

PEARSON

Foundations and Pre-calculus
Mathematics 10

TEACHER RESOURCE

Chapter 1: Measurement

Chapter 1: Measurement

Teacher Guide Contents

Planning for Chapter 1

Chapter Overview	3
Curriculum across the Grades	5
Chapter 1 At a Glance	6
Planning for Assessment	7

Teaching Notes

1.1 Imperial Measures of Length	8
1.2 Math Lab: Measuring Length and Distance	12
1.3 Relating SI and Imperial Units	15
1.4 Surface Areas of Right Pyramids and Right Cones	19
1.5 Volumes of Right Pyramids and Right Cones	23
1.6 Surface Area and Volume of a Sphere	26
1.7 Solving Problems Involving Objects	30

Black Line Masters

Master 1.1a	Activate Prior Learning: SI Units of Length	33
Master 1.1b	Activate Prior Learning: Surface Areas of Right Rectangular Prisms and Cylinders	34
Master 1.1c	Activate Prior Learning: The Pythagorean Theorem	35
Master 1.1d	Activate Prior Learning: Volumes of Right Rectangular Prisms and Cylinders	36
Master 1.2	Recording Sheet for Lesson 1.1 Try This	37
Master 1.3	Measurement Chart for Lesson 1.3 Try This	38
Master 1.4	Net of a Square Pyramid for Lesson 1.4 Try This	39
Master 1.5	Net of a Triangular Pyramid for Lesson 1.4 Try This	40
Master 1.6	Net of a Rectangular Pyramid for Lesson 1.4 Try This	41
Master 1.7	Chapter Test	42
Master 1.8	Answers	43
Master 1.9	Chapter Project: How Many Balls Fill the Room?	44
Master 1.10	Chapter Rubric: Measurement	45
Master 1.11a	Chapter Summary: Self-Assessment and Review	46
Master 1.11b	Chapter Summary: Review Question Correlation	47

CD-ROM

Masters in editable Word® and pdf formats:

Masters 1.1 – 1.11
Master 1.12 Check Your Understanding 1.1
Master 1.13 Check Your Understanding 1.3
Master 1.14 Check Your Understanding 1.4
Master 1.15 Check Your Understanding 1.5
Master 1.16 Check Your Understanding 1.6
Master 1.17 Check Your Understanding 1.7
Master 1.18 Scientific Notation
Master 1.19 Answers

Chapter Overview

Background

In grade 8, students were introduced to ratio and rate, and they set up a proportion to solve a ratio problem. Students also solved linear equations with integer coefficients and constants. In grade 9, students' work was extended to equations whose coefficients and constants include rational numbers. In grade 10, students should be able to use these algebraic strategies to solve proportions that represent ratio problems.

In grade 8, students were introduced to the squares and square roots of whole numbers which they applied to solve problems involving the Pythagorean Theorem. In grade 9, these concepts were extended to include problems involving positive rational numbers. Students entering grade 10 should be familiar with the Pythagorean Theorem and they should be able to use it to determine the length of any side of a right triangle.

In grade 8, students determined the surface areas of right rectangular prisms, right triangular prisms, and right cylinders, and applied formulas to determine the volumes of right prisms and right cylinders. In grade 9, students determined the surface areas of composite objects comprising right rectangular prisms, right triangular prisms, and right cylinders. In grade 10, students should be comfortable using formulas or personal strategies to determine the surface areas and volumes of these objects.

Rationale

Students are introduced to the imperial system of measure. They develop referents for units of linear measure in both the imperial and SI systems and use these referents to estimate lengths and distances. Students develop measurement strategies and use a variety of instruments to determine linear measures. They use proportional reasoning to convert a measurement within or between SI and imperial systems.

Students use their knowledge of both measurement systems to solve problems involving surface area and volume. Students investigate the relationship between the volumes of a right prism and a right pyramid, and the relationship between the volumes of a right cylinder and a right cone. They use these relationships and the formulas for the volumes of a right prism and a right cylinder they learned in grade 8 to develop formulas for the volumes of a right pyramid and a right cone. Students then investigate the surface area and volume of a sphere. At the end of the chapter, students apply what they have learned to solve problems involving the surface area and volume of composite objects.

Big Ideas of the Chapter	Applying the Big Ideas
<ul style="list-style-type: none"> You can use proportional reasoning to convert measurements. 	<ul style="list-style-type: none"> Converting within and between two systems of measurement allows you to measure lengths using the most appropriate unit from either system.
<ul style="list-style-type: none"> The volume of a right pyramid or cone is related to the volume of the enclosing right prism or cylinder. 	<ul style="list-style-type: none"> The volume of a right pyramid or a right cone is $\frac{1}{3}$ the volume of its enclosing right prism or right cylinder.
<ul style="list-style-type: none"> The surface area of a right pyramid is the sum of the areas of the faces and the curved surfaces. 	<ul style="list-style-type: none"> You can determine the surface area of a right pyramid by sketching a labelled diagram of its net and then calculating the area of each triangle and polygon that forms the net. For a right pyramid with a regular polygon base, the triangular faces are congruent. For a right pyramid with a regular polygon base and for a right cone, the surface area may be calculated using this formula: $\text{Surface area} = \frac{1}{2}(\text{slant height})(\text{perimeter of base}) + (\text{base area})$
<ul style="list-style-type: none"> The surface area of a sphere is related to the surface area of the enclosing cylinder. 	<ul style="list-style-type: none"> The surface area of a sphere is equal to the curved surface area of the cylinder that encloses it.

Curriculum across the Grades

WNCP Grade 8	WNCP Grade 9	WNCP Foundations of Mathematics and Pre-calculus 10
<p>Number N1 Demonstrate an understanding of perfect square and square root, concretely, pictorially, and symbolically (limited to whole numbers).</p> <p>N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).</p> <p>N4 Demonstrate an understanding of ratio and rate.</p> <p>N5 Solve problems that involve rates, ratios, and proportional reasoning.</p> <p>Shape and Space (Measurement) SS1 Develop and apply the Pythagorean Theorem to solve problems.</p> <p>SS2 Draw and construct nets for 3-D objects.</p> <p>SS3 Determine the surface area of right rectangular prisms, right triangular prisms, and right cylinders to solve problems.</p> <p>SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.</p> <p>SS5 Draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms.</p>	<p>Number N5 Determine the square root of positive rational numbers that are perfect squares.</p> <p>N6 Determine an approximate square root of positive rational numbers that are non-perfect squares.</p> <p>Shape and Space (3-D Objects and 2-D Shapes) SS2 Determine the surface area of composite 3-D objects to solve problems.</p>	<p>Measurement M1 Solve problems that involve linear measurement, using:</p> <ul style="list-style-type: none"> • SI and imperial units of measure • estimation strategies • measurement strategies. <p>[ME, PS, V]</p> <p>M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]</p> <p>M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:</p> <ul style="list-style-type: none"> • right cones • right cylinders • right prisms • right pyramids • spheres. <p>[CN, PS, R, V]</p>

Chapter 1 At a Glance

Lesson	Timing (min)	Materials and Resources	Program Support
Lesson 1.1 Imperial Measures of Length, page 8 Develop personal referents to estimate imperial measures of length.	60 - 75	<ul style="list-style-type: none"> • rulers/measuring tapes with imperial units • lengths of string • scissors • scientific calculators 	Master 1.2
Lesson 1.2 Math Lab: Measuring Length and Distance, page 12 Use measuring instruments and personal strategies to determine linear measures.	45 - 60	<ul style="list-style-type: none"> • rulers and measuring tapes with both SI and imperial measures, metre sticks, yard sticks • string • calipers • objects with different sizes and shapes to measure 	Master 1.1a
Lesson 1.3 Relating SI and Imperial Units, page 15 Convert measurements between SI units and imperial units.	60 - 75	<ul style="list-style-type: none"> • rulers/metre sticks/measuring tapes with both SI and imperial units • scientific calculators 	Master 1.3
Lesson 1.4 Surface Areas of Right Pyramids and Right Cones, page 19 Solve problems involving the surface areas of right pyramids and right cones.	60 - 75	<ul style="list-style-type: none"> • rulers with both SI and imperial units • right pyramids • 1-cm and 1-in. grid paper • scientific calculators 	Master 1.1b Master 1.1c Master 1.4 Master 1.5 Master 1.6
Lesson 1.5 Volumes of Right Pyramids and Right Cones, page 23 Solve problems involving the volumes of right pyramids and right cones.	60 - 75	<ul style="list-style-type: none"> • right prisms, pyramids, cylinders, and cones (with removable lids) • containers of sand • empty containers • scientific calculators 	Master 1.1d
Lesson 1.6 Surface Area and Volume of a Sphere, page 26 Solve problems involving the surface area and volume of a sphere.	60 - 75	<ul style="list-style-type: none"> • oranges • calipers or rulers • compasses • scientific calculators 	Master 1.18 Master 1.19
Lesson 1.7 Solving Problems Involving Objects, page 30 Solve problems involving the surface area and volume of composite objects.	60 - 75	<ul style="list-style-type: none"> • scientific calculators 	

Planning for Assessment

Purpose	Tools and Process	Recording and Reporting
Assessment for Learning	<i>Chapter Launch</i> <i>Try This or Think About It</i> Assessing Understanding: Observe and Listen <i>Check Your Understanding</i> <i>Exercises:</i> selected questions <i>Checkpoint: Assess Your Understanding</i> Question: conference; scaffold problems throughout the chapter	Master 1.10 Chapter Rubric: Measurement PM 1 Inquiry Process Checklist PM 11 Observation Record 1 PM 12 Observation Record 2 PM 13 Conference Prompts PM 14 Work Sample Records
Assessment as Learning	<i>Reflect</i> <i>Check Your Understanding</i> <i>Exercises:</i> selected questions <i>Checkpoint: Assess Your Understanding</i> Prompt student’s self-assessment Review student work; provide feedback; scaffold as needed; select key pieces	Master 1.11a Chapter Summary: Self-Assessment and Review Master 1.11b Chapter Summary: Review Question Correlation PM 2 Self-Assessment PM 3 Self-Assessment: Problem Solving PM 4 Think Mathematics PM 5 A Good Place for Learning PM 6 Before I Start PM 7 Check Your Understanding PM 8 Reflecting on My Work
Assessment of Learning	<i>Review</i> <i>Practice Test</i> <i>Chapter Test</i> <i>Chapter Project</i> Review assessment records; add chapter results to ongoing records	Master 1.10 Chapter Rubric: Measurement Master 1.7 Chapter Test Master 1.9 Chapter Project: How Many Balls Fill the Room?
Learning Skills/ Mathematical Dispositions	Observe and record throughout the chapter	PM 9 Learning Skills Checklist PM 10 Mathematical Dispositions and Learning Skills

Program Masters (PM) are in the *Program Overview* module.

1.1

Imperial Measures of Length

Lesson Organizer

60 - 75 min

Key Math Learnings

Another system of measurement is the imperial system, where measurements are recorded as mixed numbers.

Curriculum Focus

Develop personal referents to estimate imperial measures of length.

SO	AI
M1	1.1, 1.3, 1.4
M2	2.1, 2.2, 2.3

Processes

ME, PS, V, C

Teacher Materials

- demonstration ruler with imperial units and a yard stick (optional)

Student Materials

- ruler and measuring tape with imperial units
- length of string
- scissors
- scientific calculator
- Master 1.2 (optional)

Vocabulary

SI system of measures, imperial units, referent, proportional reasoning, unit analysis, conversion factor

Make Connections, SB page 4

The construction industry context offers an opportunity for students to see how measurement skills are used by tradespeople. You could begin the lesson by asking students which imperial units they are familiar with and where these units are used today. You may wish to demonstrate how to read a ruler with imperial units.

Most students know their height in feet and inches. Have a measuring tape or yard stick available to check or measure a volunteer's height.

For objects with lengths of about one foot, students may suggest the length of their textbook, or the depth of a shelf on a bookcase.

For objects with lengths of about one inch, students may suggest the length of an eraser, or the thickness of their textbook.

For objects with lengths of about one yard, students may suggest the length of a desk, or the width of a doorway.

Construct Understanding, SB page 5

It may be helpful to precut the string to 1-m lengths.

Invite students to record their measures in a chart, such as Master 1.2, then write their answers to Part D below the chart.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- How did you use your thumb to measure the length of your pencil? (*I aligned the tip of my thumb with the end of the pencil. My partner marked a point at my knuckle. I moved my thumb so the tip aligned with the point. We continued to do this until we got to the end of the pencil. The pencil was about 7 thumb units long.*)
- What was the actual length of the pencil? (*About 7 inches*)
- For which imperial unit is the thumb length a referent? (*The inch*)
- What are some potential problems with measuring lengths using referents? (*This method is time consuming, difficult to use with large or oddly shaped objects, and each person may get different measurements.*)

DI Cultural Perspectives

Take this opportunity to discuss traditional systems of measurement. Ojibwe canoe makers measured all parts of a canoe in reference to parts of their bodies. For example, the distance between the ribs of a canoe was always one hand spread. The hand was also used to measure the height of a horse. These measures differed depending on whose hand was used.

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty manipulating his thumb along the pencil.

- Suggest the student use a ruler to draw a long line segment on a piece of paper. He lays the pencil along the segment, then marks the endpoints of the pencil. The student removes the pencil and uses his thumb to measure the length of the segment between the two points.

DI Extending Thinking

Have students think about how they could estimate a longer distance. Ask, “How could you estimate the distance from your home to school?” (*I could count the number of steps it takes me to walk this distance at a normal pace. My estimate would be a distance measured in a unit equal to the length of my step.*)

Debrief Try This: Assessment for Learning

Have students compare their measures with those of their partners and then with those of other pairs of students. Ask questions such as:

- What do you notice about all the answers to each of Part D?
(*There are many different answers for each length; some are whole numbers and some are mixed numbers.*)
- Why do you think we need to have standard units of length?
(*Standard units of length would ensure that everybody gets the same measure for the same length.*)
- What are the relationships between the three units? (*There are about 12 thumb lengths in one foot length, about 3 foot lengths in one arm span, and about 36 thumb lengths in one arm span.*)

Students should recognize that:

- Each unit has a referent that can be used to estimate lengths.
- There are relationships between the units that allow for conversion between units.
- There is a standard size for each unit.

Assessment for Learning

What to look and listen for:

- Students can choose an appropriate referent for a linear measure and describe their choice.
- Students can use a personal referent to estimate a linear measure and explain the process used.
- Students can use proportional reasoning to convert between imperial units.
- Students can verify a unit conversion using unit analysis.
- Students can solve problems that involve the conversion of imperial units.

For those students who have difficulty reading the yardstick on Student Book page 6, have a yard stick available for reference.

Ensure students understand that rulers with imperial units may be different. Have different rulers available to illustrate this.

Tell students that a measure reported in several units should proceed from the greatest unit to the least unit; for example, yards, feet, inches.

Ensure students recognize that there is more than one type of proportional reasoning. Proportional reasoning is the ability to think about and compare multiplicative relationships between quantities, such as those expressed as ratios. *Example 1* uses proportional reasoning to convert between imperial units by multiplication. *Example 2* uses proportional reasoning by setting up, then solving a proportion that equates two ratios.

Ask questions such as:

- How can we use proportional reasoning to convert a measurement in yards to a measurement in inches? (*Since there are 36 in. in 1 yd., we multiply the length in yards by 36.*) To convert a measurement in inches to a measurement in yards? (*We divide the length in inches by 36. If there is a remainder, we could write it in feet and inches.*)

Some students may have difficulty with the concept of unit analysis. Remind students that when we multiply a number by 1, its value does not change.

For example, $5 \times \frac{6}{6} = 5$, because $\frac{6}{6} = 1$ and $5 \times 1 = 5$. We use the same concept when we multiply a measure by a conversion factor; the size does not change because the conversion factor is equal to 1. For example, there are 3 ft. in 1 yd., so we can write the relationship between feet and yards as a fraction in two ways: $\frac{1 \text{ yd.}}{3 \text{ ft.}}$ and $\frac{3 \text{ ft.}}{1 \text{ yd.}}$. Both fractions are equal to 1.

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty multiplying a fraction by a whole number to solve a proportion.

- Remind the student that when multiplying a fraction by a whole number, the whole number can be written as a fraction with denominator 1. The student then multiplies the numerators and multiplies the denominators.

For example, $3 \times \frac{3}{10} = \frac{3}{1} \times \frac{3}{10} = \frac{9}{10}$

DI Extending Thinking

Use the question prompts in the screened boxes of the Student Book.

<p>SB p. 7, Example 1: When you convert a measurement from a larger unit to a smaller unit, do you expect the number of units to increase or decrease? Why?</p>	<p><i>I expect the number of units to increase because I can fit more smaller units into the length than larger units; for example, 3 yd. = 9 ft.</i></p>
<p>SB p. 8, Example 2: How could you use mental math and estimation to check that your answer is reasonable?</p>	<p><i>Since there are 12 in. in 1 ft., I estimate by dividing 130 by 10 to get 13. So, an answer of 11 ft. is reasonable. Since 12 ft. must be purchased at about \$2/ft., the cost of material before taxes is about \$2 times 12, or \$24. Since \$22.68 is close to \$24, the answer is reasonable.</i></p>
<p>SB p. 10, Example 3: What conversion factor could you use to convert the units in one step?</p>	<p><i>There are 36 in. in 1 yd. I can write the relationship between yards and inches as a fraction in two ways: $\frac{1 \text{ yd.}}{36 \text{ in.}}$ and $\frac{36 \text{ in.}}{1 \text{ yd.}}$. Since I am converting 6 yd. to inches, I use the conversion factor with inches in the numerator. So, I write:</i></p> $6 \text{ yd.} \times \frac{36 \text{ in.}}{1 \text{ yd.}} = \frac{6 \text{ yd.}}{1} \times \frac{36 \text{ in.}}{1 \text{ yd.}}$ $= 216 \text{ in.}$

Assessing Understanding: Discuss the Ideas, SB page 11

Sample Responses

1. *Standard units ensure that people with different measurement tools can get the same measure for the same length. For example, suppose two people are working independently on 2 benches for a row boat. If each person makes a bench 3 ft. long and they use different measures for feet, then the benches may not fit.*
2. *I might use a referent when an exact measurement is not required or when an object is not easily measured with a ruler or a measuring tape. I might also use a referent when it is not convenient to measure; for example, I am at the store and I want to determine whether a bookcase will fit in the corner of my bedroom.*

Exercises, SB page 11

Have rulers, yard sticks, and measuring tapes in imperial units available for students who need visual assistance with some of the exercises. Remind students to use estimation strategies to check the reasonableness of their solutions.

Assessment of Learning – selected questions

AI	1.1	1.3	1.4	2.1	2.2	2.3
Question number	4	5	3	10	18	11

DI Cultural Perspectives

Old maps or maps published in the U.S. have scales in imperial units. Have students use a map of their territory or province with a scale in imperial units. They determine the distance between the city, town, or First Nation community in which they live to another city, town, or First Nation community. Students measure the distance in inches, then use the scale on the map to calculate the actual distance. For example, students could determine the distance between the Little Grand Rapids First Nation and the Berens River First Nation in Manitoba.

Sample Response: Reflect, SB page 12

To estimate a length, I think about the referent for the unit I want to use. I then count the number of referents needed to span the length of the object I want to measure.

To determine the appropriate imperial unit for a measure, I compare the referents for imperial units with the dimension of the object I want to measure. Then I select the unit that is closest in length to that dimension. For example, if I am measuring the width of a bus, I would measure in yards because a yard is closer to the width of a bus than an inch, a foot, or a mile.

Looking Ahead

In Lesson 1.2, students use referents to estimate and measuring instruments to determine linear measures in both SI and imperial units.

1.2

MATH LAB: Measuring Length and Distance

Lesson Organizer

45 - 60 min

Key Math Learnings

Linear dimensions are not simply measures of line segments; they may be measures of lengths of curves.

Curriculum Focus

Use measuring instruments and personal strategies to determine linear measures.

SO	AI
M1	1.1, 1.3, 1.4, 1.5, 1.6

Processes

ME, PS, V

Teacher Materials

- demonstration calipers

Student Materials

- ruler and measuring tape with both SI and imperial measures
- metre stick and yard stick
- length of string
- calipers
- 3 objects with different sizes and shapes
- Master 1.1a (*optional*)

Make Connections, SB page 13

Students should be comfortable with the SI units of length: metre, centimetre, and millimetre. They should be able to provide personal referents for these measures and use the referents to estimate lengths. Students should also be able to convert between the SI units. Use Master 1.1a to activate prior learning about SI units of length.

Have students list different careers that require measurement in either imperial or SI units.

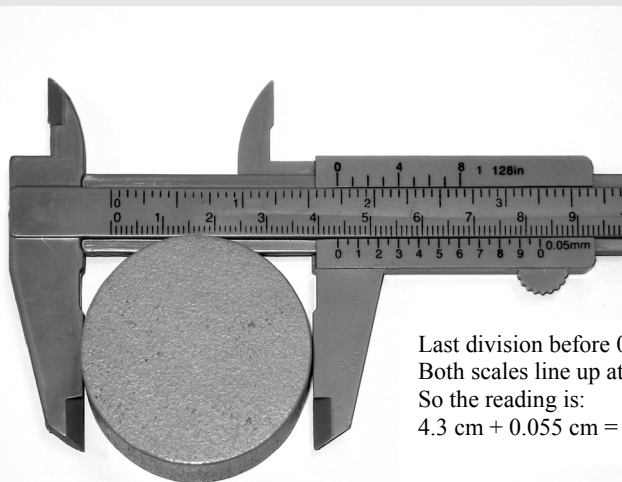
This activity provides students with the opportunity to use different measuring instruments. The instrument and strategy used will depend on the size and shape of the object being measured each time. Provide a variety of objects for students to measure (for example, tin cans, different-sized balls, cereal boxes, paper clips, a stapler or hole punch, cones, and pyramids).

DI Cultural Perspectives

Thousands of years ago, when the first craftspeople sold their wares, quality in the form of correct measurements was not standardized. By the end of the Middle Ages, craft guilds in Europe ensured members produced high quality goods. The Industrial Revolution led to tradesmen being supervised by a person who was responsible for controlling the quality of the goods produced. Today, companies have quality control departments. If an item does not meet all necessary specifications, it is not delivered to the customer.

Construct Understanding, SB page 14

Demonstrate how to measure an object with calipers. Close the jaws of the calipers lightly on the object. Read the scales on the calipers. For example, using calipers, this object has diameter 4.355 cm.



Last division before 0: 4.3 cm
Both scales line up at 0.055 cm.
So the reading is:
 $4.3 \text{ cm} + 0.055 \text{ cm} = 4.355 \text{ cm}$

Assign each pair of students three objects of different sizes and shapes. Be sure to include at least one object that has a curved surface. If possible, assign each object to two pairs of students. This will allow students to compare the accuracy of their measurements and to discuss the different measurement strategies used.

Some students may not remember the referents for the millimetre, centimetre, and metre used in previous grades. Encourage these students to develop their own referents based on the size of each unit.

Suggest students measure each object as precisely as possible. This will be determined by the measuring device used. Remind students that a fraction of an imperial unit is generally written in fraction form while a fraction of an SI unit is written in decimal form.

DI Cultural Perspectives

The objects chosen for this activity could have a cultural theme. For example, students could measure the dimensions of artwork, musical instruments, articles of clothing, or sports equipment.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- How did you decide which referent to use to estimate the dimensions of your object? (*We decided that we would measure the object, a paper clip, in centimetres. Since the width of my baby finger is about 1 cm, we used my baby finger as a referent.*)
- What strategy did you use to determine the height of your pyramid? (*I held a book on the apex of the pyramid so that the book was parallel to the base. My partner used a ruler to measure the vertical distance from the base of the pyramid to the book.*)

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty reading a ruler or measuring tape in imperial units to determine a fractional measure.

- Have the student identify which 2 whole numbers the measure is between. Then have the student count the number of divisions between the whole numbers to determine the distance between consecutive marks. For example, when there are 16 divisions, the distance between consecutive marks is $\frac{1}{16}$ of an inch. When there are 8 divisions, the distance between consecutive marks is $\frac{1}{8}$ of an inch.

The student cannot decide which dimensions to measure for each object.

- Ask the student what measurements she would need if she had to build a replica of the object. The student should start with those dimensions.

Assessment for Learning

What to look and listen for:

- Students can estimate linear measures using referents for both imperial and SI units.
- Students can describe the strategy used to determine a linear measure of an object.
- Students can use a variety of measuring instruments to determine the linear dimensions of an object.
- Students use the correct units and symbols to record measurements on a sketch of an object.

DI Extending Thinking

Have students describe a strategy they could use to determine a dimension of an object that cannot easily be measured with a ruler or measuring tape.

Ask: “How could you estimate the height of the gymnasium?” (*I could measure the height of one cinderblock, count the number of blocks from floor to ceiling, then multiply the number of blocks by the height of one block.*)

Debrief Try This: Assessment for Learning

Have pairs of students who measured the same objects compare their measurements and strategies. Together, they should determine which strategies resulted in the more accurate measures. As a class, discuss students’ findings. Ask:

- What referents did you use to estimate the SI units? (*I used the width of a pencil lead as a referent for the millimetre, the width of my baby finger as a referent for the centimetre, and the width of a door frame as a referent for the metre.*)
- Which objects were easiest to measure? (*Objects that had straight edges*) Which objects were most challenging to measure? (*Round or curved objects*)
- Describe a strategy that could be used to determine the circumference of the base of a plastic cup. (*I used a measuring tape placed flat against the outside of the cup.*)

DI Extending Thinking

Use the question prompts in the screened boxes of the Student Book.

SB, p. 14: How can you measure the perimeter of a can without using the formula for the circumference of a circle?

Wrap a string around the can and cut it to size. Unwrap the string and lay it out flat. Then use a ruler to measure its length.

Assess Your Understanding, SB page 15

Assessment of Learning – selected questions

AI	1.1	1.3	1.4	1.5	1.6
Question number	1	5	5	6	4

Looking Ahead

In Lesson 1.3, students extend their knowledge of imperial and SI units to convert measurements between the two systems.

1.3 Relating SI and Imperial Units

Lesson Organizer

60 – 75 min

Key Math Learnings

Proportional reasoning can be used to convert between imperial units and SI units.

Curriculum Focus

Convert measurements between SI units and imperial units.

SO	AI
M1	1.2
M2	2.1, 2.2, 2.3, 2.4

Processes

C, ME, PS, V

Student Materials

- ruler, metre stick, or measuring tape with both SI and imperial units
- scientific calculator
- Master 1.3 (optional)

Make Connections, SB page 16

In this lesson, students apply proportional reasoning to convert between SI and imperial units. Students again use unit analysis to verify their conversions.

Ask students if they have been driven by car in the United States. Did they notice that road signs in the United States measure distances in miles? Discuss possible strategies for determining which vehicle travelled farther from the border crossing. Students may suggest:

- converting 62 mi. to kilometres, then comparing the distances
- converting 98 km to miles, then comparing the distances

At this point, students probably do not know the relationship between miles and kilometres, so they are not expected to perform any calculations.

Have students list other examples that show how Canada and the U.S. use different units of measure. Since the United States is Canada's neighbour and largest trading partner, being able to convert measures between SI and imperial units is important.

DI Cultural Perspectives

An odometer measures the distance travelled by a vehicle. Odometers on vehicles sold in Canada display the distance in kilometres. Odometers on vehicles sold in the United States display the distance in miles. To import a vehicle from the United States to Canada, the odometer must be converted to indicate units in kilometres rather than miles.

Construct Understanding, SB page 16

To complete the table on page 17 of the Student Book, students need to know the relationships between the units of imperial measure. If necessary, review these relationships before students begin the activity. Distribute copies of Master 1.3 on which students can record their work. If students are having difficulty, allow them to work together.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- How did you estimate the number of centimetres in 1 in.?
(Since the two scales did not align at either end of my ruler, I used the ruler to draw a line segment of length 1 in. I then used the SI scale on the ruler to measure the length of the line segment. It was about 2.5 cm long.
I used two rulers and arranged them so the 0 cm mark on one aligned with the 1 in. mark on the other. Then I recorded the measure in centimetres that coincided with 0 in.)

Assessment for Learning

What to look and listen for:

- Students can use proportional reasoning to convert a length from an imperial unit to an SI unit.
- Students can use proportional reasoning to convert a length from an SI unit to an imperial unit.
- Students can verify a unit conversion using unit analysis.
- Students can solve problems that involve the conversion between SI and imperial units.
- Students can use mental math and estimation to check that a solution is reasonable.

- How did you determine the numbers to complete the table? (*I used the first estimated value in each column and the relationship between the units in each system. For example, I estimated that there are about 2.5 cm in 1 in. Since there are 12 in. in 1 ft., I multiplied to get the approximate number of centimetres in 1 ft:*

$$2.5 \text{ cm} \times 12 = 30 \text{ cm}$$

- How could you use your table to convert between SI and imperial units? (*Suppose I want to convert from metres to inches. I find the relationship between these two units in the table, then multiply. For example, from the table, 1 m is about 40 in. So, to convert 4 m to inches, I multiply:*

$$4 \times 40 \text{ in.} = 160 \text{ in.}$$

There are about 160 in. in 4 m.)

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty completing the table.

- Remind the student of the relationships between the units in both the SI and imperial systems. Then have the student use proportional reasoning to determine each number in the table. Prompt the student. For example, “How many feet are in one yard?” (3)
“So, since 1 ft. \doteq 30 cm, about how many centimetres are in 1 yd.?”
(90 cm; $3 \times 30 \text{ cm} = 90 \text{ cm}$)

The student thinks he has made an error converting between units because he does not get the same answer when he measures the length.

- Remind the student that the numbers in the table are estimates. So, when the table is used to convert between units, the answer is also an estimate. The answer obtained by measuring is likely more accurate.

DI Extending Thinking

Have students use the table and their knowledge of SI and imperial units to estimate 1 mm as a fraction of an inch and 1 km as a fraction of a mile.

Debrief Try This: Assessment for Learning

Invite students to share their tables. Here is an example of what you might see.

Imperial Units to SI Units	SI Units to Imperial Units
1 in. \doteq 2.5 cm	1 cm \doteq $\frac{2}{5}$ in.
1 ft. \doteq 30 cm	1 m \doteq 40 in. 1 m \doteq 3 ft. 4 in. 1 m \doteq 1 yd. 0 ft. 4 in.
1 yd. \doteq 90 cm 1 yd. \doteq 0.9 m	1 km \doteq 1110 yd.
1 mi. \doteq 1584 m	

Ask questions, such as:

- How did you decide if your estimate in the table was close to the actual value?

(In part D, I used the estimated value to convert a measurement from one system to the other. Then I used my ruler to get an actual measurement. I compared my answers. Since my answers were close, my estimate was close to the actual value.)

- Which column did you find easier to complete? Why? *(The first column was easier to complete because the answers could be left as decimals. I found it more difficult to work with fractions in the second column.)*

Direct students' attention to the unit conversion table on page 18 of the Student Book. Suggest students copy this table onto an index card for easy reference. Draw students' attention to the two exact conversions in the margin.

As you work through *Example 4*, point out that an exact conversion might be performed when a distance is small, or when it is a critical measure, as in a truck being able to fit under a bridge. When the distance is large, an approximation is usually sufficient; for example, the distance between Calgary and Edmonton is about 275 km.

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty converting between SI and imperial units.

- The student sets up the wrong proportion when converting between units. Have the student first include the units in the proportion. The numerators should have the same unit and the denominators should have the same unit. Then encourage the student to use algebra to solve for the variable. For example, when converting 6 ft. to centimetres, write:

$$\frac{x \text{ cm}}{6 \text{ ft.}} = \frac{30 \text{ cm}}{1 \text{ ft.}}$$

The student can use unit analysis to verify the solution.

DI Extending Thinking

Use the question prompts in the screened boxes of the Student Book.

SB p. 20, Example 3: What other strategy could you use to determine Alex's height in centimetres?

Since 1 in. \approx 2.5 cm, I could convert Alex's height to inches, then multiply by 2.5.

I could also use the exact conversion. Since 1 in. = 2.54 cm, I could convert Alex's height to inches, then multiply by 2.54.

Assessing Understanding: Discuss the Ideas, SB page 22

Sample Responses

- *I might want to convert to imperial units when I buy a carpet for my bedroom. I know the dimensions of my bedroom in metres, but the dimensions of a carpet are given in feet.*
 - *I might want to convert to SI units when I apply for a Canadian passport. I know my height in feet and inches but the height on the application must be reported in centimetres.*
- For example, one yard is a bit less than 1 m. So, when I convert from yards to metres, the number of metres will be slightly less than the number of yards.*
- I use the conversion factor with the unit in the numerator matching the unit I am converting to, and the unit in the denominator the same as the unit I am converting from.*

Exercises, SB page 22

Assessment of Learning – selected questions

AI	1.2	2.1	2.2	2.3	2.4
Question number	7	10	9	12	11

Sample Response: Reflect, SB page 23

I can use proportional reasoning to convert 7.3 m to feet.

Let x represent the length in feet.

Then set up a proportion using the conversion $1 \text{ ft.} \doteq 0.3 \text{ m}$.

$$\frac{x}{7.3} \doteq \frac{1}{0.3} \quad \text{Multiply each side by 7.3 to solve for } x.$$

$$x \doteq 24.\bar{3} \text{ or } 24\frac{1}{3} \text{ ft.}$$

So, 7.3 m is about $24\frac{1}{3}$ ft.

Students' strategies may also include multiplying by a conversion factor, using unit analysis, and using personal referents. Students should include an example of each strategy named.

Looking Ahead

In Lesson 1.4, students apply their knowledge of imperial and SI units to calculate the surface areas of right pyramids and right cones.

1.4

Surface Areas of Right Pyramids and Right Cones

Lesson Organizer

60 - 75 min

Key Math Learnings

The surface area of a right pyramid is the sum of the areas of the triangular faces and the base. The surface area of a right cone is the sum of the areas of the circular base and the curved surface.

Curriculum Focus

Solve problems involving the surface areas of right pyramids and right cones.

SO	AI
M3	3.1, 3.2, 3.4

Processes

CN, PS, R, V

Student Materials

- ruler with both SI and imperial units
- a right pyramid
- 1-cm and 1-in. grid paper
- scientific calculator
- Master 1.1b (*optional*)
- Master 1.1c (*optional*)
- Masters 1.4, 1.5, 1.6 (*optional*)

Vocabulary

right pyramid, apex, slant height, tetrahedron, regular tetrahedron, lateral area, right cone

Make Connections, SB page 26

In this lesson, students apply what they learned about the surface areas of right rectangular prisms and cylinders in grades 8 and 9 to right pyramids and cones. You may wish to use Masters 1.1b and 1.1c to activate prior learning about the surface areas of right rectangular prisms and cylinders, and the Pythagorean Theorem.

Students will be calculating the surface areas of objects whose dimensions are given in imperial units. The opening context gives students an opportunity to take a closer look at one of the seven wonders of the ancient world and to apply the math they are studying to a real-life problem.

The pyramids at Giza are one of the world's most fascinating and mysterious architectural achievements. Archeologists still wonder how the ancient Egyptians accomplished the task of covering the surface of the Great Pyramid with a limestone casing. Discuss possible strategies for determining the area that was once covered with limestone. Ensure students recognize that since the base would not have been covered with limestone, its area is not included. At this point, students are not expected to perform any calculations. However, they do know the formula for the area of a triangle and may suggest calculating the areas of the 4 triangular faces.

Construct Understanding, SB page 26

For this activity, have square, rectangular, and triangular right pyramids available. If you do not have access to these pyramids, use copies of Masters 1.4 – 1.6 to construct pyramids before the lesson. Enlargements or reductions of the masters can be made to get pyramids of different sizes. Suggest students work in groups of 4. Two students can use imperial units and two students can use SI units. Some students may require 1-cm and 1-in. grid paper.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- What strategy did you use to estimate the surface area of your pyramid? (*We worked with a square pyramid. Since all the triangular faces are congruent, we traced a triangular face and the base on 1-cm grid paper. We counted the number of squares and part squares in the triangular face, then multiplied by 4 to get the area of all the triangular faces. We counted the number of squares and part squares in the base. Then, we added to estimate the surface area of the pyramid.*)
- Which measurements did you have to make to calculate the surface area of the pyramid? (*We had to measure the length and width of the base and the height of each triangular face.*)

Assessment for Learning

What to look and listen for:

- Students can determine the surface area of a right pyramid or cone.
- Students can use the Pythagorean Theorem to determine the slant height of a right pyramid or cone.
- Students can determine an unknown dimension of a right pyramid or cone given its surface area and remaining dimensions.
- Students can solve problems involving the surface area of right pyramids and cones.
- Students can sketch a diagram to represent a problem that involves the surface area of a right pyramid or cone.

DI Identifying Common Difficulties: Possible Intervention

The student uses the height of the pyramid as the height of a triangular face.

- Have the student trace the triangular face on grid paper and measure the height of the triangle. The student should recognize that the height of the triangle is not the same as the height of the pyramid.

DI Extending Thinking

Have students determine the surface area of a hexagonal or octagonal pyramid. Students can apply the strategies used in *Try This*, but the calculations needed to calculate the area of the base may be more complicated.

Debrief Try This: Assessment for Learning

Invite students to share their strategies for calculating the surface area of a pyramid. Ask questions such as:

- How did you determine the area of the base of the pyramid?
(The base was a rectangle, so I measured the length and width of the rectangle, then multiplied.)
- How did you determine the area of a triangular face?
(I measured the base and height of the triangle, then multiplied one-half the base times the height.)
- How can you use congruent faces when you calculate the surface area of a pyramid?
(Congruent faces have the same area. Depending on the base, some or all of the triangular faces may be congruent. So, I can calculate the area of one triangular face, then multiply by the number of faces that are congruent. For example, a rectangular pyramid has 2 pairs of congruent triangular faces. I calculate the area of each type of triangular face, then multiply each area by 2.)
- Did the strategy you used to calculate the surface area depend on the type of pyramid?
(No, the surface area of any pyramid is the sum of the areas of the base and the triangular faces.)

As you work through the examples involving a right pyramid, ensure students understand the difference between the height of a pyramid and its slant height. The height of an object always refers to its perpendicular height. Fold a copy of a net of the square pyramid on Master 1.4. Insert a pencil through a hole in the centre of the base until it reaches the apex. The pencil is vertical and is perpendicular to the base. This is the height of the pyramid. Remove the pencil and place it along the surface of a triangular face from the midpoint of the base to the apex. The pencil is on a slant. This is the slant height of the pyramid. Ensure students can use the Pythagorean Theorem to calculate the height of a triangular face given the height and the appropriate base dimension, or given the edge length from the apex to the base, and the appropriate base dimension.

Ensure students realize that the formula on Student Book page 31 for the surface area of a right pyramid can only be applied to a pyramid with a regular polygon base. This formula cannot be used to calculate the surface area of a rectangular pyramid.

To introduce the surface area of a cone, sketch a square, a regular octagon, a regular dodecagon, and so on, on the board. Students should see that, as the number of sides of a regular polygon increases, the polygon looks more and more like a circle. So, we can think of a right cone as a pyramid with a circular base. This thinking is critical to the understanding of the derivation of the formula for the surface area of a cone.

Technology Note: Ensure students know how to use the square root key on their calculators. To obtain the greatest possible accuracy, ensure students use the π key on their calculators instead of a decimal approximation, and suggest they avoid approximating a square root until the last possible step. For example, in *Example 3*, $\sqrt{53}$ is not calculated until the final calculation is performed.

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty visualizing and keeping track of all the dimensions of a right pyramid.

- Encourage the student to sketch a net of the pyramid, then label the net with the dimensions given. When the slant height has to be calculated, suggest the student sketch a labelled right triangle before using the Pythagorean Theorem to write, then solve an equation to calculate the slant height.

DI Extending Thinking

Use the question prompts in the screened boxes of the Student Book.

<p>SB p. 29, Example 2: What is an advantage of using $EH = \sqrt{272}$ and $GE = \sqrt{281}$, instead of writing these square roots as decimals?</p>	<p><i>The square roots are not written as decimals to avoid unnecessary calculations and to result in greater accuracy. If the approximate decimal equivalents had been used, the area of each triangle would have been less accurate as would the surface area of the pyramid.</i></p>
<p>SB p. 32, Example 3: In <i>Example 3</i>, which part of the formula represents the lateral area of the cone? How do you know?</p>	<p><i>The surface area of a cone is the sum of the lateral area and the area of the base. The base of a cone is a circle and the formula for the area, A, of a circle is: $A = \pi r^2$ So, πrs represents the lateral area of the cone.</i></p>

Assessing Understanding: Discuss the Ideas, SB page 34

Sample Responses

1. *To determine the surface area of a right pyramid, I calculate the sum of the areas of the triangular faces (the lateral area) and the base. When the right pyramid has a regular polygon base, I can use the formula:
 Surface area = $\frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$, where s represents the slant height.*

2. The height is the distance from the centre of the base to the apex of the pyramid. The height forms a 90° angle with the base of the pyramid. The slant height is the distance from the midpoint of the base of a triangular face to the apex of the pyramid.
3. To calculate both the surface area of a right pyramid and a right cone, I determine the lateral area and the area of the base. The lateral area of a right pyramid is the sum of the areas of its triangular faces, so I can use the formula for the area of a triangle to calculate the lateral area. The lateral area of a cone is a curved surface, so I use the formula developed in this lesson:

$$A_L = \pi rs$$

Exercises, SB page 34

Remind students that a sketch showing all the given information should be the first step in the solution of any problem. Encourage students to think about the reason for calculating the surface area, then decide which faces are involved. Students should ask themselves, “Should the area of the base be included in the surface area calculation?”

Assessment of Learning – selected questions

AI	3.1	3.2	3.4
Question number	9	13	16

DI Cultural Perspectives

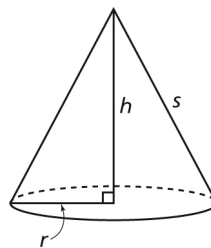
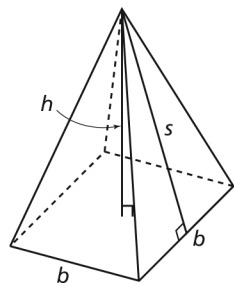
A powwow is a traditional practice in some First Nations cultures. Jingle dress dancers have small cone-shaped tin jingles sewn onto their dresses to create a jingling noise. Students could use the Internet or the library to find the dimensions of a jingle and determine its surface area.

Sample Response: Reflect, SB page 35

To calculate the surface area of a right pyramid, I need to know the side lengths of the base and the slant heights, s , of the triangular faces.

To calculate the surface area of a right cone, I need to know the radius, r , of the circular base and the slant height, s , of the cone.

If the slant height is not given, I can use the height, h , and the known length in the base of the pyramid or cone, and apply the Pythagorean Theorem to determine the slant height.



Looking Ahead

In Lesson 1.5, students extend their knowledge of right pyramids and right cones to solve problems involving the volumes of right pyramids and right cones.

Volumes of Right Pyramids and Right Cones

Lesson Organizer

60 – 75 min

Key Math Learnings

Right pyramids and right cones are related to right prisms and right cylinders with the same base and height. We can use these relationships to determine the volumes of right pyramids and right cones.

Curriculum Focus

Solve problems involving the volumes of right pyramids and right cones.

SO	AI
M3	3.1, 3.3, 3.4, 3.6

Processes

CN, PS, R, V

Student Materials

- right prism, pyramid, cylinder, and cone (with removable lids)
- container of sand
- empty container
- scientific calculator
- Master 1.1d (*optional*)

Vocabulary

volume, capacity

Make Connections, SB page 36

In this lesson, students investigate the relationship between a right pyramid and a right prism, and between a right cone and a right cylinder. The relationships students discover are used to write formulas for the volumes of a right pyramid and a right cone. In grade 8, students determined the volumes of right prisms and right cylinders. You may wish to use Master 1.1d to activate prior learning about the volumes of these objects.

Use the photo at the top of Student Book page 36 to activate prior learning about the volumes of a right prism and a right cylinder. Students should realize that the volume of any prism or cylinder is $\text{base area} \times \text{height}$. The difference is the base of a prism is a polygon and the base of a cylinder is a circle. Have students share their ideas about how right pyramids and right cones are related to right prisms and right cylinders.

Point out that the amount of liquid or gas an object contains is measured as capacity. The amount of a solid is measured as volume.

Construct Understanding, SB page 36

If the supplies required for *Try This* are limited, the activity could be done as a class demonstration. A second option is to modify the activity and have groups of students construct a cone from plain paper that has a base and height equal to that of a tin can.

If you do not have access to sand, items such as rice, salt, or sugar could be used instead. Ensure students predict the relationship between the volumes before completing the activity each time.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- What strategy did you use to fill the pyramid exactly? (*We filled the pyramid with sand, then used a straight edge to level it off.*)
- Why was it important not to overfill the pyramid? (*If the pyramid was overfilled, the volume of the pyramid would be an overestimate. This would affect the number of pyramids needed to fill the prism.*)

D1 Identifying Common Difficulties: Possible Intervention

The student finds that the number of full pyramids needed to fill the prism is not a whole number.

- Although the pyramid and prism have equal bases and equal heights, the thickness of the plastic used to make the objects may be different. Measurement errors must also be considered.

Assessment for Learning

What to look and listen for:

- Students can sketch a diagram to represent a problem that involves volume.
- Students can determine the volumes of a right cone, cylinder, prism, and pyramid.
- Students can describe the relationship between the volumes of a right cone and a right cylinder, and a right pyramid and a right prism, with the same base and height.
- Students can determine an unknown dimension of a right cone, cylinder, prism, or pyramid given its volume and remaining dimensions.
- Students can solve problems involving the volumes of right prisms, pyramids, cylinders, and cones.

DI Extending Thinking

Ask students to think about how the volumes of a prism and pyramid are related when they have equal heights but the area of the base of the prism is double the area of the base of the pyramid.

(The volume of the prism is six times the volume of the pyramid.)

Debrief Try This: Assessment for Learning

Invite groups of students to share the relationships they discovered between the volumes. Ask questions such as:

- How many full pyramids did it take to fill the prism? *(About 3)*
- How many full cones did it take to fill the cylinder? *(About 3)*
- How can this relationship be used to determine the volume of a cone given the volume of a cylinder with an equal base and equal height? *(I can divide the volume of the cylinder by 3 to get the volume of the cone.)*

As you work through the *Examples*, ensure students understand that when the height is not given, the slant height, a length in the base of the pyramid or cone, and the Pythagorean Theorem can be used to determine the height.

In *Example 4*, ensure students understand that to solve for r , we take the square root of both sides of the equation. To ensure that the numerator of the fraction is divided by 4π and not just by 4, put the denominator in brackets when entering it into the calculator.

Ask questions such as:

- Does the formula $V = \frac{1}{3}\pi r^2 h$ apply to all cones? Explain.
(Yes, the base of a cone is always a circle with area πr^2 .)
- Does the formula $V = \frac{1}{3}lwh$ apply to all pyramids? Explain.
(No, this formula only applies to a pyramid whose base is a square or a rectangle. For any other pyramid, I would calculate the area of the base, then multiply by the height and by $\frac{1}{3}$.)

DI Identifying Common Difficulties: Possible Intervention

The student gets an incorrect answer for the volume when the height is determined before the volume can be calculated.

- Remind the student to leave the height as a square root to achieve greater accuracy. The decimal equivalent of a square root is often an approximation. If this approximation is rounded, then used in further calculations, it could result in the volume being less accurate.

DI Extending Thinking

Use the question prompts in the screened boxes of the Student Book.

SB p. 40, Example 3: How can you determine if your answer is reasonable?

I can substitute numbers that are easier to work with into the formula; for example, $\pi \doteq 3$ and $h \doteq 20$, then use mental math:

$$V \doteq \frac{1}{3}(3)(6)^2(20).$$

The volume is about 720 in^3 . Since 679 is close to 720, the answer is reasonable.

Assessing Understanding: Discuss the Ideas, SB page 41

Sample Responses

- The volume of a right pyramid is one-third the volume of a right prism with the same base and the same height. The volume of a right cone is one-third the volume of a right cylinder with the same base and the same height.*
- Both formulas for the volumes of a right cone and a right pyramid are:*
$$\text{Volume} = \frac{1}{3}(\text{base area})(\text{height})$$
For the cone, the base area is the area of a circle. For the pyramid, the base area is the area of a polygon.
- I could calculate the volume of the related right cylinder or prism by multiplying the area of the base by the height. I could then divide this volume by 3 to get the volume of the cone or pyramid.*

Exercises, SB page 42

Remind students that to obtain the greatest possible accuracy, they should always use the π key on their calculators, and *not* its approximate value 3.14.

The context of question 20, an underground tank used to collect water runoff, illustrates the role of mathematics in construction and industry.

Assessment of Learning – selected questions

AI	3.1	3.3	3.4	3.6
Question number	11	8	18	7

Sample Response: Reflect, SB page 44

Each of a cone and pyramid has one base and an apex. The base of a cone is a circle. The base of a pyramid is a polygon. The volume of a cone is one-third the volume of the related cylinder, and the volume of a pyramid is one-third the volume of the related prism. Because the base of a cone is a circle, the formula for the volume of a cone contains πr^2 , the area of a circle. Because the base of a pyramid can be any type of polygon, the formula for the volume of a pyramid depends on the shape of its base.

Looking Ahead

In Lesson 1.6, students extend their knowledge of surface area and volume to include the sphere. Each pair of students will need an orange.

1.6

Surface Area and Volume of a Sphere

Lesson Organizer

60 - 75 min

Key Math Learnings

The formulas for the surface area and volume of a sphere can be developed from the formulas of other 3-D objects.

Curriculum Focus

Solve problems involving the surface area and volume of a sphere.

SO	AI
M3	3.2, 3.3, 3.4

Processes

CN, PS, R, V

Teacher Materials

- demonstration
compass and ruler
(*optional*)

Student Materials

- an orange
- calipers or a ruler
- a compass
- scientific calculator
- Master 1.18 (*optional*)

Vocabulary

sphere, radius, diameter, hemisphere

Make Connections, SB page 45

In this lesson, students develop the formulas for the surface area and volume of a sphere. Ask students to list real-life examples of spheres. Examples might include a lacrosse ball, a marble, a basketball, a bubble, a grapefruit, and a planet.

The gas storage sphere shown in the picture on Student Book page 45 is used to store highly combustible gas that is produced in the treatment of waste water from the homes and businesses in Winnipeg. The gas is used as fuel in boilers to heat the treatment plant. Invite students to share their strategies for estimating the volume of gas the sphere could hold. For example, students may say that the volume of gas in the sphere is about one-half the volume of a cube whose edge length is equal to the diameter of the sphere.

Construct Understanding, SB page 45

Before students begin the activity, demonstrate how to set a compass to a given length. Remind students to think about the measurement strategies they used in Lesson 1.2 to measure the perimeter of a can or bottle. Ensure students remember how to calculate the mean: add the diameters, then divide the sum by 3. Encourage pairs of students to work with oranges of different sizes so students understand that the relationship discovered is independent of the size of the sphere. Ensure the oranges are close to spherical and not “flattened” oranges, such as tangerines.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- How did you measure the diameter of the orange? (*I closed the jaws of the calipers lightly on the orange, then read the scale.*)
- What other strategy could you use to determine the diameter? (*I could place the orange on a ruler, then find the difference between the greater measure and the lesser measure on the ruler.*)
- What is the advantage of using smaller pieces of orange peel to fill the circles? (*Smaller pieces fit more closely together, so my answer will be more accurate.*)

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty calculating the mean diameter.

- If the student is using a calculator, make sure she adds the diameters before dividing, or inserts the sum of the diameters in brackets. It is a common error to input numbers such as $8 + 7.5 + 8.5 \div 3$, where only the last number is divided by 3.

Assessment for Learning

What to look and listen for:

- Students can calculate the surface area and volume of a sphere.
- Students can use the surface area of a sphere to determine its diameter or radius.
- Students can solve problems involving the surface area or volume of spheres.

DI Extending Thinking

Ask students to think about how the surface area of their orange would change if its diameter were halved.

(The surface area would be one-quarter its original surface area.)

Debrief Try This: Assessment for Learning

Invite students to share the number of circles filled and the formulas they created. Ask questions such as:

- Why do you think the mean diameter of the orange was used?
(An orange only approximates a sphere and the diameter of the orange is difficult to measure accurately. So, the mean diameter was used to reduce measurement errors and any variations caused by the shape of the orange.)
- How did you calculate the surface area of your orange?
(We multiplied the area of one circle by four because 4 circles were filled with peel.)
- How did you calculate the area of a circle? *(We divided the mean diameter by 2 to get the radius of the circle. We then used the formula for the area, A , of a circle: $A = \pi r^2$.)*
- What strategy did you use to create a formula for calculating the surface area of a sphere? *(Since we were able to cover about 4 circles with orange peel, the surface area of a sphere is about 4 times the area of a circle whose radius is equal to the radius, r , of the sphere. Since a circle has area πr^2 , the surface area of a sphere is about $4\pi r^2$.)*

As you work through the *Examples* involving surface area, remind students of the relationship between radius and diameter. To prompt a discussion after *Example 1*, ask, “Are all sports balls measured in imperial units?” In *Example 2*, explain that to divide by 4π and not just by 4, we place 4π in brackets when entering it into the calculator.

In *Example 3*, scientific notation is used to display a large number. Make sure students can read a number displayed in scientific notation on their calculators. You may wish to use Master 1.18 on the CD-ROM to review scientific notation.

Students who are kinesthetic learners may benefit from investigating the volume of a sphere. For example, you will need a tennis ball, a ruler, an empty 355-mL frozen juice can, scissors, water, and an overflow container.

Measure the diameter of a tennis ball.

Cut an empty 355-mL frozen juice can so that its inside height is equal to the diameter of the ball.

Place the can in an overflow container, then fill the can with water.

Place the tennis ball, soaked in water, into the can.

Remove the ball from the can, then empty the can of water.

Pour the water from the overflow container into the can. The volume of this water is equal to the volume of the tennis ball.

Measure the height of the water.

Use the results to relate the volume of the sphere to the volume of the can (a cylinder).

(Volume of a sphere = $\frac{2}{3}$ the volume of the cylinder that encloses it)

DI Identifying Common Difficulties: Possible Intervention

The student thinks that the surface area of a hemisphere is one-half the surface area of a sphere.

- Have the student peel an orange, then divide it through its centre to create two congruent halves. The student should notice that by dividing the orange in half, a circular face is exposed. So, the surface area of the hemisphere is one-half the surface area of the sphere plus the area of the exposed circular face.

The student uses the formula for surface area to calculate the volume of the sphere.

- Remind the student that the formula for surface area contains r^2 since it is an area calculation, and area is two-dimensional. The formula for volume contains r^3 since it is a volume calculation, and volume is three-dimensional.

DI Extending Thinking

Use the question prompts in the screened boxes of the Student Book.

<p>SB p. 47, Example 1: What happens to the surface area of a sphere when its radius is doubled?</p>	<p><i>I calculated the surface area of a sphere with radius 1 cm. I then doubled the radius to 2 cm and calculated the surface area.</i></p> $SA = 4\pi(1)^2 \quad SA = 4\pi(2)^2$ $= 4\pi \quad = 16\pi$ <p><i>This shows that when the radius is doubled, the surface area is $\left(\frac{16\pi}{4\pi}\right)$ or 4 times the original surface area.</i></p>
<p>SB p. 49: Why is a globe constructed from two hemispheres?</p>	<p><i>It is impossible to create a hollow sphere. Molds are used to create two congruent hemispheres, which are then glued together.</i></p>
<p>SB p. 50, Example 4: In Example 4a, why can we write $2\pi r^2 + \pi r^2$ as $3\pi r^2$?</p>	<p><i>$2\pi r^2$ and πr^2 are like terms; each contains the constant π and the variable r^2. The term πr^2 can be thought of as $1\pi r^2$. Just as we can combine the like terms $2x + x$ to get $3x$, we can combine $2\pi r^2$ and πr^2 to get $3\pi r^2$.</i></p>

Assessing Understanding: Discuss the Ideas, SB page 50*Sample Responses*

1. *The surface area of a sphere is $4\pi r^2$. The area of the circular face on one hemisphere is πr^2 . Divide the surface area of the sphere by the area of the circular face: $\frac{4\pi r^2}{\pi r^2} = 4$. So, the surface area of the sphere is 4 times the area of the circular face on one hemisphere.*

2. The formula for the volume, V , of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

The formula for the volume, V , of a cylinder is:

$$V = \pi r^2 h$$

The volume, V , of a cylinder with height equal to the diameter of the sphere, $2r$, is:

$$V = \pi r^2(2r) \text{ or } V = 2\pi r^3.$$

So, the volume, V , of the sphere is $\frac{2}{3}$ the volume of a cylinder with the same base radius and height. To determine the volume of a sphere, I could multiply the volume of the enclosing cylinder by two-thirds.

Exercises, SB page 51

Remind students that to obtain the greatest possible accuracy, they should use the π key on their calculators instead of 3.14. Encourage students to estimate to check that their solutions are reasonable.

Assessment of Learning – selected questions

AI	3.2	3.3	3.4
Question number	6	4	9

DI Cultural Perspectives

An “iglu” is the Inuit word for house. Many igloos are made from blocks of snow and are shaped like hemispheres. Inuit use math when they build an igloo. Volume and surface area are two calculations that might be made before an igloo is built.

Sample Response: Reflect, SB page 52

To remember the formula for the surface area of a sphere, I think back to the activity I did with an orange. Since I was able to fill 4 circles with orange peel, I know that the surface area of a sphere is 4 times the area of a circle with the same radius. So, the formula for the surface area, SA , of a sphere is:

$$SA = 4\pi r^2$$

To remember the formula for the volume of a sphere, I start with the formula for the surface area of a sphere, then make these adjustments. Since volume is three dimensional, I know the variable must be r^3 . I remember that when I change the exponent to 3, I also divide by 3. So, the formula for the volume, V , of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

Looking Ahead

In Lesson 1.7, students use their knowledge of the surface areas and volumes of right prisms, right pyramids, right cylinders, right cones, and spheres to solve problems involving the surface areas and volumes of composite objects. Students calculate the radius of a sphere, given its volume, in Chapter 4, as an application of the exponent laws and the relationship between a cube root and the exponent $\frac{1}{3}$.

1.7

Solving Problems Involving Objects

Lesson Organizer

60 - 75 min

Key Math Learnings

The formulas for surface area and volume of right prisms, pyramids, cylinders, cones, spheres, and hemispheres can be used to determine the surface area and volume of composite objects.

Curriculum Focus

Solve problems involving the surface area and volume of composite objects.

SO	AI
M3	3.1, 3.4, 3.5

Processes

CN, PS, R, V

Student Materials

- scientific calculator

Vocabulary

composite object

Make Connections, SB page 55

In this lesson, students determine the surface area and volume of composite objects made from right prisms, right pyramids, right cylinders, right cones, spheres, and hemispheres. Students will use strategies similar to those used in grade 9 when they determined the surface areas of composite objects made from right prisms and right cylinders.

A grain bin is used to store and dry grain after it has been harvested. The grain may be stored as winter feed for livestock or until it is transported to local markets or sold to grain dealers. Invite students to share their strategies for calculating the amount of material needed to build the bin. Discuss whether the area of the floor of the bin should be included.

Some students may suggest adding the surface area of the cylinder and the surface area of the cone, then subtracting the area of overlap. Other students may suggest adding the lateral area of the cone, the curved surface area of the cylinder, and the area of the base of the cylinder. This strategy does not require students to consider any overlap.

Construct Understanding, SB page 55

Assessing Understanding: Observe and Listen

As students work, prompt them with questions such as:

- Which objects make up the grain bin? (*A cylinder and a cone*)
- How can you determine how much grain the bin can hold?
(*I can calculate the volume of the bin.*)
- What strategy will you use to calculate the volume of the bin? (*I will add the volume of the cylinder and the volume of the cone.*)
- How will you calculate the volume of the cylinder?
(*I will use the formula $V = \pi r^2 h$.*)
- How will you determine the volume of the cone?
(*I will use the formula $V = \frac{1}{3} \pi r^2 h$.*)
- How will you determine how many truckloads are required?
(*I will divide the volume of the grain bin by the capacity of the grain truck.*)
- How could you check that your solution is reasonable?
(*I could use mental math and numbers that are easy to work with in my volume calculations, and I could divide by 500 to determine how many truckloads are required. If my answers are close, I know my solution is reasonable.*)

DI Identifying Common Difficulties: Possible Intervention

The student substitutes the measure of the diameter for r in the volume formulas.

- Remind the student that the radius is one-half the diameter.

Assessment for Learning

What to look and listen for:

- Students can recognize the objects that make up a composite object and identify any overlapping areas.
- Students can calculate the surface area of a composite object.
- Students can calculate the volume of a composite object.
- Students can determine an unknown dimension of a cylinder given its surface area and remaining dimensions.
- Students can solve problems involving the surface area and volume of composite objects.

The student does not know whether to round the number of truckloads up or down.

- Ensure the student understands that to fill the grain bin, we round the number of truckloads required up. There will be barley left in the truck when the bin is full. However, if the farmer does not want any leftover barley, she will round the number of truckloads down, and the grain bin will not be full.

DI Extending Thinking

Have students determine the surface area of the grain bin.
(About 1859 ft.^2 including the base; about 1545 ft.^2 excluding the base)

Debrief Think About It: Assessment for Learning

Invite students to share their solutions. Have a volunteer record her or his solution on the board. The solution should include a labelled sketch. What you might see:

The bin comprises a right cylinder and a right cone.

For the volume of the right cylinder:

Use the formula: $V = \pi r^2 h$

Substitute: $r = 20 \div 2 = 10$, $h = 19$

$$V = \pi(10)^2(19)$$

$$V = 5969.0260 \dots$$

For the volume of the right cone:

Use the formula: $V = \frac{1}{3}\pi r^2 h$

Substitute: $r = 10$, $h = 5$

$$V = \frac{1}{3}\pi(10)^2(5)$$

$$V = 523.5987 \dots$$

Volume of the grain bin is about:

$$5969.0260 \dots + 523.5987 \dots = 6492.6248 \dots$$

To calculate the number of truckloads required, divide by 550:

$$6492.6248 \dots \div 550 = 11.8047 \dots$$

So, about 12 truckloads are required to fill the bin.

Have a volunteer use estimation strategies to check that the solution is reasonable. For example, the student may substitute $\pi = 3$, $r = 10$, and $h = 20$ to estimate the volume of the bin.

What you might see:

$$V = \pi r^2 h + \frac{1}{3}\pi r^2 h$$

$$V = 3(10)^2(20) + \frac{1}{3}(3)(10)^2(5)$$

$$V = 6000 + 500$$

$$V = 6500$$

To estimate the number of truckloads required, divide by 500.

$$6500 \div 500 = 13$$

So, about 13 truckloads are required.

Since 13 is close to 12, my solution is reasonable.

Ensure students understand that to attain the greatest accuracy possible, they should retain all the decimal places in the calculated numbers, or do not calculate until the last step in the solution.

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty identifying overlapping faces when presented with a surface area problem.

- Suggest the student sketch a net for each object in the composite object, then colour the areas of overlap.

DI Extending Thinking

Have students work in pairs. Students sketch composite objects that comprise more than 2 objects. The sketch should include the necessary dimensions but should not identify the names of the objects. Students trade sketches and calculate the surface area and volume of their partner’s composite object.

Assessing Understanding: Discuss the Ideas, SB page 59

Sample Responses

1. *I imagine that I am painting the object. The faces that comprise the surface area of the object are the faces that I would paint. These are the exposed faces.*
2. *I might use the Pythagorean Theorem to determine the slant height when given the height and the base radius of a cone (for a surface area calculation) and to determine the height when given the slant height and the dimensions of the base (for a volume calculation). To check that my answer is reasonable, I use mental math and estimation strategies. When my answer is close to the estimate, I know my answer is reasonable.*

Exercises, SB page 59

To this point, students have added volumes to calculate the volume of a composite object. Exercises 8, 10, and 13 involve subtraction as students determine the volume of an object after another object has been removed.

Assessment of Learning – selected questions

AI	3.1	3.4	3.5
Question number	7	6	9

DI Cultural Perspectives

There are many examples of composite objects in architecture. For example, Henderson’s round barn was built in 1898 in Rabbit Hill, Alberta, and later moved to Fort Edmonton Park. This barn approximates a cylinder with a cone-shaped roof. Have students research other examples of composite objects in architecture and identify the objects from which they are made.

Sample Response: Reflect, SB page 61

I find it easier to calculate the volume of a composite object because I do not have to think about areas of overlap. To determine the volume, I identify the objects that make up the composite object, then calculate the sum or difference of their volumes.

Master 1.1a Activate Prior Learning: SI Units of Length

SI units of length include the metre, the centimetre, and the millimetre. Conversion factors are used to convert a measure from one unit to another. This table shows the relationships among some of the units of length.

1 m = 100 cm 1 m = 1000 mm
1 cm = 0.01 m 1 cm = 10 mm
1 mm = 0.001 m 1 mm = 0.1 cm

- To convert from metres to centimetres

Convert 7.3 m to centimetres.

Since 1 m = 100 cm, to convert metres to centimetres, multiply by 100.

$$7.3 \text{ m} = 7.3(100 \text{ cm})$$

$$7.3 \text{ m} = 730 \text{ cm}$$

- To convert from centimetres to metres

Convert 225 cm to metres.

Since 1 cm = 0.01 m, to convert centimetres to metres, multiply by 0.01.

$$225 \text{ cm} = 225(0.01 \text{ m})$$

$$225 \text{ cm} = 2.25 \text{ m}$$

Check Your Understanding

1. Which SI unit of length is the most appropriate unit to measure each item?

Justify your choice.

a) the width of the hallway

b) the height of the teacher's desk

c) the length of a paperclip

d) the length of an eyelash

2. Estimate each measurement in SI units.

a) the length of your shoe

b) the height of the classroom door

c) the thickness of a nickel

d) the width of an eraser

3. Convert each measure to centimetres.

a) 9 m

b) 36 mm

c) 47.3 m

d) 253 mm

4. Convert each measure to metres.

a) 845 cm

b) 4286 mm

c) 95 cm

d) 1100 mm

5. Convert each measure to millimetres.

a) 3.5 cm

b) 2.7 m

c) 43 cm

d) 0.9 m

Master 1.1b Activate Prior Learning: Surface Areas of Right Rectangular Prisms and Cylinders

The surface area of a right rectangular prism is the sum of the areas of its faces. Since opposite faces are congruent, this formula can be used to calculate the surface area:
 Surface area = $2 \times$ area of top face + $2 \times$ area of the front face + $2 \times$ area of side face

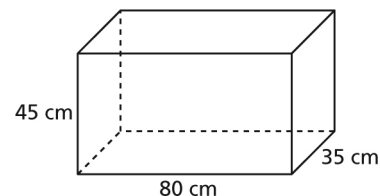
- To determine the surface area, SA , of this right rectangular prism, substitute the given measurements into the formula above.

$$SA = (2 \times 35 \times 80) + (2 \times 80 \times 45) + (2 \times 35 \times 45)$$

$$SA = 5600 + 7200 + 3150$$

$$SA = 15\,950$$

The surface area of the right rectangular prism is $15\,950 \text{ cm}^2$.



The surface area of a right cylinder is the sum of the areas of the 2 circular bases and the curved surface. Since the two bases are congruent, this formula can be used to calculate the surface area.

Surface area = $2 \times$ area of one circular base + circumference of base \times height of cylinder

$$SA = 2\pi r^2 + 2\pi rh$$

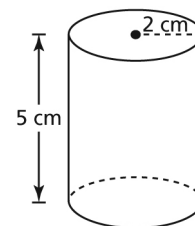
- To determine the surface area, SA , of this right cylinder:

$$SA = 2\pi r^2 + 2\pi rh$$

$$SA = (2 \times \pi \times 2^2) + (2 \times \pi \times 2 \times 5)$$

$$SA = 87.9645 \dots$$

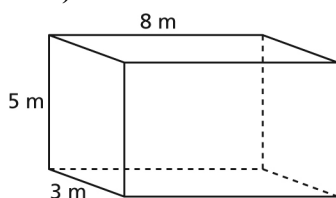
The surface area of the right cylinder is approximately 88 cm^2 .



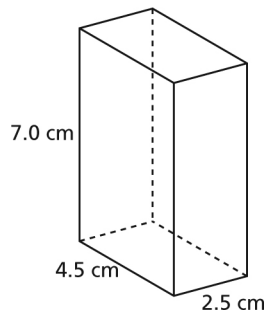
Check Your Understanding

- Determine the surface area of each right rectangular prism.

a)

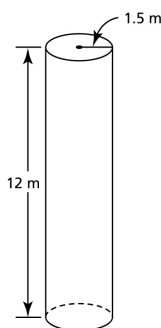


b)

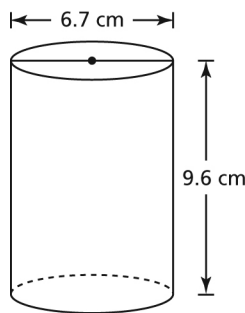


- Determine the surface area of each right cylinder to the nearest square unit.

a)



b)



Master 1.1c Activate Prior Learning: The Pythagorean Theorem

The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

We write: $a^2 + b^2 = c^2$

- To determine the length of the hypotenuse when we know the lengths of the legs,

substitute for a and b in the equation $a^2 + b^2 = c^2$.

Substitute: $a = 3$ and $b = 5$

$$3^2 + 5^2 = c^2$$

$$9 + 25 = c^2$$

$$34 = c^2$$

$$c = \sqrt{34} \quad \text{Use a calculator.}$$

$$c = 5.8309 \dots$$

The length of the hypotenuse to the nearest tenth of a centimetre is 5.8 cm.

- To determine the length of a leg when we know the lengths of the other leg and the hypotenuse,

substitute for a and c in the equation $a^2 + b^2 = c^2$.

Substitute: $a = 1$ and $c = 2$

$$1^2 + b^2 = 2^2$$

$$1 + b^2 = 4 \quad \text{Solve for } b.$$

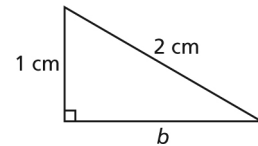
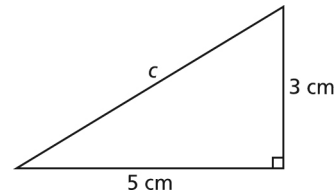
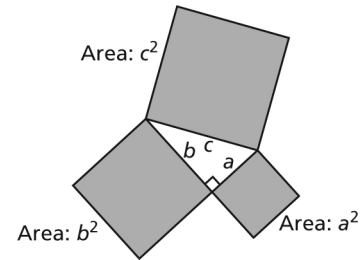
$$b^2 = 4 - 1$$

$$b^2 = 3$$

$$b = \sqrt{3} \quad \text{Use a calculator.}$$

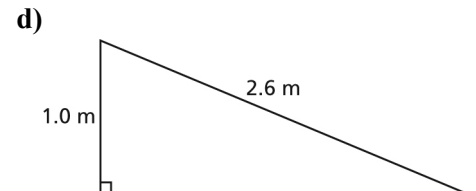
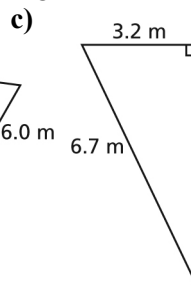
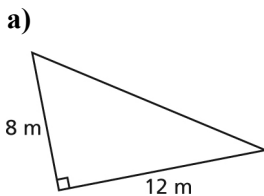
$$b = 1.7320 \dots$$

The length of the leg to the nearest tenth of a centimetre is 1.7 cm.



Check Your Understanding

1. Determine the unknown length in each right triangle to the nearest tenth of a metre.



Master 1.1d Activate Prior Learning: Volumes of Right Rectangular Prisms and Cylinders

The volume of a prism is the space occupied by the prism.

A formula for the volume of a right rectangular prism is:

Volume = area of base \times height

Let A represent the base area and h represent the height.

Then, the volume of a right rectangular prism is:

$$V = A \times h$$

- To determine the volume of this right rectangular prism:

The base is a rectangle with length 4.0 m and width 6.0 m.

$$A = 4.0 \times 6.0$$

$$A = 24.0$$

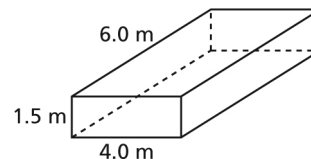
The area of the base is 24.0 m².

$$V = A \times h$$

$$V = 24.0 \times 1.5$$

$$V = 36.0$$

The volume of the right rectangular prism is 36 m³.



A formula for the volume of a right cylinder is:

Volume = base area \times height

The base is a circle with area πr^2 .

Let h represent the height.

Then, the volume of a cylinder is:

$$V = \pi r^2 \times h$$

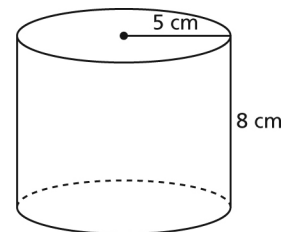
- To determine the volume of this cylinder:

$$V = \pi r^2 \times h$$

$$V = \pi(5)^2 \times 8$$

$$V = 628.3185 \dots$$

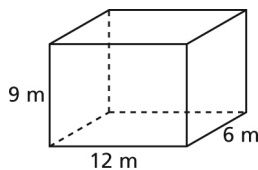
The volume of the right cylinder is approximately 628 cm³.



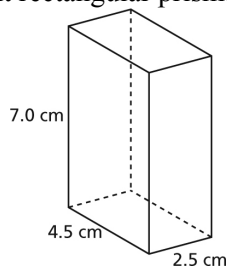
Check Your Understanding

- Determine the volume of each right rectangular prism.

a)

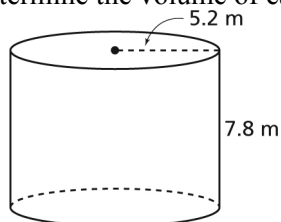


b)

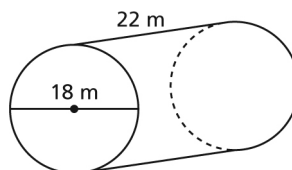


- Determine the volume of each right cylinder to the nearest square unit.

a)



b)



Name: _____

Date: _____

Master 1.2 Recording Sheet for Lesson 1.1 Try This

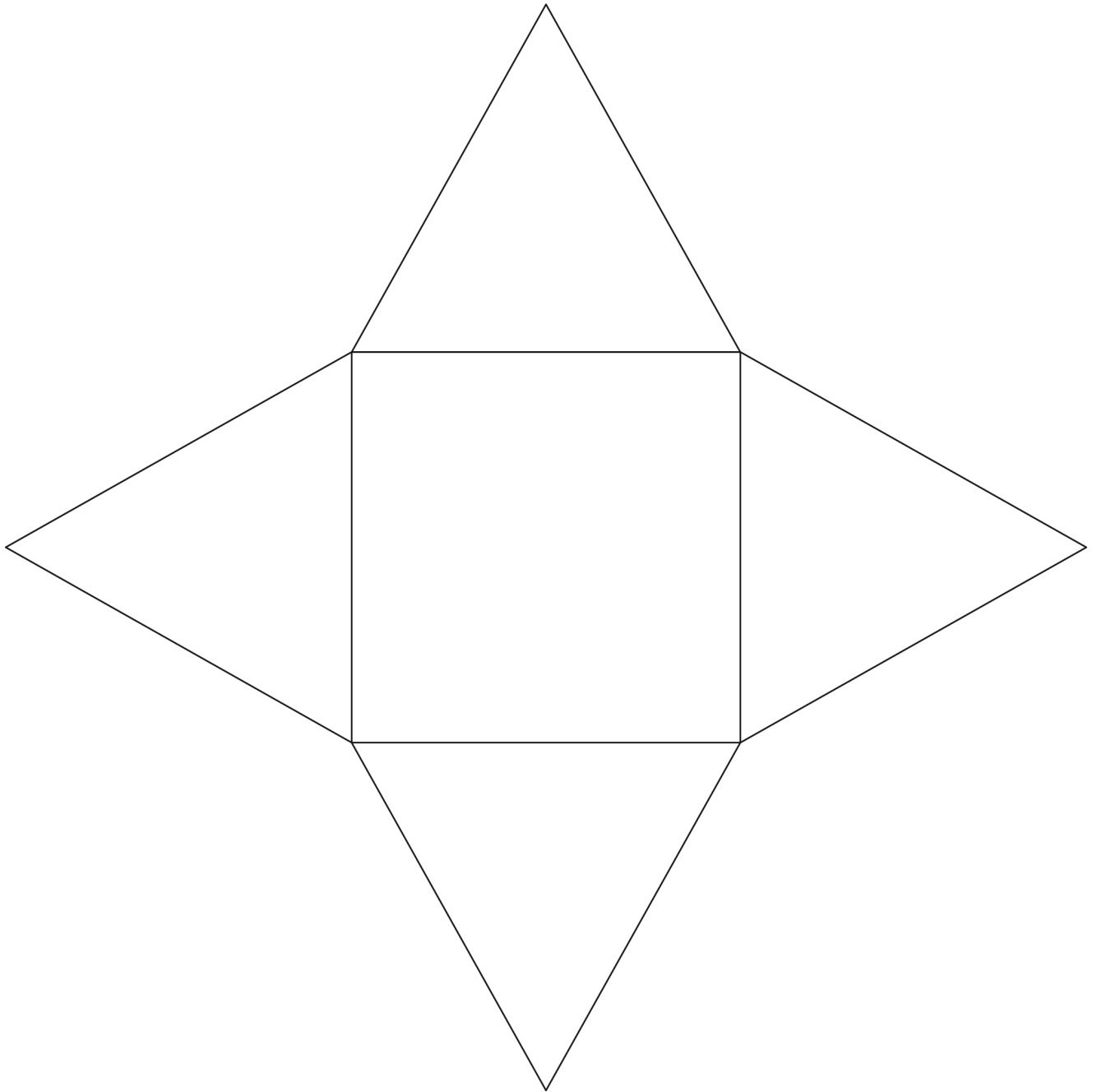
Item	Referent	Estimated Measurement	Actual Measurement
Length of Pencil			
Dimensions of Textbook			
Length of Bookcase			
Width of Bookcase			
Length of Classroom			
Width of Classroom			

Master 1.3 Measurement Chart for Lesson 1.3 Try This

Imperial Units to SI Units	SI Units to Imperial Units
1 in. \doteq _____ cm	1 cm \doteq _____ in.
1 ft. \doteq _____ cm	1 m \doteq _____ in. 1 m \doteq _____ ft. _____ in. 1 m \doteq _____ yd. _____ ft. _____ in.
1 yd. \doteq _____ cm 1 yd. \doteq _____ m	1 km \doteq _____ yd.
1 mi. \doteq _____ m	

Name: _____ Date: _____

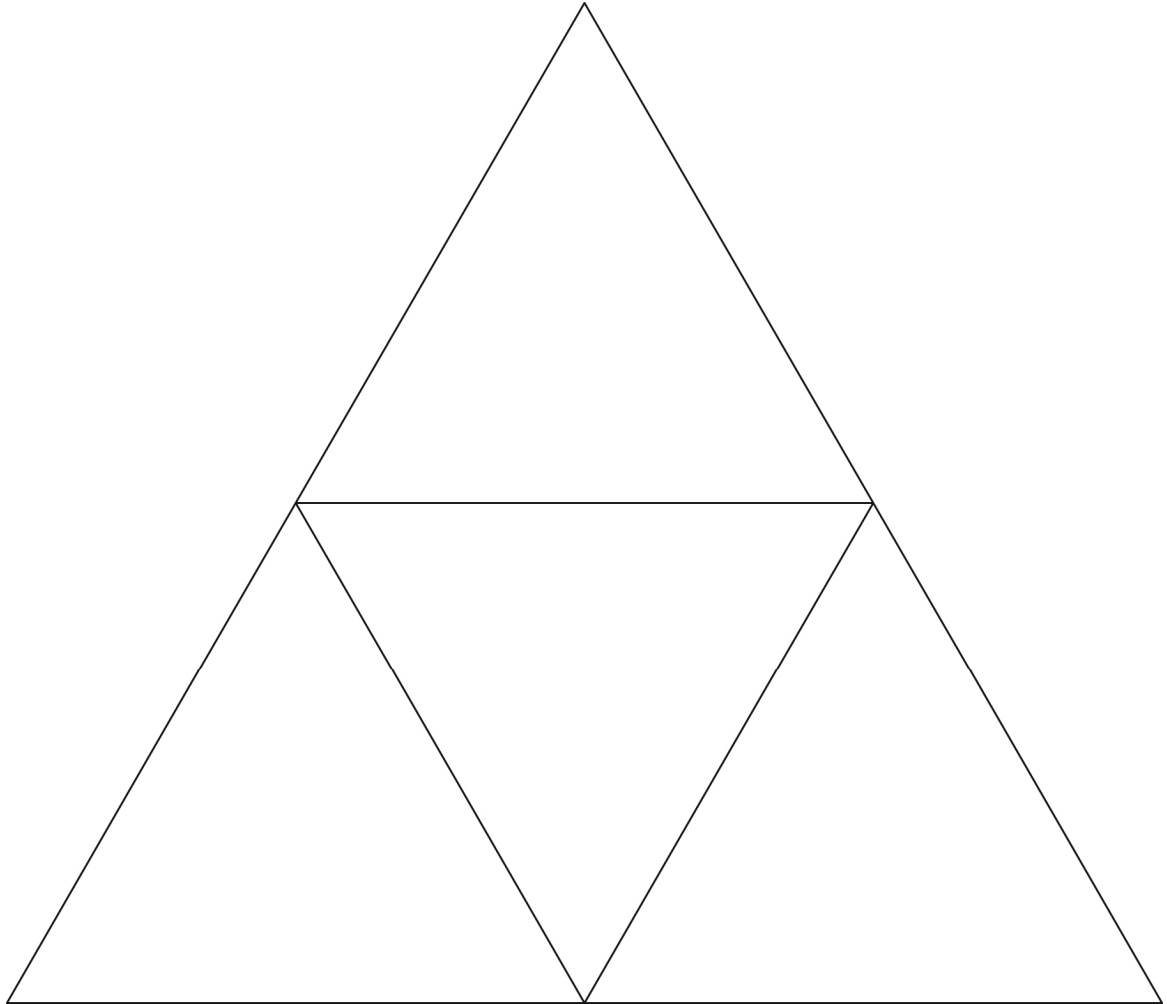
**Master 1.4 Net of a Square Pyramid for
Lesson 1.4 Try This**



Name: _____

Date: _____

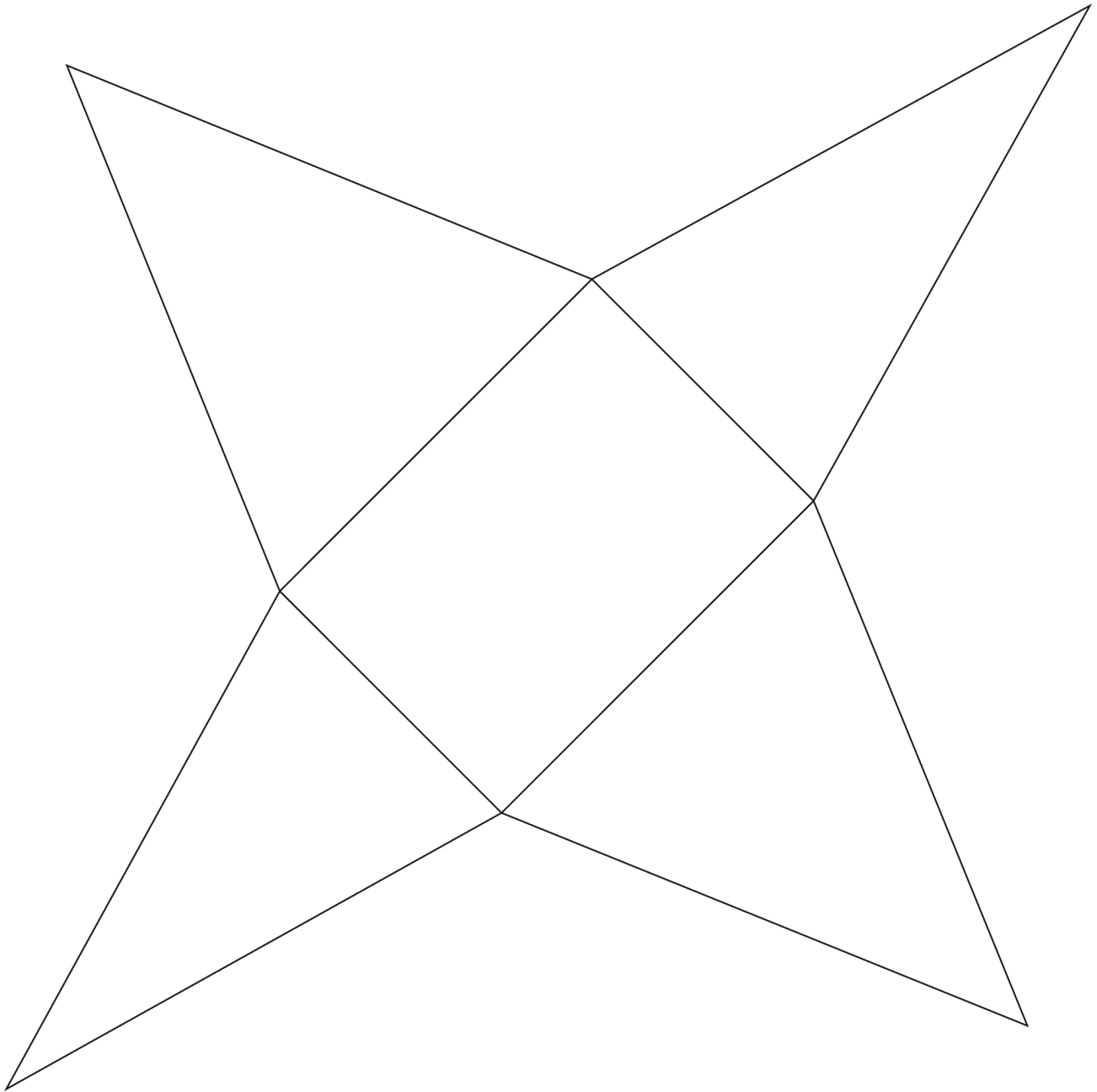
**Master 1.5 Net of a Triangular Pyramid for
Lesson 1.4 Try This**



Name: _____

Date: _____

**Master 1.6 Net of a Rectangular Pyramid for
Lesson 1.4 Try This**



Master 1.7 Chapter Test

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. Which expression would you use to calculate the lateral area of a right cone?

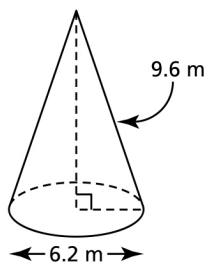
A. $\pi r^2 + \pi rs$ B. πdh C. $\frac{1}{3}\pi r^2 h$ D. πrs

2. Which equation could you *not* use to convert 5 in. to centimetres?

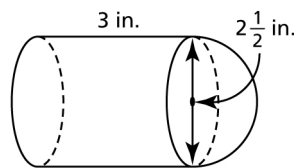
A. $5 \text{ in.} = 5 \times 2.5 \text{ cm}$ B. $\frac{x}{5} = \frac{2.5}{1}$
 C. $\frac{x}{5} = \frac{1}{2.5}$ D. $5 \text{ in.} = 5 \text{ in.} \times \frac{2.5 \text{ cm}}{1 \text{ in.}}$

3. Determine the surface area and volume of each object.
 Give your answers to the nearest unit.

a)



b)



4. A right prism and a right pyramid have the same base and the same height.
 Explain how their volumes are related.
5. The surface area of a sphere is 137.5 cm^2 . What is the radius of the sphere to the nearest tenth of a centimetre?
6. The volume of a right square pyramid is 126 cubic feet. The side length of the base is 8 ft.
 a) Sketch the pyramid.
 b) Determine the height of the pyramid to the nearest foot.
 c) What is the slant height of the pyramid to the nearest foot?
7. A student wants to determine the dimensions of the floor of his bedroom.
 a) Explain how the student could estimate the dimensions of his bedroom.
 b) Which unit is most appropriate to measure the dimensions? Explain.
 c) Which measuring device could the student use to check his estimate?
 Justify your choice.
8. Nakkita used a pedometer to count the number of steps she took on her morning walk.
 At the end of her walk, the pedometer had recorded 4498 steps.
 a) Nakkita's typical step length when walking is 0.7 m.
 To the nearest mile, how far did Nakkita walk?
 b) Use unit analysis to verify the conversion.

Master 1.8 Answers

Master 1.1a

- a) metres b) centimetres or metres
 c) centimetres d) millimetres
- Answers will vary.

a) about 28 cm	b) about 2 m	c) about 2 mm	d) about 2 cm
----------------	--------------	---------------	---------------
- | | | | |
|-----------|-----------|------------|------------|
| a) 900 cm | b) 3.6 cm | c) 4730 cm | d) 25.3 cm |
|-----------|-----------|------------|------------|
- | | | | |
|-----------|------------|-----------|----------|
| a) 8.45 m | b) 4.286 m | c) 0.95 m | d) 1.1 m |
|-----------|------------|-----------|----------|
- | | | | |
|----------|------------|-----------|-----------|
| a) 35 mm | b) 2700 mm | c) 430 mm | d) 900 mm |
|----------|------------|-----------|-----------|

Master 1.1b

- a) 158 m^2 b) 120.5 cm^2
- a) 127 m^2 b) 273 cm^2

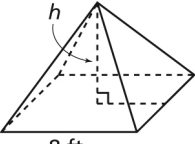
Master 1.1c

- | | | | |
|-----------|-----------|----------|----------|
| a) 14.4 m | b) 15.6 m | c) 5.9 m | d) 2.4 m |
|-----------|-----------|----------|----------|

Master 1.1d

- a) 648 m^3 b) 78.75 cm^3
- a) 663 m^3 b) 5598 m^3

Master 1.7 Chapter Test

- D 2. C
- a) 124 m^2 ; 91 m^3
 b) 38 in.^2 ; 19 in.^3
- The volume of a right prism is 3 times the volume of a right pyramid with the same base and same height.
- 3.3 cm
- | | | | |
|----|---|----------|----------|
| a) |  | b) 6 ft. | c) 7 ft. |
|----|---|----------|----------|

8 ft.
 $V = 126 \text{ ft.}^3$

- Answers will vary.
 - Since the length of a human foot is about the length of an imperial foot, the student could estimate the dimensions of the floor by walking, heel to toe, along the length and width, and counting his feet units.
 - The most appropriate unit is feet (and inches) because the dimensions of construction materials and carpeting are reported in imperial units.
 - The student could use a measuring tape because it is long enough that he can make one measure for length and one measure for width. If the student were to use a ruler or metre stick, he would have to take several measures, then add them.
- a) About 2 mi.
 b) $3148.6 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ mi.}}{1.6 \text{ km}} \doteq 2 \text{ mi.}$

Master 1.9 Chapter Project: How Many Balls Fill the Room?

Work with a partner.

You will need:

- a ruler, measuring tape, or metre stick
- a ball (for example, a volleyball or tennis ball)

Part 1

- Estimate the number of balls needed to fill the classroom.
Describe the strategy you used to estimate.

Part 2

- Measure the dimensions of the classroom.
Justify your choice of unit.
- Determine the volume of the classroom to the nearest cubic unit.
Show all the steps in your calculation.

Part 3

- Determine the diameter of the ball. Explain your measurement strategy.
- Determine the volume of the ball to the nearest cubic unit.
Show all the steps in your calculation.
- Use the two volumes to determine the number of balls needed to fill the classroom.
Do you think all the balls would actually fit in the classroom? Why or why not?
Do you think your estimate is an overestimate or an underestimate?
Use a diagram to show your thinking.

Part 4

- How many balls will fit end to end along each dimension of the classroom?
Use your answers to determine the number of balls needed to fill the classroom.
Is your answer the same as your answer in Part 3? Explain.
Do you think your estimate is an overestimate or an underestimate?
Use a diagram to show your thinking.

Master 1.10 Chapter Rubric: Measurement

	Not Yet Adequate	Adequate	Proficient	Excellent
Conceptual Understanding				
<ul style="list-style-type: none"> Shows understanding by demonstrating and explaining: <ul style="list-style-type: none"> - the process used to estimate and measure a length - the relationship between the volumes of right cones and cylinders and between the volumes of right pyramids and prisms, with the same base and height 	little understanding; may be unable to demonstrate or explain: <ul style="list-style-type: none"> - the process used to estimate and measure a length - the relationship between the volumes of right cones and cylinders and between the volumes of right pyramids and prisms 	some understanding; partially able to demonstrate and explain: <ul style="list-style-type: none"> - the process used to estimate and measure a length - the relationship between the volumes of right cones and cylinders and between the volumes of right pyramids and prisms 	shows understanding; able to appropriately demonstrate and explain: <ul style="list-style-type: none"> - the process used to estimate and measure a length - the relationship between the volumes of right cones and cylinders and between the volumes of right pyramids and prisms 	shows depth of understanding; in a general context, able to thoroughly demonstrate and explain: <ul style="list-style-type: none"> - the process used to estimate and measure a length - the relationship between the volumes of right cones and cylinders and between the volumes of right pyramids and prisms
Procedural Knowledge				
<ul style="list-style-type: none"> Applies proportional reasoning to convert a measurement within or between SI and imperial systems Applies unit analysis to verify a conversion Determines the surface area, volume, or unknown dimension of an object 	has difficulty independently: <ul style="list-style-type: none"> - applying proportional reasoning to convert a measurement - applying unit analysis to verify a conversion - determining the surface area, volume, or unknown dimension of an object 	is generally able to: <ul style="list-style-type: none"> - apply proportional reasoning to convert a measurement - apply unit analysis to verify a conversion - determine the surface area, volume, or unknown dimension of an object 	generally accurate; few errors or omissions in: <ul style="list-style-type: none"> - applying proportional reasoning to convert a measurement - applying unit analysis to verify a conversion - determining the surface area, volume, or unknown dimension of an object 	accurate; no errors or omissions in: <ul style="list-style-type: none"> - applying proportional reasoning to convert a measurement - applying unit analysis to verify a conversion - determining the surface area, volume, or unknown dimension of an object
Problem-Solving Skills				
<ul style="list-style-type: none"> Solves problems that involve: <ul style="list-style-type: none"> - linear measure - the conversion of units within or between SI and imperial systems - the surface areas or volumes of objects 	does not use appropriate strategies to solve problems involving: <ul style="list-style-type: none"> - linear measure - the conversion of units - the surface areas or volumes of objects 	uses some appropriate strategies to solve problems involving: <ul style="list-style-type: none"> - linear measure - the conversion of units - the surface areas or volumes of objects 	uses appropriate strategies to solve problems involving: <ul style="list-style-type: none"> - linear measure - the conversion of units - the surface areas or volumes of objects 	uses effective and innovative strategies to solve problems involving: <ul style="list-style-type: none"> - linear measure - the conversion of units - the surface areas or volumes of objects
Communication				
<ul style="list-style-type: none"> Records and explains reasoning and procedures clearly and completely, including diagrams and appropriate terminology 	unable to record and explain reasoning and procedures clearly and completely	records and explains reasoning and procedures with partial clarity; may be incomplete	records and explains reasoning and procedures clearly and completely	records and explains reasoning and procedures with precision and thoroughness

Master 1.11a Chapter Summary: Self-Assessment and Review

1. Read the **Description** column below.

Decide which description matches how you feel about each Review question.

Write the question number in the **Before** column based on your comfort level.

Before		After	
Review Question	Symbol	Description	Review Question
	✓	<ul style="list-style-type: none"> I can do this type of question very well. I understand this idea. I can explain to someone else how to do this question. If there are lots of questions like this on the test, I will do well. 	
	?	<ul style="list-style-type: none"> I can sometimes do this type of question. I understand some of this idea. If someone gets me started on this question, I can usually get it right. If there are lots of questions like this on the test, I will do alright but not well. 	
	✗	<ul style="list-style-type: none"> I don't know how to do this type of question. I don't understand this idea. I need help starting and finishing this type of question. If there are lots of questions like this on the test, I won't do well. 	

Spend your time reviewing the questions you identified in the bottom two rows of the table.

- Make a study plan. The table on Master 1.11b shows you where in the chapter you can find help for each Review question. You can:
 - Work through the pages where you can find help on your own. Then ask for help if you still don't get it.
 - Work with a friend who can help you.
 - Ask your teacher for help.
- When you have finished working through the pages that should help you, complete the Review questions. Write each Review question number in the **After** column based on your comfort level now. Ask your teacher for help with any questions that are still in the bottom two rows.

Master 1.11b Chapter Summary: Review Question Correlation

Where to Find Help			
Review Question	Example:	Solutions to Exercises question:	Checkpoint:
#1		#3, #4a), #5a), page 11	
#2		#4b), #5b), page 11	
#3	#1, page 7	#7, #8, page 11	#1, page 24
#4	#4, page 10	#16, #17, page 12	
#5		#4, page 15	
#6	#1, page 18	#7, page 22	#1, page 24
#7	#3, page 20	#4, #6, page 22	#1, page 24
#8		#14, page 23	#1, page 24
#9	#1, page 28; #2, page 29; #3, page 32	#5, #7, #8, page 34	#2, page 53
#10	#2, page 29	#13b), page 34	
#11	#2, page 29	#10, page 34	
#12	#1, page 28	#4b), #5b), page 34	
#13	#3, page 32	#12, page 34; #14, #15, page 35	
#14	#2, page 29	#10, page 34; #18, page 35	
#15	#2, page 39; #3, page 40	#8, #9, page 42	#2, page 53
#16		#11, page 42	
#17		#18b), page 43	
#18	#1, page 38	#12, page 42; #18, page 43	
#19	#4, page 41	#18d), page 43	
#20	#1, page 47; #3, page 49	#3, #4, page 51	#2, page 53
#21	#4, page 50	#5, page 51	
#22	#2, page 47	#8, #9, page 51	
#23		#16, page 52	
#24	#2, page 47; #3, page 49	#7, #8, page 51	
#25	#1, page 56; #2, page 57	#3a), b), #5b), page 59	
#26	#3, page 58	#7, page 60	
#27		#6, page 60	

PEARSON

Foundations and Pre-calculus Mathematics 10

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