

PEARSON

# Foundations and Pre-calculus Mathematics 10

## TEACHER RESOURCE

---

### Program Overview

Teacher Guide Modules

ISBN-13: 978-0-321-62686-8  
ISBN-10: 0-321-62686-9



Complete Teacher Guide Package

ISBN-13: 978-0-321-62685-1  
ISBN-10: 0-321-62685-0



PEARSON

**Publisher**  
Mike Czukar

**Research and Communications Manager**  
Barb Vogt

**Publishing Team**  
Claire Burnett  
Alison Rieger  
Lesley Haynes  
Enid Haley  
Ioana Gagea  
Stephanie Boragina  
Lynne Gulliver  
Cheri Westra  
Alison Dale  
Carolyn Sebestyen  
Jane Schell  
Karen Alley  
Judy Wilson

Copyright © 2010 Pearson Canada Inc.  
All rights reserved.

This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission, write to the Permissions Department.

Pages identified as line masters may be copied by the purchasing school for classroom use.

Printed and bound in Canada

3 4 5 – HI – 14 13 12 11 10



# Contents

|  |           |
|--|-----------|
| <b>Program Philosophy and Structure</b>                    | <b>4</b>  |
| Big Ideas  | 4         |
| Promoting Student Independence                             | 6         |
| <b>Assessment Background</b>                               | <b>8</b>  |
| Purposeful Assessment                                      | 8         |
| Effective Assessment Practices                             | 8         |
| Assessment for, as, and of Learning                        | 10        |
| Summarizing and Reporting Progress: Assessment of Learning | 14        |
| District and Provincial Assessment Programs                | 15        |
| Overview of Planning Assessment                            | 16        |
| References   | 17        |
| <b>Meeting the Needs of Diverse Learners</b>               | <b>18</b> |
| Success for All  | 18        |
| Diversity in the Classroom                                 | 21        |
| Strategies for Success                                     | 24        |
| Assessment and Differentiated Instruction                  | 28        |
| Differentiating Instruction                                | 29        |
| Other Examples of Diversity                                | 33        |
| Selected References  | 36        |
| Materials List   | 37        |
| <b>Program Masters</b>                                     | <b>38</b> |

# Program Philosophy and Structure

*Pearson Foundations and Pre-calculus Mathematics 10* is based on a few essential beliefs about how secondary school students learn mathematics most effectively. These beliefs influence the design, and shape the organization, of the Student Book and the supporting Teacher Resource.

## Big Ideas

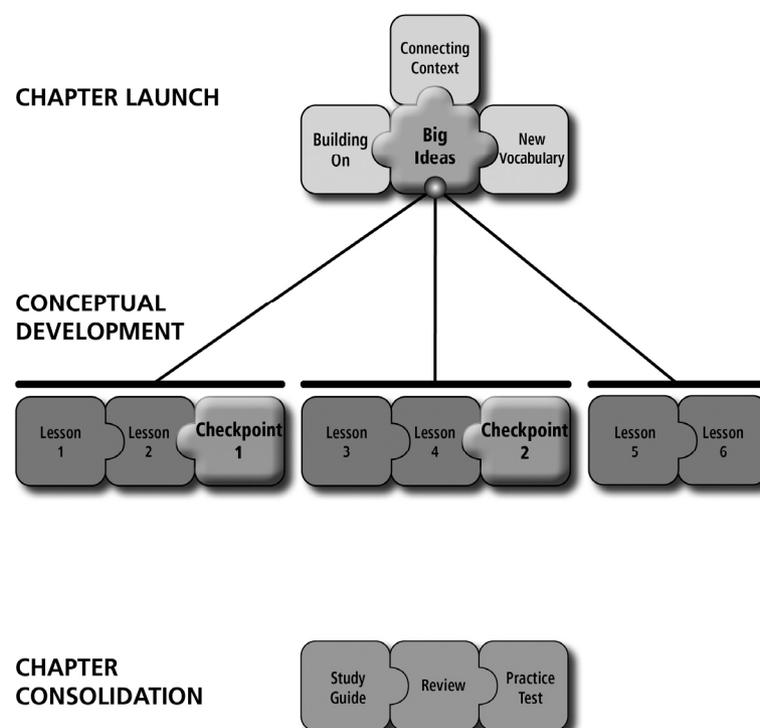
**Organizing learning around a relatively small number of big mathematical ideas allows students to develop connections and deepen their understanding of essential mathematical concepts.**

In *Pearson Foundations and Pre-calculus Mathematics 10*, the big mathematical ideas in each chapter provide a framework for the development of concepts in individual lessons. This organization helps students move through planned stages of concept development in a coherent and connected way. When students have the *Big Ideas* of a chapter in mind, they are better equipped to see connections between concepts, and with prior learning; they are better able to organize, synthesize, and summarize their conceptual understanding.

From a teaching perspective, organizing the learning around *Big Ideas* helps you to focus student attention on essential learnings, which encourages retention as you foster conceptual understanding. This organization also helps to support assessment for learning throughout the chapter.

The principle of organizing learning through the *Big Ideas* of mathematics has shaped the book structure, and its chapter organization.

*"Big Ideas should be the foundation for one's mathematics content knowledge, for one's teaching practices, and for the mathematics curriculum. Grounding one's mathematics content knowledge on a relatively few Big Ideas establishes a robust understanding of mathematics."*  
(Charles, Randall I., 2005 *Journal of Mathematics Education Leadership*, p. 10.)



The Teacher Resource supports planning in each chapter module, with:

- *Chapter Overview*
- *Big Ideas* summary chart
- *Curriculum across the Grades*
- *Activate Prior Learning* masters
- *Chapter at a Glance* planning chart
- *Planning for Assessment* chart

## Chapter Launch

The *Chapter Launch* identifies the prior learning on which the chapter builds. For students who need to review prerequisites, the Teacher Resource provides *Activate Prior Learning* line masters.

The *Chapter Launch* also identifies the *Big Ideas* around which the chapter is organized. It articulates the vocabulary introduced in the chapter, and presents a contextual application of the mathematics.

Students can use the information from the *Chapter Launch* to help them organize and assess their learning as they complete lessons and *Checkpoints*.

## Conceptual Development

*Big Ideas* guide the concept development. Lesson clusters provide connected learning opportunities that lead to a logical point of understanding, where a *Checkpoint* supports students in consolidating their learning so far.

A graphic organizer in each *Checkpoint* aids students in developing their study skills, by showing one way they might organize the main ideas of the preceding lessons. As students progress through the course, they see a range of such organizers, which can help them learn to create diagrammatic summaries of their learning.

*Assess Your Understanding* questions in each *Checkpoint* allow students to check their conceptual understanding and their facility with new strategies and procedures.

To support consolidation and assessment, the Teacher Resource offers:

- *Chapter Test* master
- *Chapter Project* master
- *Chapter Rubric* master
- *Chapter Summary: Self-Assessment and Review* master
- *Chapter Summary: Review Question Correlation* master

## Chapter Consolidation

The *Study Guide* and *Chapter Review* help students relate the development of concepts across the chapter, and reinforce their connection with the *Big Ideas*. The *Chapter Project* master, provided in each Chapter module of the Teacher Resource, provides an opportunity for students to apply the *Big Ideas* in an open-ended, relevant, problem-solving context.

Like the *Checkpoints*, the *Study Guide*, *Review*, and *Project* help students assess their understanding. Teachers can use these elements to observe student progress, offer assistance and clarification, and to make adjustments to instruction if necessary.

To support your planning, see PM 21: *Big Ideas* for an overview of the *Big Ideas* in *Pearson Foundations and Pre-calculus Mathematics 10*.

| Program Master 21 Big Ideas   |   |
|-------------------------------|---|
| Chapter                       | Big Ideas   |
| 1 Measurement                 | <ul style="list-style-type: none"> <li>You can use proportional reasoning to convert measurements.</li> <li>The area of a right-angled triangle is one-half the product of the lengths of its two perpendicular sides.</li> <li>The surface area of a right pyramid is one-half the sum of the areas of the base and the lateral surface.</li> <li>The surface area of a sphere is related to the curved surface area of the cylinder that circumscribes it.</li> </ul>   |
| 2 Similar Figures             | <ul style="list-style-type: none"> <li>Two right triangles are similar if and only if one acute angle of one is congruent to one acute angle of the other.</li> <li>The ratio of any two sides remains constant when the triangles are enlarged or reduced.</li> <li>The ratio of the area of one similar figure to the area of another is the square of the ratio of their corresponding sides.</li> <li>You can use the length of one side and the measure of one angle to determine the length of another side of the triangle.</li> </ul> |
| 3 Factors and Products        | <ul style="list-style-type: none"> <li>Arithmetic operations on integers and real numbers are closed under addition, subtraction, multiplication, and division.</li> <li>Multiplication and division are inverse operations, and a multiplicative inverse can be used to represent their inverse.</li> </ul>  |
| 4 Slopes and Powers           | <ul style="list-style-type: none"> <li>Any number can be written as the fraction <math>\frac{a}{b}</math>, where <math>a</math> and <math>b</math> are integers, <math>b</math> is not zero, and <math>\frac{a}{b}</math> is in lowest terms.</li> <li>Exponents can be used to represent powers and operations of rational numbers.</li> <li>The exponent rules can be extended to include powers with rational and real exponents, and rational exponents.</li> </ul>   |
| 5 Relations and Functions     | <ul style="list-style-type: none"> <li>A relation associates the elements of one set with the elements of another set.</li> <li>A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.</li> <li>A linear function has a constant rate of change and its graph is a non-vertical straight line.</li> </ul>  |
| 6 Linear Functions            | <ul style="list-style-type: none"> <li>The graph of a linear function is a non-vertical straight line with a constant slope.</li> <li>Knowing three of the equation of a line function, the slope and y-intercept of the graph, and the coordinates of a point on the graph.</li> </ul>   |
| 7 Systems of Linear Equations | <ul style="list-style-type: none"> <li>A system of two linear equations is solved when the set of ordered pairs that satisfies both equations is determined.</li> <li>Multiplying or dividing the equations in a linear system by a non-zero number, or adding or subtracting the equations, produces an equivalent system.</li> <li>A system of two linear equations may have one solution, an infinite number of solutions, or no solution.</li> </ul>  |

*"The main goals of mathematics education are to prepare students to: solve problems; communicate and reason mathematically; make connections between mathematics and its applications; become mathematically literate; appreciate and value mathematics; make informed decisions as contributors to society."*

(Western and Northern Canadian Protocol, The Common Curriculum Framework for Grades 10 to 12 Mathematics, p. 4)

## Promoting Student Independence

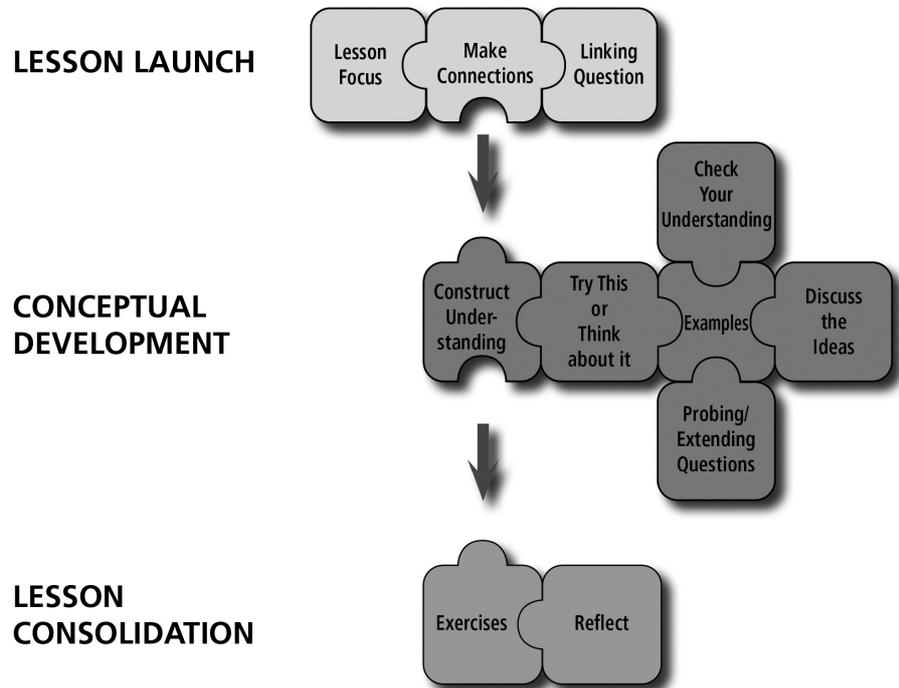
**A secondary mathematics program should, by design, promote confident, autonomous, and self-monitoring learners.**

When students are in control of their learning, they are more engaged, and more likely to succeed with understanding essential concepts and acquiring skills. Independent learners are also better equipped to contend with the challenges of higher mathematics in subsequent grades.

A well-designed student resource needs to be specifically designed for the student; it should support and encourage independent learning. This suggests a student text that provides regular opportunities for each student to make personal connections to the content, to construct personal understanding of the math, and to take charge of her or his own learning.

For teachers, the instructional emphasis shifts toward providing learning pathways and an effective learning environment, monitoring progress, and adjusting the learning process when necessary. To support your role, an effective teacher resource should provide suggestions for leading mathematical discourse in the classroom, tools to support your organization and assessment, along with practical suggestions on how you can foster engagement, confidence, and autonomy in all your students.

The program principle of developing our secondary school students as independent learners has shaped the instructional design of each lesson. The following chart illustrates the 3-part lesson design.



The Teacher Resource supports your lesson planning with:

- Lesson Organizer
- *Make Connections* suggestions for launching the lesson

Foster concept development using these suggestions from the Teacher Resource:

- Assessing Understanding
- Debrief suggestions for in-class activities
- Model questions, and sample classroom discourse
- Identifying Common Difficulties: Possible Intervention
- Extending Thinking
- Cultural Perspectives
- Sample responses to *Discuss the Ideas*
- *Check Your Understanding* solutions, on Teacher CD-ROM

To support consolidation and assessment, the Teacher Resource provides:

- Correlation of Exercises to Achievement Indicators
- Sample response to *Reflect*
- Assessment for Learning: What to look and listen for

## Lesson Launch

The context presented in *Make Connections* leads students to make personal connections from their own life experiences, or prior mathematical learning, to the mathematical content of the lesson.

## Conceptual Development

The elements in *Construct Understanding* provide an instructional path with choices and tools available for independent learning and review.

*Construct Understanding* begins with *Try This* or *Think About It*, both of which encourage students to do math together as they solve a meaningful problem.

In solved *Examples*, students see procedures and problem-solving strategies modelled. Accompanying each *Example*, a parallel *Check Your Understanding* question offers an opportunity for immediate reinforcement. Thus, students observe and then try various strategies, stage by stage, through the lesson development.

Probing questions placed in the margin promote deeper understanding of concepts. These questions challenge students to extend their thinking, hypothesize about relationships, or develop problem-solving strategies beyond those presented on the page. Students think about how the lesson fits into the context of bigger mathematical ideas.

During shared responses to *Discuss the Ideas*, students have an opportunity to further construct and refine their understanding of the mathematics. Facilitating and observing the whole-class discussion support your ongoing assessment.

## Lesson Consolidation

Exercises, organized in A/B/C categories of difficulty, allow students to self-assess as they practise skills and check their understanding of concepts, strategies, and procedures.

To bring closure at the end of each lesson, *Reflect* prompts students to clarify, summarize, and express their understanding.

# Assessment Background

## Purposeful Assessment

“A problem ... must begin where the students are ... Good [assessment] tasks should permit every student in the class ... to demonstrate some knowledge, skill, or understanding.”  
(Van de Walle 2004, p. 62)

“Assessment should support the learning of important mathematics and furnish useful information to both teachers and children.”  
(NCTM: Assessment Principles)

Assessment and evaluation in *Pearson Foundations and Pre-calculus Mathematics 10* are **purposeful** and **possible**—to ensure students are supported in the highest and deepest levels of learning, and teachers are supported with tools and strategies that enhance assessment and reporting.

The primary purpose of assessment is to improve student learning. Within that broad purpose, effective assessment planning incorporates three purposes:

- *Assessment for learning*: teachers use a range of methods to assess both *what* and *how* students are learning; they use this information for *diagnosis* and to *form* and *adjust* instructional plans.
- *Assessment as learning*: learners engage in focused and critical monitoring and reflection about their own learning—both *how* they are learning and *how well* they are learning.
- *Assessment of learning*: teachers develop balanced and dependable evidence of student learning for documenting progress and reporting.

The *Overview of Planning Assessment* on page 16, reproduced from *Rethinking Classroom Assessment with Purpose in Mind*, summarizes key aspects of assessment.

## Effective Assessment Practices

### Goal-Setting, Shared Criteria, Effective Tasks

Learning is enhanced when goals are understood and shared by teachers and students. Recognizing goals and criteria for success is crucial to both assessment *for* learning and assessment *as* learning.

To determine learning goals, we look to the curriculum for its statement of general and specific outcomes of learning. In *Pearson Foundations and Pre-calculus Mathematics 10*, the curriculum outcomes have been clustered into big mathematical ideas, to support the organization of each chapter. The *Big Ideas* provide the framework, then, for meaningful assessment of curriculum goals.

In the Student Book, the *Big Ideas* are stated in each *Chapter Launch* so students know what learning is expected of them. These ideas are supported later in the chapter, in the *Checkpoints* and *Study Guide*, so that students can monitor their progress.

When students play a role in developing criteria for assessment, they begin to understand the learning required, and are increasingly able to self-coach and direct their own learning. Effective performance tasks require complex thinking, where students make choices and decisions.

### **Self-Assessment**

Providing structured, focused opportunities for self-assessment helps students take responsibility for their learning. Students engaged in self-assessment become more interested in learning goals, criteria, feedback, and self-improvement. As developing mathematicians, students need to learn the skills of reflection and self-assessment. Provide frequent opportunities for reflection; model and provide scaffolding, in the form of coaching, prompts, and models; provide frequent and varied opportunities for students to practise applying the metacognitive skills and strategies they are developing.

### **Provide Meaningful Feedback**

Learners need regular feedback on their learning. *Descriptive feedback* should give students direction for improvement and instill confidence that they know *how* to improve. Descriptive feedback helps them to understand their own learning, and what is important in the mathematics they are learning. It helps to develop the language of learning and the ‘inner coach’ so that students begin to provide feedback to themselves as they learn. Thus students will develop their capacity for assessment *as* learning.

### **Adjust Instruction**

Feedback is also important to teachers so they know how well their teaching methods are working. Teachers who use ongoing assessment results to adjust their teaching help students achieve better results. Gather assessment data while students are working on tasks; this early assessment can be used to identify and correct misconceptions, and to identify a lesson plan that may need modification. Be open to adapt your instructional plans to reflect student needs. Select and develop assessments with a clear sense of purpose—knowing what specific information you and your students need, to work toward specific results or expectations.

### **Motivate Students**

Student motivation is one of the keys to learning and improvement. Where assessment results are conveyed in a way that supports learning (rather than questioning ability and effort), teachers have found significant learning gains. Research by Wynne Harlen and Ruth Deakin Crick found that the effects of assessment of learning could support students’ learning *when*:

- Assessments were consistent with teacher expectations and student capabilities.
- Students had opportunities to develop self-assessment skills and learning goals.
- The school took a constructive and supportive stance about testing.
- Learning goals were made explicit to low-achieving students, who were shown how to direct their effort in learning.
- Assessment results were used to convey a sense of learning progress to students.

# Assessment for, as, and of Learning

## Assessment for Learning

“Assessment for learning is the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go and how best to get there.”

(Assessment Reform Group, 2002)

Assessment *for* learning, at the heart of *Pearson Foundations and Pre-calculus Mathematics 10*, is designed to help students learn.

A comprehensive research summary concluded that assessment for learning improves student achievement substantially, particularly for low achievers (Black and Wiliam, 1998).

### Diagnostic Assessment

One function of assessment for learning is diagnostic. Diagnostic information is used to make choices about the nature and extent of the learning experiences teachers provide. Consider the varying mathematical knowledge and skills of the whole class as well as individual students.

Engage students in tasks and discussion that help to:

- identify students who may not have developed previous learning related to new concepts and procedures
- develop insights into particular needs to adjust instruction to ensure all students have the best possibility to succeed

### Diagnostic Opportunities in *Pearson Foundations and Pre-calculus Mathematics 10*

- Each lesson in the Student Book begins with *Make Connections*, designed to give students an opportunity to revisit previous content. As you discuss the content, document any gaps or misconceptions students may have.
- In the Teacher Resource, the teaching notes detail any prior knowledge students require to work on a lesson. To review this knowledge and to prepare for success with the lesson, use the corresponding *Activate Prior Learning* master provided.

### Ongoing Assessment for Learning

The purpose of ongoing assessment for learning is to:

- provide short-term feedback to students so they know how they are doing and how they might improve their learning
- gather information to adjust instruction for the group, and to provide additional support to students who need it
- ensure students are provided with purposeful practice

### Ongoing Assessment for Learning Opportunities in *Pearson Foundations and Pre-calculus Mathematics 10*

The Big Ideas underlying the organization of each chapter helps to shape your criteria for assessment. Use the *Planning for Assessment* chart in each chapter module of the Teacher Resource to create a manageable and purposeful ongoing assessment plan.

- Gather assessment data by observing and listening to students while they are engaged in a problem-solving task.
- See the *Assessing Understanding: Observe and Listen* and *Debrief* sections of lesson notes for specific cues and questions to probe student understanding.
- Listen to students’ responses to *Discuss the Ideas* questions and the margin questions in the Student Book, then adjust instruction accordingly.

- Assign *Exercises* from the Student Book to provide opportunities to observe, probe, and provide feedback about student development.
- Use *Identifying Common Difficulties: Possible Intervention* from the lesson notes to help inform your ongoing assessment.
- See the *Chapter Summary: Review Question Correlation* master as a tool to help students as they work through the *Review* section in the Student Book. Refer students to previous pages or questions in the lessons that will review or reinforce the knowledge necessary to answer the question.

## Tools in the Teacher Resource

In each chapter module, the *Chapter Rubric* master can be used for both assessment *for* learning and assessment *of* learning. It is based on the *Big Ideas* and learning outcomes for the entire chapter and can be used to monitor student development during the chapter, or adapted to assess open-ended tasks or problems.

The *Program Masters* in this module provide a variety of options that support assessment for learning. Choose to use those that best suit your classroom and assessment plans.

- *Program Master 1: Inquiry Process Checklist* provides criteria and recording forms to document student development during lessons where students develop specific problem-solving strategies.
- *Program Masters 11 and 12: Observation Records 1 and 2* are recording formats for making observational notes. They are not specific to a particular chapter or task.
- *Program Master 13: Conference Prompts* provides questions for use to probe student understanding, and to help determine the strategies students are using.
- *Program Master 14: Work Sample Records* provides a format for documenting work samples.

## Assessment as Learning

Assessment *as* learning is sometimes incorporated in descriptions of assessment *for* learning. It focuses on the role of metacognition in learning—the skills and habits of reflection and self-assessment. Ultimately, students become independent, self-directed learners who monitor their own learning, and adjust their strategies and effort to achieve the result they want. To develop the strategies and self-awareness that underpins metacognition, students need opportunities to reflect on and assess their learning, as well as frequent explicit, descriptive feedback.

### Ongoing Assessment as Learning Opportunities in *Pearson Foundations and Pre-calculus Mathematics 10 Student Book* ...

#### in the Lessons ...

- The *Lesson Focus* lets students know what learning is expected of them.
- The lesson model supports metacognition: Students investigate a key concept through a problem-solving approach. Discussion prompts help them recognize and articulate their learning, and to observe how other students deal with the same task.

Name \_\_\_\_\_ Date \_\_\_\_\_

**Master 1.10 Chapter Rubric: Measurement**

|   | Not Yet Adequate   | Adequate   | Proficient   | Excellent  |
|---|--|--|--|--|
| <b>Conceptual Understanding</b><br>• Explain understanding of the relationship between the length of a line segment and the length of its perpendicular bisector.<br>• Explain and justify the relationship between the length of a line segment and the length of its perpendicular bisector.<br>• Explain the relationship between the length of a line segment and the length of its perpendicular bisector. | Some understanding of the relationship between the length of a line segment and the length of its perpendicular bisector.                            | Generally understands the relationship between the length of a line segment and the length of its perpendicular bisector.                            | Generally understands the relationship between the length of a line segment and the length of its perpendicular bisector.                      | Shows depth of understanding of the relationship between the length of a line segment and the length of its perpendicular bisector.                            |
| <b>Procedural Knowledge</b><br>• Apply a geometric strategy to solve a problem involving the length of a line segment and the length of its perpendicular bisector.<br>• Explain the relationship between the length of a line segment and the length of its perpendicular bisector.  | Has difficulty applying a geometric strategy to solve a problem involving the length of a line segment and the length of its perpendicular bisector. | Generally able to apply a geometric strategy to solve a problem involving the length of a line segment and the length of its perpendicular bisector. | Generally applies a geometric strategy to solve a problem involving the length of a line segment and the length of its perpendicular bisector. | Accurately applies a geometric strategy to solve a problem involving the length of a line segment and the length of its perpendicular bisector.                |
| <b>Problem Solving Skills</b><br>• Apply problem-solving skills to solve a problem involving the length of a line segment and the length of its perpendicular bisector.<br>• Explain the relationship between the length of a line segment and the length of its perpendicular bisector.  | Does not use problem-solving skills to solve a problem involving the length of a line segment and the length of its perpendicular bisector.          | Uses problem-solving skills to solve a problem involving the length of a line segment and the length of its perpendicular bisector.                  | Uses problem-solving skills to solve a problem involving the length of a line segment and the length of its perpendicular bisector.            | Uses effective and creative problem-solving strategies to solve a problem involving the length of a line segment and the length of its perpendicular bisector. |
| <b>Communication</b><br>• Explain and justify the relationship between the length of a line segment and the length of its perpendicular bisector.<br>• Explain the relationship between the length of a line segment and the length of its perpendicular bisector.  | Struggles to explain and justify the relationship between the length of a line segment and the length of its perpendicular bisector.                 | Explains and justifies the relationship between the length of a line segment and the length of its perpendicular bisector.                           | Explains and justifies the relationship between the length of a line segment and the length of its perpendicular bisector.                     | Explains and justifies the relationship between the length of a line segment and the length of its perpendicular bisector.                                     |

Master 1.10  
The right to reproduce or sell this master is granted to purchasing schools.  
This master has been modified from copyright © 2010 Pearson Education, Inc.

“Assessment as learning is based on research about how learning happens, and is characterized by students reflecting on their own learning, and making adjustments so that they achieve deeper understanding.”

(WNCP 2006, p. 40)

- As students proceed through a lesson, margin questions prompt them to extend their thinking. This allows students to assess their own understanding.
- Students can complete each *Check Your Understanding* to reinforce, and assess, their learning from the corresponding *Example*.
- *Exercises* are organized by A/B/C levels of difficulty and allow students to check their skills and understanding.
- In *MathLab* lessons, students can use *Assess Your Understanding* to gauge their success with the lesson.
- *Reflect* directs students to focus on what they have learned and how well they understand the mathematics. These may be used to prompt discussion, as a demonstration, or as journal entries.

### ... in *Checkpoints* ...

- Students assess their learning from several lessons as they interpret the graphic organizer in *Connections*, or read the *Concept Development* notes, and think about how they would summarize their own learning.
- They check their understanding by solving new problems in *Assess Your Understanding*.

### ... and in *Study Guides*

- The *Concept Summary* helps students tie their learning back to the *Big Ideas* stated in the *Chapter Launch*.
- Skills Summary supports students in gauging their recall of key terms or specific strategies.
- Review questions are cross-referenced to lessons so that students can identify sections where they may need more help.
- A Practice Test helps students determine what to expect in a class test.

## Tools in the Teacher Resource

In each chapter module, the *Chapter Summary: Self-Assessment and Review* master can be used by students as they work through the *Review* section in the Student Book. Students can use this master to assess their understanding of the math content and their comfort level when answering different types of questions.

The wide selection of *Program Masters* in this module provide a variety of formats that can be used to support metacognition. Choose the ones that best suit students' needs and preferences.

- *Program Master 2: Self-Assessment* and *Program Master 3: Self-Assessment: Problem Solving* can be used to prompt reflection and self-assessment.
- *Program Master 4: Think Mathematics* and *Program Master 5: A Good Place for Learning* invite students to think about the conditions and environment that best support their learning.
- *Program Master 6: Before I Start* prompts students to think about their learning.
- *Program Master 7: Check Your Understanding* is designed to help students reflect on and assess their thinking and learning during a particular mathematics task.
- *Program Master 8: Reflecting on My Work* is intended to help students prepare for a student-led conference or other reporting activity.

*“The goal of assessment is to provide a valid set of inferences related to particular expectations for children and schools.”*

(Linn and Baker 2002, 3)

## Assessment of Learning

Assessment of learning involves both assessment (gathering and documenting evidence) and evaluation (judging student development and achievement). Effective assessment of learning balances teacher observations and conferences, and student work samples or portfolios, with more structured performance tasks. It includes assessments that are cumulative and integrated—to allow students to apply what they have learned over a period of time, and to integrate their mathematical understanding across strands.

Assessments tasks need to produce accurate and meaningful results. For this to occur, there are a number of important design considerations. Tasks used for assessment need:

- to be well written so students understand what is expected
- to vary so students have opportunities to demonstrate their understanding in the best way for them
- to reflect the learning outcomes
- to provide sufficient evidence to make an accurate judgment

### **Assessment of Learning Opportunities in *Pearson Foundations and Pre-calculus Mathematics 10***

- Students are encouraged to demonstrate and recognize what they have achieved.
- Teachers document evidence and make decisions about student achievement.

The *Planning for Assessment* chart at the beginning of each chapter of the Teacher Resource can be used to plan for assessment of learning.

- In the Student Book, the *Review* offers a set of short answer and open-ended questions, keyed to lessons in the chapter.
- The *Practice Test* in the Student Book and the *Chapter Test* in the Teacher Resource address procedural skills, conceptual understanding, and problem-solving skills.
- The three *Projects* in the Student Book and the *Chapter Project* master in each chapter module of the Teacher Resource allow you to assess the extent to which students are able to synthesize and apply their learning in an integrated way.

### **Tools in the Teacher Resource**

In each chapter, the *Chapter Rubric* master highlights strategies, concepts, procedures, and communication students are expected to develop over the course of the chapter. It describes four levels of performance, taking into account the specific outcomes for the chapter. It can also be used to assess performance tasks.

## Summarizing and Reporting Progress: Assessment of Learning

*Pearson Foundations and Pre-calculus Mathematics 10* supports a balanced approach to summarizing and reporting. Evidence of learning skills should be ongoing throughout a reporting period, rather than broken down by chapter or strands. Observations, work samples, group presentations, journals, and formal assessments should be included in making decisions about a student's progress. In considering evidence from various sources, it is important to:

- establish a tentative plan for reporting at the beginning of each chapter or term that includes identifying key evidence that will be collected
- make decisions based on a variety of evidence, collected at various times and in various ways
- ensure the evidence comprehensively addresses the learning outcomes
- establish the most consistent level of achievement, rather than the mathematical average
- assessments at the end of a chapter or at the end of several chapters should have more weight than assessments earlier in a chapter
- use a consistent rubric or definition of standards to guide decision making

It is important for students to know that if they try, practise, and take advantage of feedback, they will learn. This understanding is not developed if everything they do is given a number and every number is part of a grade. Students develop this understanding when there is a clear distinction between learning activities and assessment of learning; that is, practice and performance.

Assessment relates to the gathering and ongoing analysis of data. Evaluation is about devising a system that considers many factors and balances them in a systematic way. You may have local standards for evaluation from your jurisdiction that provide a structure to which you work. Many tools are provided in the Teacher Resource to support record keeping and to help you organize information you collected about each student.

If most students are struggling with a particular lesson, there may be a common gap in students' knowledge that should be addressed, or you may want to adjust the delivery of the lesson to reach more students. If only a few students are struggling, these students may have different learning styles. Use assessment data to try to identify the problem and then use techniques for differentiated instruction to try to overcome the problem. See the section *Meeting the Needs of Diverse Learners* in this module.

# District and Provincial Assessment Programs

In most schools, students at some grades participate in district and/or provincial assessments that are reported at the individual, school, and district level. Often, these are designed to ensure accountability—that programs are meeting the needs of students and that students are learning the required outcomes and achieving high standards. They are part of assessment of learning.

In some cases—particularly at school and district levels—this may involve aggregating results from a standard performance assessment or test administered in all classes at a particular grade. In other cases, the assessment is imposed by an external authority (for example, provincial assessments or examinations); the results are released to teachers and students.

## Impact of External Testing

Research suggests that these tests can have a powerful and (sometimes) negative effect on students' motivation and self-esteem. However, where the following instructional practices are in place, students can respond positively:

- Tests and other assessments are consistent with teacher expectations and student capability.
- Students have opportunities to develop self-assessment skills and learning goals.
- The school takes a constructive and supportive stance about testing.
- Low-achieving students are supported when teachers make learning goals explicit and show students how to direct their effort in learning.
- Assessment results are used to convey a sense of learning progress to students.

## Enhancing Student Performance on Provincial Assessments

Students who are good problem solvers and who have a strong conceptual understanding perform well, no matter what form the provincial assessment may take. Provide students with experience in answering questions in different formats to better prepare students and to reduce test anxiety.

## Overview of Planning Assessment

|                              | <b>Assessment <i>for</i> Learning</b>  | <b>Assessment <i>as</i> Learning</b>   | <b>Assessment <i>of</i> Learning</b>   |
|------------------------------|--|--|--|
| <b>Why Assess?</b>           | to enable teachers to determine next steps in advancing student learning   | to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps   | to certify or inform parents or others of student's proficiency in relation to curriculum outcomes   |
| <b>Assess What?</b>          | each student's progress and learning needs in relation to the curricular outcomes  | each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning   | the extent to which students can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes   |
| <b>What Methods?</b>         | a range of methods in different modes that make students' skills and understanding visible   | a range of methods in different modes that elicit students' learning and metacognitive processes   | a range of methods in different modes that assess both product and process   |
| <b>Ensuring Quality</b>      | <ul style="list-style-type: none"> <li>• accuracy and consistency of observations and interpretations of student learning</li> <li>• clear, detailed learning expectations</li> <li>• accurate, detailed notes for descriptive feedback to each student</li> </ul>   | <ul style="list-style-type: none"> <li>• accuracy and consistency of student's self-reflection, self-monitoring, and self-adjustment</li> <li>• engagement of the student in considering and challenging his or her thinking</li> <li>• students record their own learning</li> </ul>  | <ul style="list-style-type: none"> <li>• accuracy, consistency, and fairness of judgments based on high-quality information</li> <li>• clear, detailed learning expectations</li> <li>• fair and accurate summative reporting</li> </ul>   |
| <b>Using the Information</b> | <ul style="list-style-type: none"> <li>• provide each student with accurate, descriptive feedback to further his or her learning</li> <li>• differentiate instruction by continually checking where each student is in relation to the curricular outcomes</li> <li>• provide parents or guardians with descriptive feedback about student learning and ideas for support</li> </ul> | <ul style="list-style-type: none"> <li>• provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits</li> <li>• have each student focus on the task and his or her learning (not on getting the right answer)</li> <li>• provide each student with ideas for adjusting, rethinking, and articulating his or her learning</li> <li>• provide the conditions for the teacher and student to discuss alternatives</li> <li>• students report about their learning</li> </ul> | <ul style="list-style-type: none"> <li>• indicate each student's level of learning</li> <li>• provide the foundation for discussions on placement or promotion</li> <li>• report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning</li> </ul> |

From *Rethinking Classroom Assessment with Purpose in Mind*. Used with permission of the WNCPC.

## References

- Assessment Reform Group (1999). *Assessment for Learning: Beyond the Black Box*. Cambridge, UK: University of Cambridge School of Education.
- (2002). News/Events: *Current activities*. Retrieved from <http://www.assessment-reform-group.org.uk>
- Black, Paul, and Dylan Wiliam (1998). *Inside the Black Box: Raising Standards through Classroom Assessment*. London, UK: King's College London, School of Education.
1998. Assessment and Classroom Learning. *Assessment in Education* 5 (1): 7–74.
- BC Ministry of Education. 2000; 2002. BC Performance Standards: Numeracy. Victoria, BC: Ministry of Education. Available at [www.gov.bc.ca/bced](http://www.gov.bc.ca/bced)
- Gregory, Kathleen, Cameron, Caren, & Davies, Anne (1997–2001) Know what counts series. Merville, BC: Connections Publishing *Conferencing and Reporting* (2001) *Self-assessment and goal-setting* (2000) *Setting and using criteria* (1997).
- Harlen, Wynne, and Ruth Deakin Crick (2002). *Review: What is the evidence of the impact of summative assessment and tests on students' motivation for learning?* Presentation, International Conference, Assessment Reform Group, March 5, 2002.
- Linn, Robert L., and Baker, Eva L. (2002). *Accountability Systems: Implications of Requirements of the No Child Left Behind Act of 2001*, CSE Technical Report 567
- Marzano, Robert J., Debra J. Pickering, and Jane E. Pollock (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement*. Alexandria, VA: ASCD.
- NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- National Education Association in collaboration with Rick Stiggins and the Assessment Training Institute (2003). *Balanced Assessment*. Washington, DC: National Education Association.
- Stiggins, Rick (2001). *Making Classroom Assessment Instructionally Relevant*. Paper presented at the annual meeting of the American Educational Research Association, Seattle, 2001.
- Van de Walle, John A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally* (5th ed.). Boston: Allyn and Bacon.
- WNCP (2006). *Rethinking Classroom Assessment with Purpose in Mind* (Prepared for the WNCP by Lorna Earl and Steven Katz) Available on-line at [www.wncp.ca](http://www.wncp.ca).
- Wiliam, D. (2005). *The formative purpose: Assessment must first promote learning*. 35<sup>th</sup> National conference on large-scale assessment. San Antonio, Texas.

# Meeting the Needs of Diverse Learners

*"Equity in education is the fair and equal treatment of all members of our society .... All students and adults have the opportunity to participate fully and to experience success and human dignity while developing the skills, knowledge and attitudes necessary to contribute meaningfully to society."*

(Government of Saskatchewan)

## Success for All

One of the main goals of education is success for all learners, also known as the principle of equity in education. Coupled with that goal is the recognition that there is a great diversity among the learners in our classrooms. Equity, then, calls for meeting the needs of diverse learners.

In the past, it may have been acceptable to teach in the same manner to all students, every day. Today, educators realize that a single uniform approach does not meet the needs of all learners, and so may leave a significant portion of the student body at a disadvantage, with major consequences such as failure to meet curriculum outcomes and a narrowing of career opportunities. Equity does not mean treating every student the same, but taking into account students' unique strengths and weaknesses to ensure equal opportunity for success and equal benefit.

*"Excellence in mathematics education requires equity—high expectations and strong support for all students."*

(Principles and Standards for School Mathematics, NCTM, 2000)

The equity principle suggests a responsibility to provide supports to meet the needs of each student, so that all students find a way to succeed in mathematics. Educators and researchers have made significant advances in understanding the range of diversity that may be encountered in the mathematics classroom, and in devising methods to meet the diverse needs that result.

Providing support for diverse learners requires careful preparation, which may be considered in four broad categories.

- First, teachers need to acknowledge the diversity in the classroom and learn as much as possible about each student for whom they are responsible.
- Second, given this background knowledge of diversity, teachers need instructional practices that take that diversity into account.
- Third, teachers need assessment tasks and tools to evaluate the level of success of those practices.
- Fourth, teachers need to use that feedback to devise strategies for improving techniques for meeting diverse needs.

Teaching mathematics equitably to meet the needs of diverse learners may seem like an enormous amount of work. However, *Pearson Foundations and Pre-calculus Mathematics 10* has been structured with those needs in mind, so the work required is integrated into your regular daily preparation.

*"Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well."*

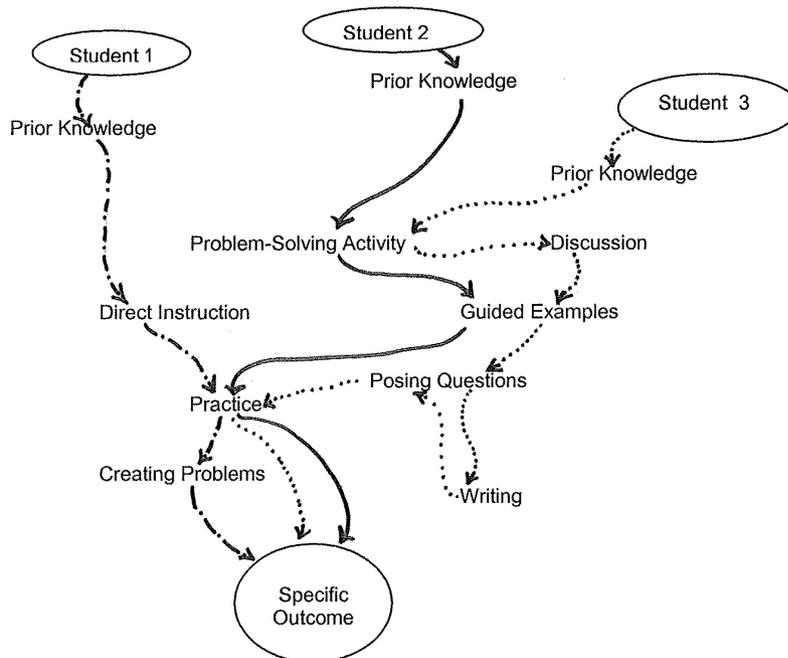
Principles and Standards for School Mathematics (NCTM 2000)

### The Role of the Teacher

The teacher in an equitable mathematics classroom has three fundamental responsibilities:

- knowing what students need to learn (knowing the curriculum);
- knowing what students know and are capable of (understanding the individual students); and
- helping students learn what they need to know given their prior knowledge and capabilities

The situation can be visualized as a sort of map, with various possible routes toward the same destination – the achievement of a particular specific outcome. Each student has a different route toward mathematical understanding.



If we could really map out student learning this way, then to an outside observer it may appear that some students take a direct route to meeting the outcome, while others take circuitous routes. From the individual student perspective, though, the route chosen makes perfect sense and may actually be the most straightforward path to understanding.

Of the teacher responsibilities mentioned above, the first, knowing the curriculum, is the most straightforward. *Pearson Foundations and Pre-calculus Mathematics 10* supports your curriculum:

- with a comprehensive program that addresses all curriculum outcomes, clustered according to the *Big Ideas*
- by stating the *Big Ideas* at the start of each chapter in the Student Book
- by stating the *Lesson Focus* in each lesson
- with *Curriculum across the Grades* in the planning pages of each chapter module in this Teacher Resource
- with *Curriculum Focus* in your Teacher Resource lesson notes, for a detailed correlation to the curriculum
- by providing *Chapter Rubric* masters that are connected to specific outcomes

The second responsibility of the teacher is to understand what students already know and are capable of. Variations in student profiles may result from variations in education, different life experiences, different retention rates, various learning styles, language abilities, and so on. The Pearson mathematics program supports your understanding of individual students with a wide range of assessment tools, such as

- *Activate Prior Learning* masters on the Teacher Resource CD-ROM
- *Make Connections* in each lesson
- *Check Your Understanding*, with each *Example*
- Probing questions, presented in the margin beside selected *Examples* and related text
- *Assessment for Learning: Observe and Listen* master, in the teaching notes for each lesson
- sample responses in your teaching notes for communication-based questions or "What If" types of questions

The third responsibility of the teacher, helping students learn what they need to know, is supported throughout the program with a rich variety of learning tasks so that each student can find a route toward each specific outcome in the curriculum.

### **Feedback**

Despite your best efforts, the methods you use will sometimes not work as well as you hoped. For example, given the great diversity in any classroom, you cannot always predict the difficulty level of a particular task for the class you have this year. In such a case, your best strategy is to observe, interpret, analyse feedback, and use that information to shape your next decisions. That is, assessment is a fundamental part of teaching in a diverse classroom.

By assessing with a wide variety of assessment tools, you'll have a richer range of feedback that more accurately reflects the great diversity of students in your program.

At times, your assessment may indicate that some students have not reached the curriculum outcome. In that case, to ensure that all succeed, it may be necessary to change the task. That is the essence of differentiating instruction: altering tasks to suit the needs of diverse learners.

No two students are exactly alike. They will differ in their current knowledge and their past experiences, so the points from which they are starting will differ. Similarly, students will differ in their skills, abilities, and preferences, so the best paths connecting them to an outcome may differ. Understanding diversity, and responding to it in an equitable manner, is essential to the role of teaching in the modern mathematics classroom.

Despite all the research on diversity in the classroom, the full scope of the variation that may occur in your classroom is ultimately unpredictable. Every class, every year, will be different. Variations will surface from week to week and from day to day. Sudden opportunities may arise in the form of insights or special events or visitors. Urgent needs may come up as you identify gaps in prior knowledge. However, by working with an expectation of diversity in your classroom, you may find that there are more strengths in a collaborative group, than there are in individual students.

## Diversity in the Classroom

Diversity is an essential fact of any human gathering. In the mathematics classroom, students will vary in ways that are visible, and in ways that are not apparent without careful observations. For example, you may encounter:

- Linguistic diversity: students for whom English is not their first language; students with regional dialects; students with large or small vocabularies
- Cultural diversity: First Nations, Métis, or Inuit students; recent immigrants
- Sex and gender diversity
- Diversity in preparation, such as prior experiences in mathematics, or former educational settings
- Diversity in learning styles
- Diversity in personal growth and maturity
- Diversity in personal circumstances, which may affect learning on a short term or long term basis

Even in a group of students of the same age and in the same community there will be variation, if not in language and culture, then in learning styles and potentially in personal growth and circumstances. Diversity in the student body should be reflected in the learning experience, including in the resources with which students are taught. In *Pearson Foundations and Pre-calculus Mathematics 10*, diversity is reflected in numerous ways:

- visually, by depicting people from many cultural backgrounds, male and female, engaged in a variety of different tasks and roles.
- contextually, with situations built around male and female characters, and a wide variety of names reflecting different cultural backgrounds
- contextually, through various geographic and cultural references throughout the exercises and problems

On a deeper level, *Pearson Foundations and Pre-calculus Mathematics 10* meets the diversity of learning styles and preferences with multiple approaches, such as:

- opportunities for hands-on and experiential learning in *Try This* activities, or in *MathLab* lessons
- opportunities to talk and write about concepts and strategies
- opportunities to work collaboratively on problem-solving tasks
- opportunities to see models of problem solving strategies
- opportunities to reflect and analyse

Students are unlikely to tell you explicitly about their unique needs, partly because they are not used to thinking about their own prior knowledge or learning styles. So, you must learn about students through indirect methods such as setting tasks that allow them to choose their own approaches to solving a problem. Their choices will tell you something about their current state of knowledge, their abilities, and their preferences. For this reason, a wide range of learning opportunities, as articulated above, can help to inform your understanding of your diverse student audience.

**Construct Understanding**

**TRY THIS**  
Work with a partner.  
You will need grid paper.

This graph shows the depth of water in a bathtub as a function of time.

**A.** What does each segment of the graph represent? Compare your description with that of your partner. Are both your stories the same? Should they be? Explain.

**B.** Sketch a graph to represent this situation:  
You put the plug in the bath and turn on the taps.  
You leave the bathroom and return to discover that the bath has overflowed.  
You turn off the taps and pull out the plug to let out some water. You put the plug back in.

**C.** Compare your graph with that of your partner. How are the graphs the same and how are they different?

---

**MATH LAB**  
**5.4 Graphing Data**

**LESSON FOCUS**  
Graph data and investigate the domain and range when the data represent a function.

**Make Connections**  
To rent a car for less than one week from Ace Car Rentals, the cost is \$65 per day for the first three days, then \$60 a day for each additional day.

| Number of Days Car is Rented | Total Cost (\$) |
|------------------------------|-----------------|
| 1                            | 65              |
| 2                            | 130             |
| 3                            | 195             |
| 4                            | 260             |
| 5                            | 325             |
| 6                            | 390             |

Who are the points on the graph not joined?  
Is this relation a function? How can you tell?  
What is the domain? What is the range?

284 Chapter 5: Relations and Functions

See pages 277 and 284 of the Student Book for examples of *Try This* activities or *MathLab* lessons.

## **Building on Prior Knowledge**

In mathematics, new concepts always build on existing knowledge. For your students to build on prior knowledge successfully, it's important to recognize that prior knowledge can differ significantly among students. For example, when you start Lesson 1.1 on imperial measures, there may be differences in both prior content and process knowledge.

In terms of content knowledge, students who have helped with home renovations may have knowledge of the lesson content, imperial measures of length. This sets them apart from other students with different life experiences: they may move quickly to extensions of the lesson, or, they may have developed misconceptions that could block their progress; they may be tempted to skip helpful new material (such as the introduction of referents) and rely heavily on their own prior knowledge which, while limited, may seem adequate for their purposes.

In terms of prior knowledge, students may have different concepts of fractions and various strategies for manipulating them. Some students may rely on calculators; some may have the misconception that fractions must always be reduced to lowest terms; some may erroneously believe that fractions must have a common denominator before they are multiplied.

This sample scenario of variation in prior knowledge demonstrates the care required to truly meet the needs of diverse learners, and it highlights how observations of students while they work can give you an understanding of the possible variations. From there, you are better equipped to determine how to respond to the variations you observe.

There may be a temptation to believe that all students should be good at one standard method as outlined in a lesson, but this would fail to recognize the strengths that individual students may bring to their learning. If students can build their understanding to arrive at the specific outcomes of the course, with the prior knowledge at their disposal, then there is no need to force them to solve problems all the same way, even if that way is regarded as more efficient. On the other hand, if a student consistently fails to meet the specific outcomes of a lesson, another approach may be required.

## Learning Styles

According to Harold Gardner (Gardner, 1985), there are multiple learning styles, also known as multiple intelligences, that all learners possess in some degree. According to Hope Martin (Martin, 1996) the eight learning styles are:

|  |  |
|--|--|
| <b>Verbal/Linguistic Intelligence</b>    | ...related to the use of language and the meaning of words. These learners are sensitive to the use of language, able to communicate effectively, and comfortable with written and oral communication. |
| <b>Musical/Rhythmic Intelligence</b>     | ...related to music. These learners enjoy and understand music, and may be talented performers or composers.   |
| <b>Logical/Mathematical Intelligence</b> | ...related to problem solving, reasoning, and symbolic abstractions. These learners have a propensity for mathematics and other forms of systematic thought.   |
| <b>Visual/Spatial Intelligence</b>       | ...related to visualization and arrangements in two and three dimensions. These learners can easily understand the arrangement of the physical world around them.                                      |
| <b>Bodily/Kinesthetic Intelligence</b>   | ...related to the ability to use the body in skilled ways. These learners can work skillfully with objects and movement.   |
| <b>Intrapersonal Intelligence</b>        | ...related to the ability to be aware of and to understand one's own feelings and thoughts. These learners are able to explain their own reasoning and thinking.                                       |
| <b>Interpersonal Intelligence</b>        | ...related to sensitivity toward others. These learners are able to work effectively with others toward a common goal.   |
| <b>Naturalist Intelligence</b>           | ...related to the ability to make distinctions between and classify objects. These learners are able to make fine distinctions among items like flora and fauna, cars, shoes, or hairstyles.           |

There is a mix of learning styles in all individuals, but it is likely that one learning style predominates. For example, all learners have some built-in bodily/kinesthetic intelligence because all people are embodied beings; but some learners will be more comfortable with movement than with any other kind of skill, and it would be helpful to base new learning on this preference.

Just as with variations in prior knowledge, variations in learning styles may affect students' approaches to a given mathematical task, and may even affect their level of success. The most practical way to accommodate variation in learning styles is to provide a rich variety of learning experiences to accommodate each of the kinds of intelligence.

## Strategies for Success

"The seven mathematical processes are critical aspects of learning, doing and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education."

The Common Curriculum Framework for Grades 10 - 12 Mathematics, WNCP, p. 6

Since students have varied needs, providing a wide variety of learning situations and environments should increase the likelihood that we reach more students, in one way or another. Using a variety of learning experiences also helps to ensure that students develop the full range of mathematical processes described in the WNCP Common Curriculum Framework, that is:

- Communication
- Connections
- Mental mathematics and estimation
- Problem solving
- Reasoning
- Technology
- Visualization

### Problem Solving

Of those seven processes, problem solving is particularly valuable in meeting the needs of diverse learners. The process of learning through problem solving encourages students to bring their individual strengths to bear on a particular problem, and allows multiple entry points, thereby supporting a wide range of students. Learning through problem solving shifts the learning focus from an algorithmic approach to a sense-making approach. In addition:

- Problem solving may be used as a framework for a rich variety of mathematical tasks
- Appropriate problems may be found for any outcome in the curriculum and any mathematical process
- Problem solving carries with it an intrinsic motivation: there is a "problem" to solve
- Success in problem solving should increase students' confidence
- Problem solving provides a wide variety of assessment data that can be used to inform your decisions about instruction
- Problem solving is fun

When asked to solve a problem that is just within their reach, students may generate many different approaches, some that you might not have imagined, and some that no one has imagined before. That creativity can flourish when we provide a task and then "let go."

In *Pearson Foundations and Pre-calculus Mathematics 10*, each lesson offers a problem-solving situation as the first part of *Construct Understanding* – either *Try This*, or *Think About It*. These opportunities allow students to approach a new problem using their own unique background knowledge. Following *Try This* or *Think About It*, other problems with more guidance arise in the *Examples*. Finally, further problems for practice are provided in the *Exercises*. The *Chapter Project* masters in this resource provide a culminating project for consolidation and/or assessment. These extended problems, by their nature, allow more choice for students as they consider not only how to approach the problem, but also on which aspects they choose to focus.

#### Construct Understanding

##### THINK ABOUT IT

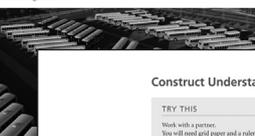
Work with a partner.  
A game uses a spinner with the number 1 or 2 written on each section.



Each player spins the pointer 10 times and records each number. The sum of the 10 numbers is the player's score. One player had a score of 17. How many times did the pointer land on 1 and land on 2? Write two equations that model this situation.

Why do we need two equations to model the situation? Is a solution of one equation also a solution of the other equation? Explain.

A school district has buses that carry 12 passengers and buses that carry 24 passengers. The total passenger capacity is 780. There are 30 more small buses than large buses.



#### Construct Understanding

##### TRY THIS

Work with a partner.  
You will need grid paper and a ruler.

Here is a problem about the hockey cards in *Kabrigoo!*.

The perimeter of each large hockey card is 34 in. The difference between the height and width is 4 in. What are the dimensions of each card?

A. Create a linear system to model this situation.

B. Graph the equations on the same grid.

C. What are the coordinates of the point of intersection, P, of the two lines?

D. Why must the coordinates of P be a solution of each equation in the linear system?

E. What are the side lengths of each large hockey card in *Kabrigoo!*?

The solution of a linear system can be estimated by graphing both equations on the same grid. If the two lines intersect, the coordinates (x, y) of the point of intersection are the solution of the linear system.

Each equation of this linear system is graphed on a grid.

$$\begin{cases} 3x + 2y = 12 & \text{①} \\ -2x + y = 1 & \text{②} \end{cases}$$



We can use the graphs to estimate the solution of the linear system.

The set of points that satisfy equation ① lie on its graph.

The set of points that satisfy equation ② lie on its graph.

The set of points that satisfy both equations lie where the two graphs intersect.

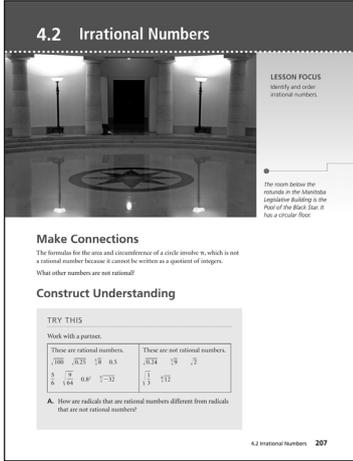
From the graphs, the point of intersection appears to be (-2, 3).

To verify the solution, we check that the coordinates (-2, 3) satisfy both equations.

See pages 395 and 404 of the Student Book for examples of *Think About It* or *Try This* activities.

## Varied Tasks

Students with varied skills and preferences will benefit from tasks that are suited to their preferences. Considering one kind of preference, learning styles, let's examine *Lesson 4.2, Irrational Numbers*, from the Student Book.



### Verbal/Linguistic Intelligence

Tasks that may appeal to students with this preference are *Discuss the Ideas* on page 211, or presenting results from *Try This* on page 207.

### Musical/Rhythmic Intelligence:

Students with this preference may benefit from investigating frequencies of notes on musical scales. In the standard scales of Western culture, notes are ordered approximately as the twelfth roots of 2, while other cultures use different roots of 2.

### Logical/Mathematical Intelligence:

Ask students to provide a convincing argument to justify their answer to the margin question on page 210. Emphasize exercises that prompt for reasoning, such as exercise 14 on page 211, or that extend thinking, such as C exercises.

### Visual/Spatial Intelligence:

Students can benefit from using diagrams as in *Example 2* on page 209, or for *Check Your Understanding* on page 210. They may enjoy questions 18 and 19 on page 212, involving the golden rectangle in both two and three dimensions.

### Bodily/Kinesthetic Intelligence:

Students could physically place numbers on a number line, or even create a large-scale number line and position themselves on it. Both approaches require creative problem solving when additional numbers call for greater degrees of accuracy. As a project related to exercise 18, students could investigate the claim that the golden ratio appears in the ratios of measurements of the human body.

### Intrapersonal Intelligence:

Assignments like *Reflect* on page 212 could be the basis for a personal journal that allows for further writing on a metacognitive level (reflecting on thinking), or in the affective domain (how one feels about a concept or a particular task). Exercises that prompt for reasoning can also appeal to these students.

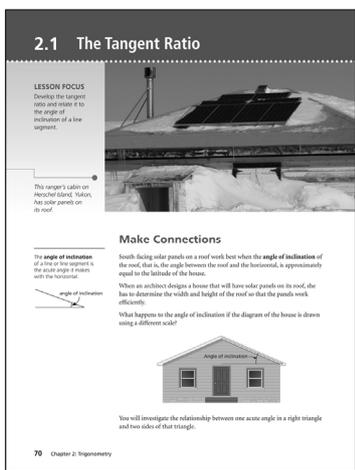
### Interpersonal Intelligence:

Students with this preference learn well in group situations, like *Try This* on page 207. They benefit personally, and may aid other students, when allowed to work through examples or to complete exercises with others.

### Naturalist Intelligence:

Students with this preference will readily take to the notion of classifying numbers, as defined on page 208 and illustrated on page 209. They may enjoy questions like exercises 15 and 16 on pages 211 to 212.

In summary, there is an approach to the material in Lesson 4.2 suited to every learning style. Of course, there is not enough time in the year to assign this range of tasks in any one lesson. But to appeal to diverse learners, using a wide variety of different tasks throughout the year can help all students base some new learning on the skills with which they are most comfortable. Students then grow to realize that there is something in mathematics that is appealing and of value to them, and they can be encouraged to develop their own approaches that build on their own particular skills.



Student page 70 shows a margin note with a key term definition.

*"A positive attitude ... has a profound effect on learning. Environments that create a sense of belonging, support risk taking and provide opportunities for success help students to develop and maintain positive attitudes and self-confidence. Students ... are likely to be motivated and prepared to learn, to participate willingly in classroom activities, to persist in challenging situations and to engage in reflective practices."*

The Common Curriculum Framework for Grades 10 - 12 Mathematics, WNCB, p. 3

## Communication With and Among Students

An important strategy for success, related to one of the seven mathematical processes, is encouraging good communication with and among students.

The Student Book was designed with careful attention to communication, which in turn helps to support the needs of diverse learners. Every page has been edited for clarity and reading level. New vocabulary is identified in each chapter opener, and new terms appear in bold-face type at their first use. Tinted margin notes provide the meaning of a key term, where appropriate, and the *Glossary* further supports communication. In the Teacher Resource, potential vocabulary issues are highlighted in the Lesson Organizer section of each lesson.

To foster effective communication in your classroom, you will want to model effective communication: clear, simple sentences; no unusual vocabulary; speaking aloud what is written; writing nearly everything that is spoken; and seeking feedback from students on their interpretations.

When students speak with you, active listening is a powerful strategy for ensuring that you have clearly heard what the student is trying to convey. Active listening requires periodic feedback to the speaker, for example, a statement of the form, "What I hear you saying is ...."

As with all the mathematical processes, it is important for students not only to use their skills, but to improve them. Opportunities for students to practice oral communication skills arise as you assign tasks to groups, or lead whole-class discussion. In written work, occasionally correcting grammar and spelling can be useful for students, and it demonstrates that you value clear communication as an essential part of doing mathematics.

### Group Work

Having students work together regularly has many advantages, particularly when students are uncovering new ideas or techniques. Group work:

- draws on pre-existing social skills
- requires students clarify and explain their thinking to one another, which is a powerful way for students to learn
- allows students to bring complementary abilities to bear on a problem
- is often more enjoyable than working alone, and so can improve students' attitudes toward mathematics
- mirrors the kinds of situations in which students are likely to use mathematics in the real world

Experiment with different kinds of groupings. Ensure groups reflect diversity in mathematical achievement, learning styles, and other skills. For example, for a task in which communication is important, pair an English language learner with a bilingual learner. For a project that requires a written report, assign groups that include a student with interpersonal skills, a student with reasoning skills, a student with visual skills, and a student with verbal skills.

In contrast to the group assignments described above, ability groupings would be demeaning to those not included in the "high ability" group; it diminishes learning opportunities for all students, and, it fails to recognize the diverse range of skills and abilities in the classroom. Self-selected groupings may result in the same groups each time and narrows the potential for developing students' interpersonal and communication skills.

## The Physical Environment

The physical environment of the classroom may also affect learning, with different environments better for different kinds of learners and different learning situations. Some desk arrangements are conducive to group work, while others are better for listening to a single speaker. No single environment is best for all different situations; think of ways to vary the environment in time and in space. For example, shift the desk arrangement according to the task (variation in time); or set aside part of the classroom as a resource area with reference materials or portfolios of student samples from previous years (variation in space).

Consider also the visual and sonic environment of the class. Displays of posters that illustrate important ideas or formulas could be changed when you move from one chapter to another. Near silence may be appropriate for some situations such as formal tests, while raucous group work may be appropriate for others. Natural lighting or recordings of natural sounds might be appropriate for some situations. Occasionally, you may want to leave the classroom altogether and go outside for applications like the one in *Lesson 2.3 Measuring an Inaccessible Height*.



## Variety and Choice

Two general principles should guide your efforts to meet the needs of diverse learners: variety and choice. As a teacher, you provide students with a variety of different tasks (problems or projects; straightforward, or complex and open-ended; presentations or written work or journals) and a variety of different methods for tackling those tasks (manipulatives or diagrams; techniques for using fractions, or arithmetic with decimals, or calculators, or mental arithmetic to obtain numerical results) and then allow each student to choose the method that best suits her or him.

The principles of variety and choice encourage flexibility. Two other important principles encourage constancy: these are high expectations and the value of tested methods. High expectations means that you expect all students to reach all the curriculum outcomes, and to develop proficiency with the seven mathematical processes. Tested methods are those that have proven to be superior in past experience, such as: communication using clear, simple language; active listening; group work for discovery activities; and treating all students with respect.

Students will have expectations of you, just as you have expectations of them. So there is a tension between flexibility and constancy: too much variety and students don't know what to expect from day to day; too much constancy and teaching becomes rigid, effective for some students but not meeting the needs of all. There is a fine line between those two alternatives that is difficult to navigate without appropriate feedback, which highlights the fundamental importance of assessment.

## Assessment and Differentiated Instruction

This module presents a full section on Assessment, on pages 8 to 17. Assessment surfaces here because of its critical contribution to your decision-making as you think about, and plan for, the diverse learners in your classroom.

Assessment data that can inform your decisions are best gathered while an activity is in progress – assessment for learning. If assessment data hint at a certain misconception for one student, new tasks could be assigned that might help the student overcome the misconception. If assessment data indicate that most students are not achieving an expected outcome from an assigned task, the task could be modified or a new task identified. Used in this way, assessment can promote mathematics learning and equity for diverse learners.

The goal of assessment for learning is to gather data that can help each student in the learning process. Such data can consist of observations, impressions, and intuitions about the process of concept formation in a student’s mind, and can be used to modify a learning process while it is in progress, rather than at the end. In other words, assessment for learning can be part of a feedback process that includes “correction” in the form of differentiating instruction.

Assessment as learning is also a critical factor in meeting the needs of diverse learners because, with self-assessment, students can develop a personal understanding of unique strengths, and personal challenges. By fostering assessment as learning, students start to acquire the skills and habits of mind to develop as self-aware, independent thinkers.

## Differentiating Instruction

It is not uncommon for a particular instructional task to work well for some students but not for others. When such instances arise, it is up to the teacher to alter the task so that all students can succeed. This process of adjustment to meet student needs is known as *differentiating instruction*.

There are two main strategies to differentiate instruction: accommodation and modification. Accommodation is a change to teaching and the environment to support students, such as writing instructions as well as giving them orally. Accommodation does not alter the assigned task. Modification, on the other hand, is a change in the task itself, such as replacing a rectangular pyramid with a square pyramid, in a problem involving the surface area of a pyramid.

Since accommodation does not alter the assigned task, students who complete a task with accommodation will still achieve the outcomes associated with the original task. In other words, successful accommodation leads to equitable results. In contrast, modification changes the task, and so it is not guaranteed that students completing a modified task will reach the specified outcomes associated with the original task. Thus, modifications need to be made carefully, with consideration of the corresponding curriculum outcomes, to ensure equitable outcomes.

Methods of accommodation or modification include:

- identifying multiple entry points
- differentiating tasks
- scaffolding
- the use of technology

As you experiment with these strategies, you may find that accommodations and modifications can be beneficial for the majority of students, not just a few. As a result you might consider using altered tasks regularly.

Your Teacher Resource provides specific content support for modifications in the lesson notes. Look for the **DI** icon and related suggestions.

### **DI** Identifying Common Difficulties: Possible Intervention

The student uses direct measurement.

- Allow students to use a scale diagram for an estimated answer. Suggest a simple scale, such as 1 cm to represent 10 cm, to simplify the work.
- Some students may benefit from creating a full-scale diagram, using the board, a metre stick, and a large protractor.

Ask, "How can you use what you learned in Lesson 2.1 to check your answer?" (*I could use the length I get for RQ by measuring and the given length for PQ to write the tangent ratio for  $\angle P$ . I could*

## Identifying Multiple Entry Points

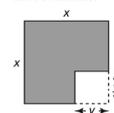
Most mathematical tasks have multiple entry points: there are typically many different ways to approach the problem and obtain a correct answer. For example, here are ways to approach the problem of factoring a difference of squares such as  $x^2 - 25$ .

- use an algebraic solution, connecting to prior knowledge about factoring trinomials, as in the Student Book on page 192
- use a geometric approach (either diagrammatic, or using materials) as in the Student Book on page 193
- use a geometric solution (either diagrammatic, or using materials) as in question 9 on Student Book page 194
- substitute integers for  $x$  and look for patterns in the factors of  $x^2 - 25$
- introduce a change of variable with  $x = y + 5$ , factor the resulting expression by familiar methods, then reverse the substitution

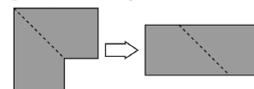
9. a) Cut out a square from a piece of paper. Let  $x$  represent the side length of the square. Write an expression for the area of the square.

Cut a smaller square from one corner. Let  $y$  represent the side length of the cut-out square. Write an expression for the area of the cut-out square.

Write an expression for the area of the piece that remains.



- b) Cut the L-shaped piece into 2 congruent pieces, then arrange as shown below.



What are the dimensions of this rectangle, in terms of  $x$  and  $y$ ?

What is the area of this rectangle?

- c) Explain how the results of parts a and b illustrate the difference of squares.

Given the factoring problem and no help, it is possible that every student might hit on one of these solutions, and hence arrive at a deep understanding of factoring a difference of squares, using her or his special knowledge and abilities. However, it might take a long time for some students to arrive at the answer. At the other extreme, all learners might find out “what to do” by watching the teacher demonstrate a preferred process on the blackboard. However, in this case, understanding and retention for some students might be tenuous since the method provided may not connect with their prior knowledge and abilities.

Between those two extremes lies a middle way, identifying multiple entry points to the problem. Students are not presented with a single method of solving the problem, nor are they left on their own to struggle and become frustrated. Instead, you may choose to present the problem along with a variety of suggestions about possible ways to proceed (identifying entry points), and then encourage students or groups to pick the way that appeals the most or makes the most sense for them.

### Differentiating Tasks

Preparing differentiated tasks is a form of modification in which you plan a task with multiple versions of varying difficulty. You can introduce variations in the size of the numbers chosen, or the number of variables that appear. To continue with the example of factoring a difference of squares, here are multiple versions of the problem, ordered roughly by level of complexity. Each one would be appropriate for some learners and not for others. Factor:

- $49 - 25$
- $x^2 - 25$
- $4x^2 - 25$
- $20x^2 - 125y^2$
- $2x^2 - 8y^2$
- $7^2 - 5^2$
- $x^2 - y^2$
- $4x^2 - 25y^2$
- $ca^{2x^2} - cb^{2y^2}$
- $4x^4 - 9y^6$

Based on your observations of students, you can assign selectively from this list, so that students encounter problems within their ability to solve in a reasonable length of time.

A challenge with differentiating tasks, as with any modification, is ensuring that each student reaches the curriculum outcome associated with the lesson. In the variations above, students who complete just one or both of the first two problems will not reach the outcome, while students who move on to complete the third problem are working with the outcome but on a limited basis. Students who complete selections from the second half of the range above are demonstrating a strong facility with understanding of outcome AN5, and the Achievement Indicator that articulates work with differences of squares.

These considerations highlight a dilemma: choosing a simple version of a task may bring it within the reach of a student having difficulty but may not satisfy the curriculum outcome, while choosing a less simple version of a task may satisfy the curriculum outcome but take the task beyond the reach of the student. There is a solution to this dilemma: scaffolding.

#### Specific Outcome

AN5. Demonstrate an understanding of common factors and trinomial factors, concretely, pictorially and symbolically.

#### Achievement Indicator

AN5.3 Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where  $b = 0$ .

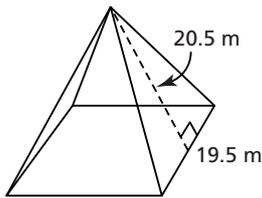
(WNCP Common Curriculum Framework, Grades 10 - 12 Mathematics, page 51)

## Scaffolding

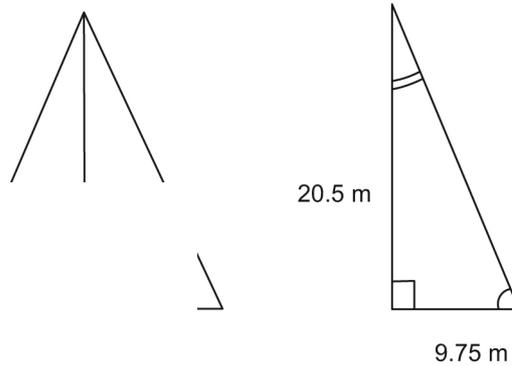
A valuable technique to support struggling learners is scaffolding, in which one develops a sequence of modifications of a task, tailored to the learner's needs. The sequence ends with a task that meets the curriculum outcome which is the focus of the lesson. In effect, scaffolding is a sequence of modifications which together become an accommodation.

For example, consider Lesson 2.7 question 7 on page 118.

At the Muttart Conservatory, the arid pyramid has 4 congruent triangular faces. The base of each face has length 19.5 m and the slant height of the pyramid is 20.5 m. What is the measure of each of the three angles in the face? Give the measures to the nearest degree.



As presented, it is a problem that requires a good understanding of the applications of trigonometry. The problem can be broken down into a series of simpler tasks with the introduction of appropriate “construction lines” which literally look like scaffolding. Scaffolding can be seen in a visual, obvious way in this problem.



The planks of the scaffolding in this sequence are two-dimensional problems that are more readily solved than the original problem.

The main challenge in scaffolding a question is identifying the intermediate steps that will make sense for a student. Scaffolding can be created from differentiations of the original problem. Returning again to the question of factoring a difference of squares, consider *Example 3b* on page 193:

$$\text{Factor } 5x^4 - 80y^4.$$

It is helpful to consider what makes this example challenging. There are several factors: each term is a product of a number and a power of a variable; the expression is not a difference of perfect squares as it stands, although it can be factored to yield a difference of perfect squares; there are two variables; and,  $x^4$  and  $y^4$  do not have exponent 2, and so may not be immediately recognized as “squares.”

From the analysis above, we develop a sequence of differentiated tasks that help to scaffold the original problem:

- Factor  $u^2 - 16$ .
- Factor  $u^2 - 16v^2$ .
- Factor  $5u^2 - 80v^2$ .
- Write  $x^4$  and  $y^4$  as perfect squares.
- Factor  $x^4 - y^4$ .
- Factor  $5x^4 - 80y^4$ .

**Technology:** the methods, knowledge, and theory needed to create and maintain tools and other types of equipment, and such tools, etc., viewed collectively:

(The Oxford Companion to the English Language)

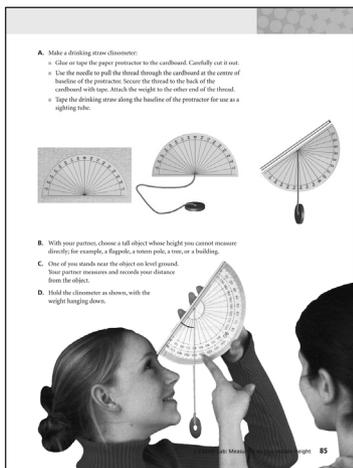
## The Use of Technology

Technology is a powerful tool for equity in our society. Think, for example, of myopia which would be seriously disabling in many individuals if it were not for the power of corrective lenses. The technology of corrective eyewear equalizes the visual abilities of a wide range of individuals, enabling nearly everyone to drive a car, for example. Technology in the mathematics classroom can have the same equalizing power.

The term “technology” in mathematics education is often taken to mean electronic tools. However, it is helpful to think of a continuum of technology beginning with counters and algebra tiles and running through straightedges and compasses; rulers and calipers, to electronic tools like calculators and computers. Using a broad definition of technology has several implications. It situates one item in a range of tools: a graphing calculator is not a mysterious black box but one tool in a continuum of human invention. A broad definition of technology encourages respect for the technologies of other cultures. It empowers learners by encouraging them to make or modify their own technology, taking control of it rather than letting it control them.

For example, it is empowering for students to make their own clinometers. Some may simply follow the directions set out in the Student Book; others may copy the design and personalize it with drawings or inscriptions. Some may devise useful modifications, such as labelling the angles decreasing from  $90^\circ$  to  $0^\circ$  so that angle measures can be read immediately, or labelling the angles with their corresponding tangents. In these ways, students develop a sense of ownership of the technology they use.

With respect to electronic tools consider that for students who have difficulty with arithmetic calculations, calculators can give them access to concepts they wouldn't otherwise be able to explore. With a Common Curriculum Framework that indicates the importance of technology as one of the fundamental processes of mathematics, it is clear that students need to develop an understanding of how, and when, to use electronic technology tools like calculators to support their learning.



## Other Examples of Diversity

In light of the above discussion on meeting the needs of diverse learners, let's now consider a number of specific kinds of diversity in greater detail.

- First Nations, Métis, and Inuit learners (FNMI)
- Francophone learners
- English language learners (ELL)
- Sex and gender variation
- Students with learning disabilities
- Mathematically promising students

Students from these groups may have skills, life experiences, and cultural perspectives that are significantly different from your own. While the suggestions given earlier in this section apply to all learners, including those described above, you may find it beneficial to have additional background understanding of these learners.

The information below is not intended to characterize a whole group, but to offer guidelines about approaching student members of diverse groups respectfully and productively.

### First Nations, Métis, and Inuit (FNMI) Learners

First Nations, Métis, and Inuit learners make up a significant part of the population in many areas served by the Western and Northern Canadian Protocol. In the Student Book and Teacher Resource, effort has been made to include FNMI contexts and perspectives, both to appeal to FNMI learners and to teach non-FNMI learners something about FNMI perspectives.

With respect to overall approach, *Pearson Foundations and Pre-calculus Mathematics 10* respects Aboriginal ways of learning and acknowledges the importance of oral tradition:

- in the use of a problem-based lesson model, with *Try This* or *Think About It*
- by acknowledging the value of multiple approaches, and multiple solutions, to the same problem
- through an emphasis on community, as indicated by activity opportunities in *Try This* and *Project*
- with oral communication prompts such as *Discuss the Ideas*

To increase the number of opportunities for explicit contexts that relate to First Nations, Métis, and Inuit backgrounds, you can localize examples to reflect your students' environment and community. Localizing, at its simplest, could mean using names of local communities in distance problems, or referencing species of local trees in trigonometry problems. On a deeper level, localizing resources could mean tapping into the ethnomathematics of a particular cultural group, such as their knowledge about constructing homes or designing art, or the games traditionally played. When localizing resources in this manner, it is important to have the advice of an Elder or other resource person. Many schools and school boards have dedicated FNMI resource people who could help you localize resources.

*"First Nations, Métis and Inuit students in northern and western Canada come from diverse geographic areas and have varied cultural and linguistic backgrounds. ... Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students"*

The Common Curriculum Framework for Grades 10 - 12 Mathematics, WNCP, p. 6

## **Francophone Learners**

Francophone learners differ from mainstream English-language students in more ways than just language. Francophone Canadians have a heritage that is reflected in a unique history, in special celebrations and the foods associated with them, and in creative endeavours such as music and dance.

As with FNMI learners, one approach to ensure inclusivity is to localize learning tasks. Moving beyond the local community, you can also consider current events on a provincial or national scale for examples of Francophone culture in music and entertainment, and look for ways to draw those examples into the mathematics classroom.

## **English Language Learners (ELL)**

In the case of English Language Learners, while there may be many cultural factors at play as well, language is the central issue that could pose a barrier to learning if not accommodated successfully. English Language Learners may appear in your classroom as a few students, or many, or all students; as a homogeneous representation (all arriving from common countries of origin) or with a high level of language diversity. You will want to adapt some or all of the strategies suggested here, and in the teaching notes for specific lessons.

When your classroom includes English Language Learners, some of the basic communication guidelines for any classroom have a higher degree of importance:

- use language clearly and precisely
- use consistency of presentation
- offer simultaneous oral and written communication
- use active listening
- ensure explicit introduction and review of vocabulary

Consider using oral reading more regularly in the classroom, using diagrammatic approaches more frequently, and pairing students so that bilingual students can support ELL students, when possible. Your ELL students may also want to build a personal mathematics dictionary that allows them to access familiar words, and their translated forms. In classrooms where many students have common countries of origin, such a resource could be shared as a learning reference and stored in a shared space in the classroom.

## **Sex and Gender**

Sex means differentiation into the categories of male and female on the basis of physical differences. Gender variation includes sex but also social roles, self-conception, cultural expectations according to gender, sexual orientation, and so on. Gender variations can therefore be much more complex than sex variation alone.

Although research in the area is constantly growing and changing, there is evidence that sex differences may be correlated with differences in learning styles, developmental rates, and issues within the “affective domain.” For example, some girls may be needlessly turned off mathematics, which presents a serious barrier to gender equity. This situation offers a compelling reason for using the strategies mentioned in this module to meet the needs of diverse learners, male and female.

### **Learners with Learning Disabilities**

Your classroom may include students who are formally designated as having learning disabilities. For such students, you should be able to find more information and suggested strategies for accommodation in his or her individual education plan. Remember that a learning disability need not define the whole student, but is one factor in the profile of that student, who may have other strengths that contribute to the classroom as a whole.

### **Mathematically Promising Learners**

Mathematically promising learners have special educational needs that should be accommodated to ensure equity. Such learners may be advanced in some domains and not others. For example, they may have advanced abilities with mathematical reasoning, but poor communication skills, or, they may have difficulty with non-mathematical subjects. As with all students, mathematically promising students should be encouraged to work on all the mathematical processes, and to work in groups with fellow students at all other levels, so that students benefit from each others' unique perspectives.

In the Student Book, there are challenging problems that may provide enrichment for mathematically promising students, particularly the C exercises in each lesson, and the *Extending Thinking* notes for each lesson in the Teacher Resource.

Support for working with mathematically promising students is also available through most schools and school boards, and through the mathematics departments in many colleges and universities. Some universities, like the University of Regina, hold regular Math Camps and Math Problem Solving Sessions. There is also enrichment material on the internet; some web references are included in the Selected References section that follows.

## Selected References

- Bishop, Alan (1988). *Mathematical Enculturation: a Cultural Perspective on Mathematics Education*. Norwell, MA: Kluwer Academic Publishers.
- Charles, Randall (2005). *Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics*. *Journal of Mathematics Education Leadership*, 7(3), 9–24.
- Gardner, Howard (1985). *Frames of Mind: the Theory of Multiple Intelligences*. New York: Basic Books.
- Government of Saskatchewan (1997). *Our Children, Our Communities, and Our Future: Equity in Education, a Policy Framework*. Retrieved from <http://www.sasked.gov.sk.ca/equity/>
- Manitoba Education, Citizenship and Youth (2006). *Rethinking Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Retrieved from [http://www.edu.gov.mb.ca/k12/assess/wncp/rethinking\\_assess\\_mb.pdf](http://www.edu.gov.mb.ca/k12/assess/wncp/rethinking_assess_mb.pdf)
- Martin, Hope (1996). *Multiple Intelligences in the Mathematics Classroom*. Arlington Heights, IL: IRI/Skylight Training and Pub.
- NCTM (1995). *Assessment Standards for School Mathematics*. Reston, VA: Author.
- NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.
- Van de Walle, John, and Sandra Folk (2008). *Elementary and Middle School Mathematics: Teaching Developmentally* (2nd Can. ed.). Toronto: Pearson Education, 2008.
- WNCP (2008). *The Common Curriculum Framework for Grades 10-12 Mathematics: The Western and Northern Canadian Protocol*. Retrieved from <http://www.wncp.ca/media/38771/math10to12.pdf>
- Art of Problem Solving*. (2008). Available at <http://www.artofproblemsolving.com/>
- Canadian Mathematics Olympiad (CMO). (2009). Available at <http://cms.math.ca/Competitions/CMO/>
- Doolittle, Edward (2009). Available at <http://www.math.uregina.ca/~doolittl/FNMI.html>
- International Mathematical Olympiad. (n.d.). Available at <http://imo.math.ca/>
- Math Central. (n.d.). Available on the University of Regina Web site at <http://mathcentral.uregina.ca/>
- Stocker, David (2007). *Math that Matters: A Teacher Resource Linking Math and Social Justice*. Ottawa, ON: CCPA.
- Waterloo Mathematics Contests. (n.d.). Available on the University of Waterloo Web site at <http://cemc.uwaterloo.ca/contests/contests.html>

### Note

Internet references are subject to change. All references were accurate at time of printing.

## **Materials List for *Pearson Foundations and Pre-calculus Mathematics 10***

Recommended quantities of materials other than program resources provided on Black Line Masters.

| For classroom use  | For each group of 3 or 4 students   | For each pair of students  | For each student   |
|--|---|--|--|
| <ul style="list-style-type: none"> <li>• overhead projector</li> </ul> <p><b>Technology</b></p> <ul style="list-style-type: none"> <li>• graphing software</li> <li>• spreadsheet software</li> <li>• Internet access</li> </ul> | <ul style="list-style-type: none"> <li>• right pyramids, cylinders, prisms, and cones</li> <li>• sand</li> <li>• empty containers</li> <li>• 100 congruent square tiles</li> <li>• 100 linking cubes</li> <li>• old math textbooks</li> </ul> | <ul style="list-style-type: none"> <li>• metre stick</li> <li>• yard stick</li> <li>• scissors</li> <li>• string</li> <li>• calipers</li> <li>• objects of different sizes and shapes</li> <li>• an orange</li> <li>• compass</li> <li>• heavy cardboard</li> <li>• drinking straw</li> <li>• glue</li> <li>• adhesive tape</li> <li>• needle and thread</li> <li>• small washer or weight</li> <li>• algebra tiles</li> <li>• dictionary</li> <li>• length of rope</li> </ul> | <ul style="list-style-type: none"> <li>• ruler and measuring tape with imperial units</li> <li>• ruler and measuring tape with SI units</li> <li>• protractor</li> <li>• scientific calculator</li> <li>• graphing calculator</li> </ul> |

# Program Masters

| Contents |  | Page |
|----------|--|------|
| PM 1     | Inquiry Process Checklist .....                    | 39   |
| PM 2     | Self-Assessment.....                               | 40   |
| PM 3     | Self-Assessment: Problem Solving.....              | 41   |
| PM 4     | Think Mathematics .....                            | 42   |
| PM 5     | A Good Place for Learning .....                    | 43   |
| PM 6     | Before I Start .....                               | 44   |
| PM 7     | Check Your Understanding.....                      | 45   |
| PM 8     | Reflecting on My Work.....                         | 46   |
| PM 9     | Learning Skills Checklist.....                     | 47   |
| PM 10    | Mathematical Dispositions and Learning Skills..... | 48   |
| PM 11    | Observation Record 1.....                          | 49   |
| PM 12    | Observation Record 2.....                          | 50   |
| PM 13    | Conference Prompts.....                            | 51   |
| PM 14    | Work Sample Records.....                           | 52   |
| PM 15    | 0.5-cm Grid Paper .....                            | 53   |
| PM 16    | 1-cm Grid Paper .....                              | 54   |
| PM 17    | $\frac{1}{2}$ -Inch Grid Paper.....                | 55   |
| PM 18    | 1-Inch Grid Paper .....                            | 56   |
| PM 19    | 3-Column Chart .....                               | 57   |
| PM 20    | Isometric Dot Paper .....                          | 58   |
| PM 21    | Big Ideas.....                                     | 59   |



**Program Master 2**

**Self-Assessment**

1. What was the main mathematical concept, procedure, or strategy you learned?

2. Give an example to show how it works. You can use words, diagrams, symbols, or sketches.

3. Why is this mathematical concept important? Why might it be useful to you or others?

4. Check the options below that describe how well you understand it.

- I could teach it to someone else.
- I know when and how to use it on my own.
- I can use it with a bit of help or reminding.
- I still need a lot of help and practice.

**Program Master 3**

**Self-Assessment: Problem Solving**

**Step 1. Understand.**

State the problem in your own words. Make sure you tell exactly what you are supposed to find or solve.

What information are you given?

How well do you understand the problem? not at all  partly  fully

**Step 2. Plan.**

How will you solve the problem? What strategy will you use?

How confident are you that your plan will work?  
not at all  partly  completely

**Step 3. Solve.**

Solve the problem (in your notebook or on paper).  
Explain your solution.

**Step 4. Look Back.**

Is your solution reasonable? no  not sure  yes

How certain are you that your solution is correct?

not at all  somewhat  a lot

Describe another way to solve this problem. Compare it to the method you used.

**Program Master 4****Think Mathematics**

Use the first box in each section to answer the question. After a month or a few weeks, use the second box to update your answer. Revisit your answers at least twice. Notice what changes and what stays the same.

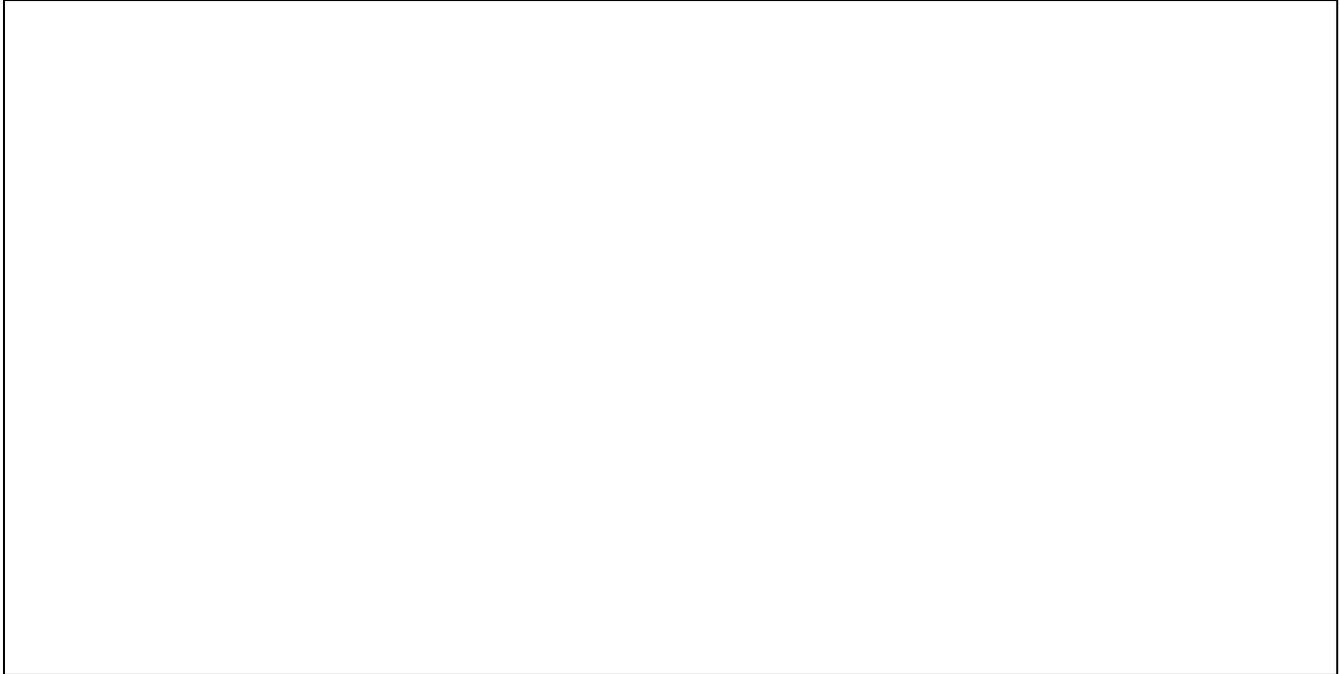
| Date | What kinds of mathematics do you like to do? |
|------|--|
| 1.   |  |
| 2.   |  |
| 3.   |  |

| Date | What math strategies or goals are you working on or trying to improve? |
|------|--|
| 1.   |  |
| 2.   |  |
| 3.   |  |

**Program Master 5**

**A Good Place for Learning**

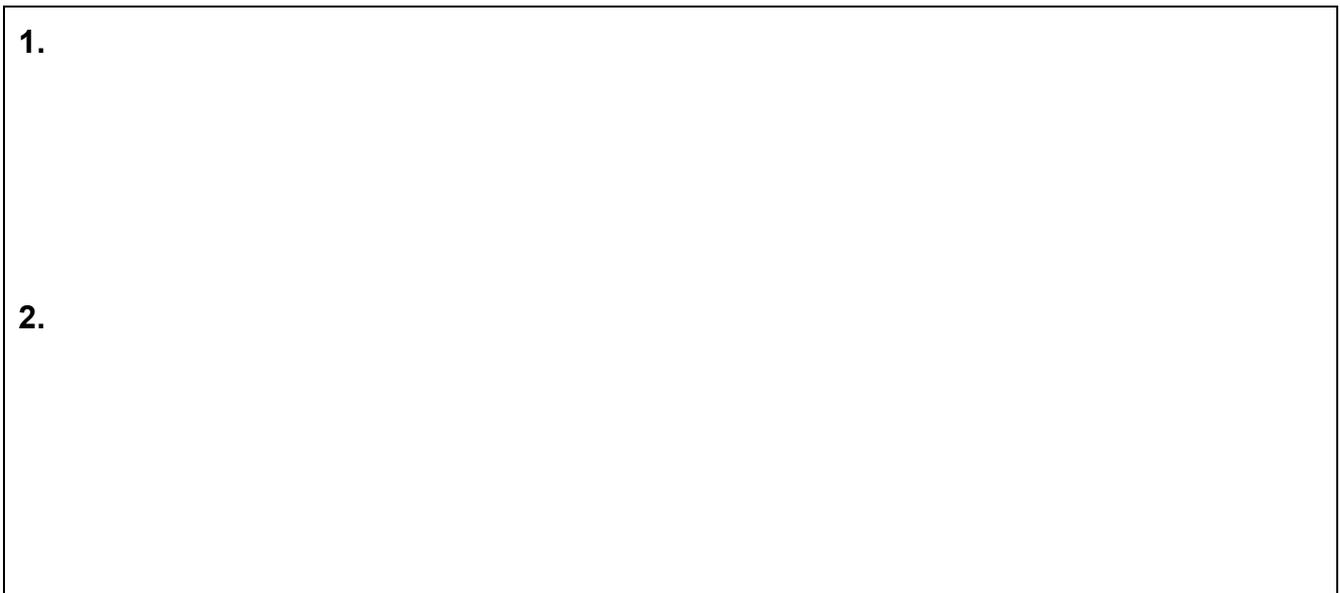
1. Describe or draw a classroom where you could do your best math learning.



2. Describe 2 things in your classroom that would help you learn mathematics.

1.

2.



Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 6**

**Before I Start**

Before I start a math problem, I get ready by \_\_\_\_\_

---

---

---

---

When the teacher or other students are explaining, I help myself focus and remember what I hear by

---

---

---

---

I can sometimes help other students by \_\_\_\_\_

---

---

---

---

**Program Master 7**

**Check Your Understanding**

1. What math did you learn about today?

---

---

---

2. What did you do to make sure you understood the math?

---

---

---

3. Which parts did you have to go over more than once? Tell about what you did.

---

---

---

4. What did you do to make sure you would remember what you learned?

---

---

---

**Program Master 8**

**Reflecting on My Work**

1. What would you like others to notice about your work in math?

---

---

---

2. What part of your math did you work the hardest on? How did it turn out?

---

---

---

3. Tell about one important way your work in math has improved.

---

---

---

4. What would you like to work on and improve in math? How could your teacher or someone else help you?

---

---

---



**Program Master 10**

## Mathematical Dispositions and Learning Skills

Record observations about students' dispositions and learning skills once or twice during each unit by noting a rating (for example, 1 to 4, or E = Excellent, G = Good, S = Satisfactory, N = Needs Improvement) along with brief comments.

| Learning Skill  | Observation 1 |               | Observation 2 |               |
|---|---------------|---------------|---------------|---------------|
|   | Rating*       | Date/Evidence | Rating*       | Date/Evidence |
| <b><i>Independent work</i></b><br><ul style="list-style-type: none"> <li>• shows self-responsibility</li> <li>• completes work on time and with care</li> </ul>   |               |               |               |               |
| <b><i>Initiative</i></b><br><ul style="list-style-type: none"> <li>• takes risks in performing mathematical tasks</li> <li>• exhibits curiosity</li> <li>• shows enjoyment of mathematical experiences</li> <li>• approaches new tasks confidently</li> </ul>   |               |               |               |               |
| <b><i>Work habits</i></b><br><ul style="list-style-type: none"> <li>• puts forward consistent effort</li> <li>• perseveres in mathematical tasks and projects</li> <li>• shows flexibility</li> </ul>   |               |               |               |               |
| <b><i>Class participation</i></b><br><ul style="list-style-type: none"> <li>• contributes to mathematical discussions</li> <li>• contributes to co-operative problem solving</li> <li>• shares responsibility</li> </ul>  |               |               |               |               |
| <b><i>Problem solving</i></b><br><ul style="list-style-type: none"> <li>• mathematically represents and organizes information</li> <li>• devises plans</li> <li>• carries out plans</li> <li>• self-checks to verify solutions</li> <li>• makes connections among different problems and solutions</li> </ul> |               |               |               |               |
| <b><i>Goal setting</i></b><br><ul style="list-style-type: none"> <li>• assesses own work; evaluates success</li> <li>• identifies goals</li> <li>• identifies specific steps or actions needed to improve</li> <li>• uses criteria</li> </ul>   |               |               |               |               |

\*Use notes, or locally or provincially approved levels, symbols, or numeric ratings.

**Program Master 11**

**Observation Record 1**

This form can be used to observe three students who are working together or who are in close proximity.

Chapter \_\_\_\_\_ Lesson \_\_\_\_\_ Activity \_\_\_\_\_

|  | Observation Notes |
|--|-------------------|
| <p>Name _____<br/>                     *Focus: CU, PK, PS, CM</p> <p>Observed strengths:</p><br><p>Observed needs:</p> |                   |
| <p>Name _____<br/>                     *Focus: CU, PK, PS, CM</p> <p>Observed strengths:</p><br><p>Observed needs:</p> |                   |
| <p>Name _____<br/>                     *Focus: CU, PK, PS, CM</p> <p>Observed strengths:</p><br><p>Observed needs:</p> |                   |

\*Circle one or more to indicate the focus of your observations and notes:

**CU** – Conceptual understanding  
**PS** – Problem-solving strategies

**PK** – Procedural knowledge  
**CM** – Communication

Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 12**

## Observation Record 2

Use this form to record ongoing observations over a period of time. You can record more than one date/observation in each category.

Observation period from \_\_\_\_\_ to \_\_\_\_\_

| Observation Notes  |
|--|
| <b>Conceptual understanding</b>  |
| <b>Procedural knowledge</b>  |
| <b>Problem-solving skills</b>  |
| <b>Communication</b>   |
| <b>Summary</b><br><b>Strengths:</b><br><br><b>Needs:</b><br><br><b>Next steps:</b> |

**Program Master 13**

**Conference Prompts**

Select and develop questions and prompts to use during conferences or interviews with students. Answers will often provide evidence of more than one category.

*Note:* The questions are not intended to provide a complete outline. They are examples.

|  |  |
|--|--|
| <p><b>Problem-solving skills</b></p> <ul style="list-style-type: none"> <li>➤ Explain the problem to me.</li> <li>➤ What strategy did you try?</li> <li>➤ Is there another way to do this?</li> <li>➤ Which strategy do you prefer?</li> <li>➤ How did you decide where to start/what to do?</li> <li>➤ What is the first step in ...?</li> <li>➤ Were there any places where you got stuck? How did you get going again?</li> <li>➤ Why did you choose ...?</li> <li>➤ How did you solve ...?</li> <li>➤ Show/tell me about your thinking.</li> <li>➤ Show me another way ...</li> <li>➤ What other ways could someone solve this problem?</li> <li>➤ Have you found all possible solutions/answers? How do you know?</li> <li>➤ What advice would you give someone else who had to solve a problem like this?</li> <li>➤ Can you make up another problem like this for me to solve?</li> <li>➤ Here's what I saw you do ...</li> </ul> | <p><b>Conceptual understanding</b></p> <ul style="list-style-type: none"> <li>➤ Tell me what you know/learned about ...</li> <li>➤ Tell me about your thinking ...</li> <li>➤ How do you know ...?</li> <li>➤ Why does ...?</li> <li>➤ Tell me how you could ...</li> <li>➤ Show me ...</li> <li>➤ What do you predict/think will happen if ...? Why?</li> <li>➤ Does that make sense to you? Tell me why/why not.</li> <li>➤ How could you explain this to someone who has not learned it yet?</li> <li>➤ Explain what you need to do ...</li> <li>➤ About how much/how many ...? Tell me about your thinking—how did you decide on your estimate?</li> <li>➤ What is the same/different ...?</li> <li>➤ What questions do you have about ...?</li> </ul> |
| <p><b>Procedural knowledge</b></p> <ul style="list-style-type: none"> <li>➤ How many ...?</li> <li>➤ Show me how to ...</li> <li>➤ What answer/solution do you have?</li> <li>➤ How did you get that answer/solution?</li> <li>➤ Is your answer reasonable? Why or why not?</li> <li>➤ How could you check?</li> <li>➤ Have you answered all the parts?</li> <li>➤ Why is this important? How could you use ... outside of school?</li> <li>➤ How is ... connected to ...?</li> <li>➤ Have you done work like this before? Tell me about it.</li> </ul>  | <p><b>Communication</b></p> <ul style="list-style-type: none"> <li>➤ Is there another way to say/show that?</li> <li>➤ What do you call that? Does it have another name?</li> <li>➤ Explain the difference between ...</li> <li>➤ How could you tell/show someone else what you learned/found out?</li> <li>➤ Tell me what you did.</li> </ul>   |

Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 14**

## **Work Sample Records**

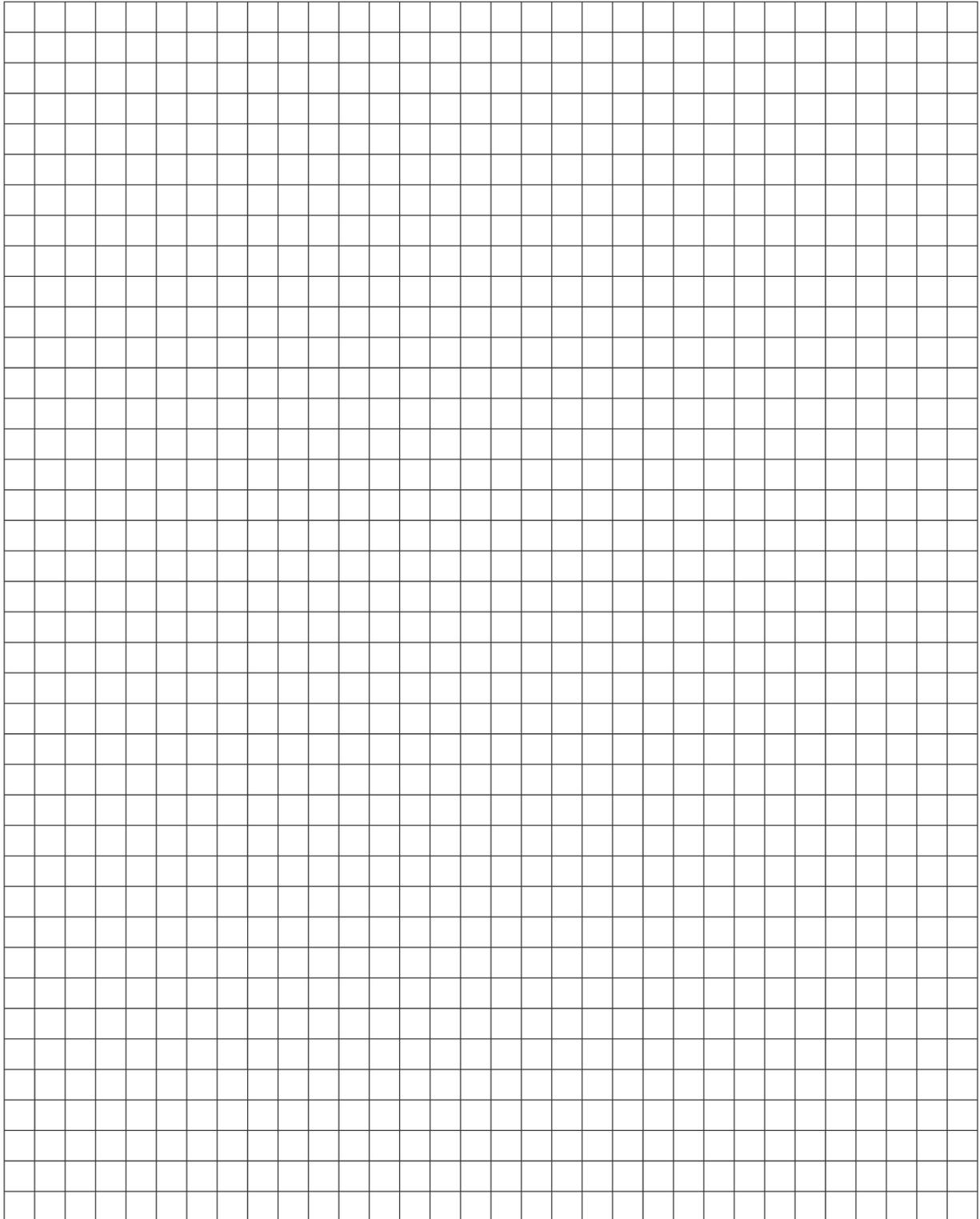
Use this form to record key information about work samples, such as those selected for a portfolio.

| <b>Date and strand<br/>Description of task</b> | <b>What this sample shows about student learning</b> |
|--|--|
|  |  |
|  |  |
|  |  |

Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 15**

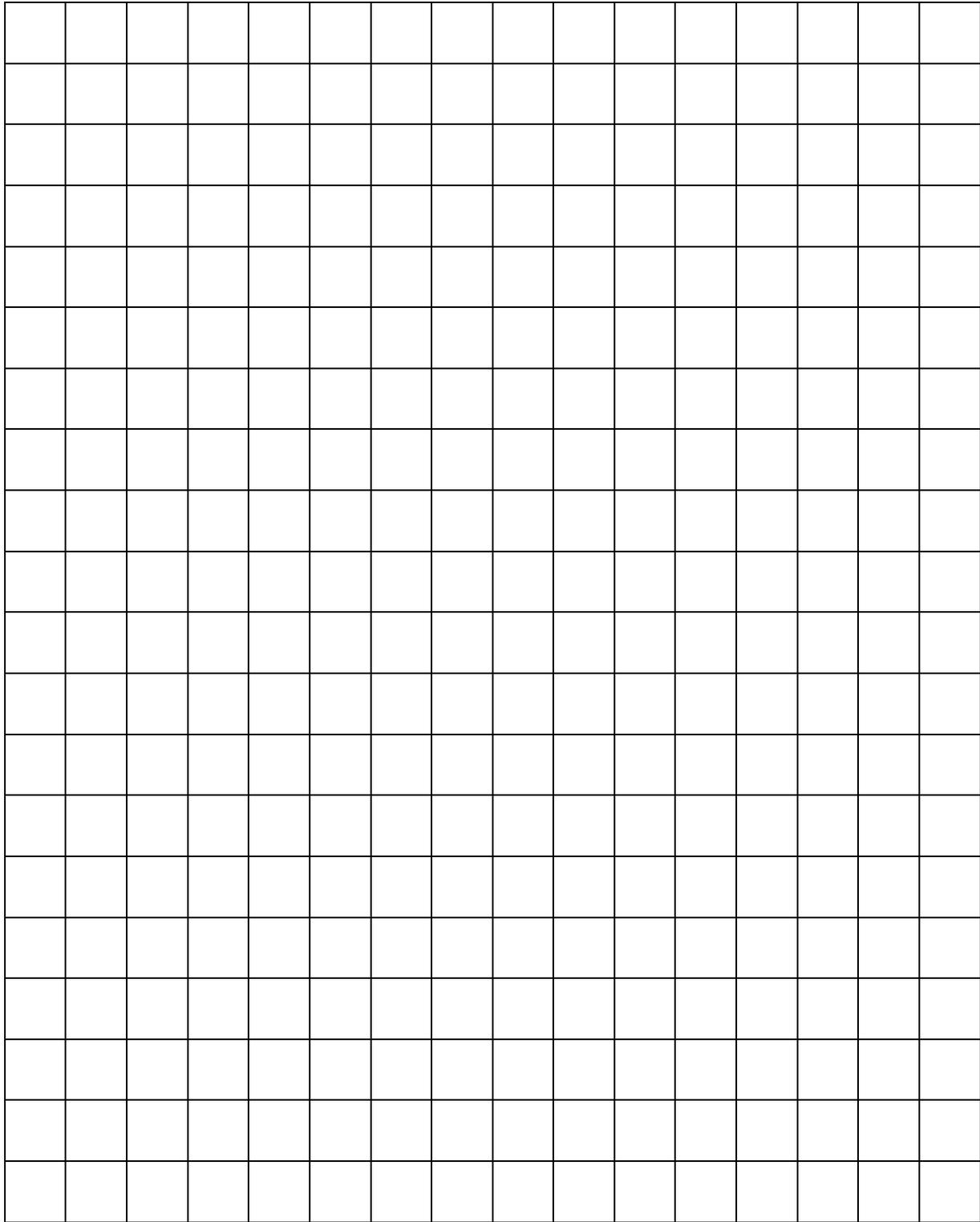
## **0.5-cm Grid Paper**



Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 16**

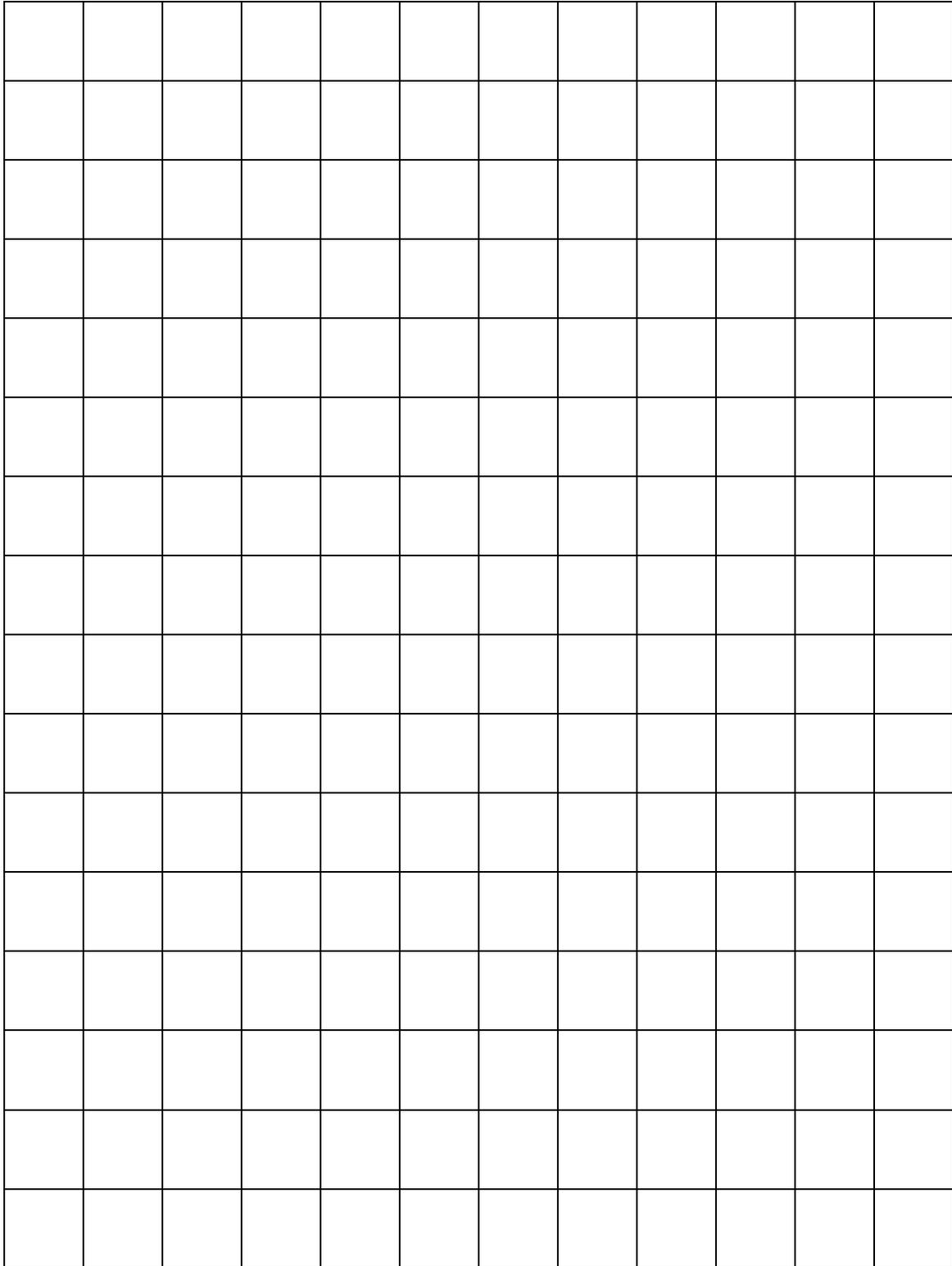
## **1-cm Grid Paper**



Name \_\_\_\_\_ Date \_\_\_\_\_

Program Master 17

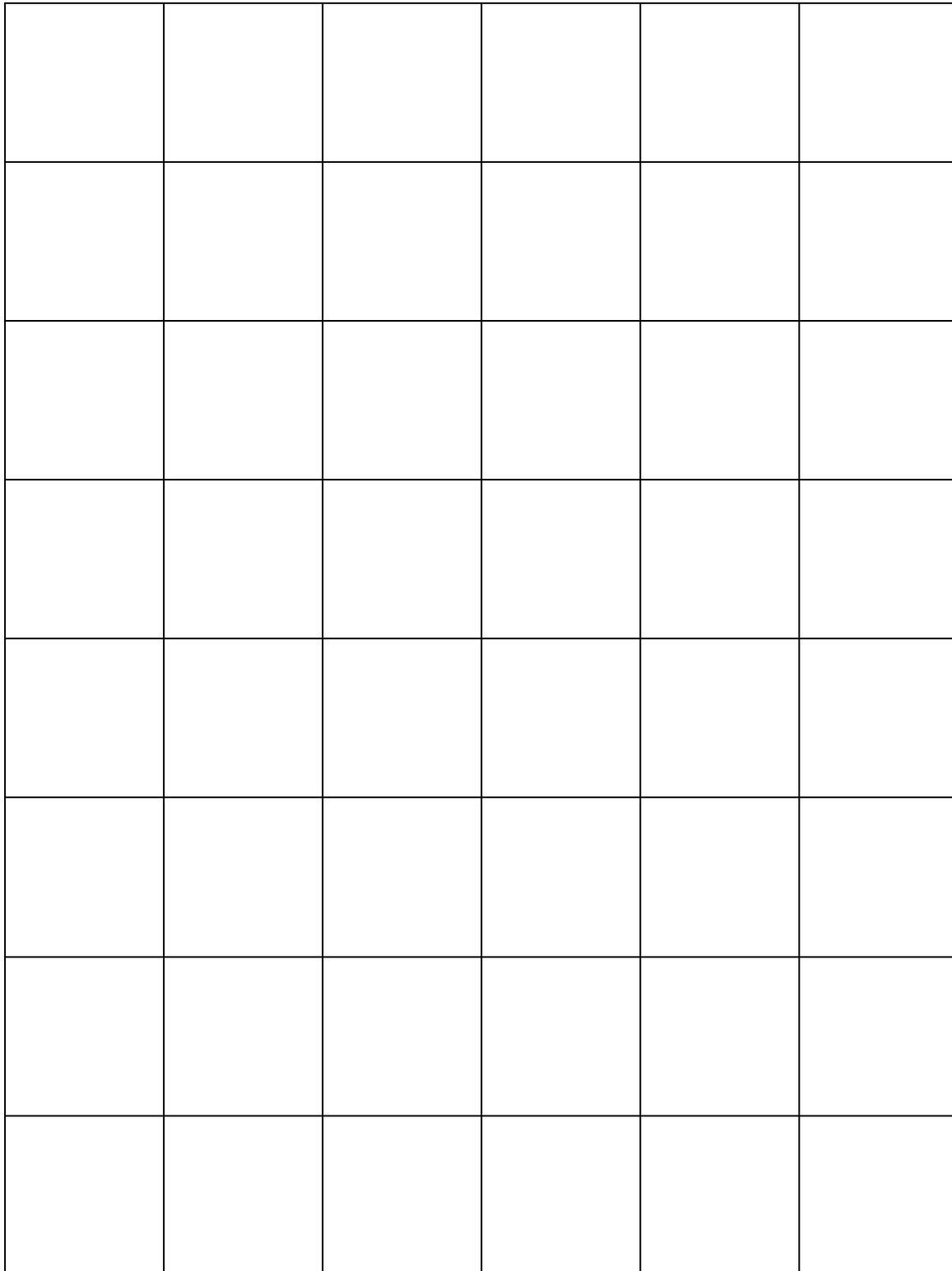
# $\frac{1}{2}$ -Inch Grid Paper



Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 18**

## **1-Inch Grid Paper**



Name \_\_\_\_\_ Date \_\_\_\_\_

**Program Master 19**

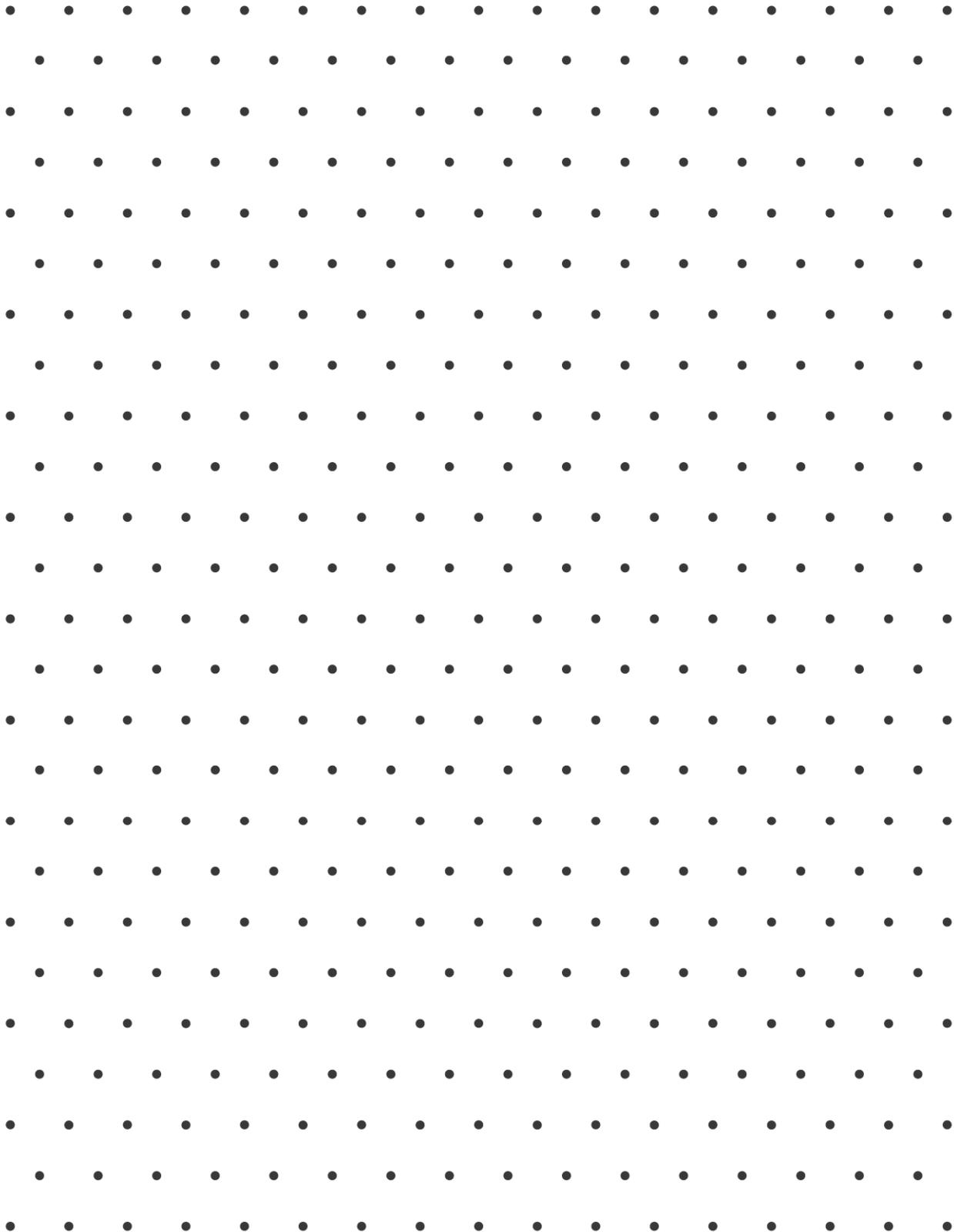
**3-Column Chart**

|  |  |
|--|--|
|  |  |
|  |  |
|  |  |

Name \_\_\_\_\_ Date \_\_\_\_\_

Program Master 20

# Isometric Dot Paper



**Program Master 21**

**Big Ideas**

This table presents the Big Ideas that were used to organize the content across *Pearson Foundations and Pre-calculus Mathematics 10*

|                                    | <b>Chapter</b>                | <b>Big Ideas</b>  |
|------------------------------------|-------------------------------|---|
| <b>Measurement Strand</b>          | 1 Measurement                 | <ul style="list-style-type: none"> <li>You can use proportional reasoning to convert measurements.</li> <li>The volume of a right pyramid or cone is related to the volume of the enclosing right prism or cylinder.</li> <li>The surface area of a right pyramid or cone is the sum of the areas of the faces and the curved surfaces.</li> <li>The surface area of a sphere is related to the curved surface area of the enclosing cylinder.</li> </ul> |
|                                    | 2 Trigonometry                | <p>In a right triangle,</p> <ul style="list-style-type: none"> <li>the ratio of any two sides remains constant even if the triangle is enlarged or reduced.</li> <li>you can use the ratio of the lengths of two sides to determine the measure of one of the acute angles.</li> <li>you can use the length of one side and the measure of an acute angle to determine the length of another side of the triangle.</li> </ul>                             |
| <b>Algebra and Number Strand</b>   | 3 Factors and Products        | <ul style="list-style-type: none"> <li>Arithmetic operations on polynomials are based on the arithmetic operations on integers, and have similar properties.</li> <li>Multiplying and factoring are inverse processes, and a rectangle diagram can be used to represent them.</li> </ul>  |
|                                    | 4 Roots and Powers            | <ul style="list-style-type: none"> <li>Any number that can be written as the fraction <math>\frac{m}{n}</math>, <math>n \neq 0</math>, where <math>m</math> and <math>n</math> are integers, is rational.</li> <li>Exponents can be used to represent roots and reciprocals of rational numbers.</li> <li>The exponent laws can be extended to include powers with rational and variable bases, and rational exponents.</li> </ul>                        |
| <b>Relations and Number Strand</b> | 5 Relations and Functions     | <ul style="list-style-type: none"> <li>A relation associates the elements of one set with the elements of another set.</li> <li>A function is a special type of relation for which each element of the first set is associated with a unique element of the second set.</li> <li>A linear function has a constant rate of change and its graph is a non-vertical straight line.</li> </ul>  |
|                                    | 6 Linear Functions            | <ul style="list-style-type: none"> <li>The graph of a linear function is a non-vertical straight line with a constant slope.</li> <li>Certain forms of the equation of a linear function indicate the slope and y-intercept of its graph, and the coordinates of a point on the graph.</li> </ul>   |
|                                    | 7 Systems of Linear Equations | <ul style="list-style-type: none"> <li>A system of two linear equations is solved when the set of ordered pairs that satisfies both equations is determined.</li> <li>Multiplying or dividing the equations in a linear system by a non-zero number, or adding or subtracting the equations, produces an equivalent system.</li> <li>A system of two linear equations may have one solution, an infinite number of solutions, or no solution.</li> </ul>  |









PEARSON

# Foundations and Pre-calculus Mathematics 10

Robert Berglind  
Garry Davis  
Edward Doolittle  
David Ellis  
Jack Hope  
Delcy Rolheiser  
David Sufrin  
Chris Van Bergeyk  
David Van Bergeyk  
David Zimmer

Copyright © 2010 Pearson Canada Inc.  
All rights reserved.

This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission, write to the Permissions Department.

Pages identified as line masters may be copied by the purchasing school for classroom use.

Printed and bound in Canada

3 4 5 – HI – 14 13 12 11 10

PEARSON

The Pearson logo consists of the word "PEARSON" in a bold, white, sans-serif font, centered within a black rectangular box. Below the text, a thin white curved line arches across the width of the box.