand, that quickly produced displays can be very informative both for their own understanding of the data, and when trying to give others a "snapshot" impression of trends and relationships. On the other hand, they see that there are times when precision is essential so that they and others do not misinterpret, arrive at erroneous conclusions, or mislead. Therefore, they realize that data can be distorted accidentally or deliberately to reach inappropriate conclusions. They consider the impact of technological change on the handling of data. They also consider ethical issues in the representation of data and act responsibly in this regard.

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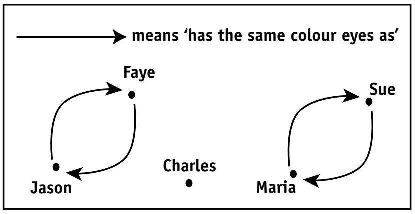
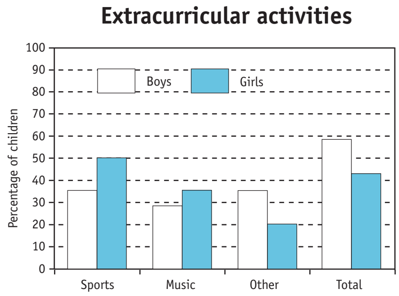
BACKGROUND NOTES

Types of Data Displays

Arrow Diagrams

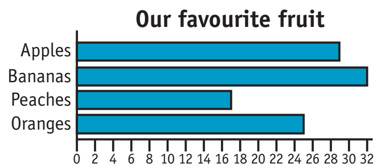
Used to organize ideas, categories, and relationships: Symbols for objects, people, and so on, are spread out on a page and arrows are used to link the symbols in meaningful ways. The display needs to have a legend or key that shows how the arrows should be interpreted.

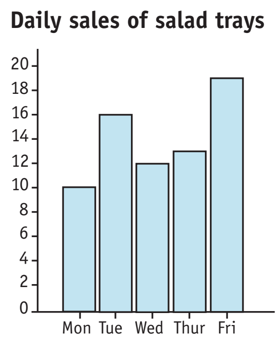




Bar Graphs

A common graph type that uses the lengths of columns or rows to represent frequencies or measurements of categories or groups: A wide range of data can be represented, with either axis being used for categories or groups, and the other axis calibrated as a scale to show a count, a percentage, or a measurement. The lengths of the bars should be proportional to each other and where the data is about discrete categories, the columns or rows (bars) must be separated. Different sources of the same kind of data can be compared by putting two or more bars side by side and providing a key to show the meaning of adjacent bars. *See also Histograms.*





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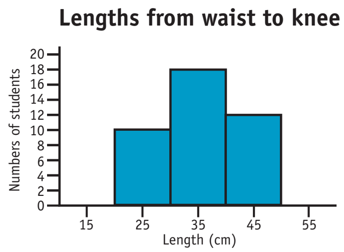
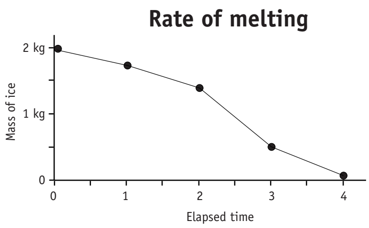
Histograms

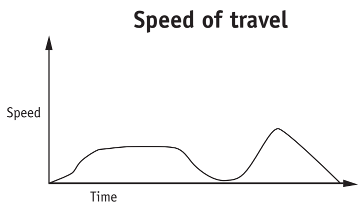
A variation of a bar graph used for continuous quantities, or categories that can be thought of as naturally ordered in time or quantity: Bars are generally vertical, with the columns touching to represent the continuity between the groups of data. If different intervals of data are used, the bars may be different widths, so that the area of the bars is proportional, as well as the length.



Line Graphs

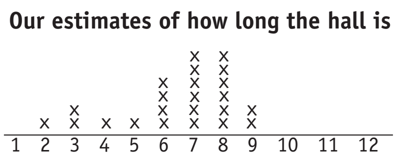
Used when it is meaningful to think of the frequency or measurement varying, usually over time: Points are plotted at intervals and the points joined to represent how the quantity changes between the data points. The base axis must be calibrated as a measurement scale so that every point on the line has meaning.





Line Plots

Used to record or display frequency data. Dots, crosses, or other equal-sized marks are used to represent each piece of data. They are placed above a baseline that has been labelled for each category or number. A second axis is not needed because each mark represents one piece of data.



One-way Tables

Sets out related information in adjacent lists: Only the columns in one-way tables are labelled. The list of names or information in the first column links in rows to other information across the page.

Name	Eye colour
Mai	blue
Freya	brown
Peter	blue
Kim	brown

Eye colour	Frequency
blue	////
brown	//
green	/

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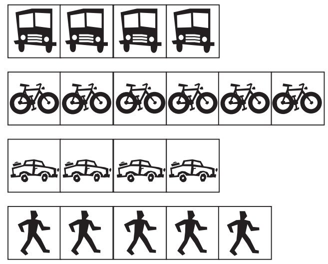
Pictographs

Small pictures or icons that relate to different categories of data are placed equidistant from each other in rows or columns. Each icon may represent one or more pieces of data. A key will indicate if an icon means a quantity of data, in which case in-between quantities are represented by parts of icons.

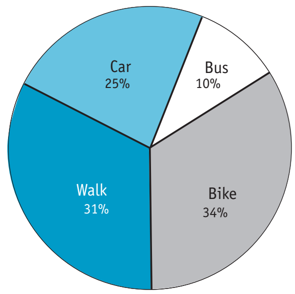
How we come to school



Pie Graphs (also called Circle Graphs)

Circle graphs in which the area of the slices relates proportionally to the quantities in the categories: The use of a pie graph makes sense only when each piece of data belongs in exactly one category of a clearly defined whole set of data. The quantities are normally shown as percentages.

How our class came to school this morning



Stem Plots (also called Stem and Leaf Plots)

Used to display numerical data that ranges from zero into the hundreds. The tens are listed in a column, with the units of each piece of data listed in order in rows to the right and/or to the left of the respective tens number. A stem plot enables the full data set to be visible, while creating a graph of the data grouped in tens. This form of display makes it easy to see the mode(s) and to work out the median.

Girls		Boys
2 1	0	3 4
6	1	0 3 5
7 4 4 0	2	1 1 3 4 7
8 8 4 2 2 1 1	3	0 2 2 3 4 5 7 9
3 0	4	0 5 6
9 4 4 4 3 1	5	1 3 3 3 4 9
7 6 1 1 1	6	3
	7	0

0	4 3
1	5 0 3
2	1 1 7 4 3
3	9 3 7 0 2 2 4 5
4	0 6 5
5	3 4 1 9 3 3
6	3
7	0

Two-way Tables

Information is shown in two or more categories in columns and rows, which may or may not need to be totalled. A two-way table is useful for showing how different types of frequency data might be related, for example, the different ways that boys and girls travel to school.

	Girls	Boys	
Year 6	15	13	28
Year 7	12	18	30
	27	31	58

	Heads	Tails
Heads	HH	HT
Tails	TH	TT

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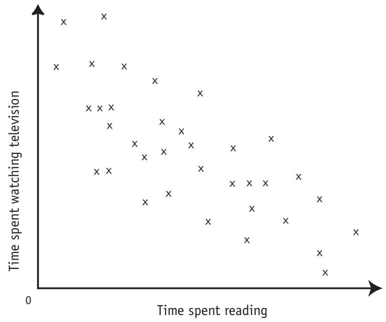
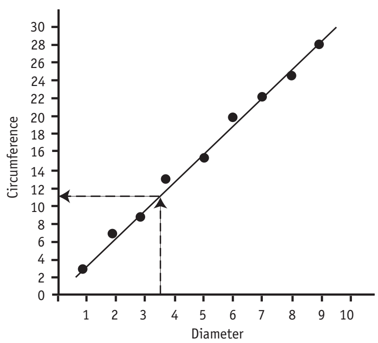
Relationship between time spent reading and time spent watching television



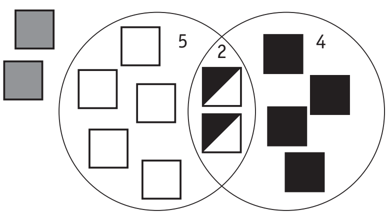
Relationship between diameter and circumference of lids



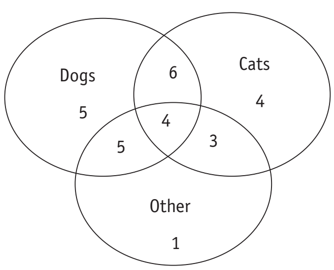
Scatter Plots

Demonstrates visually how two different types of data are related: Two different measures for the same person or thing are paired and plotted on a two-axis grid. Each axis relates to one type of measure, and a mark shows where the two measures intersect. A "line of best fit" is sometimes used to emphasize strong relationships between the data.

Comparing shapes and colours of blocks



Children in our class who have pets



Venn Diagrams

Used to visually represent data that have overlapping categories: The number in each category is shown within an oval, and the number that is in more than one category is shown in the relevant overlapping sections.

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Collect and Process Data (Part B):
Key Understandings Overview

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of this family of concepts. The learning experiences should connect to students' current knowledge and understandings rather than to their grade level.

Key Understanding		Description
KU1	We can display data visually; some graphs and plots show how many or how much is in each category or group.	page 156
KU2	We can display data visually; some graphs and plots show how one quantity varies over time.	page 174
KU3	We can display data visually; some graphs and plots show how two quantities are related.	page 182
KU4	We use tables and diagrams to organize and summarize data in a systematic way.	page 190
KU5	How we display our data depends on the kind of data we have and our purpose.	page 198
KU6	We can use words and numbers to summarize features of a set of data.	page 210

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Grade Levels— Degree of Emphasis			Sample Learning Activities
K-3	3-5	5-8	
★★	★★★	★★★	K-Grade 3, page 158 Grades 3-5, page 161 Grades 5-8, page 165
★	★★	★★★	K-Grade 3, page 176 Grades 3-5, page 178 Grades 5-8, page 180
★	★	★★	K-Grade 3, page 184 Grades 3-5, page 185 Grades 5-8, page 186
★★	★★★	★★★	K-Grade 3, page 192 Grades 3-5, page 194 Grades 5-8, page 196
★	★★	★★	K-Grade 3, page 200 Grades 3-5, page 201 Grades 5-8, page 204
★	★★	★★★	K-Grade 3, page 212 Grades 3-5, page 213 Grades 5-8, page 215

Key

★★★ **Major Focus**
The development of this Key Understanding is a major focus of planned activities.

★★ **Important Focus**
The development of this Key Understanding is an important focus of planned activities.

★ **Introduction, Consolidation, or Extension**
Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

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Key Understanding 1

We can display data visually; some graphs and plots show how many or how much is in each category or group.

The first aspect of this Key Understanding is that we can display data visually. It is common to Key Understandings 1, 2, and 3 and develops gradually from related work for all three. The second aspect of this Key Understanding relates to the plots and graphs used to represent one-variable data where a frequency or measurement is the thing that varies, as in line plots, stem plots, pictographs, bar graphs, histograms, and pie graphs. With the exception of pie graphs, these plots and graphs are based on columns or rows, each labelled with a category or group, where the length of a column or row represents the frequency or measurement associated with that category or group.

Initially, the need to use a common baseline and to space objects evenly to ensure a match between number and length will not be obvious to students. To understand why this is important, they will need plenty of opportunities to make their own displays using objects or pictures that they have made or collected, rather than simply filling in spaces on provided grids. Through exploring what their displays show (or seem to show), they will gradually see the need to think about appropriate placement.

Using one thing to stand in for another is not obvious, and students need help to make the transition from displaying actual things to representing these with tokens or pictures. For example, after lining up according to eye colour, they might each write their name and draw their eyes on a sticky label, and use one-to-one correspondence to build up a graph. Such pictographs are a way for students to begin to abstract or simplify information, and this development should not be rushed. Students often continue to want to show the identity of each piece of data in their displays but, with help, they will gradually learn to represent data where information about individual values is increasingly summarized and therefore some information becomes lost or obscured. For example, they can replace their personal drawings with a more abstract cross above the appropriate column on a line plot, or colour in a corresponding square to form a pictograph. As students move from Kindergarten to Grade 5, they should represent data with increasingly higher levels of abstraction. For example, they should make pictographs or bar graphs using one square or one symbol to represent more than one unit (e.g., one square for 10 people), and make bar graphs to represent other measures such as time or mass.

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There is a conceptual leap required for students to begin using the vertical scale to produce bar graphs directly from frequency data, rather than colouring or counting squares one at a time. They must switch from seeing the data as individual pieces of information to seeing it as aggregate information about a group and understand that length is used to represent an amount.

In Grades 6 to 8, students should also learn how and for which purposes other representations, such as stem plots and pie graphs, are constructed. They should use bar graphs for grouped data and with more complex scales on axes. They should also make use of computer graphing programs to investigate the effect of varying the groupings or the type of graph on the impression we gain of the data.

Links to the Phases

Phase	Students who are through this phase in Data Management…
Emergent	■ can display their collections physically or using their own pictures
Matching and Comparing	■ can represent one thing with another and display their data in different ways
Quantifying	■ can represent more than one thing with squares or pictures ■ make use of a simple vertical scale to produce bar graphs from frequency data ■ know that lengths in bar graphs can represent other things besides length, such as mass or amount of money raised
Measuring	■ can appropriately represent categories in their bar graphs as either discrete or continuous ■ can use frequency scales based on multiples
Relating	■ can use quite complex scales on axes to produce the full range of graphs required

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Sample Learning Activities

K-Grade 3: ★ ★ Important Focus

Student Groups

Have students group themselves into various categories such as wearing or not wearing red; liking or not liking watermelon. Students then line up in their groups and make comparisons using matching and one-to-one techniques. Ask: How can we tell if one group has more than another? Can we tell just by looking, or do we need to count?

Birthdays (1)

Form groups of students with the same birth month. Using a rope tape on the floor or a wall as a baseline, have students line up in their groups. Take a digital photo of this display, then ask students to look at the photo and say which month has the most students' birthdays. Ask: How can we find out? Can we tell by just looking, or do we need to count?

Birthdays (2)

Draw up a large sheet of paper showing the months of the year. Ask students to write their name on a card, then place the card on the paper on their birthday month. (Allow them to place the cards in a pile if they choose.) Ask: Which month has more birthdays? Are there months with no birthdays? How could we place the cards on the paper so that we could tell just by looking? Help them reorganize the cards into evenly spaced rows and columns. Ask: Do we need to know which card belongs to which person?

Birthdays (3)

After the previous activity, have students use sticky notes with their names on to produce a class pictograph representing the months in which students' birthdays occur. Have each student take back his or her sticky note, replacing it with a cross as the note is taken off, producing a line plot. Ask: Can we use the crosses to tell us which month has the most/fewest birthdays?

Physical Graphs

Use connecting cubes to create physical class graphs for data of interest, such as favourite colours. Initially, allow students to choose cubes of the colour they like best and construct a graph to show the most-liked colours in the class. Then collect other data, such as the type of fruit brought in that day, and ask students to assign a colour to a category, and build up a physical graph.

Colour Graph

Have students consider how to position objects of different sizes when lining them up to show how many in each category or group. For example: Repeat the

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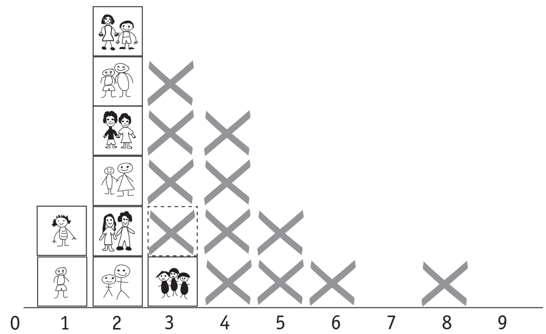
previous activity, replacing connecting cubes with different-sized coloured blocks (small green blocks, large yellow blocks, etc.). Help students line up the blocks to find out which colour is the most popular. Focus on the need to space the blocks evenly. (See Case Study 1, page 171.)

My Family

Invite students to use a line plot as a quick way to represent data. For example: Have each student draw the children in their family on a sticky note, then place the note on a line plot showing numbers along the base from 0 to beyond the largest number of children. Ask each child to remove his/her note and put a cross in its place, to produce a line plot. It may help to first have a discussion of what a family is, since this understanding will be culturally specific.



Comparing Heights

Cut lengths of paper tape that match students' heights. Arrange a number of these tapes on the board, placing them vertically so that they still represent height, but using different starting points that make it difficult for students to compare the lengths. Ask: Can you tell from our display who is the tallest? The highest tape belongs to Ravi, but he is not as tall as Fiona. How could we make it easier to see quickly who the tallest person is? Rearrange the tapes so that they begin on the floor (and therefore are as high as the students). Ask: Is it easier to see now? Then rearrange again so that they are lined up with the bottom of the board. Ask: Can we still tell who is tallest?

Tree Size

Repeat the previous activity, using a horizontal layout instead of vertical. For example: Use strings to measure the distance around trees in the playground. Then ask students to pin the strings on the display board so they can compare the lengths. Allow them to choose how they pin them up. If they pin them at angles and with different starting points, ask: Can we see which is the longest/shortest when they start in different places or are crooked? Why? Draw out the need for all to start on the same line and all to be evenly laid out.

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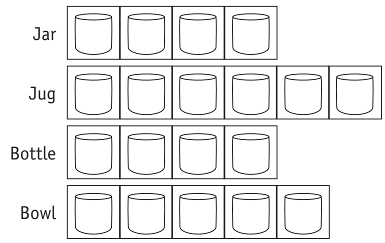
K-Grade 3: ★ ★ Important Focus

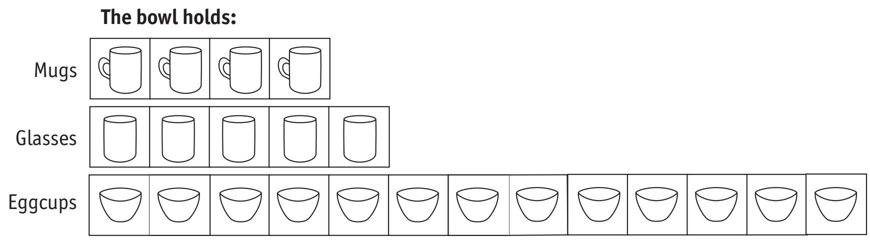
Measurements

After students have recorded their measurements of different objects using the same unit, they can produce a pictograph of the data. For example: Have students use a cup as a unit to measure the capacity of a variety of containers. They then draw a picture of a cup on a number of squares of paper, according to how many cups each container holds. They arrange the squares of paper next to a label for each container tested. Ask: Can you use the graph to say which container held the most, without counting each cup?



Comparing Units

Ask students to measure one object using different units, and produce a pictograph of their data. For example, they can use various objects to fill a bowl with water. Ask: Which object did we need to use the most times to fill the bowl?



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Sample Learning Activities

Grades 3–5: ★ ★ ★ Major Focus

Pictographs

After students have made physical displays of data, have them represent what they have done on paper by drawing pictures or listing names. Compare their representations with the original physical display. Ask: Can you tell just by looking at your drawings which fruit we had the most of? On Nathalie's graph, it looks like we had more bananas, but on Lyle's, it looks like we had more oranges. If we look at our fruit, we had more oranges. How could Nathalie change her graph? Draw out the importance of spacing the pictures evenly. Ask students to do another graph of their data on squared paper and to draw pictures or write names one to each square.

Pictographs Extended

Extend activities like the one above where students draw a picture or write a name for each object, to have them use more abstract representations of data. For example: Use connecting cubes to create physical class graphs for data of interest. Have students assign a category to a colour—e.g., let the blue blocks be the plums—and together build up a physical block graph. Encourage students to show on paper what they see in the three-dimensional graph. Ask: What needs to stay the same on the paper? What could be different?

How Many Dogs?

Extend activities like the ones above by focusing attention on the number of students in each category before they draw their graph. Use large grid paper and ask: How many students have pet dogs? How many blocks will I have to shade to show this?

Pictographs from Data

After students have shaded squares to match a physical graph, ask them to produce graphs directly from unsummarized data. For example: Collect the names of students who fit into specific categories (have no pets at home, one pet, two pets, etc.). Provide students with squared paper and help them to produce a horizontal line under which they write the names of the categories. They then shade squares one to one to represent each piece of data in a block graph, proceeding through a series of levels of abstraction over time and with various topics:

- Initially they name each square above the appropriate category for the relevant student and produce a block graph by shading named squares.

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Grades 3–5: ★ ★ ★ Major Focus

- Next they cross off the students' names one at a time as they shade or mark a square. The result will be a block graph in which individual students are not identifiable.
- Then they count how many students are in each category and shade the appropriate number of squares.

In each case, students name their graph and the horizontal axis.

Pictographs with Scales

Extend the previous activities by providing students with squared paper on which vertical and horizontal axes have been drawn and a simple scale (e.g., 0, 1, 2, …, 10) has been marked on the vertical axis. Repeat the sequence of levels in the activity above, but in each case link the number of squares shaded to the number on the frequency scale. Children name the graph and label each axis. The frequency axis says "number of [children]."

Make Your Own Scale

Extend the previous block graph activities. Provide students with squared paper and ask them to produce a pictograph by one-to-one shading. Ask them to draw in the vertical axis so that another person could see how many were in each group without having to count.

Using the Scale

Provide students with pre-formatted scales as in *Pictographs with Scales* above. Ask them to count how many are in each category from their original data and use the scale to work out how high to shade without counting the squares one by one. Start with a context in which there are nine or ten squares in some categories (it is tedious to draw one square at a time, and mistakes are likely to occur). Draw out the efficiency of using the scale to determine the number of squares.

Bar Graphs (1)

Have students construct simple bar graphs on grid paper from a tally, using the scale on the vertical axis to define the length of each bar, rather than counting squares. Help students to focus on the frequency scale and the length of the bar used to represent the data, not the individual squares. Ask: How can we see the number of squares without counting them?

Bar Graphs (2)

Following the previous activity, change the axis to make 1 square = 2, so students need to take account of this length. Ask: What would happen if we just counted the squares on this graph? How can we work out where 5 finishes when each square means 2 things? What if we made 1 square equal to 5 things? What would change in the graph? (*The length of the vertical axis*) What would still be the same? (*How many things the bar represents*)

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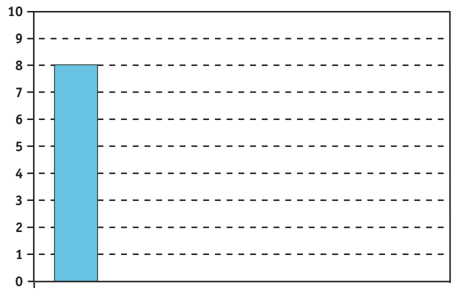
Bar Graphs (3)

Provide students with a pre-formatted axis on paper, where the squares are not visible but lightly dotted horizontal lines are. Ask students to shade columns to the appropriate heights using the vertical scale. Initially, all counting numbers should be on the scale, so students do not need to read between calibrations. Make the point that we usually make the bars the same width to show that the height relates to the different amounts.



Bar Graphs (4)

Have students construct bar graphs with scales calibrated every 2 or 5. Ask them to work out how high to shade a frequency of, say, 3. Repeat with scales calibrated every 2 or 5, but no squares showing, that is, only horizontal dotted lines at labelled calibrations.

Measuring My Length

Ask students to produce bar graphs displaying actual lengths. For example: They cut paper tapes to fit specific body parts (head diameter, length of little finger), write their names on the strips and produce a bar graph to display head diameters (or finger length) of students in their group of eight, and then in the class as a whole.

Graphing Measurements

Have students collect measurement data other than length and construct bar graphs in which the height of the columns represents the measurements. For example: Ask students to fill identical paper cups with different materials (flour, water, sand, metal shavings) and find the mass in grams. Help them to choose a scale, perhaps 1 g to 50 g, and cut paper strips to represent the mass of each material. Use the strips to produce a bar graph.

Measurement Bar Graphs

Extend activities such as the two previous ones by helping students to make a vertical scale to represent the measurements (e.g., label each centimetre as 50 g). Ask students to make a bar graph of their data by reading across from the vertical axis and drawing columns of a suitable height.

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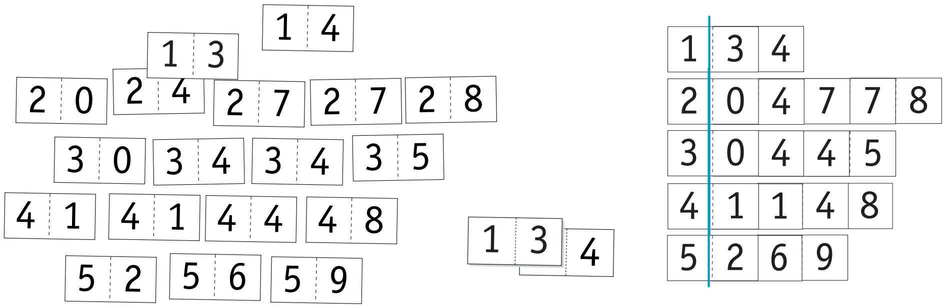
Grades 3–5: ★ ★ ★ Major Focus

Stem and Leaf Plots

Have students produce some data that results in a wide range of two-digit numbers, for example, each student throws a 10-sided number cube 10 times and adds the results to find the score. Have students write their score on a rectangle of paper with the tens digits on the left and the ones digit on the right. As a class, sort the rectangles into the tens and put the groups in order.

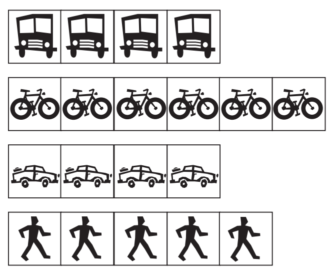


To construct a stem and leaf plot, paste the rectangles into strips for each group of tens, gluing the ones part of the first rectangle on top of the tens part of the next rectangle. Glue each strip on a page, lining them up in order of the tens. Draw a line down the page between the tens and ones to complete the stem and leaf plot. Explain that the stem is the tens digits, and the leaves are the rows of ones digits. Ask: Can you still say what all the different scores were, even though parts are covered? What does the four in the first row mean? What does the four on the left of the fourth row mean? Can you see from the plot where most of the scores are? Do you think you could create a stem and leaf plot on grid paper from the scores alone, without any cutting or gluing?

Pictographs

Find examples of pictographs used in newspaper and magazine reports and use them as a guide for students to construct their own pictographs for suitable data. Ask: What sort of little pictures get chosen? How many things are represented by each little picture? Have students draw, duplicate, and cut out pictures to represent data produced from class surveys and produce their own pictographs.

How we come to school



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Sample Learning Activities

Grades 5–8: ★ ★ ★ **Major Focus**

Frequency Data

Have students produce bar graphs of frequency data based on their own investigations, without props such as pre-formatted axes. Model the appropriate choice of graph names and labels on each axis. Draw out that with frequency bar graphs, one axis shows categories while the other shows the frequency (number) in each category, for example, This axis should be labelled "number of ...." Show students that the bars can be vertically or horizontally laid out. Ask: How does this affect how the axes are labelled?

Large Frequencies

Prompt students to produce bar graphs on grid paper from frequency data, such as census data on the Internet, where the frequencies are sufficiently large to require a calibrated scale that goes up in numbers other than 1, for example, 5 or 10. Ask: What is the largest frequency? What would be a good scale to make the graph fit the height and width you have chosen for it?

Measurement and Price

Invite students to produce bar graphs of measurement or price data, based on their own investigations, such as the results of a survey to find the average height of the students in each grade level at school, or the cost per gram or per kilogram of various foods. Draw out that one axis shows categories while the other shows the measurement or cost. Say: This axis should be labelled "mass in kilograms" (or "price in dollars") to indicate the quantity. Have students choose a suitable calibration, thinking about the size of the graph wanted and the largest quantity they need to represent.

Pictographs

Have students construct their own pictographs for suitable data. Ask: What should we do when we want to show a part of a quantity? How can we decide how much of the little picture to show?

Graphing Predictions

Invite students to use a graph to organize the predictions they make. For example: What do most of us predict will be the number of babies in the guinea pig's litter? On a class line plot constructed on grid paper, students fill in a square above the number they predicted and also write their number next to their name on a class list. Use the graph to see what the majority of the students predicted. Ask: Can we just look at the class list and get the same information at a glance? Why not?

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Grades 5–8: ★ ★ ★ Major Focus

Height of Vertical Bars

Graph data to help students understand that the height of the vertical bars need not reflect the height of things in the real world. While "up" always means more, in the real world this might be "lower," as in more depth or more depressed. Have students graph a range of measures, such as time, depth, mass, and strength of feeling, as well as create their own scales for other attributes. For example, in science they could develop a scale to rate the absorbency of different types of paper, or the strength of different kinds of thread. Have students create a bar graph using the vertical axis for their absorbency or strength scale, and listing the types of paper or thread along the horizontal axis. Ask: What does the length of the bars mean?

Finding the Key

Have students inspect bar graphs and pictographs for the key to interpret the colours, shadings, and pictures. Ask: Have you included a key on your graphs? Encourage students to experiment with ways to do this and then assess ease of reading.

Scale and Labels

Ask students to inspect the scale and labels on the frequency axis of published bar graphs and use this information to help create graphs from their own data sets. Ask: What is the highest value in your data? How can you make sure all your data will fit on your graph? How will you segment the axis?

Comparing Graphs (1)

Have students construct graphs to compare two sets of data that are very different in quantity but not in range, such as food sales over a week at the school cafeteria. Have one set of data showing sales between 10 and 20, the other showing sales between 100 and 110. Ask students to construct a bar graph for each, adjusting the vertical frequency scales to be the same length. (On page 167, see graphs, left and middle.) Ask: Can you see from each graph the quantity sold each day? (*Yes*) Do the graphs enable you to easily compare the daily variation in sales for the two foods? (*No*) Do they give the impression that salad tray sales vary more than grilled cheese sales? (*Yes*) Could we draw a graph that better illustrates the variation in sales for the two foods? Have students redraw the grilled cheese graph, using a scale from 90 to 110, and introduce the convention for showing an incomplete scale. (See graph, above right, page 167.) Ask: How do the graphs compare now? What is emphasized? (*The similarity in the pattern of variation*) What is obscured or less obvious? (*The difference between the quantities of each food sold*) Draw out that this graph could be misleading because at first glance, the total sales for the two foods also look similar.

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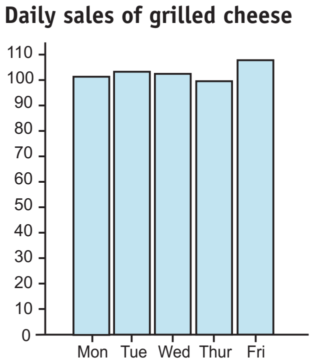
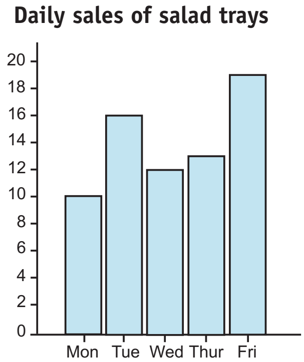
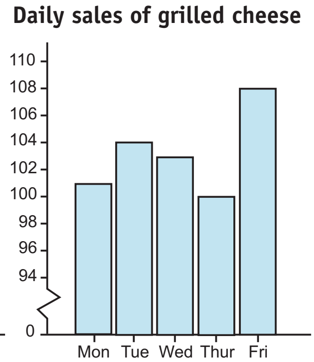
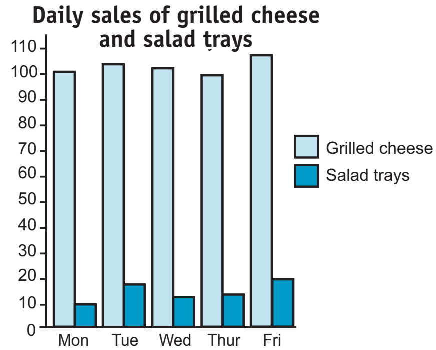






Have students then construct a graph with side-by-side columns showing the sales of both foods each day. (See *Comparing Data*, page 168.) Ask: How has this display made both the total sales and the range of variation easier to interpret?



Comparing Graphs (2)

Extend the previous activity by asking students to find bar graphs in newspapers, in magazines, and on the Internet and having them redraw the graphs, changing the scale in different ways to see the effect on the overall impression gained by the graph. What is emphasized if we do not begin the scale from zero? (*The differences between the categories*) How might this distort the data? How might this clarify the data?

Grouped Data

Invite students to produce bar graphs for data that is grouped, but which they can think of in categories (e.g., measure students' lengths from waist to knee), then group the measures into short, medium, and long for costumes to be made for the school production. Draw out that each category contains a range of leg lengths but the costumes will be cut to one length for each size range. We can represent them in our graphs as three categories. (Here, the vertical bars are kept separate.)

Histograms

Extend the previous activity to introduce simple histograms, where the horizontal axis is a continuous scale for the range and the vertical bars touch to indicate the range of measurements included in each group. Thus the previous data could be grouped as 20 cm–29 cm, 30 cm–39 cm, and 40 cm–49 cm and represented in columns accordingly. Note that some groups of data in the range may have zero frequencies.

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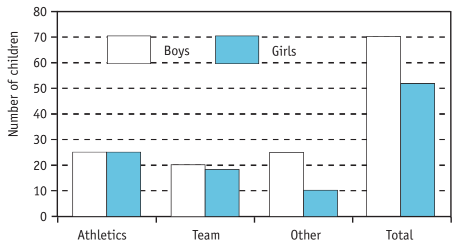
Grades 5–8: ★ ★ ★ Major Focus

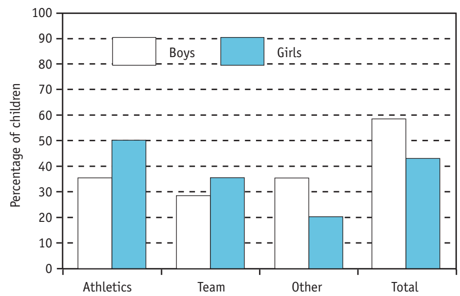
Bars and Histograms

Obtain a range of bar graphs and histograms from various sources, including students' own constructions. Prompt students to sort the graphs into those in which the columns or bars are separate and those in which they are touching. Ask: Can you write instructions as to when it is appropriate for the data to be represented by bars that touch? Do you think any of the graphs might be incorrect? Why?

Comparing Data

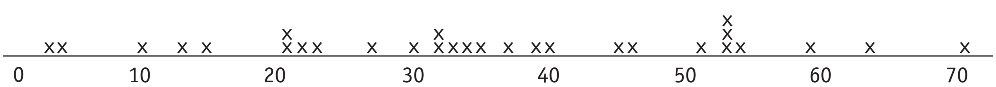
Ask students to use a computer graphing program to create graphs that compare two different sets of data. For example: After surveying all the Grade 7 boys' and girls' preferences for sporting activities, students make a bar graph that compares boys and girls. Show how the bar for girls can be distinguished from the bar for boys, using a key, and consider if percentages rather than frequencies are appropriate. Draw out that percentages are needed when the total number of boys surveyed is different from the girls.

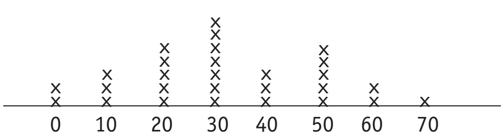




Stem Plots (1)

Have students represent suitable data in a stem and leaf plot, and compare it to the same data shown in line plots. Ask students to construct a line plot for data that has a range of at least 50, for example, points earned by their class. They then round the data to the nearest ten and create a second line plot.





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Help students to then produce a stem and leaf plot, first listing the tens digits down the page and then adding the ones digits of each student's points to the correct row. Students then sort the digits within each tens row to create the stem and leaf plot. Draw out that the list of tens digits forms the stem and the leaves are formed by the rows of ones digits. (The stem can continue beyond nine 10s to ten 10s and so on, for data into the hundreds.) Each piece of data can be read by combining the tens digit in the stem with one of the leaves (a ones digit). Ask: What can you tell about the data in the stem and leaf plot compared to the line plot? What can you no longer see in the plot of the rounded points? Why does rounding to the nearest ten give a different graph profile than is produced in the stem and leaf plot?

0	4 3
1	5 0 3
2	1 1 7 4 3
3	9 3 7 0 2 2 4 5
4	0 6 5
5	3 4 1 9 3 3
6	3
7	0

0	3 4
1	0 3 5
2	1 1 3 4 7
3	0 2 2 3 4 5 7 9
4	0 5 6
5	1 3 3 3 4 9
6	3
7	0

Stem Plots (2)

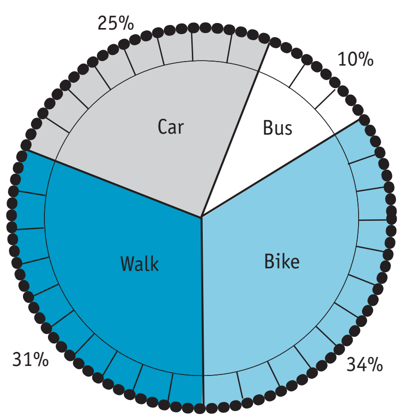
Have students compare two sets of data in a back-to-back stem and leaf plot. Note that the tens digits form the stem down the centre for both sets of data. The ones digits are listed to the left for one set of data, and to the right for the second set of data. For example, the stem and leaf plot shown here compares Grade 6 boys and girls for the number of points they have earned. To find each piece of data, combine the tens (stem) with the ones (leaves). Ask: Can you read the number of points each girl earned in the twenties row? (*20, 24, 24, 27*) How many did each boy earn in the twenties row? (*21, 21, 23, 24, 27*) How is this way of displaying data more helpful than using a table?

Girls		Boys
2 1	0	3 4
6	1	0 3 5
7 4 4 0	2	1 1 3 4 7
8 8 4 2 2 1 1	3	0 2 2 3 4 5 7 9
3 0	4	0 5 6
9 4 4 4 3 1	5	1 3 3 3 4 9
7 6 1 1 1	6	3
	7	0

Pie Graph

Help students use whole-class data, such as ways of travelling to school, to construct a pie graph that shows how percentages relate to the "slices." First, string together exactly 100 beads and tie to form a circle. Cut a strip of card the same length as the bead circle's circumference and divide it into the same number of segments as students in the class. Colour the segments to match the number of students in each category. Draw a circle with the same circumference and place the cardboard strip around the edge to mark off the categories. Join the marks to the centre and then use the bead circle to approximate the percentage of students in each category.

How our class came to school this morning



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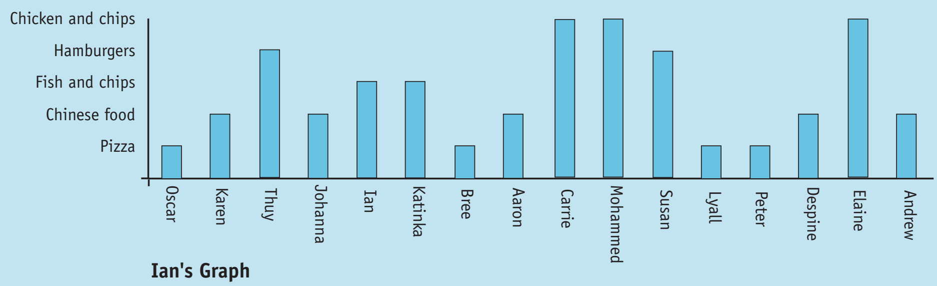
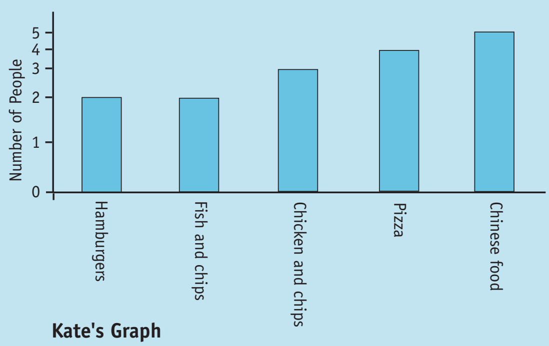
Did You Know?

Ian and Kate (Grade 7) produced these bar graphs, using the same data on their group's preferences for take-out food.

In Ian's graph, we can see Aaron's favourite take-out food and find the frequencies by counting the bars that are the same size. So, if Ian believes the bars in his graph are meant to simply point to or label the type of take-out food liked by each student, it is not surprising that he thinks his graph is correct. However, in his graph the relative lengths on the frequency scale have no meaning—liking chicken and chips is in no sense more than liking fish and chips.



Ian's Graph



Kate's Graph

Kate's graph shows frequencies, and it appears that she has just made a trivial error in not spacing the scale marks correctly. However, Kate may be unaware that the relative heights of her columns should be meaningful. While you can read that four like pizza and two like hamburgers, you cannot see just by looking that twice as many like pizza as hamburgers.

Their teacher was surprised to realize that Ian and Kate, both capable students, had missed the point of the frequency axis. Often we tell students exactly how to produce each graph or always provide them with pre-drawn and labelled axes. By doing this, we may prevent them from learning to develop the axes and scales for themselves and we may persuade ourselves that they understand more than they do!

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CASE STUDY 1

Sample Learning Activity: K–Grade 3—Colour Graph, page 158

Key Understanding 1: We can display data visually; some graphs and plots show how many or how much is in each category or group.

Working Towards: Emergent (late) phase and Matching and Comparing phase

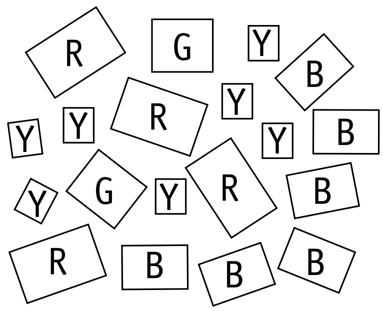
TEACHING PURPOSE

I wanted my Kindergarten students to have some early experience in making physical displays of data that they collected in the form of real objects. In particular, I wanted to draw their attention to how they might place their objects to make it easier to see which group has most.

ACTION AND REFLECTION

I prepared a bucket of blocks (large red, medium blue, medium green, and small yellow) and asked the students to each choose a block of the colour he or she liked best. I asked them to place all their blocks together in the centre of our floor mat so that we could all see the blocks chosen.

I asked, "Which colour do most of us like best?" Jamie replied: "Yellow's the most littlest! Children all like the littlest yellow ones best. They make lots of them for all the little children." I explained that we were thinking about only the blocks that we had chosen to help us find out which colour block most of the children in our group liked best. Then I asked, "What could we do to find out which colour most of *us* like best?"



I realized this was a difficult question for the students. I had to help them understand that we needed to know which colour had the *most* blocks to find out which colour was liked best by *most* people. So I asked, "How could we sort the blocks to help us see which colour has the most?" I was satisfied that the students could see the need to sort the colours when they made four piles—red, yellow, blue, and green. However, when I asked which colour had the most now, Yasmeen quickly answered, "Red. They are big. Really, really big." Many of the other students agreed.

I could see that some of the children were using the word "most" to mean biggest. This is a reasonable meaning, especially since I had asked, "Which colour has the most?" In response, they were attending literally to the amount of redness, and that was influenced by the size of the blocks. However, to

I deliberately chose to use blocks of different sizes as I wanted to draw my students' attention to the numbers of blocks rather than other attributes such as size or length.

One student removed her block from the central pile, saying: "That one's mine. I don't want my block in the pile." This did not surprise me because I knew young children often want to be able to identify their own piece of data in a display. I needed to persuade these students to consider their blocks as part of the information about the whole group before we could proceed with the lesson.

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answer our original question we needed to attend to the *number* of red blocks, since this was a proxy for the number of people who chose red. I reminded students that we wanted to know what colour most people chose, and rephrased my question: "Which colour has the most *blocks*?"

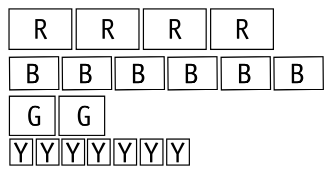
After a pause, I asked, "I wonder if it might help us to see which colour was picked the most if we lined up the blocks?" Liking this idea and eager to help, Simon placed the blocks in one long line, with all the red blocks followed by the blue, green, and yellow blocks. This linear arrangement seemed to prompt Hadi to count. He began counting from one end of the line, disregarding the colours, attempting to count all blocks. After counting 10 blocks, he stumbled over the next few numbers and stopped.

I would not expect the students to be able to do this independently. I knew I would need to focus my students' attention on the need for a common baseline in a future measurement lesson.

DRAWING OUT THE MATHEMATICS

I suggested we could line up the blocks in another way and asked the students to help me arrange the blocks in four separate lines, one for each colour. The children placed the blocks within each line so they were touching and I structured the situation to ensure the lines had a common baseline.

"Can we see which colour has the most now?" I asked.

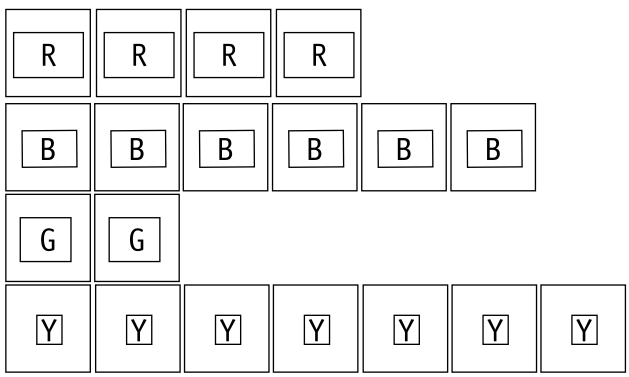


"Blue!" responded Damien.

"The blue line is the longest but let's think about how many blocks there are," I said.

Even though there were fewer blue than yellow blocks, the line of blue blocks was longer due to their larger size.

To draw the students' attention to number, I placed each block on a piece of identically sized square card and had the children help line up the cards in the same way as before. My intention was to make it easier for the children to compare the collections of each colour, using one-to-one matching, to enable them to see which collection had the largest number.



"I know, it is yellow," said Georgia. "It has got the most."

"How could we check to see if yellow has the most blocks?" I asked.

As I had hoped, this prompted Hadi to want to count again. He carefully counted the blocks one colour at a time and said: "It is yellow. That has got seven, and blue has only got six, so it is yellow."

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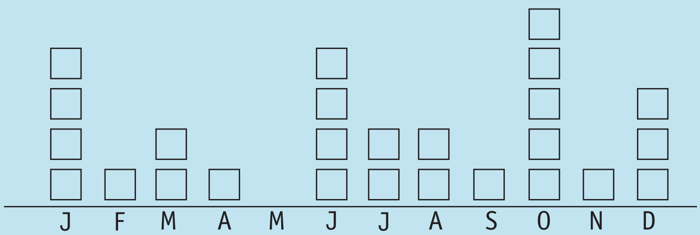
Hadi saw the usefulness of counting as a strategy to solve this problem and had shown the rest of the class. Most students had not yet developed sufficient understanding of number to recognize when counting is useful, but many could use their emerging ideas about one-to-one matching to see which is more when set out in this way.

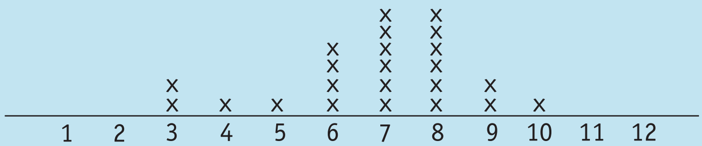
Did You Know?

The term "plot" can refer to graphs that involve plotting each piece of data separately, often, although not always, "as it comes in." For example, young students might draw a self-portrait on a sticky note and then stick their pictures above the month in which they were born, thus making a *line plot*. This, in effect, is the beginning of being able to understand *bar graphs*, although line plots are themselves valuable and frequently used tools. Similarly, older students could make an estimate of the length of the hall in metres and each plot her or his estimate directly onto the display.

Our birthdays



Our estimates of how long the hall is



Recording data in this way quickly shows how the attribute of interest is distributed, leading to interpretation and explanation. Recording data "as it comes in" means that a distribution can change before one's eyes (as when different boxes are opened on election night). This highlights the effect of chance variation on sample data—an important point to be drawn out with students. At times, our plot will have all the data in it (population data) so we know what the distribution is, but at other times it will remain a sample of data from the population. Students need to think about how to take this into account when drawing conclusions. (Link to Collect and Organize Data, Key Understanding 5; and Interpret Data, Key Understanding 3.)

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Key Understanding 2

We can display data visually; some graphs and plots show how one quantity varies over time.

Plots and graphs showing how a quantity changes with the passage of time are probably the most familiar and readily understood form of two-variable display. For this form of data, we usually use line graphs that are based on using straight lines to join points plotted at discrete intervals. Alternatively, we draw continuous curves that model how we think the quantity changes between the data points we know.

These types of graphs are used when it is meaningful to think about what the frequency or measurement is at a particular time, and to think of it as changing continuously so that it is possible to interpret "in-between" times. For example, it is meaningful to think of and plot the height of a child at any particular time, one's hunger level at any time of the day, or the total distance one has travelled. It does not make sense to think of or try to graph the amount of rain that falls at any particular moment, since to measure rainfall we would have to do so over a span of time. However, it does make sense to think of the cumulative rainfall at any particular time of the day or month or year.

From Grades 3 to 5, students should begin to plot trend data that is based on their own data collection as it occurs in measurement activities and across the school curriculum. This could include, for example:

- their measurement of the height of a pole bean plant
- the cumulative rainfall for a three-month period based on data collected daily over the Internet (and perhaps compared with the cumulative rainfall from the school's rainfall gauge)
- the money they have raised to send to a charity
- the distance they have travelled through the day as measured on a pedometer on their belt
- their hunger level estimated on a scale of 1 to 10 at 15-minute intervals throughout the day

During Grades 6 to 8, students should also begin to make sketch graphs that represent familiar experiences. They could, for example, sketch curves to show how their mood varies through the day or to estimate the noise level in their classroom at different times of the day. The aim is for students to understand that graphs are intended to help us get a feeling for how variables are related to each other.

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Links to the Phases

Phase	Students who are through this phase in Data Management…
Quantifying	■ use the height of bars to represent measurements they have made at equal intervals over a period of time, e.g., a student uses the height of bars to represent the height of a sweet pea plant at 10 a.m. each day
Measuring	■ can produce a graph in which the horizontal axis shows the progression of time ■ use a vertical scale to plot data points at equidistant times, which are clearly marked on the horizontal axis ■ understand when and why they can join the data points to produce a simple line graph
Relating	■ will be able to produce sketch graphs that "give a feel for" how familiar things change over time without recourse to careful data collection or point plotting ■ can also plot available time data using more complex scales, where each time may not be represented on the horizontal axis

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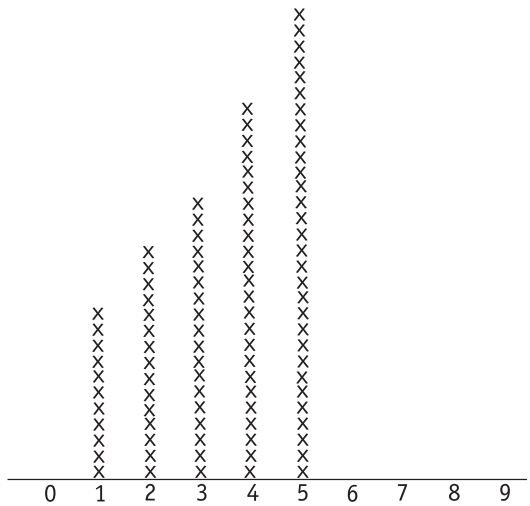
Sample Learning Activities

K-Grade 3: ★ Introduction, Consolidation, or Extension

Collecting Cans (1)

When students are confident in producing simple pictographs and dot plots, have them collectively produce a cumulative dot plot over time. For example: As the class collects cans for recycling, count/compute the total at the beginning of each week and record as a large class dot plot. Focus attention on the top dot or mark in each column and ask: What does the height of these show? Could they go up and down? Why? (*The upward trend shows more and more cans as the weeks go by.*)



Collecting Cans (2)

Extend the previous activity by helping students to plot only the top point. For example, at the end of week 6, say: As the number of cans increases, it is going to take a long time to put in all the marks each week. Sometimes to save time, we work out where the top dot would be and just put that in. Highlight in felt pen the top mark in each column. Ask a student to indicate where he or she thinks the next top mark will be and plot that. Repeat over successive weeks.

Growth Measurement

Build up class graphs over time using growth measurements from science activities (e.g., the number of marbles that balance each baby guinea pig each week, the height of the sunflower seedling each day). Help students produce bar graphs of their data. Use a felt pen to highlight the top of each column. Ask: What does this show? What do you think would be happening to the guinea pig/sunflower plant between the times we measured it?

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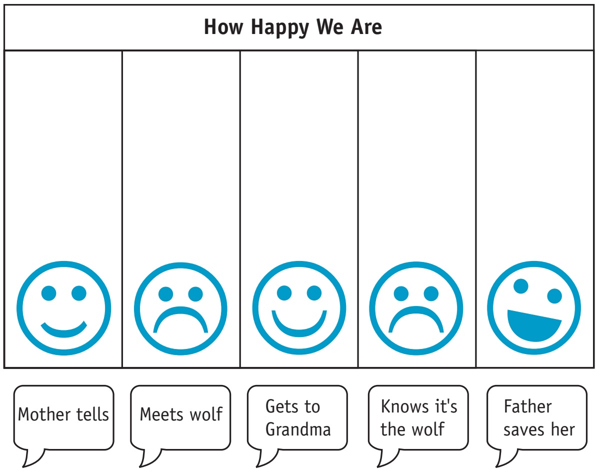
Feelings (1)

Talk to students about easily recognized feelings such as anger, happiness, or sadness and what it means to be not angry, a little angry, quite angry, or very angry. Use drawings of faces showing different levels of these emotions for students to rate how happy or angry different incidents in a story make them feel. During a second reading, stop after every incident so students can choose a face and paste it in order across the page.

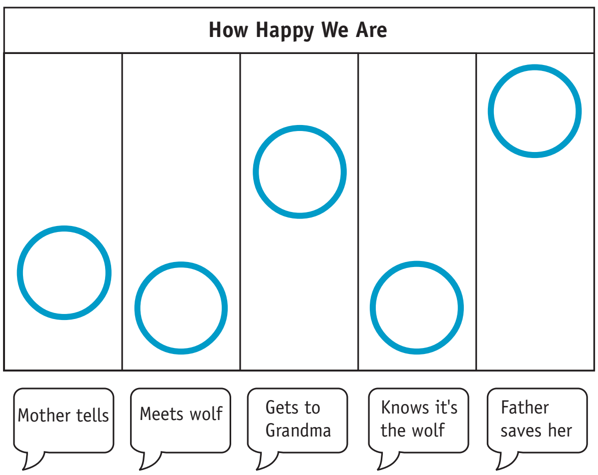


Feelings (2)

After the previous activity, relate strength of feeling to height, asking students to sit, kneel, stand, or stand very tall, according to the strength of a particular feeling as a story is told. Make sure the focus is on the relative strength of a single emotion, so they know that if it is about feeling sad they stand tall, just as they would stand tall if it is about feeling happy.

Feelings (3)

After the previous activity, ask students to think about how they might use the position of objects on a page to indicate different strengths of feeling of happiness. Ask: What if we did not look at the face? If the faces were circles without mouth and eyes, is there a way to tell how you feel by where we put the circles? Refer to the previous activity to draw out that we could put them higher or lower on the page to show more or less happiness (e.g., “very happy” would be near the top of the page, “not at all happy” would be low on the page).



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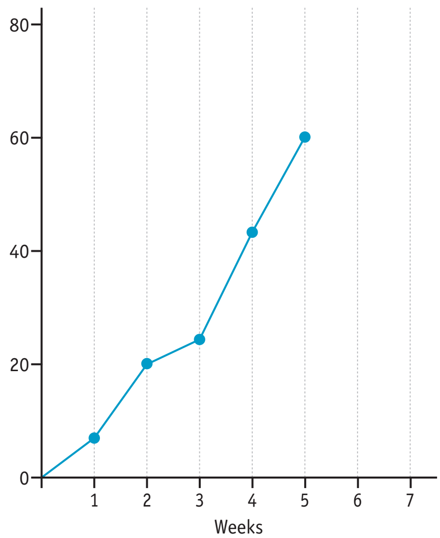
Sample Learning Activities

Grades 3–5: ★ ★ Important Focus



Collecting Cans

Extend *Collecting Cans*, page 176. As a class, plot the cumulative number of cans collected at regular intervals. Count/compute the total at the beginning of each week and show this on a line plot. Mark a single point at the top of each column. Join the points and ask: What might this line show? (*The upward sloping line shows more and more cans as the weeks go by.*) What would a mark on the slope halfway between two of the weeks show? (*If the cans are collected each day, it shows about how many cans we might expect in the middle of the week.*)

Changing Data

Have students consider data that refers to growth or change over time, such as the height of the wheat plant each day, the length of a shadow every half hour, or the mass of a melting block of ice every five minutes. Create a class bar graph so that each bar represents the measurement at each time interval. Have large gaps between each of the bars. Ask: Could we put other bars between the ones we have made? What would those bars tell you? How long would we expect them to be?

Story Map

As a class, make a story map showing incidents in a story by listing these incidents in order below the baseline of a graph. Have students decide on a 1 to 10 scale to show how exciting the story is during each of the incidents, and then plot these points using a vertical axis labelled 1 to 10.

Story Graph

Invite students to individually develop their own story graph from a class story they all know very well, using a scale that involves a strength of feeling. As in the previous activity, have students list the main incidents in a story sequentially below the baseline of a graph. Have different students choose different emotions to plot, for example, excitement, happiness, boredom, or fear. Ask them to think about the numbers on the vertical axis meaning more or less strength of whichever feeling they choose. Compare graphs and discuss. Ask: Why does the excitement graph go down at the same place as the boredom graph goes up?

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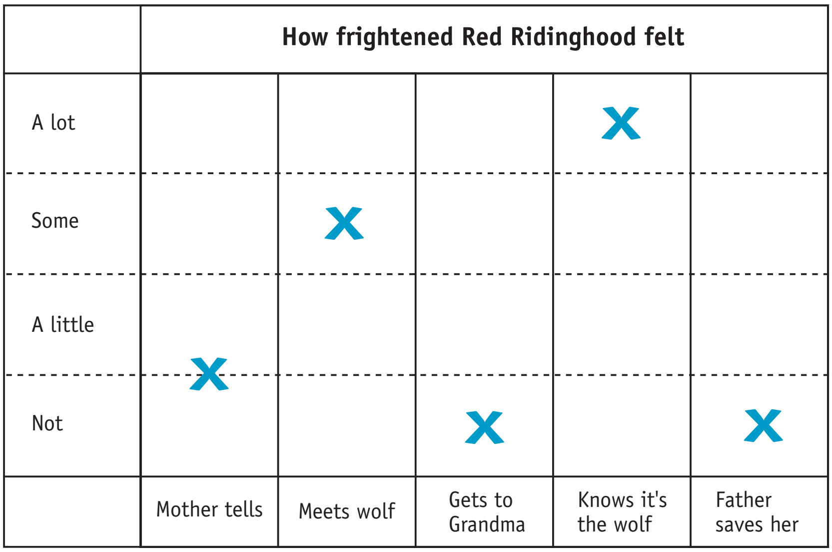
Strength of Feelings

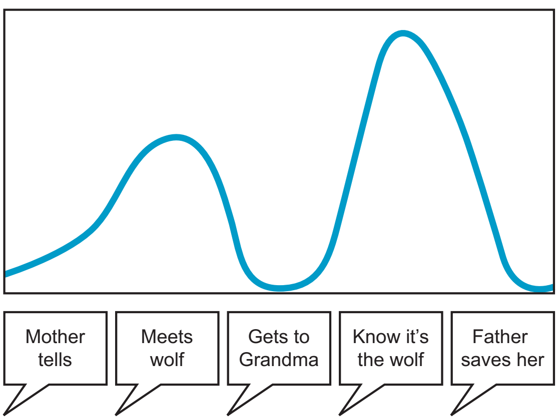
As a class, establish a scale for different feelings (e.g., not, a little, some, a lot). Have students divide up a page horizontally above a baseline, with each section relating to increasing strength of feeling. List story incidents along the baseline, and have students mark how they (or different characters) feel about the incident—the higher up the page the greater the strength of feeling.



Ask: Would it make any sense to join the marks? Why? Why not? Draw out that there may not be a gradual change of feeling—characters might change feelings suddenly, or be feeling different between incidents.

Up and Down

Tell a short story or incident that stimulates a build-up of excitement or fear. Have students help you to draw a line from one side of a page to the other to represent their feeling as the story progresses—the line slopes up as fear or concern increases, and down as fear or concern decreases. After the story, retrace the line and recall its meaning. Ask: What does the line tell you when it goes up?



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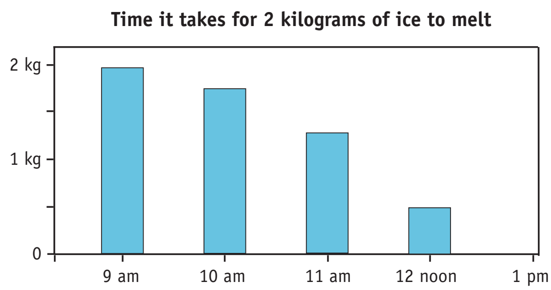
Sample Learning Activities

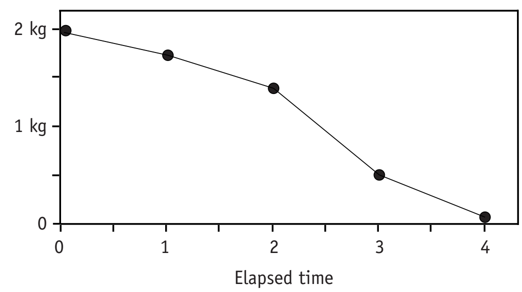
Grades 5–8: ★ ★ ★ Major Focus

Collecting Cans

Build on *Collecting Cans*, pages 176 and 178. Have students plan their own axes and scale and keep their own cumulative record as data comes in. They join the weekly points. Ask: Can you estimate about how many cans you will have collected by the end of term if you keep collecting at the same rate?

Growth and Change





Have students consider straightforward data that refers to growth or change over time, such as the height of the wheat plant each day, the length of a shadow every half hour, or the mass of a melting large block of ice every hour. Ask students to create a bar graph so that each bar represents the size at the time each measurement was taken. Help them understand that the change or growth continues between the times the measurements are taken, and this can be shown on a graph by making the horizontal axis into a timeline (time scale) and joining up the measurements. What can the height and slope of the line between the measurements tell us?

Line Graph

Have students create line graphs using data obtained from time intervals of growth, such as the length of shadows over a day, the growth of a plant over a week, or the mass of a baby guinea pig over several weeks. Ask them to use a time scale on the horizontal axis to plot every second measurement and join the dots. Invite students to use their graphs to estimate the missing measurements and compare the actual measurements taken at those times. Ask: Do they match? Why? Why not?

Height Measurements

Arrange for students to visit a Kindergarten or Grade 1 classroom each month and take careful height measurements of the children (each student can be partnered with a child whose height they measure each month). Ask students

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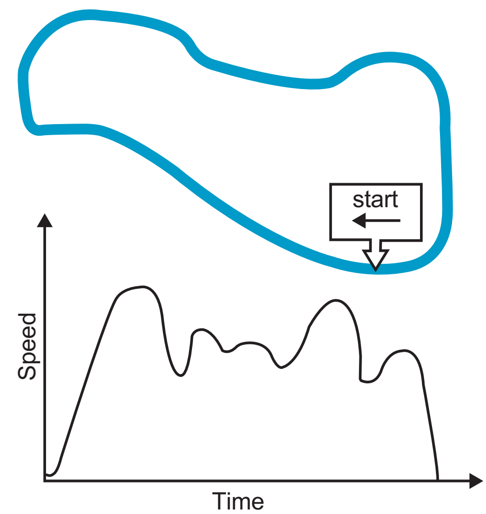
to graph their partner's growth over the year, joining each new growth point with a straight line. Towards the end of the year, ask: What does the slope between the measurements mean? Does the slope change after some of the points? Does this mean the child suddenly changes his or her growth rate, or would the change be more gradual? Show how joining the points with a curving line is more likely to reflect the child's true growth pattern.

Informal Line Graphs

Have students sketch informal line graphs to show how various things change over time. They explore the idea of more and less of some attribute in relationships, such as the height of water in different-shaped jugs as they are filled from a steady flow, the excitement level in a story as it is told, or the circumference of a balloon as it is blown up. Focus on what increases or decreases, when there is more and when there is less of it, and when the quantity may stay the same.

Racing

Extend the previous activity by asking students to make a sketch graph representing the speed of a toy electronic car travelling around a circuit. Have one student call out every 10 seconds "very slow," "slow," "medium," "quite fast," or "very fast," as they all watch the car slow down around corners and speed up on the straights. They record the calls. Have students use this data to sketch a line graph of the speed of the car over time. Compare the graph with the shape of the track. Invite students to draw different circuits, then sketch speed/time graphs that match the circuits. Mix and distribute several circuits and graphs to different groups and then have them come to a consensus about which graph matches which track.



Feelings (1)

Read or tell a short story that stimulates a build-up of tension or excitement and have students draw a line from one side of a page to the other to represent the strength of this tension or excitement as the story progresses—the line slopes up as tension increases and down as tension decreases. At the end of the story, ask students to retrace their line and recall its meaning.

Feelings (2)

Extend the previous activity by having students construct graphs of the build-up of tension or excitement over time in short stories they read using the same tension scale on the vertical axis. Ask: Are there typical patterns in the graphs for different types of stories?

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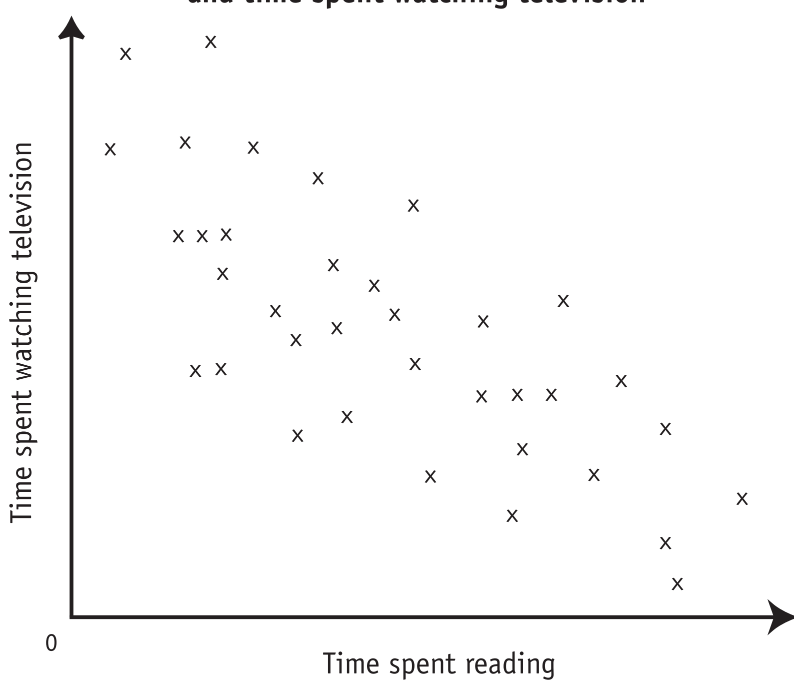
Key Understanding 3

We can display data visually; some graphs and plots show how two quantities are related.

Often we wish to display data in a way that shows us whether or how two variable quantities are related. In order to be able to display data involving two variables, students need to be able to coordinate the information on two axes at once. Games that involve placing objects in specific squares in an array or using coordinates to locate things can help with this process, and can begin during Grades 3 to 5. There are many computer games that help students to develop this skill.

From Grades 6 to 8, students should begin to use plots and graphs involving two quantities. For example, to investigate whether there is a relationship between how much television children watch and how much they read, students could collect information from a sample of children of the hours spent in each of the two activities each week. They could then make a graph by labelling one axis with the variable television time and the other with the variable reading time and plot an ordered pair for each child. The resulting scatter plot would then give some idea about the extent and nature of any relationship. (Link to Interpret Data, Key Understanding.)

Relationship between time spent reading and time spent watching television



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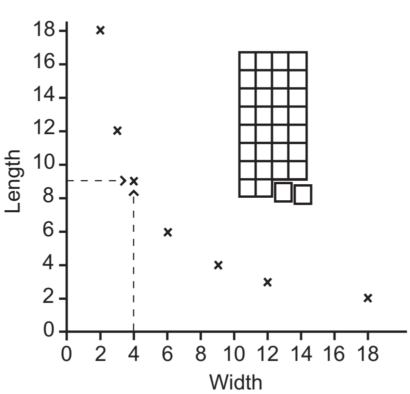
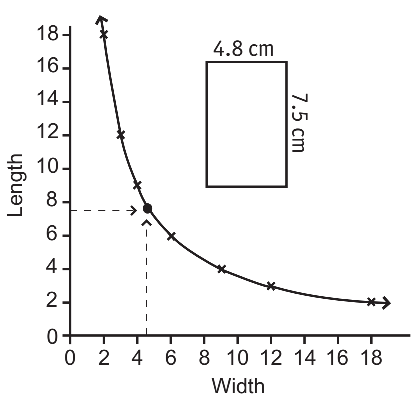
Students should also use graphs to investigate measurement relationships. They might measure various circular lids to find their diameter and circumference, or the height and head circumference of students in their class. In each case, they can graph the pairs of values obtained. When plotting the circumference of circular lids, the points should, theoretically, lie on a line but the chance variation introduced through measurement means that the points are likely to lie close to, rather than exactly on the line. As long as the measurement is reasonably precise, the underlying relationship will still be evident and students should be able to predict the circumferences of other lids. (See Interpret Data, Key Understanding 2.)

Sometimes one variable can be predicted from another. For example, when a student, after playing Guess My Rule, says that the rule is *double the number and add one*, he or she is describing a quite precise relationship between the input number and the output number: output number = 2 × input number + 1. In this case, plotting the ordered pairs (input, output) will show that all the ordered pairs are on a line, enabling the prediction of other pairs of values.

Students should learn to decide whether it makes sense to join the points on their graphs, and to explain their decisions. For example, they know that if they were plotting pairs of points that showed the length and breadth of rectangles composed of 36 centimetre squares, only whole number values would be sensible on the axes and joining points would not make sense. If they were plotting pairs of points that showed the length and breadth of *any* rectangle of area 36 cm^2, any numbers would be sensible on the axes and joining points of the graph would make sense.





Students who have achieved the Relating phase can represent two-variable data in scatter plots and simple coordinate graphs.

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Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

Ordered Measurements

Extend *Measurements*, page 160, by repeating the activity for containers that go up in size systematically. For example: Collect soft drink containers clearly labelled as 1 L, 2 L, or 3 L. Have students measure and record how many cupfuls fit into each bottle. They then can produce a block graph on squared paper using the container size as categories. Encourage them to put the containers in order on the horizontal axis and equi-spaced. What happens to the columns as the container size goes from 1 L to 2 L to 3 L?

Two-way Grids

In the context of various games and activities, encourage students to begin to place things in a two-way grid. This is the beginning of being able to plot points, although students would generally not be expected to hold two variables in their minds at once.

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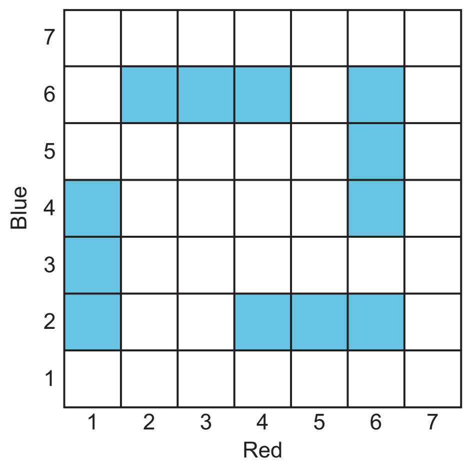
Sample Learning Activities

Grades 3–5: ★ Introduction, Consolidation, or Extension

Battleships

Have students play games where they need to coordinate two axes at once. For example, have them use numbers to mark grid references on squared paper, distinguishing vertical columns and horizontal rows by colours (see diagram), labelling the spaces between the grid lines. They then secretly place four battleships onto their grids by choosing three adjacent squares for each ship. Players have three turns to locate their partner's ships by calling out grid references (e.g., red 3, blue 2). Two "hits" means a ship sinks.



Physical Grid

Set up a physical grid on the pavement with chalk or rope or based on square paving stones. Along the bottom of the grid write Height, and on the side write Mass. Show pictures of a variety of animals of varying proportions. Ask students which part of the grid would a snake go into? What about a giraffe? an elephant? and a raccoon?

Physical Scatter Plot

Set up a physical scatter plot using ropes or the edges of the classroom as the two axes. Label one axis "Not very hungry to very hungry" and the other axis "Not very tired to very tired." Ask students to position themselves along the horizontal axis according to how hungry they are, and then walk forwards until they have reached the appropriate point on the vertical axis to show how tired they are. Ask: Where should you be if you are very hungry, but not very tired? Where would you be if you are not very hungry, but very tired? What about if you are very hungry and very tired? Draw out what the different parts of the grid mean.

Coordinate Grid

Following the two previous activities, set up a coordinate grid on the display board, and use it with the students to explore a variety of familiar relationships. Have students make a labelled pin with their name on it, and use it to show their position on the grid. Encourage students to contribute ideas for the grid (e.g., going-to-bed time and getting up time, liking sports and liking reading). Ask: If you like both sports and reading a lot, where will your pin go? If you like reading a lot but not sports, where would you put your pin?

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Sample Learning Activities

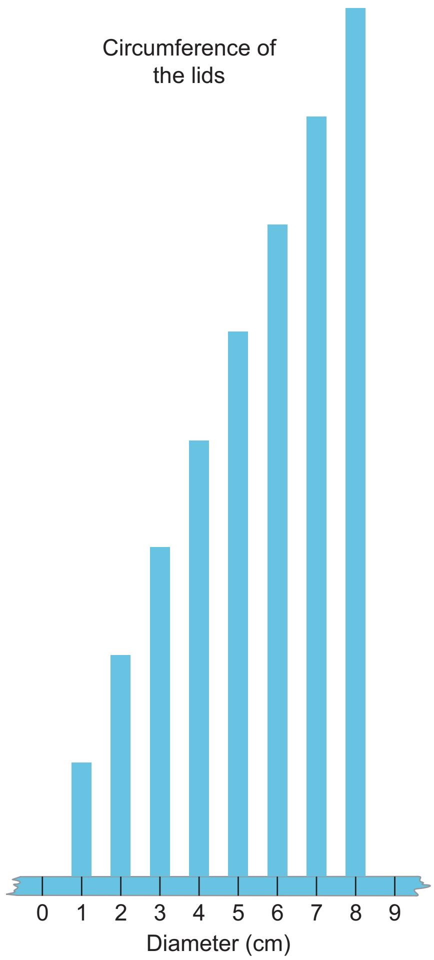
Grades 5–8: ★ ★ Important Focus

Skeletons

When human bones are found, forensic specialists can estimate the height of the person from the length of a leg bone. Ask students to produce two pieces of measurement data on themselves: their height and the length from their ankle to knee. Set up a class scatter plot using a large sheet of one-centimetre grid paper on a pin-up board, labelling the axes in two-centimetre increments on each grid line. Label the horizontal "Ankle to knee" and the vertical "Height." Have students place a pin on the plot at the appropriate grid position for their two measurements. Encourage them to be systematic in finding the appropriate location, first using the horizontal axis and then matching the vertical value. Ask: Are there patterns in the placement of the pins? Do you think ankle-to-knee length is related to height? Do the taller people generally have longer legs from the ankle to knee? Do the shorter people generally have shorter legs?

Measuring Lids (1)

Have each student measure the diameter of a different-sized lid to the nearest centimetre and then cut a piece of tape the length of the circumference. Create a class display of the data by attaching a paper strip with a centimetre scale from zero to beyond the longest diameter across the bottom of a pin-up board. Have students attach their circumference paper strips vertically above the respective diameter length. Ask: What can you say about the graph we have created? Why do the circumference lengths go from shortest to longest along the diameter strip?



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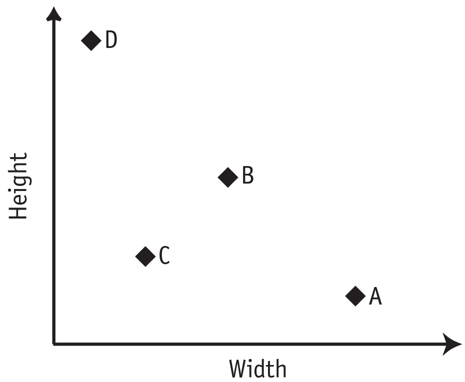
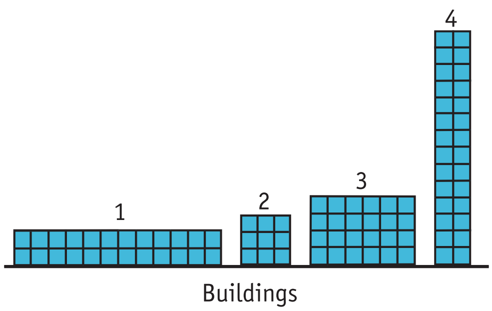
Measuring Lids (2)

Extend the previous activity to produce a scatter plot. Have students draw a pair of axes on centimetre grid paper. Label the horizontal axis Diameter and number the centimetre grid lines by ones. Label the vertical axis Circumference and number the centimetre grid lines by twos. Invite students to plot points on their coordinate grid to represent data from the previous activity. Draw out that each point represents a lid, and provides two pieces of information about the lid. Ask: How is your scatter plot similar to the paper tape display? Would it make sense to draw a line from the lowest point to the highest point on both displays? Why are our points not all exactly on the line? (*Our measurements of the lids are not accurate enough; they were rounded to the nearest centimetre.*) If we measured a lid with a diameter of, for example, 3.5 cm, could we use the line to approximate the circumference? Why is the slope of the diagonal less on our scatter plots than on the wall display? (*We used different scales on the axes.*)

Buildings

Have students look at a picture of four buildings, represented by rectangles of different proportions, and try to match them to points marked on a simple coordinate graph. Explain that the horizontal axis shows how wide the building is, and the vertical axis shows how tall it is.





People Graph

Provide students with pictures of four people—two old and two young. In each age group, one person is tall and one is short. Have them informally plot the points on a provided pair of axes.

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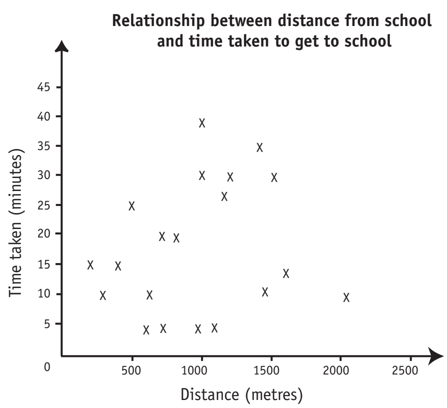
Grades 5–8: ★ ★ Important Focus

Ordered Measurements

Have pairs of students collect different-shaped containers for which they know the capacity (e.g., soft drink bottles, milk or juice cartons, measuring cylinders, and cones). Ask them to fill each container with water and find the mass of the water (weigh the empty container and full container and find the difference, or pour into a scale). Have each student in each pair draw a bar graph, one showing the mass and the other showing the capacity of their containers. Have them compare their graphs. Ask: Is there a relationship between mass and capacity? How do you know? Then invite each pair to construct a scatter plot, combining the two types of data. Ask: Is it easier now to see the relationship between the two measures? Why? Would a line joining the lowest to the highest point make sense in this relationship? (*Yes, because there is a direct relationship between the millilitres and grams of water, 1 mL = 1 g water.*) Should any of the points fall outside the diagonal line? (*No*) Can you say what the mass of water would be for other containers of known capacity?

Exploring Relationships

Have students brainstorm a wide range of data types (e.g. mass, height, length, thickness, distance, time spent, scores, ratings of attitudes and feelings, ratings of physical characteristics of objects). Invite students to consider and predict various relationships that they can later explore by taking two kinds of measurements and producing scatter plots. For example: They might create a scatter plot to see if there is a relationship between the distance they live from the school and the time taken to get to school. Ask: What does your scatter plot look like? Are the points spread out all over the grid? (*No relationship*) Do they seem to cluster about an imaginary diagonal line? (*Some kind of relationship*) What can you say about the relationship between the data when the points slope up from left to right? (*When there's more of one measure, there's also more of the other.*) What if the points slope down from left to right? (*When there's more of one measure, there's less of the other.*) What if the points seem to cluster in a bunch in one place on the plot? (*There is not much variation in the data, or the scale on the axes covers too much range.*) Can you explain why the points are so spread out on the scatter plot for distance and time taken to get to school? Why do those living farthest from the school not take the longest to get here?



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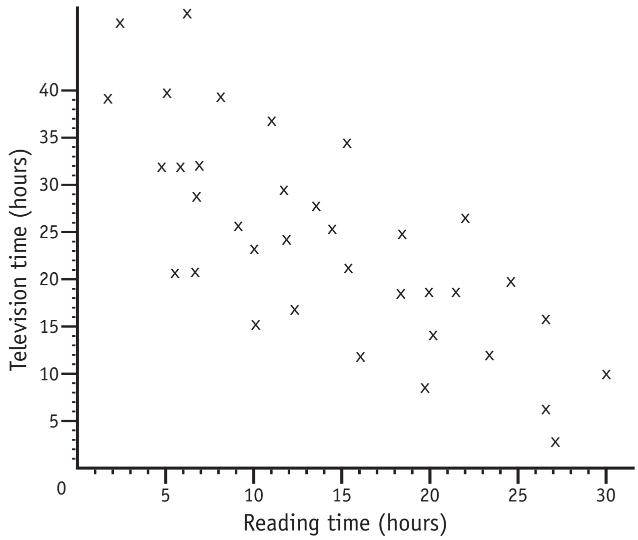
Time Relationships

Have students explore time relationships by making scatter plots and examining the clusters. For example: Over a week, collect data about how much time students spend watching television and reading. Draw a graph, labelling one axis "Television time" and the other axis "Reading time" and plot the ordered pair for each student. Ask: Can you say just by looking whether there is a relationship? Would it make any difference to the look of the graph if the axes were reversed? Why?

Relationship between time spent reading and time spent watching TV

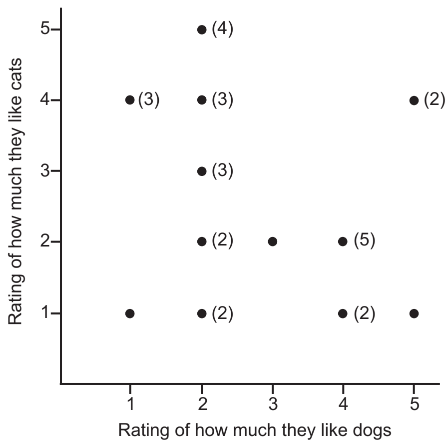


Ratings Data

Have students construct scales to record strength of feelings for two different things. They can collect data from others and produce a scatter plot to help them look for relationships to test their predictions. For example: They could ask their classmates to rate their degree of liking for dogs and cats separately on a five-point scale to test their prediction that cat lovers do not like dogs and dog lovers do not like cats. Ask: How will you label the axes? Does it matter if two or more points fall on the same spot? Draw out the need to distinguish between one piece of data at a point and a number of pieces of data at the same point. Model the convention of showing a bracketed number next to the point when more than one piece of data is represented by one dot or cross.

Tick the box.	Dislike them 1	Don't like them much 2	They're OK 3	Like them quite a lot 4	Love them 5
How much do you like cats?	☐	☐	☐	☐	☐
How much do you like dogs?	☐	☐	☐	☐	☐

Relationship between liking cats and liking dogs



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Key Understanding 4

We use tables and diagrams to organize and summarize data in a systematic way.

Much of the information we collect or respond to comes in an unstructured way. We record the colour of cars as they arrive at the intersection, the numbers on the number cube as they appear, and the answers to the questions as people give them. Lists, tables, and diagrams enable us to organize data to enhance its accessibility and meaningfulness. Sometimes we plan ahead and use pre-formatted tables and diagrams to organize information as it comes in, perhaps producing a tally or recording information directly into a computer database. Sometimes we organize it later. For example, we may collect information about different holiday options and then sort this information into a table to help us make comparisons.

Tables are used to exhibit data in a definite and compact form or scheme where the arrangement of the information is significant in interpreting it. The simplest table is a list, which may or may not be ordered. An alphabetical list of names and the "six times table" are examples of organized lists. Slightly more complex are one-way tables that require students to place information in the right position in relation to other information. This structure is not always immediately obvious to students and it needs explicit attention.

Name	Eye colour
Mai	blue
Freya	brown
Peter	blue
Kim	brown

Eye colour	Frequency
blue	////
brown	//
green	/

Two-way tables are more complex and require students to coordinate two constraints at once. Using the table at the top left of page 191 requires them to understand the grid cell structure and find the appropriate intersections of columns and rows. To produce the right-hand table, they need also to understand how frequencies in rows and columns may be summarized to show totals, often without the word "total" being used explicitly or the column or row being labelled.

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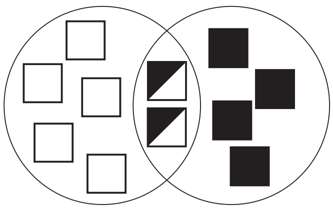
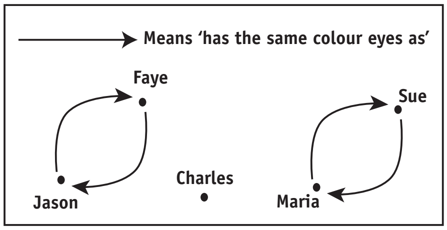
	Heads	Tails
Heads	HH	HT
Tails	TH	TT

	Girls	Boys	
Grade 6	15	13	28
Grade 7	12	18	30
	27	31	58

Other diagrams, such as Venn diagrams and arrow diagrams (see below), may highlight relationships between categories or things.



Venn diagram



Arrow diagram

Students should organize data in a range of lists, tables, and diagrams with specific attention being drawn to the way the structure of these representation methods helps us to organize information.

Links to the Phases

Phase	Students who are through this phase in Data Management…
Matching and Comparing	■ can use organized lists and one-way tables to organize information
Quantifying	■ can use Venn diagrams involving two overlapping categories ■ can place information into the correct location in simple two-way tables
Measuring	■ can easily use Venn diagrams and two-way tables ■ can also construct arrow diagrams
Relating	■ can display information in tables involving provided class intervals

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Sample Learning Activities

K–Grade 3: ★ ★ Important Focus

Equipment

Have students draw the individual pieces of play equipment stored in the gym and involve them in making a representation of how many of each item of equipment is stored (e.g., draw two toy tractors, four shovels, seven skipping ropes). Help students to set their drawings out in a table. They then use the table to help them check whether any items are missing after packing away equipment.

Modelling

Model the process of producing simple lists and tables incidentally in school activities. For example: When students are reporting on their progress on a task, produce a table with labelled columns (Have Not Started, Written Story, Collected Pictures, Finished). As students report their progress, record their name on the table. After the first day, have students rub their name off the table when their progress changes and record their name in the appropriate part of the table.

Language Groups (1)

Prompt students to collect data about their classmates and decide how to organize it. For example, after they have recorded the different languages spoken in the class, ask: How could we organize the information so we can quickly find all of the students who speak Chinese? Draw out that lists and tables help us find information quickly if organized in a systematic way.

Language Groups (2)

Extend the previous activity to include data where there are likely to be students who belong in more than one group. For example, ask: How can we show the students who speak more than one language? Could we write their names on both lists?

Food Groups

During health lessons, ask students to create a list of the different food types and add items to each list whenever they eat one of the foods. Ask: How does the list help you see what you have eaten most of? Could you organize the information in a different way to help you see things at a glance?

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Where Do You Fit?

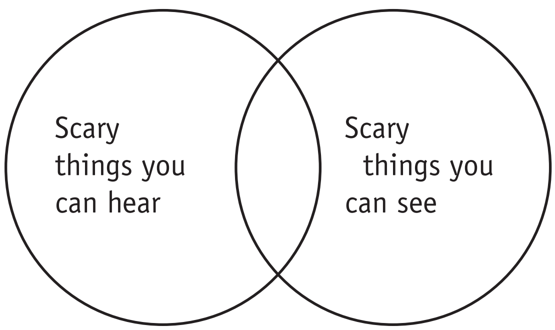
Divide the classroom in half lengthwise by running a piece of paper tape down the centre (see diagram below). Label each half of the room so that all students fit into one category or the other (e.g., boy/girl, wearing sneakers/not wearing sneakers). Invite students to decide where to stand. Ask: What made you decide to stand on that side of the room? What do we know about Claire if she is standing over here?

Wearing sneakers	Not wearing sneakers

Ask: How could we record this information to show someone what we have found out? Draw a table on the board to represent the classroom and list the students who stood in each section. Ask: Where was Tom standing? Where will we write his name? Draw out that the order of the names is not important — if Tom was at the front of the classroom, he does not need to be at the top of the grid — but putting them in the right section is important.

Venn Diagrams

Have students use two hoops to sort categories of data that overlap. For example: After reading a scary story to the students, ask them to find out what scares most children in their class. They draw what scares them and then use two hoops to show two categories of scary things. Show how overlapping the hoops allows a new space for pictures that show both scary things. Ask: Where would we put pictures that are about neither of those things?



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Sample Learning Activities

Grades 3–5: ★ ★ ★ Major Focus

Modelling

Model the process of producing organized lists and tables incidentally in school activities. For example: Produce a form for groups to use when working out a roster for sharing sports equipment. Have students enter relevant data for their own group.

Name	Monday	Tuesday	Wednesday
Fiona	football	basketball	tennis gear
Paul	basketball	tennis gear	football
Jade	tennis gear	football	basketball

Favourite Fast Foods

As you help students to plan their survey data collection, model the process of producing an appropriate data collection table by talking aloud as you sketch a table on the board. For example:

Our Favourite Fast Food—Room 5		
Food	**Tally**	**Frequency**
Pizza		
Chinese		
Hamburgers		
Chicken		

Provide pairs of students with a copy so that they can record data appropriately as they produce it.

Food Preferences

Have students compare alternatives when producing tables for data collection. For example: Compare the following possibilities for collecting and/or displaying data on food preferences.

Name	Food liked
Karen	Pizza, hot dog, spaghetti
Aaron	Hot dog, spaghetti, grilled cheese

Food liked	Name
Pizza	Karen, Lisa, Ian, Tom, Cassandra, Bill, Sam
Hot Dog	Karen, Aaron, Petros, Lisa, Ian, Cassandra

Food liked	Tally
Pizza	𝍸 ||
Hot Dog	𝍸 |

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Where Do You Fit? (1)

Extend *Where Do You Fit?*, page 193. Introduce a second pair of categories and have children re-sort themselves. For example: After students have sorted themselves into groups for wearing and not wearing sneakers, divide the groups in half again at right angles and label the rows (see diagram below). Ask students to decide which square they belong in now. Record the students' names in each section on a two-way table drawn on the whiteboard.

	Wearing sneakers	**Not wearing sneakers**
Light-coloured shoes		
Dark-coloured shoes		

Where Do You Fit? (2)

Have students produce their own version of the table in the previous activity. Working in pairs, one student reads the names in one cell from the table on the board and the other produces a tally. Change over for next cell. Ask students to produce a final table of data using the frequencies from their tally. Help them to add up the columns and rows and explain what they show.

Useful Tables

Prompt students to examine a range of tables containing information ordered in different ways, such as sports team player lists, names by alphabet, frequency of lotto numbers by size. Ask them to answer questions like these: "Which player is number 14?" or "How often did number 15 come up?" Explore what made it easy to find the information wanted. Build on your observations: "Raffi, you found the answer to that question very quickly—what helped you? Did you have to read the whole table, or did you know where to look to find the information?" Ask some questions that are less well facilitated by the table (e.g., which number a certain player wears). Ask: Did the table help? How could you rearrange the information to make it easier to find? Draw out that useful tables are ordered by the information people are most likely to want.

Venn Diagrams

Extend *Venn Diagrams*, page 193, by having students draw and label the circles. Have them count how many are in each category and write the numbers in the appropriate section. Ask: Why do we not need to label the overlapping sections?

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Sample Learning Activities

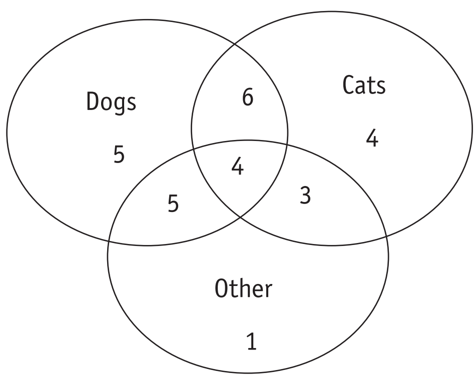
Grades 5–8: ★ ★ ★ Major Focus

Flight Information
During science and technology activities, have students produce tables to record information about their designs. For example, have students make airplanes out of different materials and test them to see which flies the farthest. Ask: What are some choices in the way a table can be organized? Where will you write the name of the plane? How many columns will you need to allow for your information? What will you label the columns? Is there a particular order that is better than another? Draw out that labelling such columns in the same order as they will be used is sensible.

Studying Tables
Collect a range of tables from publications for students to study. Draw out features that are common and those that are different. Ask students to copy different table forms and insert their own data obtained from science, health, or chance investigations. Ask: Do the tables show totals? How are they indicated on the tables?

Changing Headings
When students are constructing or analyzing tables, invite them to try swapping the headings of the columns and rows. Ask: Is the information still correct? How does changing the headings change the ease of reading the tables?

Venn Diagrams
Extend *Venn Diagrams*, page 195, and have students produce their own three-circle diagrams. Ask them to collect data, such as on pet ownership, and plan what each circle represents. They can first use sticky notes with names to put the data on the correct part. They then count how many and label each section. Ask: What do the overlaps mean? How can you check that you have included everyone? Why do you not need to label the overlapping parts?



Children in our class who have pets

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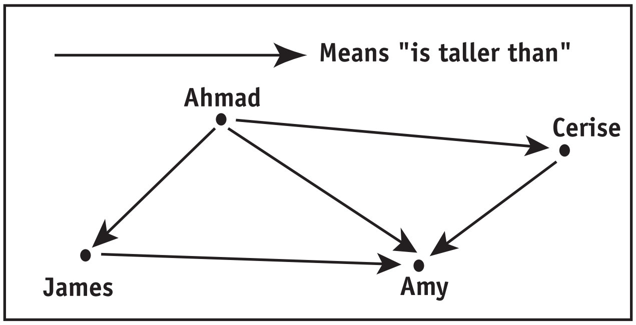
Arrow Diagrams

Invite students to represent relationships using arrow diagrams (e.g., to represent relationships between people in their family, or various physical characteristics).



Maps and Pictures

Explore a range of tables and charts that organize information in different ways, such as populations shown on maps or pictures on which information is listed. Have students try organizing their own data using these techniques. Ask: What use are the pictures or maps? How do they convey information differently from an ordinary table? For example, showing population for each province and territory on a map of Canada means you can find the information quickly without needing to look through a list of places.

Totals Tables

Ask students to decide whether their two-way tables need a total column and/or row. Draw out that it depends on the purpose of the table—for example, would we be interested in the total height, mass, or age of a group of students?

Name	Height (cm)	Mass (kg)	Age (yrs)
Karen	102	23	8
Jemma	112	29	9
Colin	98	22	7
Ahmad	106	26	8
Maria	110	27	9

Would we be interested in the total spent in a month by a group of students?

Name	Books ($)	Snacks ($)	Movies ($)	Total ($)
Karen	5	8	8	21
Jemma	10	10	16	36
Colin	15	12	0	27
Ahmad	5	4	16	25
Maria	20	15	10	45
Totals ($)	**55**	**49**	**50**	**154**

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Key Understanding 5

How we display our data depends on the kind of data we have and our purpose.

As students extend their repertoire and experience with activities for Key Understandings 1 to 4, they should be helped to develop the capacity to make conscious decisions about how to represent data. Students should learn that graphs, tables, and diagrams are neither an end in themselves nor simply appealing or artistic presentations of data. Rather they serve quite specific purposes in enabling us to more readily understand, analyze, and interpret our data and to communicate it to others. Thus, our decisions about how to represent data should take into account the type of data, the messages to be conveyed, and the context and audience for the display.

As described in Key Understandings 1 to 3, different types of data require different data handling and display techniques. For example, if we plotted the height of a particular child on his or her birthday each year, it would make sense to join the points to form a line graph, since the line consists of points that indicate the approximate height of the child between years. However, if we selected a number of individual children at each age and plotted their heights, it would make no sense to draw lines connecting the data points, as these lines would have no meaning. Students should be helped to choose methods of presentation that suit the type of data they have.

Different forms of representations (tables, graphs, plots) will highlight different aspects of the data and they should be chosen with care so as to give a quick, lasting, and accurate impression of the significant information. The particular type of table, graph, or plot needed will also depend on the aspects of the data to be illustrated or highlighted. Line graphs, for example, are useful for tracking trends over time, while pie charts make it easy to see relative proportions. In bar graphs (or histograms), different groupings of data, and even the order in which groups are shown, will convey different messages.

As students perceive the need for increasingly sophisticated forms of data representation, the teacher can help them by introducing new methods of representation. Little is likely to be achieved by providing a collection of data out of context and having students practise drawing graph types in isolation. Students should compare the same data displayed on different types of graphs and with different groupings in order to understand the impact on the messages conveyed and the potential of different displays to mislead. Computer graphing programs free students from the tedious and time-consuming chore of producing graphs manually, allowing them to produce different graphs of the same data and make rapid comparisons between them.

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Whether intended for personal use or to communicate information to others, data displays are intended to enhance understanding and communication. At times, all that is required to clarify or make a point is a quick sketch graph. Under such circumstances, fussing with accurate plotting and labelling is a waste of time. However, at other times, accurate plotting and attention to detail may be needed for differences and patterns to become evident. While there are some conventions associated with different types of graphs, such decisions need to be made by thoughtfully taking context and audience into account, rather than adhering to sets of rules about the "correct" way to produce them.

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Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

Lost Teeth

When students have collected data about those in the class who have lost teeth, ask them to create their own display of the information. Have students compare their displays and say what information they can get from each other's. Ask: Which display helps you to see who has lost the most teeth? Which tells you how many teeth the boys in our class have lost? Ask students how they would change their display to show the different totals.

Favourite Sports

After students have physically arranged themselves or objects to make a display like a bar graph, ask a question that encourages them to represent a back-to-back graph. For example, invite students to line up according to which sport they prefer. Ask: I wonder if more boys than girls prefer baseball? How can we tell? The boys and girls in our groups are all mixed up. Draw out that the different question made it necessary to use a different type of representation.

Matching Collections

Invite students to select a handful of different coloured beans from a jar. Ask them to sort and represent their collection on paper to make it easy for someone else to re-create an identical collection. The paper is then given to a partner who attempts to make the collection. Ask: Did your partner's collection match yours? How could you make it easier for someone to make the same collection?

Measuring Height

Have students cut strips of paper tape to match their height. They then write their names on one side of the strip of paper and display the strips without these names showing. Ask: We know this is the smallest strip, but how do we know who it belongs to? If we were out at recess and someone else came into our room, how would he/she know who is the shortest? What would we need to do? Draw out the importance of labelling the data when you are communicating with others.

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Sample Learning Activities

Grades 3–5: ★ ★ Important Focus

Fitness Facts
Have students brainstorm types of data they could collect during daily fitness sessions to find out whether their fitness is improving. After they have collected their information over a period of weeks, ask them to suggest ways to represent the information to find out if they have improved.

Book Data
Ask students to sort the classroom books, then to make a chart to display near the bookshelf. Ask: What kind of data about our books would be useful to display? What would be the best way to present this information?

Zero Values
Invite students to make decisions about how to represent data values of zero. For example, they may have discovered that no students in the class have green eyes. Ask: Should we still include a column for green eyes on our graph? What would happen if we did not include green eyes? Draw out that whether or not we include zero values depends on the question we want to answer.

Newspaper Graphs
Have students collect graphs used in the newspaper and compare the differences and similarities between them. Ask: Why do you think the writer chose to represent the data using a line graph? Could we use a bar graph to show the same information? Which would be better? Why?

Graphs and Tables
Direct students to compare the use of a table and a graph when representing data they have collected. After students have looked at the points scored by each team at the sports day, ask: What would be the best way to represent this data if we wanted to see quickly how many points a certain team scored? How would we represent the information if we wanted to see quickly which team scored the most? Draw out that tables are often useful for finding specific information quickly, and graphs are often useful for making comparisons and examining relationships.

Basketball Tournament
In a class basketball competition, ask students to decide which information about teams' previous performances might help them make their predictions. Ask: How could we represent the information about previous performance to help us pick our teams for next week?

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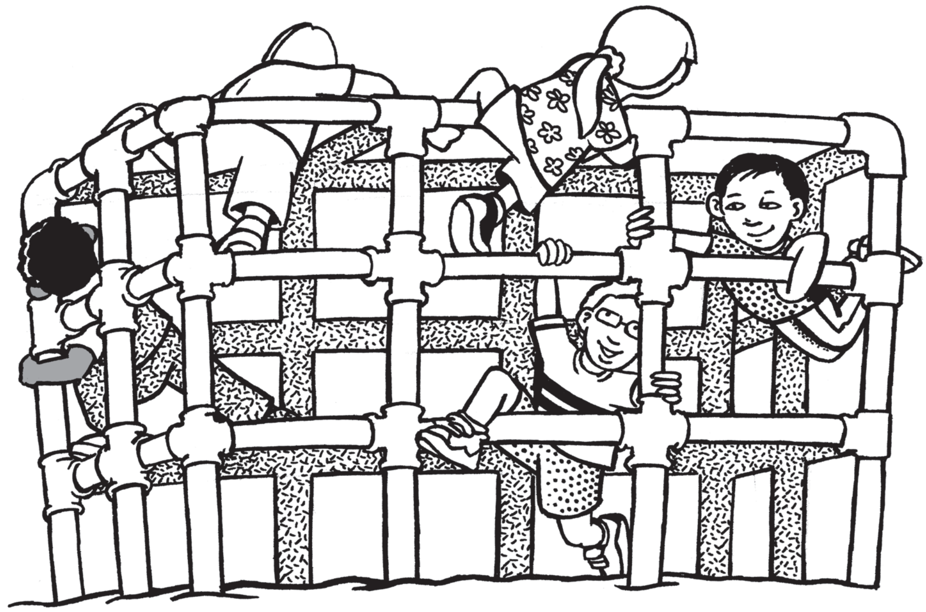
Grades 3–5: ★ ★ Important Focus

Playground Equipment

Ask students to consider the purpose of their data display and the message they are trying to convey when making decisions about how to represent their data. For example: After collecting data about which piece of playground equipment is the most popular, ask students to decide how to present this data to the parents' council to convince them to buy more of the equipment they need.



Class Results

Encourage students to use simple plots as a direct means of exploring some data they have collected, rather than constructing formal tables and graphs. From a class list of scores in a game or test, draw a baseline and label it with the range of scores, then put a mark for each score above its spot, building up a profile of the class results. Ask: In what ways is this helpful?

Chocolate-chip Cookies

Have students use real data to investigate the usefulness of dot plots for exploring emerging patterns in data. For example, have them count the chocolate chips in chocolate-chip cookies, adding their data for each cookie to a class line plot as they finish counting the chips (and eating) each cookie. Ask: How does the plot change as each new lot of data is added? What makes using a line plot particularly useful? (See Case Study 2, page 207.)

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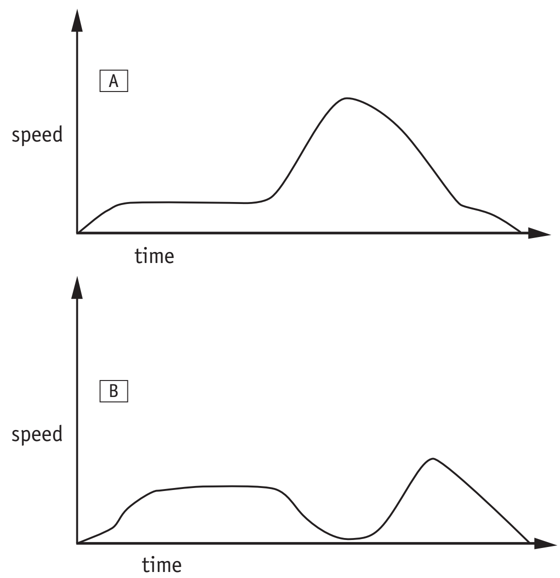
Sixty-four Grade 7 students were asked to say which graph (see right) best matched the story. Seventy-six percent said A, 13 percent said neither, and only 11 percent gave B, the correct response. Typical reasons for choosing A or neither were as follows:

- In graph B there are two hills and in the story there is only one hill, and it did not say anything about a ditch in the road.
- Christie does not go down a hill after the long flat bit; she goes straight to a steep hill, but A has a little hill at the start and that's not in the story either.

Christie went for a ride on her bike. After she started riding, she went along a flat road at about the same speed for a while, and then she came to a hill. Her bike slowed as she climbed the hill, but when she got over the top, she sped up as she rode down the other side. She got so fast she had to put her brake on. Then she slowed right down until her bike stopped.



Many students come to think of graphs as pictures and will look for a direct match between the shape of a plot and real-world physical features. As a consequence they have difficulty reading graphs that represent more abstract information.

Graphs are visual representations *of the information we record* about objects, events, or experiences. They are *not* pictures of the objects, events, or experiences. Students need considerable experience in plotting data, including information such as distance, speed, and time, in order to understand what graphs show.

For example, students could take a walk that includes a steep hill and other events that influence their walking pace (such as a traffic crossing or stopping at a store). Using a pedometer they could record their distance from school at equal time intervals and make notes as they go. Plotting the actual data points can then act as a starter for discussing what graphs represent.

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Sample Learning Activities

Grades 5–8: ★ ★ Important Focus

Which Type of Graph?

Ask students to suggest which type of graph is more effective to get a quick overview of how the data is distributed. For example, they may choose to make a frequency graph or a frequency line plot to get an overview of the distribution of the measures of how far the balloon cars in their science experiment travel.

The Average Height

Have students measure their height to the nearest centimetre and record, pinning a label to themselves. Ask them to line up from shortest to tallest (by look) to consider what the "typical height" is. (Refer to *The Average Height*, page 214.) They then can rearrange themselves into a line plot with each column made up of students of the same height. Ask: What can we learn about our heights from this? Regroup into columns made up of various height ranges of, say, 2 cm, 5 cm, and 10 cm and consider what each of the line plots do and do not tell us about our heights.

Computer Graphing

Ask students to use a computer graphing program to explore different ways to group and graph measurements and compare the results. For example: Have them measure arm spans of students in Grade 7 and Grade 3, then produce four graphs grouping data in 1 cm, 5 cm, 10 cm, and 20 cm intervals and compare. After they have experimented with groupings for several different sets of data, draw out that if the intervals are too small, it is difficult to see a pattern in the differences, but if the intervals are too large, it can look as if there are no differences.

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Allowance

Invite students to discuss when it makes sense to have the bars in a graph touching and when it does not. For example: The bars in a cumulative graph of allowance saved month by month could touch, as could a height graph where heights are in intervals, but it would not make sense for the bars in a graph of people's favourite food to touch. Have them brainstorm kinds of data and sort them into those that could sensibly have touching bars and those that need to have separate bars.

Sports Preferences

Ask students to explore which kinds of graphs make sense for a set of data, such as sports preferences of boys and girls. They can enter their data in a computer graphing program and have the computer construct a bar graph, a pie graph, and so on. Have students print and compare the graphs and discuss which are helpful and which do not make sense.

Pie Graphs

Invite students to consider which type of data can sensibly be displayed in a pie graph. For example: They could collect the names of their classmates' favourite bands and singers and organize the information into the percentage of the class who preferred each band. When trying to construct a pie graph, they may realize that the percentages add to more than 100%. Draw out that a circle graph for this data can make sense only if each person in the class chose only one favourite band or singer. Whereas a bar graph makes sense even if more than one favourite can be chosen by each child, a pie graph must represent data that are about a complete whole (e.g., all the class) divided up into independent categories: each child is represented in only one category.

Publishing Reports

Provide opportunities for students to publish reports that include graphic representation of their investigations. Ask them to consider which graph to use and how much information to include.

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Grades 5–8: ★ ★ Important Focus

Labelling Graphs

Prompt students to consider when and what sorts of labels are appropriate when creating graphs. Ask: Would you need a title and labels for the line plot you used to explore your data? Why? Would you need a title and labels for the graph you are going to put into the parents' newsletter with your report? Why?

Balloon Power

Have students consider which type of graph best suits their data about the distance their balloon-powered cars travelled. Have them use a range of sketch graphs or tallies to represent their data, and to describe the data from their graph. Ask: Are there any differences in what you can see about the data in the different graphs? Which graph gives you a better overview of the distance travelled by all the balloon-powered cars? Which one was the quickest and easiest to produce? Why? Draw out that a line plot is most suitable for discrete data that has a small range. Stem and leaf plots enable a large range of data to be quickly represented. A scatter plot could have been useful if students wanted to see if there is a relationship between distance travelled and some other measure, such as mass, size of balloon, or diameter of the wheel.

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CASE STUDY 2

Sample Learning Activity: Grades 3–5—Chocolate-chip Cookies, page 202
Key Understanding 5: How we display our data depends on the kind of data we have and our purpose.
Working Towards: Quantifying phase

TEACHER'S PURPOSE

I wanted my Grade 5 class to use line plots as a tool to allow them to observe and identify emerging patterns in data.

ACTION

I held up a package of chocolate-chip cookies and asked: "How many chocolate chips would you expect to find in one of these cookies?" After drawing their attention to the range of their estimates, between 5 and 40, I issued a challenge: "Well, how are we going to find out?"

The students quickly decided that they could produce some data by eating the cookies after counting the numbers of chocolate chips. As they had previously had some experience in representing data in different ways, I was pleased that they suggested they could organize their data in a tally or table and then make a bar graph.

However, I wanted to focus their attention on choosing a method for a specific purpose, so I asked: "Could we use a line plot instead?" William was not sure: "But isn't that just a graph anyway?"

"Line plots are graphs, but there are some important differences," I said as I drew a line across the board, labelling 5 towards one end and 40 towards the other. "Think about how a line plot might be different."

Maria said, "You can do it faster."

Abram said, "It does not have to be as neat."

"Yes, both those things," I said. "So it is particularly good when we are organizing our data. We can just build our plot as we go along—we do not have to make a tally or count how many in our categories first."

The students each ate a cookie and took turns to enter a mark, representing their number of chocolate chips, above the number line that I had drawn.

The quickly produced line plots allowed the students to see the data accumulate "before their eyes." They were able to observe the changing "shape" of the data as the sample size increased. They could not easily do this when constructing a bar graph.

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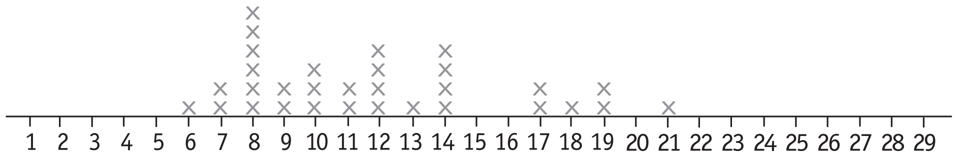


DRAWING OUT THE MATHEMATICS

"So what does our data tell us about the number of chocolate chips we could expect to find in a cookie?" I asked.

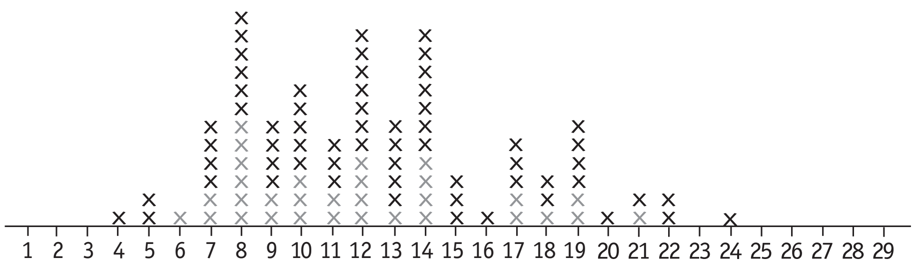
Blair: "I think it is between 6 and 21."

Te-Lin: "Eight had the most, so you would mostly expect eight in a cookie."

The students had begun to consider the risks of drawing conclusions from a sample.

However, Julian was not convinced. "Yeah, but if we ate another cookie, it might be different." I asked Julian to elaborate, and he said, "Well, it is just luck how many. If you ate another 2 and they had 14, then that would be more."

I suggested we add some more data to our line plot and see what happens. They were more than happy to eat another cookie each, adding their new data to the line plot—this time using a different colour so we could distinguish between the two lots of data.



To further explore the idea of chance variation in a sample, I decided to use line plots in a different way in the next lesson. The students could compare three separate line plots that each represented data produced from one cookie per student, rather than combine the data into one line plot.

I wanted to draw my students' attention to the changing distribution of the data so I asked: "Is what we think now about the number of chocolate chips the same as when we ate our first cookie?"

The students were able to make some comparisons.

Stella: "Now it goes down to 4 and up to 24."

Nigel: "And 14 is the same as 12. They are both the most."

Denise: "And it is still the numbers between 7 and 14 that have the most."

"So, what do you think we would find if we added more data?" I asked.

Peter answered: "I think it would be the same; some of our marks would be on the same numbers. But there might be some new numbers because the cookies might have different numbers of chocolate chips."

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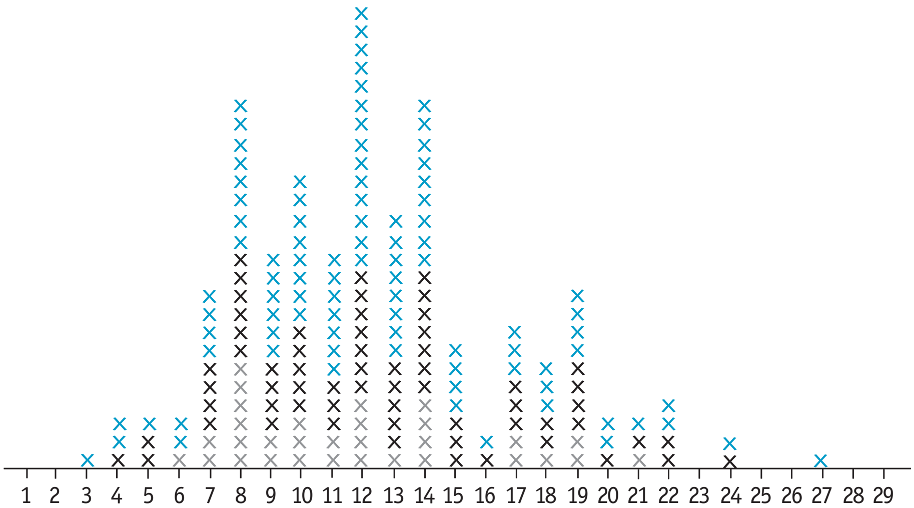
The students were eager to each eat another cookie and find out. They quickly recorded the additional data on the line plot using a third colour.

Gary: "Wow, look at that! It has changed. Twelve really is the most common number and there are more new numbers now."

Denise: "But still 7 to 14 have the most; it looks more bunched up."

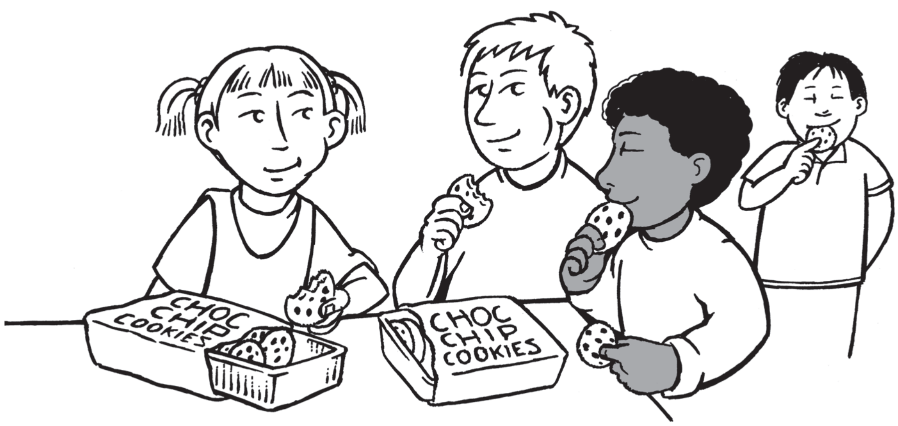
"Yes, I can see a pattern there," said Peter.



If a graph is to be interpreted without its producer being present, it must be sufficiently well labelled. However, fussing with details may not be necessary when we are using a graph for our own personal data analysis. Students need to take context and audience into account when making decisions about the type of data display they will use.

REFLECTION

After watching the line plot grow with each cookie, students were able to better understand why this graph was more useful than a bar graph for exploring emerging patterns in frequency data such as this. But in discussion, they decided they would use a bar graph in their report about this activity because it would be properly labelled and easier for someone else to understand.



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Key Understanding 6

We can use words and numbers to summarize features of a set of data.

During the elementary years, students should learn basic approaches to summarizing data. To summarize data means to describe, usually numerically, features of a whole group. Students need to learn that these summary measures do not tell us about individual scores, but rather tell us something about the data set as a whole. Thus, when we summarize data, we always lose some information.

At its simplest, summarizing data may involve a simple statement: "There are three blue birds and five yellow ones." During Kindergarten to Grade 5, students will learn to make tallies within various categories and describe the frequencies obtained. Later, they should learn to summarize data with fractions and percentages (e.g., two-thirds of the class prefer dogs to cats; 30% of children and 50% of parents think school should go until 4 p.m.). This is often necessary when we wish to compare two or more different-sized groups.

An important feature of a set of data relates to the idea of being typical or average. Students need to understand that measures such as mean, median, and mode do not tell us about individual scores or pieces of information, but describe characteristics of the group thought of as a whole. This concept is not easy to learn and simply working out these averages using a formula is unlikely to help students understand their meaning. Exploring sets of data can help them develop a sense of what is typical of it. Initially, students will concentrate on individual pieces of data and find it hard to make sense of questions that ask them to consider measures of the group as a whole. They may pick the largest number when asked about what is typical or average, or list the complete set of scores. Later they may invent measures that make sense, for example, suggesting that a typical score on a game was "between 0 and 7" or "it is mostly 3 or 4" or choose a score roughly in the middle of the data. Over time, they should come to understand the distinction between the three common measures of average: mode, median, and mean—what each tells you and when it is useful. They should also informally investigate the effect on each of the outliers (pieces of data that are very different from the rest) and zero scores.

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Another important feature of a set of data is how spread out it is. Initially, students may simply describe highest and lowest scores, which together with the median (the middle score) give a sense of the distribution of the data.

Links to the Phases

Phase	Students who are through this phase in Data Management…
Emergent	■ can summarize information by making simple counts
Matching and Comparing	■ will efficiently count to describe and compare how many are in a number of different categories
Quantifying	■ report numerically on the result of making tallies
Measuring	■ use fractions, means, and lowest, highest, and middle scores to summarize what their data shows, e.g., a student may say that about two-thirds of the children choose to play tennis, or that while the mean distance around students' heads was 55 cm, they varied from 49 cm to 60 cm. A student could also suggest why average head size may not be helpful for designing hats!
Relating	■ use fractions, percentages, means, and medians to describe and compare their results, e.g., having found that 22 out of 40 parents and 45 out of 60 students thought that teenagers should earn their allowance, a student could convert each to a percentage to make comparison easier.

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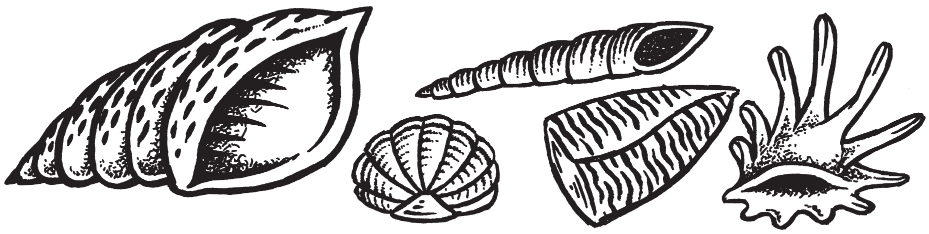
Sample Learning Activities

K–Grade 3: ★ Introduction, Consolidation, or Extension

Describing the Group

Prompt students to make statements that describe characteristics of groups of objects. For example, after a collection of shells has been sorted, ask students to describe the types and sizes of shells in the whole collection. Ask: What can you say about the collection of shells? Which shells in our collection have very different sizes? Which group are nearly all the same size?



Breakfast Foods

After students have represented data about favourite breakfast foods, ask questions that require them to make statements about the data using, for example, a pictograph. Ask: How many students liked scrambled eggs? Encourage students to make their own summary statements without having to respond to your questions. Ask: What else can you tell from our graph/table?

Mode

Build on students' intuitive notions of mode as a measure of average by posing questions such as these: Which of these is the most common? Which do most people use? What is the most popular? How did you know?

Television Programs

Scaffold summarizing data into frequencies. Have students list the class names and record each person's preferred television programs alongside. Ask: How can we rearrange this information so that we can see which program most students liked? What if we put the names of the programs on this side, and the names of the students on the other side? It is taking us a long time to write all of these names. Do we want to know who likes each program, or just how many of us like it? Is there a quicker way to record how many? Could we just count them?

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Sample Learning Activities

Grades 3–5: ★ ★ Important Focus

Number Cube Tossing

Have each group of students produce a large-scale line plot on the same topic, for example, the results of tossing a number cube 30 times. Pin up the plots and ask students to pick one of the line plots and to think of how to summarize what the set of data looks like. Have students, one at a time, describe the data without saying which line plot they are describing. Other students try to decide which line plot is being described. Draw out informally that it helps to talk about where the data clusters, which scores appear the most, and which are the smallest, biggest, and about in the middle.

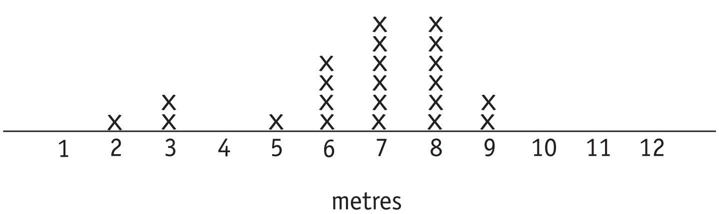
Describing Data

After producing line plots of data, encourage students to invent ways to describe the general features of the set of data. They may say, *All the scores were between 2 and 10 and most were around 3 or 4*. Compare different students' descriptions, asking which they think are the best.

Estimating

Invite students to summarize line plots, trying to give a sense of the "typical" result. For example: Given the plot below, ask students to describe the data to each other. Ask: Did your description enable your partner to understand which were the most common estimates, or what was the range of estimates? Were there any estimates that you did not include in your summary because you knew they were "way off"?

Our estimate of how long the hall is



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Grades 3–5: ★ ★ Important Focus

The Average Height

Have students record their heights to the nearest centimetre. Ask: What is the average height in our class? After some discussion, have students line up around the room from tallest to shortest. Discuss where the average height might lie. Ask: Which student is in the middle of the line (the middle height in our class)? What is his/her height? Mention incidentally that this is often called the median, but it just means the middle result or score or height. Suggest grouping the height data by forming a human line plot. On the floor write the height ranges (e.g., in 3 cm ranges such as 130 cm–133 cm) on an axis and ask students to form lines behind their range. Ask: Which line is longest? So more students in the class are between … and…. Does it contain the middle score? Have students write an answer to the original question. Share descriptions. Draw out which descriptions focus on the middle score and which heights are the most common.

Extend Average Height

Encourage students to think of "typical" in different ways. Using sticky notes rather than themselves, repeat a similar activity for other aspects of students' lives so that different children become average or typical for different things (e.g., family size, sport played, how they come to school).

Arm Spans

Have students cut lengths of paper tape to match their arm span. Ask them to sort the tapes into piles of the same length (to the nearest centimetre). Ask: Which pile has the most strips? Which arm span is the most common? (This is the mode.)

Identifying the Mode

Introduce the idea of mode as the most common result or score. Have students inspect several line plots and bar graphs and identify the mode. Provide them with unsorted data of, say, 20 scores and suggest that they organize the data in a way to make finding the mode easy.

Stem and Leaf Plots

Have students look at data displayed in stem and leaf plots and determine the median and mode. Ask: How can we find the middle (median) amount? (*Count how many scores and then count halfway, or mark off pairs from each end of their data until we find the middle.*) How can we quickly see which is the mode or modes? (*The value that occurs most often*) Use back-to-back stem and leaf plots to compare two sets of data and consider how the mode and median do, or do not, provide a useful way to compare the data.

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Sample Learning Activities

Grades 5–8: ★ ★ ★ **Major Focus**

School T-shirts

Have students consider proportion when summarizing data from different-sized groups. For example, 16 out of 20 Grade 1 students and 16 out of 32 Grade 6 students voted for a particular design for a new school T-shirt. Discuss how to make a reasonable comparison. Ask: Is it more helpful to say 16 students in each class voted for the design, or that 80% of the Grade 1 class but only 50% of the Grade 6 class voted for it?

Mean Averages and Fair Shares (1)

Suggest a scenario of buying jelly beans in bulk for a party and pouring a pile into each child's hand as he or she arrives. Is this fair? How can you redistribute so it is fair? Give each student in a group a handful of tokens (from 5 to 15). Ask them to make a note of how many they originally got and then find a way to redistribute their tokens fairly. How many did each get? (Groups will differ.) Each group reports to the class about how they did it. Most will physically put tokens together and redistribute, although some may count and divide. Ask: How sure are you that it is now fair? If they all have the same (acknowledging some "part tokens"), it is fair. How many tokens did your group have to start with? And how many students? How can you relate the total your group had to the number of students and the amount of each share? If the same total number of jelly beans were handed out to the same number of students, but distributed with different starting amounts, would the fair shares still be the same, or would they be different? Why? Ask students to test with their tokens if they are not convinced that the fair shares must be the same no matter how the total was distributed.

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Grades 5–8: ★ ★ ★ Major Focus

Mean Averages and Fair Shares (2)

Extend the previous activity to link the "fair share" to the mean. Have students compare the share they got with their original amount. Draw out that some went up and some went down. The share was "sort of" in the middle of their original amounts. Have students write the number of tokens they got on a sticky note and produce a line plot. Again, draw out that the fair share was "sort of" in the middle of the original amounts. Tell students that the "fair share" they have worked out is another typical score, or average, and it is called the mean.

Mean Averages and Fair Shares (3)

Extend the previous activities to emphasize the underlying meaning of the mean. Informally discuss the difference in amounts students started with and ended with. Draw out that some went up, some went down, but the fair share removed the differences between students. Ask students to make a table listing their names and showing the number of tokens they started with and the number they finished with. Each calculates the difference, either using + and – , or the words "more" and "less" to signify the direction.

Name	Start Amount	Fair Share	Difference
Terri	8	9	+ 1
Tony	11	9	– 2

Have students combine the differences, taking into account whether they need to add or subtract. (*The result should be 0 if the mean was exact; or if there were a few left over, the total will reflect that.*) Emphasize that the mean balances out the differences—it is another way of thinking about the middle or typical amount. Help students find the median (middle score) and the mode for the data and compare to the mean. What information has been lost in these summaries? How might this kind of information be useful? (*Comparing large quantities of overlapping data to see if we can say there is an overall difference between groups*)

Mean Heights

Put students into groups of four or five and challenge them to find a way to get their average (mean) height without calculating. Offer paper tape to help. (*They might lie on the floor in a line head to toe, mark the beginning and end, stretch the tape along it and fold into four or five equal parts OR they might use the paper tape to find each height, stick them together and fold.*) Have students in each group line up in order of height and put their mean height in its right place. Discuss where it sits and again draw out the idea of balancing the differences.

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Line Plots (1)

Provide each student in a group with a different line plot, each containing 20 to 25 dots spread over a similar but not identical range. Select line plots so that, say, two have the same mode but different other averages, two have the same mean, and so on. Ask each student to write a description of his or her data set, keep the description, and then put the plot into a pile in the middle of the table. Each student in turn reads a description of the data set to the group, and the group decides which plot it describes.

Line Plots (2)

Vary the previous activity by having each student work out the mode, median, and mean for their data set and write it on a separate sheet of paper. The groups make two piles, one of the plots and one of the list of three averages for each, and then together sort them so that the line plot is with the correct set of averages. Prompt students to discuss how they could tell. Pin the plots and summaries around the room and have a class discussion to draw out features of the plots. Ask: For which plots were the three averages about the same? (Look at the shape or distribution of the data.) What is the effect of single scores that are "odd"? (Look at outliers.) How would it help to know the range, that is, the highest and lowest values?

Comparing Data (1)

Ask students to summarize data to compare the same information from different populations. They could collect data (such as height, time spent playing video games) from the whole school and represent the data in a way that allows comparison. Ask: Is the data from your class like the whole school? Where does most of the data lie for each group? How is it the same or different? Where is there no data? Is this the same for each of them? Is there some data out on its own? Is this the case for each of the sets of data? How would finding the mode, median, or mean help with this?

Comparing Data (2)

Extend the previous activity and use census and other population data found on the Internet to compare their class data to data from schools both nationally and internationally.

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Grades 5–8: ★ ★ ★ Major Focus

Balloon Power

Have students use the highest, lowest, and middle scores or measures to describe data. Ask them to record the distance travelled by each student's balloon-powered car on a word-processed table or spreadsheet on the computer. Sort the distance column from shortest to longest distance. Find the middle measure (median). If there is an odd number of measures, one will be in the middle; if there is an even number of measures, the median is halfway between the two "middles."

Families (1)

Have students produce data sets that result in a specific value for the mean. For example, invite them to draw a series of families so that the mean number of children in the families is three. Compare drawings. Ask: Is there more than one way to draw these families? Draw out that different data sets can result in the same mean.

Families (2)

Extend the previous activity by asking students to include a family with no children. Ask: If this family did not have any children, what changes would we have to make to the rest of the families?

Families (3)

Extend the previous activities further by using a mean of, say, 2.5. Ask: Is it possible for any of the families to have 2.5 children? If not, how can we make sure that the mean number of children is 2.5? Draw out that the mean does not need to represent the value of any one particular member of the data set.

Sports Equipment

Ask students to consider how the median helps describe only some data. For example: Have students try to find the middle measure for sports equipment or other sets of data with categories that do not have a set numerical order. Ask: Why is it possible that one student says "balls" is the middle when someone else says it is "bats"? Is the median helpful when the categories of sports equipment can be in different orders? Draw out that because the categories could be listed in any order, a middle measure does not make sense. Students indicate whether a median is a helpful descriptor as they deal with other data.

Predictions

Use a class frequency graph of the students' predictions for future events to compare mode and median. First, ask whether we can see which was the most popular prediction. For example: How many babies will be in the guinea pig's litter? How many goals will the winning team get? Is there more than one popular prediction? Then have students find the middle (median) prediction. Ask: Which best described the "average" prediction?

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More Chocolate Chips

Extend *Chocolate-chip Cookies*, page 202, by recording the quantity of chocolate chips in each cookie on a sticky label. Use the data to explore and compare the mean, mode, and median. (See Case Study 3, page 220.)

The Average Height

Repeat *The Average Height*, page 214, recording each height on a sticky note. Put them in order and find the middle height (median). Reorganize the notes to form a line plot and identify the height or heights with the most students (the mode or modes). These measures provide a sense of the typical or average measure for the whole group. Ask: Is the mode the same as the median? Close? Then have students round their heights to the nearest 5 cm and make a new sticky label and line plot, finding the median and mode(s) as before. Ask: Has the median changed? Has the mode(s) changed? Why?

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CASE STUDY 3

Sample Learning Activity: Grades 5–8—More Chocolate Chips, page 219

Key Understanding 6: We can use words and numbers to summarize features of a set of data.

Working towards: Measuring phase

Soon after the students began to produce their data, they started to pose questions such as: Do I count this half chocolate chip as a one in my tally? I used the opportunity to talk about the need for consistency while collecting data.

TEACHER'S PURPOSE

My Grade 7 students were able to organize data in frequency graphs and identify the most common category. To develop their sense of what was typical of a set of data, I wanted to focus their attention on the meanings of and the differences between mean, mode, and median.

CONNECTION AND CHALLENGE

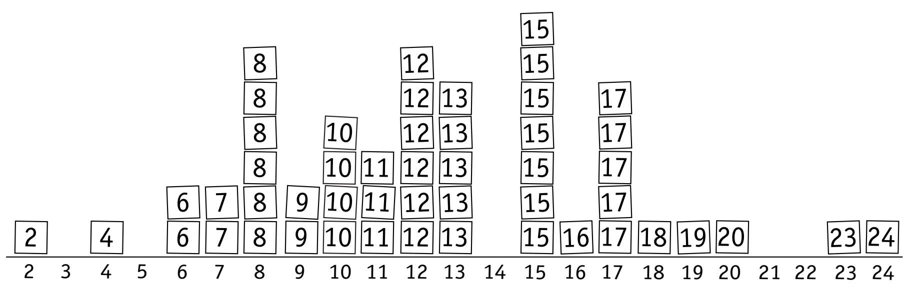
During language arts classes, the class had begun to question the validity of some claims made by manufacturing companies. They showed interest when I suggested we could use mathematics to help evaluate such claims. Holding up a package of chocolate-chip cookies, I asked them to imagine they were the manufacturers of the product and to decide what advertising claims they could make about the number of chocolate chips in their cookies.

I suggested two cookies per student to ensure that there was sufficient data to allow trends to emerge.

ACTION

The students decided that they would need to find the number of chocolate chips in some cookies. So, at my suggestion, they were happy to each carefully nibble two cookies and record the number of chocolate chips in each cookie on separate sticky notes. When I asked if there was a way we could organize our data, the students suggested a line plot.

I deliberately chose to use sticky notes to collect and organize our data so we could physically rearrange the data later in the lesson.



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After constructing our line plot, I asked, "What advertising claim could we make about our chocolate chip cookies if we used this data?"

"Most of our cookies have 15 chocolate chips," said Penny.

"No, they do not!" argued Hason. "Seven out of 50 is not most. Fifteen is just the most common number. We could only say that our cookies are more likely to have 15 chocolate chips than any other number."

I continued: "We can use the most common number as an average or typical number to give people an idea about the whole set of data without telling them all the individual numbers."

"But I thought the average was 13!" Jerome argued. "Thirteen is in the middle of 2 and 24."

"We could have a look to see if 13 is the number in the middle of our data," I said.

DRAWING OUT THE MATHEMATICS

I invited some students to rearrange our set of sticky notes into one long line, in numerical order, across the board.

2 4 6 6 7 7 8 8 8 8 8 . . . 12 12 12 12 12 13 13 13 13 . . . 17 17 17 17 18 19 20 23 24

When they were finished I asked, "How could we find the middle score?" We decided we could remove the sticky notes in pairs, one from each end, until we got to the middle. "What would that actually be doing? Is there an easier way?" I asked. We established that halving the number of notes and counting from one end would do it.

"Twelve is the number in the middle!" said Carlos.

"Yes," I said. "What does the 12 tell us about the number of chocolate chips in our cookies?"

I was pleased when Carlos ventured: "It is like the halfway number. Half the cookies have 12 or fewer chocolate chips and half the cookies have 12 or more chocolate chips."

Nasrat then said, "Well, that would not make the cookies sound very good in an advertisement!"

Inja replied, "Yes, but it tells us what they are like."

I agreed. "That is right, we can also use the middle number as an average or typical number to tell people about the set of data."

Jerome was satisfied. "I get it now. I did not know you had to get the middle of all the numbers. I thought it was just the middle of 2 and 24, like on a ruler."

Another student, however, had a query. "I thought the average was when you added up and divided."

I recognized that I would need to return to these ideas of chance in future lessons.

Some students have a misconception that the "average" is the middle number of the range of data.

Students often do not realize it is possible to find the mode, median, and mean of the same set of data.

We ended up with two "middle numbers" because we had an even number of sticky notes. In this case both middle numbers were 12. If the two middle numbers had been different, I would have discussed the notion of an imaginary middle number halfway between the two.

Many students (and often the media) use the word "average" to refer to the mean.

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When the class computed the mean to be 12.25, some students were perplexed, saying, "But that's silly! You would not count out exactly 12.25 chocolate chips for each cookie." I explained that the mean is a calculated, imaginary number that we use to give us some information about the group of cookies as a whole rather than about individual cookies. Many students were convinced that the mean had to be one of the numbers of chocolate chips.

I replied: "You might be thinking about another average or typical number of a group called the mean. Imagine our 50 cookies before they were cooked. How could we find the number of chocolate chips we would have had if we had removed all the chocolate chips from the cookies?"

Pointing to the sticky notes, Carlos answered, "You would have to add up all those numbers."

I continued: "Then, imagine we decided to put the chocolate chips back into the cookies. But this time we want every cookie to have exactly the same number of chocolate chips so each cookie got its fair share. How could we find the number of chocolate chips for each cookie?"

Hasan offered an answer: "We would have to get that number we found when we added and then share it into 50 bits."

REFLECTION

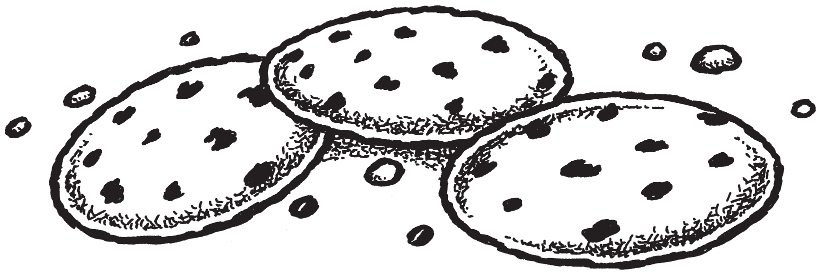
"So would we use the most common number, the middle number, or the mean in an advertising claim?" I asked.

"Well, the mean does not tell people very much," Irene suggested. "It is not what is really in the cookies. I think the most common number is better because that is what people are most likely to get."

"But the median tells the truth more," argued Nasrat.

I was pleased. The students had begun to compare the information provided by the mean, mode, and median. I now needed to provide opportunities for them to consider which average was the most useful in describing other sets of data to answer particular questions.

At this stage, I wanted to help the students to understand that there are three common measures of the average—mode, median, and mean—and to understand each of them. It was not my intention to provide them with formulas for their computation.



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Chapter 5

Interpret Data

Locate, interpret, analyze, and draw conclusions from data, taking into account data collection techniques and chance processes involved.

Overall Description

Students interpret and report on their own data (about age and height, pet preferences, allowances, fitness levels, attitudes to smoking, a weather simulation) and data taken from a variety of secondary sources (magazines and newspapers, sports results, farm records). They locate and use databases about Canada and Canadians, such as those available through Statistics Canada. They distinguish between census and sample data and understand that considerable care needs to be taken in selecting samples and in forming conclusions about the whole group from sample data. In this, they realize that the uncertainty involved in drawing conclusions from data is the essence of the connection between probability and data.

Students know that when making use of data they should question its quality and credibility, as well as the way in which data is organized and represented, before evaluating the conclusions drawn by others. They recognize that good-quality data and a knowledge of probability processes can help them assess risks, form opinions, and make decisions, such as whether to immunize children. However, they also know that while mathematics and data can each contribute to our decision-making, they cannot determine what we should or should not do in any particular circumstance, the latter being matters of personal and/or community judgement.

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Interpret Data:
Key Understandings Overview

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of this family of concepts. The learning experiences should connect to students' current knowledge and understandings rather than to their grade level.

Key Understanding		Description
KU1	Graphs, tables, and diagrams display data about the real world, although they are not pictures of the real world. We need to learn how to read them.	page 226
KU2	When we analyze and interpret data, we are deciding what it says and what it means. There is a difference between the data itself and what we think it means.	page 240
KU3	We need to evaluate the data we are using in order to be confident about the conclusions we have drawn.	page 252

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	Grade Levels—Degree of Emphasis			Sample Learning Activities
	K-3	3-5	5-8	
	★★	★★★	★★★	K-Grade 3, page 228 Grades 3-5, page 230 Grades 5-8, page 233
	★	★★★	★★★	K-Grade 3, page 242 Grades 3-5, page 244 Grades 5-8, page 246
	★	★★	★★★	K-Grade 3, page 254 Grades 3-5, page 256 Grades 5-8, page 258

Key

★★★ **Major Focus**
The development of this Key Understanding is a major focus of planned activities.

★★ **Important Focus**
The development of this Key Understanding is an important focus of planned activities.

★ **Introduction, Consolidation, or Extension**
Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

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Key Understanding 1

Graphs, tables, and diagrams display data about the real world, although they are not pictures of the real world. We need to learn how to read them.

There are three related aspects to this Key Understanding:

- To make sense of (that is, to read) data displays, we need to know what aspect of the real-world situation the data refers to. (Link to Collect and Organize Data, Key Understanding 2.)
- Data displays are not pictures; they do not look like the real-world situation from which the data was produced. (Link to Summarize and Represent Data, Did You Know? page 203)
- We need to learn how to read graphs, tables, and diagrams.

Throughout their schooling, students should learn to read a range of diagrams, tables, and graphs that go beyond those they can readily produce for themselves. The range should reflect the variety one would find in general use, including those that provide very good models of data presentation, through those that are flawed in some way, to those that are downright misleading. Older students should locate and use data from a wide variety of media sources.

Visual and tabular displays of data require specific reading techniques that students must learn. For example, tables are generally based on a cell or grid structure and conventional left-to-right, top-to-bottom scanning may be unhelpful. In reading a table such as the one after this paragraph, students need to understand that the Input and Output numbers are intended to be thought of as pairs (relating back to the situation from which the data comes). Therefore, the table needs to be read in an order that connects the pairs, perhaps noting that the output number is always three times the input number. However, certain aspects of the relationship will also be highlighted by reading along the rows, perhaps noting that the input goes up by ones while the output goes up by three. Often the meaning of tables is lost because students do not read them either flexibly or as intended. Depending on how they are read, however, the relationships could be apparent or not be noticed at all.

Input	1	2	3	4	5
Output	3	6	9	12	15

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Students need to understand the conventions about how the content of a table cell is determined and labelled. They should practise reading one-way and two-way tables, including those where both cell frequencies and row and column totals must be read and where some grouping of data is involved. They could be asked, for example, to extract particular pieces of information from tables (How many girls play a particular sport?) or to explain what a particular number in a cell reveals (What does that 11.6 tell us?).

Many students are unable to separate their reading of a graph from their personal knowledge of the situation. Thus, students may ignore the obvious information, such as red is the favourite colour, in favour of statements about their own preferences. To overcome this, students should read graphs about situations for which they are not already privy to the key information.

Graphs also have their own conventions, and students need structured activities that require them to describe or recount the information provided in a graph. Even reading bar graphs requires that students read labels properly and read frequencies and measures from a range of scales, including reading between calibrations. Students seem to respond to some graphs intuitively and can read them without specific instruction, such as where bar graphs represent the heights of children. Perhaps the most difficult for students to read are those that involve two variables and when the graph's appearance does not directly match the idea it represents. Students should learn to write brief stories to describe what is represented by such graphs. The aim is for students to understand that such graphs are intended to help them get a feeling for how variables are related to each other; they are not pictures of situations.

Links to the Phases

Phase	Students who are through this phase in Data Management…
Matching and Comparing	■ can extract frequencies from lists and one-way tables
Quantifying	■ can read a tally, extract data from simple one- and two-way tables, and work out frequencies from a pictograph or a bar graph where each unit is marked on the axis
Measuring	■ read frequency and other types of information from a range of tables and bar and line graphs (including those where data has been grouped)
Relating	■ are able to extract information from a wide variety of descriptions, tables, diagrams, and graphs produced by others, including informal graphs showing the relationships between two quantities

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Sample Learning Activities

K-Grade 3: ★★ Important Focus

Shoelaces
Encourage students to read lists used to display information in the classroom as a natural part of other activities. For example: Display a list of the names of students who can tie shoelaces and encourage students to find someone on this list when their shoes need to be tied.

Colour Graph
Invite students to read block graphs produced collectively by the class. For example: Have students create a graph of favourite colours in their class by placing coloured blocks in lines. Ask: How can we tell which colour most people like? Can we tell by just looking? What if we wanted to know how many people liked red? How can we tell?

Reading Pictographs (1)
Ask students to read pictographs produced by their classmates. Have students produce a pictograph that represents information on a topic familiar to the class. They exchange graphs with a partner, who describes what the graph says.

Reading Pictographs (2)
Ask students to read pictographs produced by their classmates, but this time the pictographs should contain unfamiliar information. For example, each group works on a different question for their graph. They then hand their graph over to another group, which reads the graph to decide what it is about. Each student states one piece of information from the graph.

Reading Pictographs (3)
Invite students to read pictographs in which one picture stands for one object, (e.g., where faces are used to represent the number of students in the class who belong to a particular sports team). Using a label for each sport, students place their pictures next to where they belong. Ask: How many students play soccer? Can we tell from our graph which sport has the most people?

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One-way Tables

Have students read frequency information from one-way tables produced by their classmates. For example: Ask students to collect information about the number of each colour of jelly bean in a bag and record the information in a table. They can swap with a classmate who reads the information. Ask: How many red jelly beans were in your partner's bag? Which colour did she or he have most of? Pin up a table of data, point at a number, and ask: What does this number tell us?

Lunch (1)

Ask students to produce a simple table that lists the names of students in their group (of six or eight) and indicates whether they brought lunch from home or bought lunch. Have one student in the group say a name, and another read to find what that child did. Draw out that it can be quite slow to find the right name. Ask: How could we make it quicker? Draw out that listing alphabetically might help.

Lunch (2)

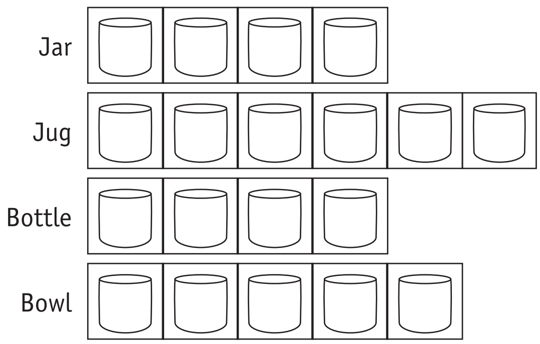
Extend the previous activity by having students exchange reworked lists. Call out names and have students find whether the name is on their list, and read the relevant information. Ask: What if it were a whole class list? Draw out that alphabetical order would help.

Tallies

Have students read tallies to say how many in each category. For example, while on a zoo trip, ask students to use a tally to record each time they see a particular animal. Afterwards, ask: How many monkeys did you see? Encourage children to skip count by 5s. Extend this to include tallies that are not their own.

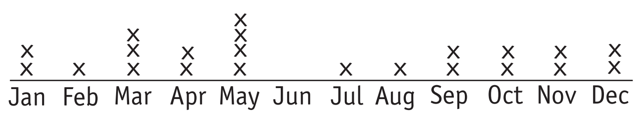
Measurements

Have students record measurements of different objects using the same unit in a table or in a pictograph, and help them to interpret it. For example, use a cup to measure how much water various containers hold. Ask: Which container holds the most water? Which holds the least? How much more does the jug hold than the bottle?



Birthdays

Invite students to read line plots to compare how many children have birthdays in different months. Ask: How many birthdays in April?



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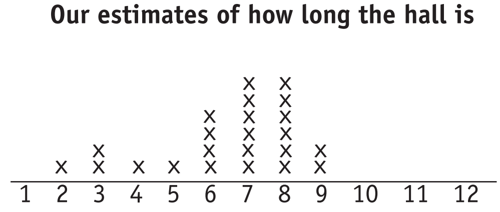
Sample Learning Activities

Grades 3–5: ★★★ Major Focus

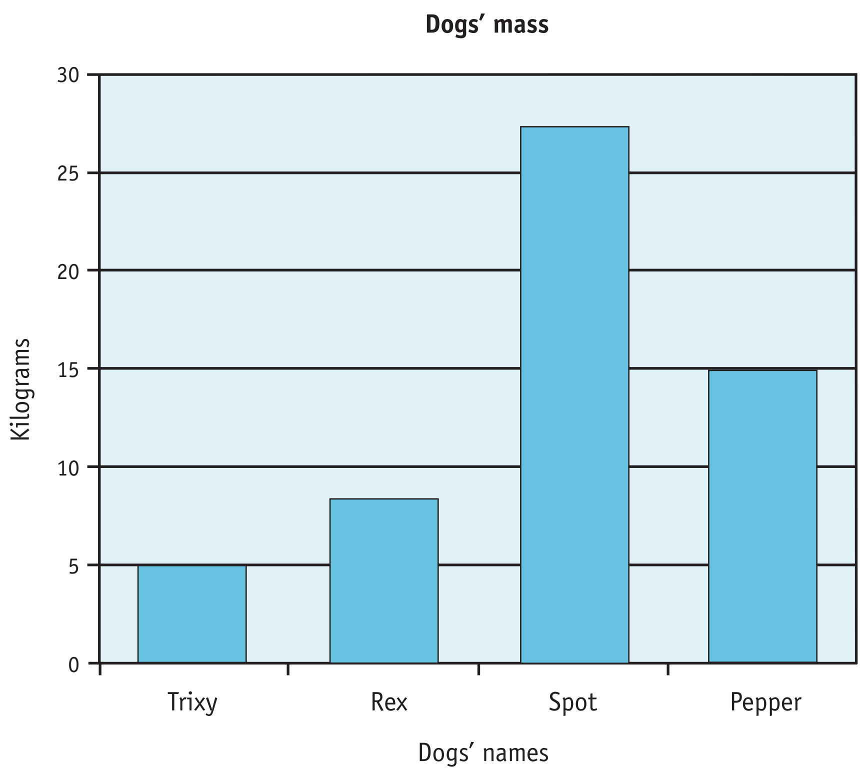
Estimating

Give students practice in reading line plots. For example, given the plot shown at the right, ask them to say what it is about. Ask: How many children think the hall is 5 m long? 8 m long? 11 m long? 4 m long? What is the longest estimate? the shortest?



Dogs

Have students read simple bar graphs in which scaffolding, in the form of horizontal lines, assists in the reading. For example: Model the process of reading the graph below by asking students to tell their partner what the heading and each axis tell you. Prompt them to say, without referring to the numerical scale, which dog has the greatest mass. Ask: How can you tell just by looking? Students then look at the numerical scale to determine the mass of each dog. Provide new graphs and ask students to describe to their partner what they see.



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No Props

Ask students to read simple bar graphs in which the reading props are not provided. Repeat the previous activity, but with bar graphs in which horizontal lines are not drawn in.

Pie Graphs

Invite students to interpret simple pie graphs by comparing the sectors and saying which is the largest/smallest.

Counting Cars

Have students skip count to read pictographs where one picture stands for more than one object. For example, students count by 10s to work out how many. Ask: If one car stands for 10, how do we count half a car? How can we tell which has the most? Do we need to count?

Hockey

Ask students to read simple one-way tables from published sources to find specific information. You could use two separate tables, one showing heights and the other masses of hockey players. Ask: What does this table tell you about? *(The mass of each player)* Who has the greatest mass? Do different players have the same mass? Are there three players who have the same mass and also the same height? How can we tell? *(Look at other table)*

Shoe Size

Invite students to extract the information they need from a frequency table produced by others to produce a pictograph or bar graph. For example:

Survey of children's shoes sizes	
Size	**Number of Students**
4	4
$4\frac{1}{2}$	6
5	0
$5\frac{1}{2}$	7
6	3

Getting to School

Have students read simple two-way tables created by others, and answer questions that require them to interpret the contents of various cells. For example:

	Grade 4	Grade 7
Walk to school	6	12
Driven to school	18	8
Ride to school	4	10

Ask: What does the 18 tell us? How many in Grade 4 walk to school? Do more students walk to school in Grade 4 or in Grade 7? How do you know? How do most students get to school in Grade 7?

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Grades 3–5: ★ ★ ★ Major Focus

Venn Diagrams (1)

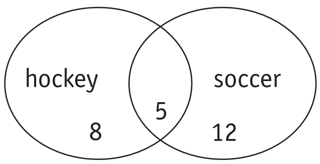
Ask students to read Venn diagrams involving two circles. You could have them place their names in a Venn diagram according to whether their family has boys or girls. Ask: Which circle are you in? Who else is in the same circle as you? What about the people in the middle? What does their position tell you about their families?

Venn Diagrams (2)

Extend the previous activity by replacing names with frequencies and have students explain what each number says. Draw out that the number of families having boys is the number having only boys, added to the number having boys and girls. Repeat for girls.

Venn Diagrams (3)

Provide two circle Venn diagrams for students to read frequency information from, for example, the number of students who play each of two sports, hockey and soccer.

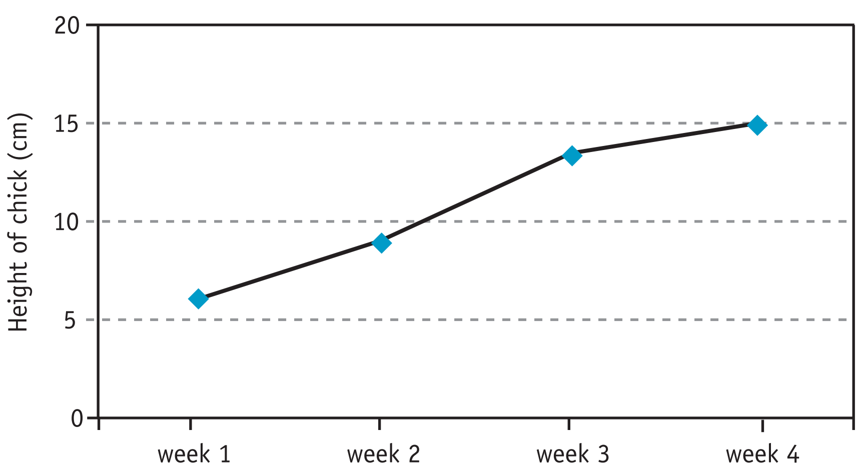


Point to the frequency for hockey alone and ask: What does this number tell you? How many people play hockey in total? How many play soccer in total? Point to the 5 and ask: What does this number tell you?

Growth of a Chick

Have students read simple line graphs to say how a quantity varies over time, (e.g., the growth of a chick at weekly intervals). Ask: What do the labels on each axis show? How tall was the chick after week 2? after week 3? So, how much did it grow between weeks 2 and 3?

Growth of chick at weekly intervals



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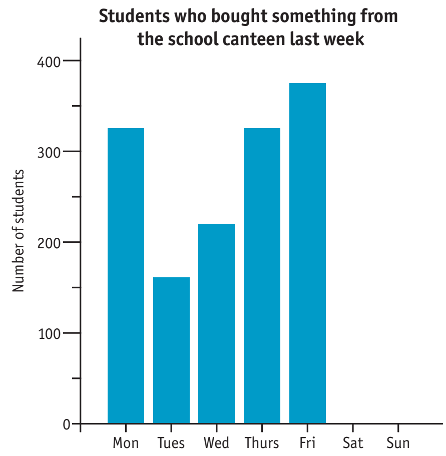
Sample Learning Activities

Grades 5–8: ★ ★ ★ Major Focus

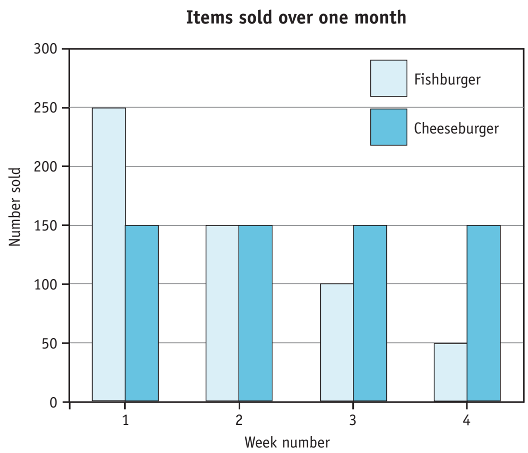
Estimating Values

Ask students to read simple bar graphs where all the calibrations on the frequency axis may not be marked. Help them to estimate values between calibrations.



Double Bar Graphs

Have students read horizontal and vertical double bar graphs. For example:



Ask them to describe to their partner what the heading tells them. They then read the information on each axis and explain what each shows. Ask: What do the two different-coloured bars tell you? Which burger was more popular in the first week? Did it continue to be popular? What do you notice about the sales of cheeseburgers over the four-week period?

No Props

Repeat the previous activities with bar graphs that do not have props in the form of horizontal bars.

Statements/Collecting Graphs (1)

Ask students to collect bar graphs from newspapers, magazines, encyclopedias, and the Internet. Ask them to select a graph and make two factual statements about what it shows. Ask their partner to agree or disagree about whether the graph actually says that.

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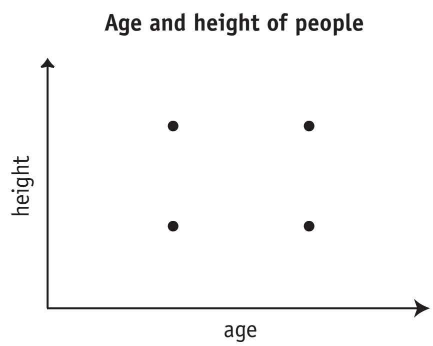
Grades 5–8: ★ ★ ★ Major Focus

Collecting Graphs (2)
Extend the previous activity and ask students to select a graph and write two questions for their partner to answer from the graph.

Collecting Graphs (3)
Ask students to work in groups to sort their collected graphs into those that were easy and those that were hard to read. Ask: What made the difference?

People Graph
Provide students with two-variable graphs with just a few points to read. For example: Present a picture of four people (short and old, short and young, tall and old, tall and young) and a matching graph (see right). Explain that the four points represent the four people and challenge the students in groups to write the label of each person on the graph. Prompt students to justify their placement to the whole class. Draw out that the graph shows the height and age of the four people. Ask: What does the vertical axis show? the horizontal axis? What happens to the heights as you go up the vertical scale? What happens to the ages as you go to the right? Describe each person's age and height.



Dot Plots
Provide students with a range of two-variable dot plots, each with just a few dots on it, and ask them to state what each point shows, for example, how tired and happy four students were after the sports day. Have students match the points with alternative forms of the information (e.g., a drawing such as in the previous activity or a table showing the data).

Hockey Table
Ask students to read one-way tables from published sources, including those with several columns of information. For example, use a table showing height, mass, and age statistics for hockey players. Ask: What does this table tell you about? (*Each player's mass, height, and age*) What is the mass of player X? Whose mass is the greatest? Do players with the same mass have the same height?

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Hockey Graph

Invite students to extract data from one-way tables, such as in the previous activity, to produce a two-variable dot graph. For example: Students find the height and weight of each player and plot points in a two-way coordinate grid to represent the extracted information. Ask them to compare what the table and graph highlight. Repeat for weight and age, and height and age.

Allowance

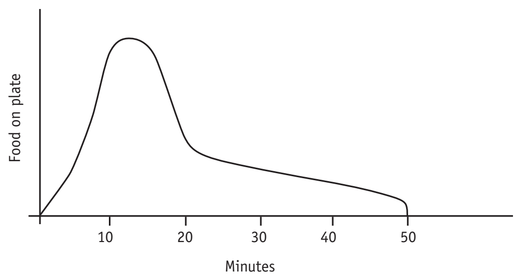
Have students read two-way tables where the data is grouped into categories.

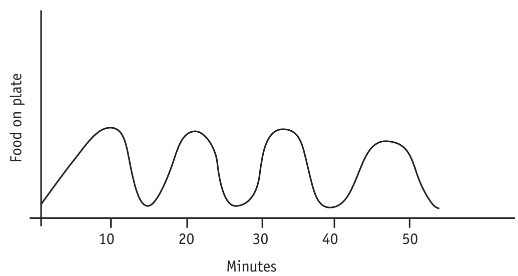
Allowance earned by students in Grade 5 and 6		
Amount($)	**Number in Grade 5**	**Number in Grade 6**
0 to 5	8	10
6 to 10	12	1
11 to 15	3	6
16 to 20	0	1

Ask: What does the 10 mean? What does the 12 mean? How do the table headings help you to know? What other questions can you ask of the data?

Eating Patterns

Ask students to read and report on graphs where the appearance does not directly match the idea it represents. For example: These graphs were drawn by students to describe their eating patterns at an all-you-can-eat restaurant.





Ask: What does the graph show about each child's eating pattern? How did you use the information on both axes to help you tell the story?

Junk Food

Have students bring in published graphs. Choose a graph that has not been fully labelled. Enlarge it and, as a class, try to work out its meaning. Ask: What might the columns be about? How does looking at the main title help? What else do you need to know to interpret the graph? (See Case Study 1, page 237.)

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Lunch

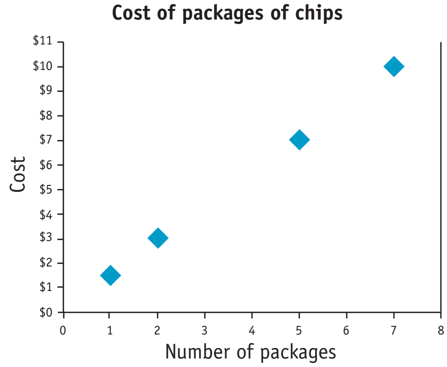
Have students read two-way tables where rows and columns are totalled. For example:

Lunch orders for the sports day				
	Grade 5	Grade 6	Grade 7	Total
Pizza	5	7	4	16
Hot dogs	7	3	5	15
Grilled cheese	10	10	15	35
Lasagne	4	5	7	16
	26	25	31	

Ask: How many of each item of food needs to be ordered for the sports day? Write the number of items that need to go into the basket for each grade so we can check that the number of items is correct.

Buying Chips

Invite students to read two-variable (coordinate) dot graphs. Ask: What does this graph tell you about? How much does it cost to buy five packages? How many packages can you buy for $2? Is there a discount for buying seven packages? How do you know?

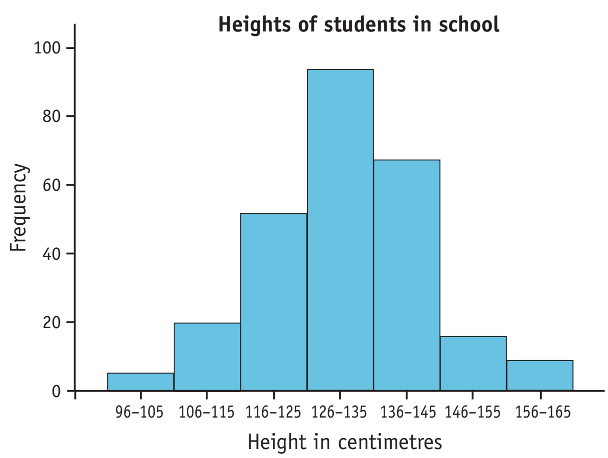


Venn Diagrams

Have students extract information from Venn diagrams as on page 232, but involving three circles.

Heights of Children

Ask students to read graphs where the data is grouped and where reading between the calibrations on the frequency axis is required. Ask: How many students are between 126 cm and 135 cm high? How did you work it out?



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CASE STUDY 1

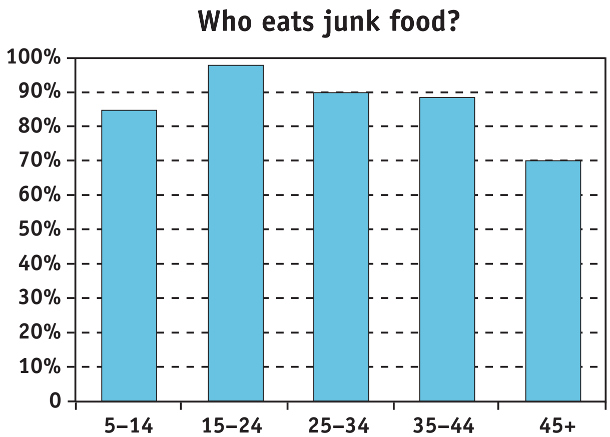
Sample Learning Activity: Grades 5–8—Junk Food, page 235

Key Understanding 1: Graphs, tables, and diagrams display data about the real world, although they are not pictures of the real world. We need to learn how to read them.

Working Towards: Quantifying phase

TEACHER'S PURPOSE

During a health lesson on nutrition in my Grade 5 class, Amanda brought in a graph that she had found.



She wanted to include it in her report about junk food, but she was not sure what some aspects meant.

ACTION

I enlarged the graph on the photocopier, placed it on my bulletin board, and invited students to comment on it. Amanda said, "I do not know what the numbers mean—they did not put anything on the side or bottom to tell you."

Amanda did not bring in the accompanying article, so I asked the students what the columns might be about.

"It could be how much they eat," ventured Minu.

"Or it might be how much it costs," said Prajit.

I suggested they look at the main title for a clue. "It says, 'Who eats junk food?' so it is more likely to be about groups of people."

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Students can make comparisons between columns without understanding what exactly the data represents. They often try to relate the size of the columns to real-world quantities—in this case, the quantity of junk food.

Amanda then correctly reasoned that the numbers across the bottom might be ages. I clarified this for those who did not understand. "The bar labelled 15 to 24 is about the group of people who are at least 15 years old but not more than 24 years old."

"It is how much junk food the people eat," Rod said confidently.

"Yes, the 15 to 24 group eat more junk food than kids," said Amanda, "and old people eat the smallest amount."

Although the students were comparing the relative heights of the columns, their language suggested they were thinking about the bars as simply representing amounts of junk food. I wanted to help them think more broadly about the kind of data the bars could represent.

DRAWING OUT THE MATHEMATICAL IDEA

I realized at this point that Ariel's idea that it was about how many people were in the survey was not surprising. For most of the surveys the students had undertaken, their bar graphs showed exactly that—they asked a question, categorized the answers, and graphed the number in each category. This was possibly the first time they had needed to think about the bars representing a proportion or part of a whole.

"But is the graph about how much junk food they eat, or is it about who eats junk food?" I asked.

"If it is about people, then the numbers could tell us how many people were in each of the groups," suggested Ariel. "Like there are 80-something kids, nearly 100 in the next one, 90 in the middle ones, and 70 old people—if you added it up you would get how many people were there altogether."

I realized that even if they understood the vertical axis was about people rather than food, a lack of understanding of percentages made it difficult to make sense of what the numbers on the vertical axis meant. However, I thought they might be able to understand that the height of the columns showed how many people out of every hundred in each age group ate junk food. "You can think of the 85% as out of every hundred children from 5 to 14, 85 of them eat junk food."

"So there are a hundred people in each group, and the columns show how many eat junk food. Does it mean the rest do not eat junk food?" asked Amanda."

"Yes, out of *every* hundred people in each group, that is how many eat junk food, so what is left from 100 is how many out of *every* hundred do not eat junk food."

Few saw the full implications of "every hundred" in my explanation but thinking of percentages as "out of a hundred" would be sufficient for them to compare the categories.

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REFLECTION

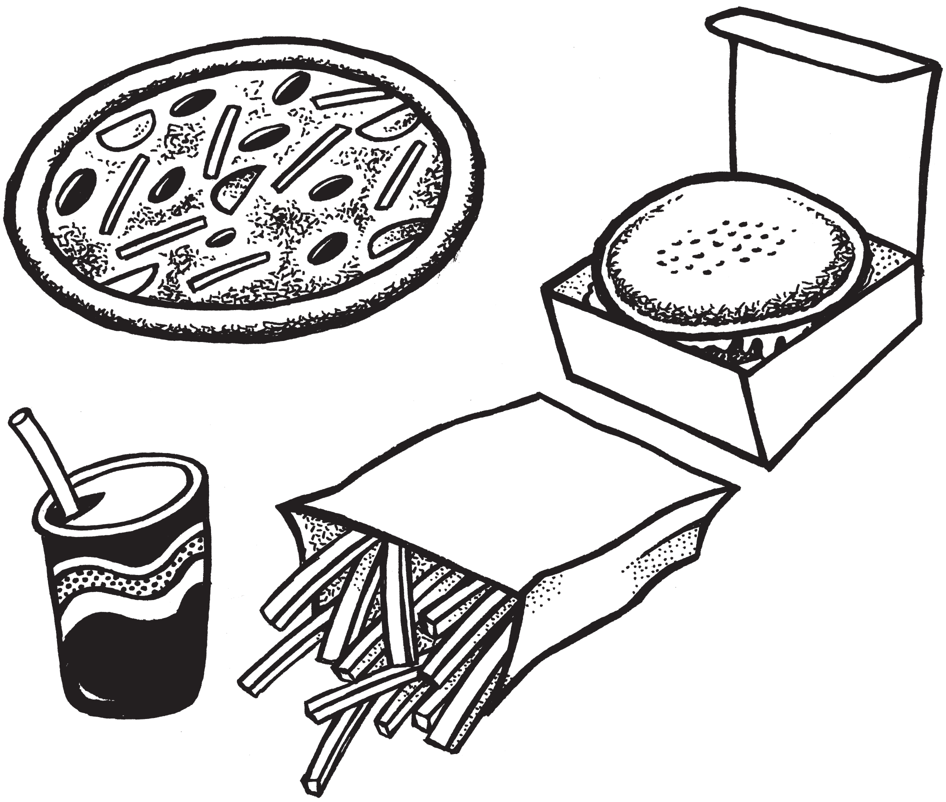
To focus their attention back on the possible conclusions they might draw from the data display, I asked: "Do all people eat junk food?"

"No," said Evan, "most people in all the age groups eat junk food, but some do not." Others agreed and I was satisfied most students had developed some awareness of the percentage scale.

However, I realized we had not talked about what aspect of the situation the data might refer to. "How do you think it was decided if a person ate junk food or not?" I asked.

This stimulated discussion about the information we could not find out from the graph alone (e.g., which foods were considered junk food, what sort of survey was carried out, which questions were asked). Amanda said there was a "story" with the graph, so I asked her to bring it in the next day to see if it would help us make better sense of the data.

Understanding percentages is not expected until the Operating/Relating phase, but I had noticed that many graphs and tables in the texts and reference books they were using did in fact involve percentages. Helping students to think of 85% as "85 out of every 100" (even if initially they ignore the "every") will enable them to get some meaning from these displays. This will not conflict with their later proportional understanding of 85% as 85/100.



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Key Understanding 2

When we analyze and interpret data, we are deciding what it says and what it means. There is a difference between the data itself and what we think it means.

Analyzing and interpreting data goes beyond direct reading and involves deciding what the data tells us and what we think it means. This includes interpreting the data produced and interpreting beyond the data produced.

This Key Understanding is involved when we interpret the data that is available. Students may ask their classmates which pets they like best and, on finding out that eight children say rabbits, four say dogs, and fewer say other pets, conclude that rabbits are the most popular pet. Students need to be involved in two processes of interpretation:

First, rather than simply reporting the information (eight children say rabbits, four say dogs,…), students need to compare the frequencies for each category in order to say which has more. In other circumstances, they may also need to combine and integrate information, perhaps calculating totals or differences or reorganizing data to enable a question to be answered.

Second, either implicitly or explicitly, students have to infer popularity from pet liked best. Had they used a different measure of popularity (e.g., asking which pet children would get if given a choice), they may have reached a different conclusion. Often we do not distinguish clearly between the data itself and our interpretation of it, and this can lead to miscommunication and inappropriate actions.

This Key Understanding is also involved when we draw inferences and make predictions that go beyond the data we have.

First, we generalize to a group bigger than the sample for which we have data. Students should begin to make inferences on the basis of samples—informally considering whether it is reasonable to generalize. To do this, they need to tap into their existing knowledge for information about whether this sample is likely to be a good predictor of the population. For example, they may decide to use the pet preferences of their own class and to generalize to local children of similar age, but not to much older or younger children or to children from different backgrounds.

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Second, we may generalize a sameness, difference, pattern, or trend in our data, drawing an inference about the nature of the relationship between the variables, and hence about what will happen in other cases or in the future. We might look at a scatter plot, for example, and search for trends—to see if there is a positive relationship between the two measures, a negative relationship, or no relationship at all.

Through the elementary years, students should begin to take chance variation into account in sensible rather than technical ways. They should learn not to assume that what happens in a sample will exactly predict what happens in the whole population, and to develop an everyday sense of what is normal variation and what is unusual. For example, they might find that 50% of boys and 47% of girls in their class prefer watching television over other pursuits. If the 50% represents just one more boy, it probably does not imply a meaningful difference between boys and girls. The uncertainty involved in sampling is what causes the uncertainty in drawing conclusions. This is the reason we should be more conservative in interpretations based on data collected on samples than if the whole population of interest has been surveyed or tested. This is what links probability and data.

Links to the Phases

Phase	Students who are through this phase in Data Management...
Emergent	■ each simple conclusions based on counting, e.g., a student may say, "There are more strawberries, so strawberries are the best-liked fruit"
Matching and Comparing	■ can describe what their own data collection shows and can comment on the reports of their peers
Quantifying	■ interpret tables, diagrams, bar graphs, and pictographs produced by themselves and others, including their peers, drawing sensible conclusions from them ■ comment on their predictions in light of their collected data
Measuring	■ interpret fractions and means, informally commenting on trends they notice in their own and others' data
Relating	■ describe the results of their data collection, talking informally about relationships they see in the data, e.g., a student may note that faster readers also seem to read the most, but state that they do not know which causes which

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Sample Learning Activities

K-Grade 3: ★ Introduction, Consolidation, or Extension

Fruit

After students have lined up their fruit to form a simple pictograph of fruit types, model questions to help them to interpret their display. Ask: Are there more bananas or more apples? Which fruit is there most of? Which fruit is there least of? Which two lines of fruit are the same length? What does that tell you? Tell your friend one other thing our display of fruit shows. How many more are there in this row of bananas? So how many more people brought bananas? Select a fruit not in the display (say, pears) and ask: How many people brought pears? How can you tell? Repeat regularly with other similar displays of collected objects.

Fruit Graphs

After students have produced pictographs that represent, for example, their fruit or other collected items, repeat the previous activities, modelling the processes of interpretation of simple graphs (as distinct from physical displays of objects). For example, you could say that there were more bananas than apples. Ask: What do you think this means? Does it mean bananas are more popular than apples? What else might it mean?

Interpreting Displays

After students have carried out activities to interpret their own visual displays, have them exchange and interpret each other's graphs.

Birthdays

After students have produced a class pictograph representing the month of their birthday (see *Birthdays*, page 229), model simple questions as for *Fruit* (above). Extend by helping them interpret ages from the graph. Ask: Does being in the same month mean you are the same age? Which students are likely to be the oldest? Which students have already turned 7? How many have not had their birthday yet?

Measurements

Extend *Measurements*, page 229, by asking: If you were really thirsty, which container would you drink from? If the containers were full of juice that you did not like, which container would you choose?

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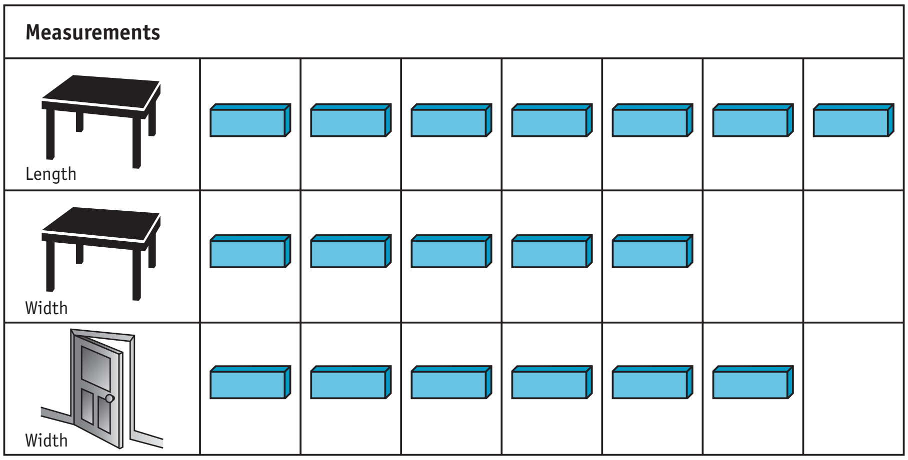
Does It Fit?

Have students record measurement data as graphs, then help them to draw simple conclusions. For example, direct them to measure a table and a doorway using blocks. Ask: How many blocks fit across the table? How many fit along the table? So is the table wider or longer? How many blocks fit across the doorway? Is the doorway wider or is the table wider? So would the table fit through the door?



My Family

Have students produce a pictograph showing the number of people in their family, then help them to interpret it. Ask: How many families have four children? Which are more common, families with two children or with four children? Which is the most common family size? Do any of the columns have the same number of pictures in them? What does that mean? How many families have no children in them? Does that mean there are no families without children? What do you mean by "family"?

Predictions

After students have read frequency information from one-way tables or simple bar graphs, ask them to make predictions. You might have students collect information about the number of each colour of jelly bean in a bag. Ask: Does this mean that all bags of jelly beans will have more red ones?

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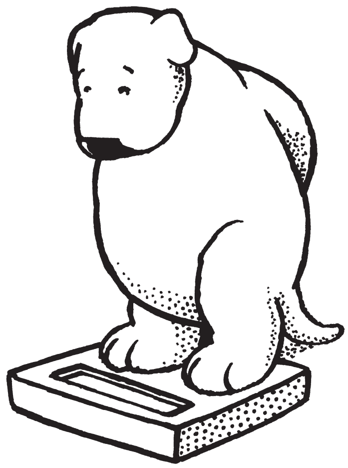
Sample Learning Activities

Grades 3–5: ★★★ Major Focus

Estimating
Have students interpret line plots produced in *Estimating*, page 230. Ask: What are the most common estimates? Does this graph give you any ideas about what the real length of the hall might be? What is the difference between the longest and the shortest estimates? Would you expect people to disagree by that much? What does that suggest to you? (*Perhaps people have different ideas of how long a metre is*.)

Dogs
Provide students with simple bar graphs and model the process of interpreting the data. For example, have students read the basic information from the graph of dogs' masses (see *Dogs*, page 230). Ask: Which two dogs are closest in size? Which dog is likely to eat the most? Why do you think that? Do you know it from the graph, or are you deciding this for yourself? Can you be sure?



Zero Frequencies
Provide students with bar graphs in which at least one category has a zero frequency. For example, in interpreting a graph of sport preferences, ask: Were any sports not preferred by anyone? How does the graph show that? Why did they not just leave that sport off the graph? Refer to a sport not on the graph and ask: How many people prefer that? Can you tell? If it is not on the graph, can you interpret that?

Venn Diagrams
Extend *Venn Diagrams (3)*, page 232, by asking students to compare the frequencies in the different categories. Ask: So, do more people play hockey or soccer? Do more people play one sport or do more play two sports? How many people play at least one of these two sports?

Getting to School
Extend *Getting to School,* page 231, by having students combine the information in the simple two-way tables to answer questions. Ask: Do more students walk to school, get driven to school, or ride to school? Are there more students in Grade 4 or Grade 7? How do most of these Grade 7 students get to school? What is the least common method of getting to school?

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Playground Equipment

Have students gather data by collecting or counting things. For example, ask students to count the number of people on different pieces of equipment at recess. Ask: Which equipment was used the most? Which equipment do people like the best? Draw out the difference between the two types of answers. One is factual information we can be certain about; the other is a conclusion we can say is probably true but we are not sure about. Ask: Are there other reasons people may have played on the equipment? Do you sometimes play on equipment that is not your favourite?

Growth of a Chick (1)

Invite students to compare measurements in simple line graphs that show how a quantity varies over time, for example, the growth of a chick at weekly intervals. Ask: Did the chick grow more between weeks 1 and 2 or between weeks 3 and 4? How much has the chick grown in total?

Growth of a Chick (2)

Extend the previous activity by comparing two graphs showing the growth of different chicks. Ask: Which chick grew the most? Which grew the fastest? What is the difference in the height of the two chicks?

Dog Food

Invite students to draw inferences from graphed data to answer a question (e.g., "Which food do dogs like best?") Ask them to look at their graphs showing which foods dogs mostly eat. Ask: Does this mean that most dogs prefer to eat chicken? What else might influence what a dog eats? Have students write to a dog food company, describing their results.

Cafeteria Orders

Have students make statements of likelihood based on data in a simple bar graph that shows more children order sandwiches than meat pies from the cafeteria. Ask: Is it likely that children prefer to eat sandwiches, or is there another reason why more children order them? What might the graph look like if the cafeteria's bread order did not arrive? What might the graph look like on a cold day?

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Sample Learning Activities

Grades 5–8: ★ ★ ★ Major Focus

Pictographs

Ask students to examine a pictograph, such as one that shows different kinds of vehicles passing the school; each picture should represent more than one unit. Ask questions to help students interpret the data: What type of vehicle passed the school most/least often? How many more trucks than cars passed the school? What did you have to do to work it out?

Ask questions that require students to think beyond the data: At what times would you expect more bikes to pass the school? Would you expect the same results every day of the week or every hour of the day? Draw out the difference between the data and the conclusions we draw from the data.

Lunch

Extend *Lunch*, page 236, to interpret the data. Ask: How many of each item of food needs to be ordered for the sports day? How many lunches need to go into each year's lunch basket?

Have students say which numbers they used and what they did to the numbers to work it out. Ask questions that require students to interpret beyond the data: Would you be able to use this data to say what to order for the summer swim meet? How would you change the menu for the next winter carnival?

Heights of Children

Have students interpret graphs where the data is grouped and where reading between the calibrations on the frequency axis is required (see *Heights of Children*, page 236). Ask: What height are most children in the school? How many are taller than 145 cm? How many are between 146 cm and 165 cm? How did you work it out? Can you see a trend in the height of the children in the school? Who might this information be useful for? (*People ordering chairs and desks*)

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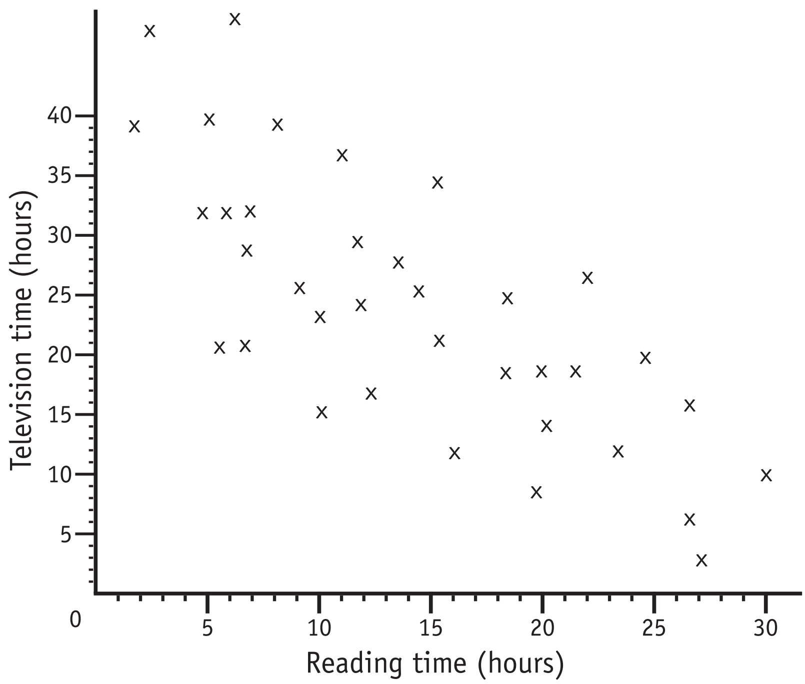
Ordered Pairs

Encourage students to look for trends and make predictions from data. You could ask them to make a graph, labelling one axis for the television time and the other axis for the reading time, and plot an ordered pair for each student in the class. Have students describe the relationships they can see. Ask: Given television time, can you exactly predict reading time? Why? Why not?

Relationship between time spent reading and time spent watching TV



Draw out that although there appears to be a slight negative relationship—as television hours go up, reading time tends to go down—you could not predict one exactly from the other because there is a lot of variation within this general trend. This individual variation introduces an element of uncertainty (or chance) into the process of prediction, even though there is an overall trend.

What Is Average?

When reading newspaper articles, books, or information on the Internet, ask students to consider what is meant by the word "average." For example: If they read that the average life expectancy for a lion is 20 years, prompt them to consider what this means. Does this mean that most lions will live for 20 years?

Skeletons

Have students examine the dot plot made in *Skeletons*, see page 186, and make simple descriptive statements about the relationship between height and length from ankle to knee. Ask: Are there patterns in the placement of the pins on the dot plot? How do you know? Do you think leg length is related to height? Do the tallest people always have the longest lower leg?

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Grades 5–8: ★ ★ ★ Major Focus

Fractions

Invite students to use fractions or percentages to compare data in categories. For example, consider the table below:

Girls' and boys' preferences for lemonade or orange juice		
	Boys	Girls
Lemonade	15	5
Orange juice	15	15

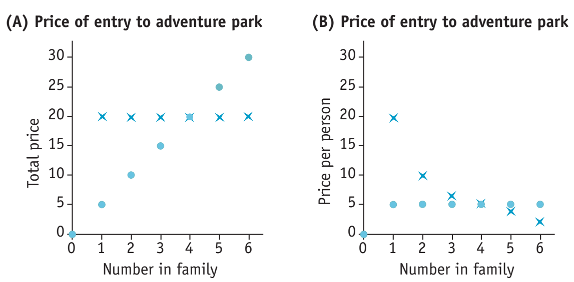
Ask: Could you use the numbers to say what drinks to order for the school camp? Could you use the numbers to say that the girls and boys liked orange juice equally as much? Do more girls out of the total number of girls like orange juice? Have students find the fraction of boys who like both drinks out of the total number of boys. Repeat this for the girls. Ask: How does changing the numbers into fractions change the results of comparing the two groups? Draw out that if there is the same number of girls as boys, it is possible to compare the numbers as is, but if not, we need to convert to a percentage or fraction to judge.

Comparing Variation

Have students collect data from their class science investigations. For example: Does fertilizer make wheat plants grow taller? Have students plant seeds and measure the heights of at least 30 or 40 wheat plants regularly over a number of weeks. Only half the plants are fertilized. Have students plot all of the growth patterns and determine whether fertilizer makes a difference. Ask: What if some of the unfertilized plants grow taller than some of the fertilized plants? Does this mean the fertilizer makes no difference, or is there another way to compare our data? (See Case Study 2, page 249.)

Holidays

Present the two graphs below and the scenario of families on a trip to an adventure park. You can buy a family entry ticket for $20, or you can buy individual tickets for $5 each. Ask: Which graph shows which of the following? How can you tell?



- Cost per person if family ticket is bought, for different-sized families (*B*)
- Cost per person if individual tickets are bought, for different-sized families (*B*)
- Cost per family if family ticket is bought, for different-sized families (*A*)
- Cost per family if individual tickets are bought, for different-sized families (*A*)

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CASE STUDY 2

Sample Learning Activity—Grades 5–8, Comparing Variation, page 248

Key Understanding: When we analyze and interpret data, we are deciding what it says and what it means. There is a difference between the data itself and what we think it means.

Working Towards: Measuring phase

TEACHER'S PURPOSE

When my class of 11 and 12 year-olds interpreted some data that they had produced to investigate their television show preferences, I noticed that they considered small differences between the preferences of boys and girls as significant and that they were happy to base conclusions on these small differences. I wanted them to think more carefully about differences that we might normally expect to find between groups, and differences that are unusual and therefore might really be significant.

ACTION

The class chose to investigate this question: *Does fertilizer make wheat plants grow taller?* Showing an awareness of the need for fair testing, the students worked in pairs to grow two plants, one with and one without fertilizer. They planned to keep all other variables identical for both plants: light, depth of seed in soil, amount of water, soil type, and temperature. The plants were watered and their heights measured and recorded twice a week for five weeks. I asked the students to graph the results of height against time for each plant on a separate set of axes. To facilitate comparisons between plants, I suggested that all students use the same scales on the axes of their graphs.

As I wanted to focus my students' attention on the interpretation of the data rather than the organization or summarization of the data, I needed to make these decisions for the students.

DRAWING OUT THE MATHEMATICS

I began by asking the students to compare the graphs of the growth of their two plants.

"The one with fertilizer ended up much taller," said Akiko.

"Not for us," countered Paul. "There is not much difference at all."

"The fertilizer made ours grow worse. It's not as tall as the one without fertilizer," observed Masao.

"Would you use the fertilizer if you were a farmer?" I asked.

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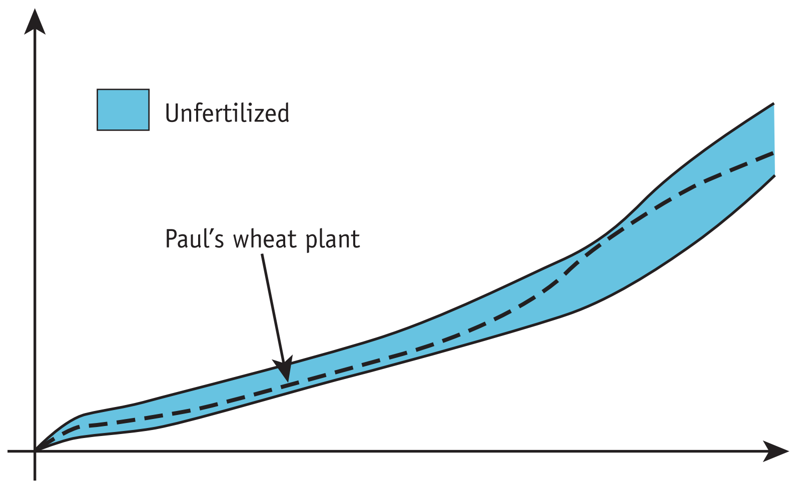
"No," Santokh answered. "If I was a farmer who had to pay for the fertilizer, I would want to be sure it was going to make a real difference."

"But I think most of us found it made our plants grow more," offered Petra.

"So how can we find out if the fertilizer does make a difference overall?" I asked.

I used the same scales as the students on the axis of the transparency, so we could place it over the students' graphs and make comparisons.

The students saw the need to somehow combine the data of their individual plants before we could compare the fertilized and unfertilized plants. I suggested that, beginning with the unfertilized plants, we could plot the smallest and largest individual height measurement at each time point and so show the total variation in growth. By superimposing this transparency over their own graphs the students confirmed that, although the growth patterns and final heights of their individual plants varied greatly, the growth of all their unfertilized plants came within the range shown on the graph on the transparency.



The students then superimposed the transparency over their individual graphs of the fertilized plants. They were surprised when many of these also fitted within the range of the unfertilized plants.

"See, our fertilized plant fits in that graph of all the unfertilized plants. I told you the fertilizer was no good," remarked Paul.

"Our fertilized plant started off in the graph of all the unfertilized plants, but then it gets taller near the end," commented Masao.

We then made a second overhead transparency showing the range of all the fertilized plants. We put one transparency on top of the other.

"They all grew about the same to start with," noted Akiko.

"But they were still different," said Maryam.

"But it is sort of the 'same' different," said Yvonne.

"After that, the fertilized plants were more different—the plants got taller, but some were still the same," said Joss.

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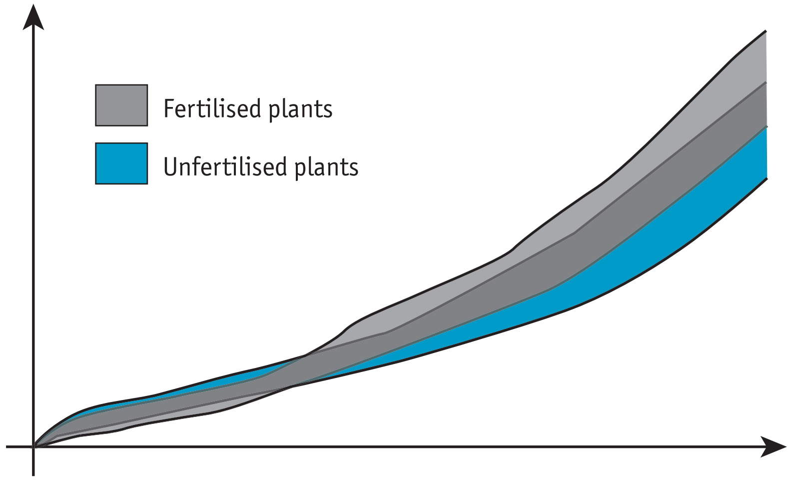

Students struggled with the idea that although the variation *within* each group meant there was overlap (and you often could not tell just from one plant's growth whether or not it was fertilized), we could still say there was an overall difference *between* the two groups.

"So, even though my unfertilized plant grew taller than my fertilized plant, when you put them all together, it does not show like that," said Masao.

REFLECTION

"So, do you think the fertilizer made a real difference, or do you think those plants might have grown like that anyway, without any fertilizer?" I asked the students.

"It might have," said Joss. "Some could have just grown taller."

"But they started the same and then only the fertilized ones were the highest—they were a lot more different at the end," said Akiko.

"We might have just had unusual plants—they might have grown tall anyhow," commented Paul.

"Why don't we do it again with more plants," said Santokh, "so we can really tell?"

I was satisfied that, when interpreting their data, they were beginning to think about what differences are due to normal variation and what differences are greater than normal.

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Key Understanding 3

We need to evaluate the data we are using in order to be confident about the conclusions we have drawn.

An important aspect of numeracy for everyday life is being able to judge the quality of the data and the inferential processes upon which we and others have reached conclusions. Students should begin to evaluate the quality and suitability of data (collection, organization, and display) for answering particular questions. During Kindergarten to Grade 6, this will mostly consist of students reporting on their own data collection and handling, and reading and listening to the reports of their peers. For example, looking back over their own processes (or those of their peers), they should ask questions such as these:

- When we refined our question, did we still find out what we originally wanted to know? Is there anything we left out?
- How helpful were our measurements? Were they accurate and reliable? What of our survey questions? Were they ambiguous or confusing, biased or leading? What of our collection methods? Were we careful or a bit sloppy? What effect might it have had on our data?
- Was the sample appropriate? Did we ask the right people? enough people? Did we collect enough cases? the helpful cases?
- Did we record all the information we needed to be able to answer our original question? What else did we need? Were the categories that we decided to use the right ones?
- Was our graph or table helpful? Did we need to redo it? What did we learn?
- Have we under- or over-interpreted our data? For example, did we assume that two things being related meant that one caused the other?
- Can we answer our original question? If not, why not? What would we do differently next time?

By noting the strengths and weaknesses in their own and their peers' work, students should improve their data collection and handling over time. However, this does not mean that they should expect to be able to design an error-less data collection process and then simply implement it. The process of formulating and refining the question, producing data, summarizing and representing it, and then interpreting it is an iterative one, with the

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relationship between the data and the original question the focus of what is happening along the way. Each step may need to be reviewed and redone; indeed many experiments involve a trial (or pilot study) to see whether the processes set up will do the job.

Older students should begin to consider the credibility of data sources and to question the meaning and legitimacy of conclusions drawn from data that is not their own. Some of this data may be drawn from everyday contexts (such as advertisements on television or in children's magazines); other data may relate to topics in various parts of the school curriculum (a report on bicycle accidents from the health unit or a history database). Students should also be helped to reflect on the importance and use of their knowledge of probability and data in helping them form opinions and make decisions. Some of these opinions and decisions may be quite informal (forming views about famous people), and others more formal (undertaking research to decide what time of day school should begin).

Links to the Phases

Phase	Students who are through this phase in Data Management...
Measuring	■ make sensible comments about how well their collected data answers their questions. *For example:* After surveying fellow students about their food preferences, a student may say, "We thought asking people what food they liked would help plan the camp, but we did not ask it very well and so we could not classify it. Next time we would...."
Relating	■ will comment on the quality of their data collection and handling processes, and suggest how they might improve them

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Sample Learning Activities

K-Grade 3: ★ Introduction, Consolidation, or Extension

Asking Questions

After students have collected information by asking for it, have them check whether they have included everyone they should. Ask: Have you asked everyone? Should you ask the adults in the class? Are they included in this group?

Yes/No

Hang cards with "yes" on one side and "no" on the other next to a list of students' names. Have students turn their card to the appropriate side to show whether they have completed work or need to do a particular thing. Display the question above the names, for example, "Who needs to change their books?" Ask: Does the table help us to see who needs to change their books?

Who needs to change their books?

Names	Yes/No
Sherry	Yes
Tom	No
Ali	Yes

Long Jump

When students draw conclusions from their own data, ask them to say what steps they took to make sure their data was accurate. For example, have students use paper tapes to measure their long jumps, then compare tapes to see who jumped the farthest. Ask: How do we know our paper tapes really show us how far we jumped? Who can remember what we did to make sure we measured all of a jump, and no more?

Popular Foods

Have students refer back to their initial question to explain what their displays of data show. For example, take this question: What is the most popular lunch food in our classroom? Ask students to watch what everyone eats for lunch and display the information. Ask: What does our graph tell us about the most popular foods? Invite students to consider whether the data they have collected could be generalized across different groups. For example, ask them to collect data and create a table to show what food students in their class prefer. Ask: Would other children like the same foods? If we asked adults, would we get the same results? What if we asked teenagers?

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Pet Food

Encourage students to think about the source of data. For example, have them look at a graph that shows the number of people who buy a certain pet food. Ask: Who could have drawn this graph? Where would their information come from? What are they trying to tell us?

Scary Things

Have students make statements that link their data to their methods of collecting and sorting information. After reading a story such as *Scaredy Squirrel* by Melanie Watt, prompt students to decide whether to write or draw what scares them the most. Ask: Would it make a difference if we had sorted our pictures using different categories? If we collected and sorted what people wrote instead of these pictures, would we have found out the same thing?

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Sample Learning Activities

Grades 3–5: ★ ★ Important Focus

Comparing Data

When comparing collected measurement data, have students decide whether the results are accurate. Ask: Did we all measure in the same way? Do you think it matters? Why? Draw out that to be able to compare their data, they need to be accurate, and that to get an accurate measure, they need to start and finish all their measurements at the same spot.

Favourite Foods

While students are reporting on and discussing their survey data, draw attention to difficulties they have in reaching conclusions. For example, students might ask about favourite fast food and have some people name a type of food (pizza, Chinese) and others name a company. Ask: You are saying that people answered the question the wrong way—how could we avoid that if we did it again? Could it be that we asked the wrong questions? Perhaps our questions were not clear?

Computer Games

Invite students to review data they have collected and say whether it answers their original question. For example, have students collect data to show which computer games children play. Ask: Does this data show us which game is the most popular? If not, why not? Was it the way that we collected the information, or the way that we organized the information? Did we collect the wrong information to answer our question?

Recycling

Have students make predictions based on data and then say how sure they are of their conclusion. You could prompt them to review a graph of the recycling habits of people in their community and say whether more or fewer items will be recycled in the future. Ask: Does the graph show a trend over time? Can we use this trend to make a prediction about the future recycling habits of the community?

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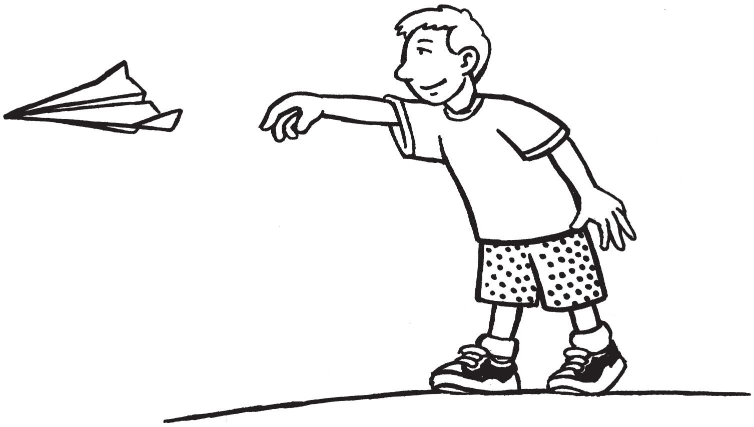
Paper Planes

After students have collected data to answer a question of interest, have them reflect on their data representation method. For example, have them collect, then represent data on the best design for a paper airplane. Ask: Was this the best way to represent the data? Would it help to draw a table/graph instead? Could we redraw/redo it to better show a relationship in the data?



Eye Colour

Invite students to consider two different representations of the same data and say how they are the same and different. For example, compare the tables below:

Name	Eye colour
Mai	blue
Freya	brown
Peter	blue
Kim	brown

Name	Eye colour
Brown	///
Green	//
Blue	/

Ask: Are there more people with brown eyes or green eyes? Which table helps you to see this more easily? What colour eyes does Freya have? Which table tells you this?

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Sample Learning Activities

Grades 5–8: ★ ★ ★ Major Focus

Examining Data

When examining data obtained by their own survey, encourage students to think about the source of the information. Ask: Did we ask the right question? Should we have used fixed-choice instead of open-answer questions? Did we ask the right people? Did we ask enough people? Draw out that the way the survey was conducted may affect the results.

Who Were Asked?

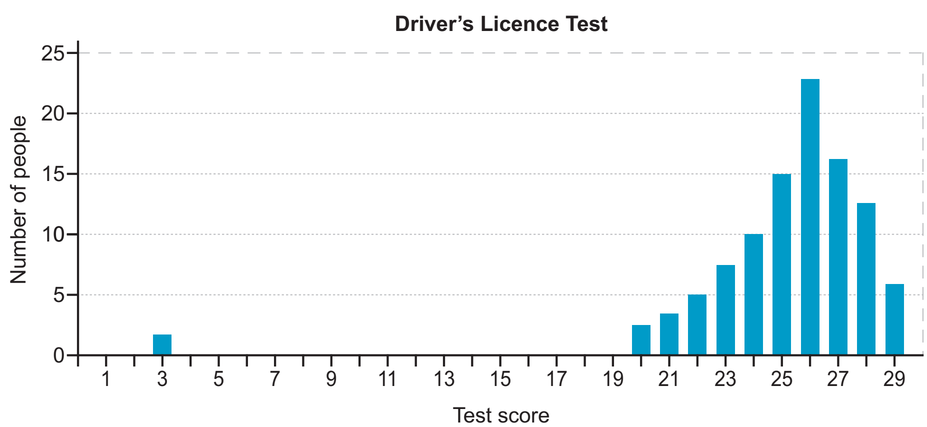
When examining data presented in newspapers, magazines, books, or on the Internet, encourage students to think about the people who were asked in order to produce the information. Ask: Were these sensible people to ask? Is it reasonable to generalize from these people, or is the source stretching the information?

What Is Missing?

When students are reading newspaper articles, books, or information on the Internet that use data to illustrate a point, ask them to consider whether any information that may affect the data is missing. Have students brainstorm questions they would like to ask the people who produced the data. For example, with data showing the average age of lions as 20 years, they might want to know whether the information was collected on zoo animals or wild animals, and whether zoo animals live longer than animals in the wild.

Outliers

Present students with data that contains obvious outliers, or results differing greatly from others in the same sample (see diagram below), and ask them to explain how this might have occurred. Focus on the fact that sometimes we make mistakes when collecting data, although sometimes these outliers are part of our data.



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Balloon Power (1)

Have students say whether the data they collected enabled them to answer their question(s). For example, prompt them to consider their data about the balloon-powered cars. Ask: Did the data help you answer the reworked questions you asked? Did the data help you answer your original question? Would collecting different data help you to better answer your original question? Can you now build a car that travels farther? What would you do differently next time? Why?

Balloon Power (2)

Extend the previous activity and discuss which type of data display helped them to see if there was a relationship between two measures. Students could compare the ways they displayed the weight of each car with the distance travelled. Ask: How did you use the display to work out that there was or was not a relationship between the measures?

Favourite Drinks (1)

Encourage students to see how asking a question in a different way can give different results. Ask: Which do you prefer—water or apple juice? Have them record the result. Now prompt them to write down the name of their favourite drink. Ask: How do the results compare? Draw out that results obtained from a multiple-choice and an open-ended question will produce different results.

Favourite Drinks (2)

Extend the previous idea. When they are interpreting data, ask students to consider how a question was asked, by whom, and in whose interests the survey was collected. Ask them to think about the following different conclusions:

- Eight out of 10 children prefer water to apple juice.
- For those who expressed a preference, 8 out of 10 children preferred water to apple juice.
- Eight out of 10 parents said that their children prefer water to apple juice.

Ask: What is different about these statements? What questions might have been asked in each case? What effect would the word “apple” have in the question, rather than just “‘juice”? Why would it be in someone’s interest to ask the question that way?

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Grades 5–8: ★ ★ ★ Major Focus

Relating Data

Have students review data that is being used to illustrate a particular point, such as a newspaper article on the effect of greenhouse gases on the atmosphere, or the rate of logging in the Amazon rainforest. Invite them to say how the data is related to the issue. Ask: Can the conclusion drawn in the article be made on the basis of the data presented? Can we be sure that one event actually causes the other?

Tables and Graphs

When students have individually produced various tables and graphs to reinforce science reports, have them exchange tables and graphs (without the reports) with other students. They study the tables and graphs, and report what they think they can conclude from the information. Ask: How do these conclusions compare to those in the original reports?

Misleading Representations

Invite students to discuss the effect of misleading representations of data on the way information is interpreted. Provide examples from newspapers, magazines, and the Internet of bar graphs or histograms that can be misleading because of one or more of the following:

- The vertical axis does not start at zero.
- Bar lengths or widths are not proportional to each other.
- The horizontal axis is made much narrower.
- The values of intervals are not equal.

Ask: What effect might each have on the way you interpret the information?

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Appendix

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Planning Master

Classroom Plan for Week ________, Term ________ **Grade Level: ____________**

Curricular Goal/Key Understanding	Mathematical Focus	Activities	Focus Questions	Observations/ Anecdotes

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Tracking Master **Ongoing Progress through the Data Management and Probability Diagnostic Maps**

Record the date that students move into each developmental phase of the Diagnostic Maps. Copies of this sheet (one for Probability and one for Data Management) can be placed in each student's math portfolio to chart individual growth over time.

Student Name	Emergent Phase	Matching and Comparing Phase	Quantifying Phase	Measuring Phase	Relating Phase

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Diagnostic Map: Probability

In Probability, students extend what they know about measuring *perceptual* attributes to measuring the *chance* something is likely to happen.

In this way, it follows the same development as Measurement: students become aware of the attribute (Could it happen? Won't it happen? Is it likely?); they compare and order from more to less of that attribute (Which is more likely? Equally likely?); they measure the attribute by comparison with a unit (being able to say how likely), and then to the standard unit (placing events on a scale from 0 to 1); they understand the relationships that enable us to indirectly work out the likelihood.

However, in another way, Probability does not follow the same development as Measurement. The fact that measuring chance is not a perceptual attribute makes it more abstract. As a result, when it comes to this new attribute, there is a lag when students move through the phases of development for Measurement.

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Emergent Phase

Number Sense Phase: *Matching*

Enter: 3–5 years **Exit: 5–6 years**

Students use one-to one matching to judge quantity.

Measurement Phase: *Emergent*

Enter: 2–3 years **Exit: 5–7 years**

Students initially attend to the overall appearance of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

Number Sense Phase: *Quantifying*

Enter: 5–6 years **Exit: 6–9 years**

Students trust the count to describe quantity without variance.

Measurement Phase: *Matching and Comparing*

Enter: 5–7 years **Exit: 7–9 years**

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to *use counting to directly compare* things so as to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Probability: Measuring Chance, *Emergent*

Enter: 2–3 years **Exit: 7–9 years**

Students develop awareness that some things are more and less likely to happen and begin to use some of the comparative language of their communities to describe likelihood.

As a result, they use this type of language themselves and describe familiar, easily understood events as being more or less likely, e.g., Mom said we *might* go to grandma's after school; we are *more likely* to go home than to grandma's; we *usually* go home after school.

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Emergent Phase cont.

By the end of the Emergent phase in Probability, students typically

- are beginning to show that they recognize an element of chance in many things that are a part of their lives
- understand expressions such as "will happen," "won't happen," and "might happen"
- are able to distinguish impossible events from events that are possible but unlikely

But, as they enter the Matching and Comparing phase in Probability, they

- distinguish between certain and uncertain events, but may not realize that certainty must also include events that are certain not to occur
- may be unable to distinguish equally likely events, e.g., may assume all colours are equally likely to appear when given a four-colour spinner with unequal sectors
- may understand that some things are more likely than others, but not be able to provide relevant reasons why events might be more or less likely to occur (e.g., believe they will spin a 6 because 6 is their favourite number)

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Matching and Comparing Phase

Number Sense Phase: *Partitioning*

Enter: 6–9 years **Exit: 9–11 years**

Students use additive thinking to deal with many-to-one relations.

Measurement Phase: *Quantifying*

Enter: 7–9 years **Exit: 9–11 years**

Students connect the two ideas of directly comparing the size of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Probability: Measuring Chance, *Matching and Comparing*

Enter: 7–9 years **Exit: 9–11 years**

Students draw on their experience to describe familiar things as more or less likely. They use expressions such as "very likely," "less likely," "equally likely," and "quite unlikely."

As a result, they are able to directly compare and order events from more to less likely and are able to justify their decision with relevant reasons.

By the end of the Matching and Comparing phase in Probability, students typically

- construct simple experiments and use counting to determine which event is more likely
- understand that that certainty includes those events that must happen and those that cannot happen
- understand what it means for simple events to be equally likely, e.g., can see why a spinner with four equal sectors is equally likely to stop on any colour, and that one divided into unequal sectors will not
- can list all possibilities for straightforward situations when prompted

But, as they enter the Quantifying phase in Probability, they

- may be uncritically influenced by other dominant features when ordering objects by likelihood, e.g., may be influenced by personal preference or personal experience and so say, "It is less likely to rain tomorrow because it never rains on my birthday" or "I'm more likely to roll a 6 because I always roll a 6"
- may construct an experiment to determine likelihood, but be casual about ensuring fairness, even to the point of altering or fixing outcomes to produce the predicted results

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Quantifying Phase

Number Sense Phase: *Factoring*

Enter: 9–11 years **Exit: 11–13 years**

Students think both additively and multiplicatively about numerical quantities.

Measurement Phase: *Measuring*

Enter: 9–11 years **Exit: 11–13 years**

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Probability: Measuring Chance, *Quantifying*

Enter: 9–11 years **Exit: 11–13 years**

Students connect the idea of likeliness to the frequency of an event. They come to understand that repeated trials provide a count that can predict future likeliness. This count enables two things to be compared without directly comparing them.

As a result, they trust information gained from repeated trials as an indicator of probability and are prepared to use this to order events and determine how likely they are.

By the end of the Quantifying phase in Probability, students typically

- draw on personal experience to compare and order a variety of chance-related events and order them along a continuum that acts as an informal scale
- draw on numerical information alone to decide whether two simple events are or are not equally likely to occur
- are careful to ensure that an experiment is fair, e.g., that the sections of a spinner are equal or that the coin toss is consistent
- systematically list all possibilities, unprompted, to work out numerical probabilities for one-stage actions
- use experimental results to determine a range of possible outcomes and informally use relative frequency to estimate probabilities

But, as they enter the Measuring phase in Probability, they

- may be unable to create devices such as spinners or bags of coloured balls to produce specified orders of probability, e.g., make a spinner which is most likely to come up red and equally likely to come up blue or green
- may trust numerical information but may not be able to accurately order events where the total number of trials is not the same, e.g., might say that an event that occurs 8 out of 12 times is more likely to occur than an event that occurs 5 out of 7 because 8 is greater than 5

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Measuring Phase

Number Sense Phase: *Operating*

Enter: 11–13 years **Exit: — years**

Students can think of multiplication and division in terms of operators, and reason proportionately.

Measurement Phase: *Relating*

Enter: 11–13 years **Exit: — years**

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students work with measurement ***information*** and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

Probability: Measuring Chance, *Measuring*

Enter: 11–13 years **Exit: 15+ years**

Students begin to quantify the chance of events occurring using probability as a measure for "how likely" or "how much more likely." They realize they can produce ratios by comparing the total number of occurrences to the total number of trials (experimental probability) or by comparing the number of desired outcomes to the number of possible of outcomes (theoretical possibility).

As a result, students understand that probability is the way we measure chance and that probability statements give a measure of how likely something is to happen.

By the end of the Measuring phase in Probability, students typically

- use a range of information sources to put things in order from least likely to most likely, e.g., use research data or experimental data to form conclusions
- understand that the greater the number of trials, the greater its reliability as an indicator of likelihood
- use their understanding of equivalent fractions to judge equally likely events
- interpret the 0 to 1 scale in general usage and understand why the probability that a toss of a fair die will produce 5 is one-sixth
- identify all the outcomes for two- or three-stage situations, e.g., rolling a die and tossing a coin

But, as they enter the Relating phase in Probability, they

- may not be able to overcome deep personal instincts about the likeliness an event should occur in spite of experimental or theoretical data
- may not recognize or trust calculations that would determine all possible outcomes for multiple-stage situations
- are able to simulate a situation where it would be difficult, costly, or inappropriate to generate real data, by designing simple experiments that replicate a significant aspect of the situation; they use their understanding of ratios and numerical probabilities

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Diagnostic Map: Data Management

In Data Management, students use their understanding of Measurement to gather, compare, represent, and interpret data. In this way, students' understanding of Data Management is dependent upon their understanding of Measurement, which is dependent upon their Number sense.

As a result, the phases outlined in the Measurement Diagnostic Map should be considered when interpreting students' responses to Data Management activities. Doing so will help in understanding why some students may struggle to achieve certain outcomes while others do not.

The accompanying chart shows what students will be in a position to do and understand as they move through an appropriate program in Data Management, phase by phase.

Ultimately, Data Management becomes a quest to determine the likelihood or chance of something happening; it is intricately connected to Probability.

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Emergent Phase

Number Sense Phase: *Matching*

Enter: 3–5 years **Exit: 5–6 years**

Students use one-to one matching to judge quantity.

Measurement Phase: *Emergent*

Enter: 2–3 years **Exit: 5–7 years**

Students initially attend to the *overall appearance* of size, recognizing one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated and absolute way (big/small), rather than as describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison, differentiating between attributes that are obviously perceptually different.

Implications for Data Management

By the end of this phase, students are in a position to

- look at physical displays of familiar data and say which is most or has more
- sort and arrange data they have collected into familiar groupings
- count when asked to say how many in each group in a data display

But, students

- may count when asked to say how many in pre-arranged data, but focus on overall physical size rather than the numerical size of the group e.g., count 3 bananas and 5 strawberries, then say there are more bananas
- may focus on overall physical size rather than the numerical size of the group
- may say one group has more than another, but cannot say how much more
- may lay out objects, but lack the intention to compare

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Matching and Comparing Phase

Number Sense Phase: *Quantifying*

Enter: 5–6 years **Exit: 6–9 years**

Students trust the count to describe quantity without variance.

Measurement Phase: *Matching and Comparing*

Enter: 5–7 years **Exit: 7–9 years**

Students use one-to-one matching to directly compare things. They match in a conscious way to decide which is bigger by using familiar, readily perceived and distinguished attributes, such as length, mass, capacity, and time. They also repeat objects, amounts, and actions to decide how many fit (balance or match) a provided object or event. Until students understand the significance and invariance of the count, they cannot really understand the use of counting to measure size.

As a result, students learn to use *counting to directly compare* things: to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions such as these: How long? How tall? How wide? How heavy? How much time? How much does it hold? And they know what to do when explicitly asked to measure something.

Implications for Data Management

By the end of this phase, students are in a position to

- suggest counting as a way of answering data questions that focus on comparing collections, e.g., will suggest counting our pets to answer the question "Which pets are more popular?"
- use skip counting to say how many in a tally
- suggest direct comparison when prompted to record growth data, e.g., we can cut a streamer to match the sunflower plant each week to see how much it grows
- use counting to help construct their data display, e.g., construct a block graph by counting how many in each group, then counting how many squares to colour in
- understand the need for a baseline and space blocks regularly to allow comparisons to be made
- place direct measurement data in sensible sequences using a baseline, e.g., cut paper strips to fit around their heads and make a bar (column) graph by lining up the bottom of strips
- choose to count to compare the sizes of groups, without prompting
- look at a bar graph and say which bar has more based on its length

But, students

- may not attend to equal units when grid lines are not provided, e.g., they may create the correct number of pictures for each group, but not use the same size for each picture
- cannot construct a scale on the vertical axis to represent frequencies or measurements (although they can use a common baseline and label the horizontal axis with the groups)
- may not realize that the relative lengths of the bars relate to quantities in the collected data
- may not use a scale on the axis to tell how many, instead preferring to count

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Quantifying Phase

Number Sense Phase: *Partitioning*

Enter: 6–9 years **Exit: 9–11 years**

Students use additive thinking to deal with many-to-one relations.

Measurement Phase: *Quantifying*

Enter: 7–9 years **Exit: 9–11 years**

Students connect the two ideas of directly comparing the sizes of things and of deciding "how many fit," and so come to understand that the count of actual or imagined repetitions of units gives an indication of size and enables two things to be compared *without directly matching them*.

As a result, students trust information about repetitions of units as an indicator of size and are prepared to use this in making comparisons of objects.

Implications for Data Management

By the end of this phase, students are in a position to

- see that, when organizing data, the categories can be reorganized without changing the overall total, e.g., to combine two brands of hamburgers, they need to add the two brands, or if they separate boys' and girls' responses for favourite take-out foods, the total must match the combined result
- see that they need to ensure that the data they collect is consistent
- produce and read pictographs or block graphs where each unit represents more than one piece of data
- produce simple two-way tables and Venn diagrams, partitioning totals between the cells or sections for straightforward data
- recognize that the length of bars in a bar graph can represent any numbers or measurements of data
- represent whole-number data in different ways, e.g., after measuring everyone's height, can produce a measurement graph that shows each child's height, or a frequency graph that shows the number of people of each height
- recognize that the length of bars and simple whole-number scales on the axis refer to quantities in the data collected, e.g., given a frequency graph about people's favourite take-out food, they know that the lengths of the bars represent the number of people naming each food in the survey
- use column and row headings to interpret what the numbers in simple two-way tables represent

But, students

- may be able to represent only whole numbers of units
- may not be able to work out how to represent the data when it is not a multiple of the unit
- may not realize that measurement data can be grouped
- may be unable to represent or read data using a continuous scale

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Quantifying Phase cont.

- may not be able to convert to proportional measures to make comparisons
- may be unable to interpret the meaning between marked intervals on scales of frequencies or measures
- reading frequency data in two-way tables may not realize that to make sensible comparisons, the total frequencies need to be taken into account, e.g., students may say that more girls than boys like orange juice because 15 girls and 12 boys say they like it, but not realize that this information is misleading if there are 30 girls and 16 boys in the sample

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Measuring Phase

Number Sense Phase: *Factoring*

Enter: 9–11 years **Exit: 11–13 years**

Students think both additively and multiplicatively about numerical quantities.

Measurement Phase: *Measuring*

Enter: 9–11 years **Exit: 11–13 years**

Students come to understand *the unit as an amount* (rather than as an object or as a mark on a scale) and to see the process of matching a unit with an object as equivalent to subdividing the object into bits of the same size as the unit and counting the bits.

As a result, they see that part-units can be combined to form whole units and they understand and *trust the measurement as a property or description* of the object being measured, something that does not change as a result of the choice or placement of units.

Implications for Data Management

By the end of this phase, students are in a position to

- think carefully about the accuracy of their data and recognize that data collection is about measuring different aspects of a situation
- understand that they can group measurement data in their display
- create axes that show discrete or continuous quantities, including time scales
- use simple proportional comparisons when interpreting data in tables and graphs, e.g., half as many people prefer pizzas to hamburgers; the height of the wheat is three times higher than it was at the end of week 2

But, students

- may not recognize when they need to convert their data to fractions or percentages to make sensible comparisons

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Relating Phase

Number Sense Phase: *Operating*

Enter: 11–13 years **Exit: — years**

Students can think of multiplication and division in terms of operators, and reason proportionately.

Measurement Phase: *Relating*

Enter: 11–13 years

Students come to trust measurement information, even when it is about things they cannot see or handle, and to understand measurement relationships, both those between attributes and those between units.

As a result, students *work with measurement information* and can use measurements to compare things, including those they have not directly experienced, and to indirectly measure things.

Implications for Data Management

By the end of this phase, students are in a position to

- choose from a wide range of measurement options when planning data investigations, using indirect measurements and creating measurement scales for non-standard attributes, e.g., they may create a scale from 1 to 5 to measure people's concern for environmental issues
- plan complex scales on axes to produce a wide range of graphs, including using class intervals, fractions, and percentages
- represent growth or change data over time by using a time scale, and approximate value within the intervals by joining the points when appropriate
- see that both axes can be made into number or measurement scales and used to show relationships between data, e.g., they could sketch a line graph to show how the amount of food on a plate varies over time at a buffet lunch as the plate is repeatedly filled with food, which is then eaten
- interpret displays showing relational information between measurements or frequencies
- interpret complex scales on graphs where not all scale markings are labelled

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