Mathematics Success for All

Pearson Canada Research Overview

Illustration by: Lucy Vigrass

Introduction

Mathematics Success for All

Welcome to Mathematics Success for All, our research overview that has informed the development of our new family of math resources for K–9 Mathematics: Pearson Mathology.

Created with a deep understanding of Math learning and the needs of teachers, Mathology is a comprehensive math program with real-world applications that helps educators plan lessons, and engage and teach students across all skill levels. Co-developed with Canadian teachers, Mathology offers a differentiated learning program rooted in classroom reality, as well as effective teacher support. Based on an easy-to-understand Math learning progression, it combines insights from teacher interviews, focus groups, and classroom observations, with the best of pedagogical approaches and the academic research presented in this report.

Very early in the development process for Mathology, Pearson Canada surveyed the educator community in Canada to identify key research areas in Canadian mathematics that are influencing mathematics instruction (K–9) today.

This document provides an overview of the topics that these educators stated were crucial to high-quality mathematics instruction. Key quotes from research articles and reference materials are presented for each topic, and connected to the development of Mathology, to show how these ideas inform the resource.

We are confident this research overview will provide useful insight into the Canadian mathematics landscape as it exists today.

Table Of Contents

Planning and Assessment

Balanced Math

In a balanced mathematics program, students are engaged in a variety of instructional approaches and learning contexts involving purposeful exploration and practice. This approach balances important concepts and strategies, and mathematical ideas, reasoning, and skills.

What the Research Says

> “An important key to developmentally appropriate mathematics instruction, at any age or grade level, is achieving balance between teaching for conceptual understanding and teaching for procedural fluency. When students learn procedures without meaning, they are only memorizing discrete pieces of information that are difficult for them to remember.”
>
> (Protheroe, 2007)

“[Students] need a balanced program of understanding, skills, and problem solving and they need a flexible set of thinking and reasoning tools they can call on to pull all of these pieces together.”
(Seeley, 2009)

“...numerous studies have shown that rote learning alone produces a narrow and brittle form of knowledge, whereby the individual can reproduce – or recite – what has been learned (and thus can pass the test) but does not necessarily understand the new information and is unable to make practical use of it.”
(Devlin, 2010)

“The art of teaching is to balance the need for surface knowledge with deep processing of this knowledge.”
(Hattie, 2015)

“The integrated and balanced development of ... mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) should guide the teaching and learning of school mathematics.”
(National Research Council, 2001)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need a mathematics program that balances skills, concepts, strategies, and thinking. When students are engaged in a balanced program, they can apply their computational skills and reasoning abilities to solve real-life problems, and communicate their understanding to others.	Resources should encourage students to build computational skills at the same time as they develop problem-solving abilities, deepen conceptual understanding, expand their ability to communicate thinking, and demonstrate understanding.

Why Early Math Matters

High-quality and engaging mathematics learning at an early age allows children to develop a strong foundation upon which to build in the later years. A large body of research demonstrates a strong connection between early success in mathematics and later success in school, not only in mathematics, but also on many other indicators. This research presents a strong case for rich early experiences in mathematics.

What the Research Says

> … young children require high quality, challenging, and accessible math education experiences in order to build a strong foundation for their future learning.
>
> (Foundations for Numeracy, 2010)

"… preschool and first grade mathematical ability are positive and highly significant predictors of mathematics achievement through age 15."
(Watts, Duncan, Siegler, & Davis-Kean, 2014)

"There is good evidence that spatial reasoning experiences at an early age contribute to children's "development of both numerical and spatial/ geometrical concepts.""
(National Research Council cited by Moss, Bruce, Caswell, Flynn, & Hawes, 2016)

"The relation between early number competence and mathematics achievement was strong and significant."
(Jordan, Kaplan, Ramineni, & Locuniak, 2009)

"A focus on establishing the foundations of mathematics knowledge early in formal schooling, then, "seems to be an essential first step for achieving equity.... Early intervention is now viewed as one key step toward ensuring a level playing field.""
(Baroody et al. cited by Bruce, Flynn, & Moss, 2012)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need access to resources that will engage young children and help them build a strong foundation in mathematics. Teachers need effective teaching strategies that will allow them to meet the diverse needs of their students. When students have established a strong foundation in mathematics in the early years, they are set up for future success.	Resources should establish a strong foundation in mathematics in the early years to sustain learning throughout school and life. Resources should help teachers identify and close learning gaps early.

Assessment

Assessment includes a wide variety of tools that allow teachers to continually observe, measure, and document a student's learning and understanding. Teachers then use what is observed to plan for next steps.

What the Research Says

> The most powerful single modification that enhances achievement is feedback.
>
> (Marzano, Pickering, & Pollock, 2001)

"… student self-assessment, defined as a dynamic process in which students self-monitor, self-evaluate, and identify correctives to learn, is a critical skill that enhances student motivation and achievement."
(McMillan & Hearn, 2009)

"Assessment should guide teaching. It should be continuous and provide information about the 'zone of proximal development.'"
(Storeygard, Hamm, & Fosnot, 2010)

"… formative assessment helps to guide students to make improvements during the course of learning. It also informs teachers as to how to support individual students or to alter classroom instruction."
(Suurtamm, 2010)

"Because too many culturally different students are not scoring well when evaluated, it is important that teachers create and use tests and assessments that are culturally responsive."
(Ford, 2010)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need tools to help them assess students and suggestions on how they can use the information gained to respond to students' learning needs. When students experience high-quality, meaningful assessment practices, including processes that support self-assessment, they are consistently and appropriately supported throughout their mathematical learning journey.	Resources, such as teacher instructional materials and assessment tools, should address all facets of assessment that can be employed across the mathematics curriculum: assessment *of* learning (e.g., summative evaluation), assessment *as* learning (e.g., teacher feedback and student self-reflection), and assessment *for* learning (e.g., ongoing data to inform instruction). As much as possible, criteria should be co-constructed with students. Resources should provide in-the-moment tools and supports to help teachers observe and recognize students' strategies and thinking, understand what they see or hear, and then choose the appropriate next steps.

Selection and Use of Quality Resources

Quality resources reflect the current research and provide all students with an entry point for learning. These resources help educators better understand mathematical content and pedagogy, thus enhancing their teaching practices.

What the Research Says

"Architects wish to design beautiful buildings and environments, but they must also apply many foundational principles of engineering and adhere to structural principles. If they do not, their buildings, however beautiful they may be, will not stand. Similarly, a teacher seeks to design lessons that stimulate students and entice them to learn – lessons that are sometimes a beauty to behold. But if the lessons are not based in the science of pedagogy, they, like poorly constructed buildings, will fail."

(Stanovich & Stanovich, 2003)

"Efforts to develop textbooks and other instructional materials should include research into how teachers can understand and use those materials effectively."
(National Research Council, 2001)

"Implications of this research for resource development include ensuring teachers possess a bank of tasks linked to contexts known to be realistic, purposeful, of high interest and effective in supporting students' mathematical learning."
(Harvey & Averill, 2012)

"Appropriate tasks have at least three features … students see the task as an interesting problem … the tasks must connect with where students are … [and] the tasks must engage student in thinking about important mathematics."
(Hiebert et al., 1997)

"Teacher support: Do support materials have the potential to enhance the quality of mathematics instruction?"
(Tarr, Reys, Barker, & Billstein, 2006)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources that support their understanding of mathematical concepts, differentiated instruction, and their teaching practices. When resources provide engaging and meaningful tasks and are designed to meet the needs of all learners, teachers can effectively support students' mathematical learning.	Resources should effectively address the learning needs of all students; they should support a range of instructional approaches and ways of learning. Resources should be engaging, accessible, and user-friendly.

 |

Effective Lesson Model

A flexible lesson model allows for various teaching approaches and ensures that teaching is intentional and effective. It allows educators to shift and respond to a child's needs.

What the Research Says

> "... the phases of the lesson plan structure emerge from a multitude of decisions made about the curriculum learning expectations, student experiences and interests, task development, and assessment tools."
>
> (Van de Walle, Karp, Bay-Williams, McGarvey, & Folk, 2015)

"Frameworks for effective teaching to support children's conceptual understanding also emphasize the need for tasks that are mathematically challenging and significant."
(Askew et al., 1997)

"Students learn from the kind of work they do during class, and the tasks they are asked to complete determines the kind of work they do."
(Hiebert et al., 1997)

"The success with which a teacher conducts a lesson is often thought to depend on the effectiveness with which the lesson was planned."
(Richards, 1998)

"The selection of a problem for the problem solving activity... is extremely critical for teachers when they plan a lesson."
(Takahashi, 2006)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need opportunities to incorporate problem-solving tasks and consolidation techniques into their mathematics teaching to allow students to develop a deeper understanding of the mathematics involved. When students are given opportunities to solve problems and consolidate their learning, they develop deeper conceptual understanding and become better thinkers and communicators.	Resources should provide a lesson model with a flexible and adaptable structure. Resources should provide opportunities for students to solve rich problems and consolidate learning before moving on to new concepts.

Curriculum and Big Ideas

Big ideas of the curriculum help facilitate mathematical understanding by focusing on key concepts and processes/competencies. When educators frame content around big ideas, they are often able to encompass content from other strands and make crucial connections between mathematical concepts.

What the Research Says

> Big ideas 'invite students to look beyond surface features of procedures and concepts and see diverse aspects of knowledge as having the same underlying structure.'
>
> (Baroody, Feil, & Johnson, 2009)

"Much research indicates that children from diverse backgrounds can learn mathematics if it is organized into big coherent chunks and if children have opportunity and time to understand each domain deeply ..."
(Ontario Ministry of Education, 2003)

"Understanding the contexts of instruction and curricula used are necessary to be an effective educator."
(Clements & Sarama, 2009)

"If the teacher had a deep, rich, and connected understanding of the particular concept, or 'big idea', then s/he would likely be prompted to further investigate and probe the child's thinking."
(Hurst & Hurrell, 2014)

"When one understands Big Ideas, mathematics is no longer seen as a set of disconnected concepts, skills, and facts. Rather, mathematics becomes a coherent set of ideas."
(Charles, 2005)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources that are organized around the big ideas of mathematics to allow students to explore mathematical concepts in depth and to make connections between the concepts. When resources are organized around the big ideas, students are able to develop an understanding of the connections between mathematical concepts and procedures.	Resources should cluster curriculum outcomes/expectations around the big ideas of mathematics, and provide tools for teaching and learning that align closely with the Pearson Canada Mathematics Learning Progression, allowing students to make connections instead of seeing mathematics as compartmentalized sets of disconnected ideas.

Differentiated Instruction

Differentiated instruction allows teachers to meet students where they are, and to help students progress from one understanding to another. It strives to meet the diverse needs of every student by matching different strategies to different student needs and strengths. Differentiation personalizes the learning process so that every child has the best chance of success.

What the Research Says

"Today's classrooms are filled with learners whose strengths are as diverse as their needs. In order to teach all students effectively … teachers must have an extensive repertoire of strategies."

(Literacy and Numeracy Secretariat, Ontario Ministry of Education, 2008)

"Students are more likely to be successful if the assessment system encompasses a broad spectrum of abilities and modes of expression."
(Benjamin, 2006)

"Differentiated instruction (DI) is based on the idea that because students differ significantly in their strengths, interests, learning styles, and readiness to learn, it is necessary to adapt instruction to suit these differing characteristics."
(Ontario Ministry of Education, 2013)

"Differentiated instruction is at least as important for students with significantly higher ability as for students with significantly lower ability."
(Lawrence-Brown, 2004)

"… when instruction is culturally responsive, teachers modify their teaching styles to accommodate and affirm learning styles."
(Ford, 2010)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need to differentiate instruction and assessment to meet the diverse needs of their students and to help them learn efficiently in various ways. When students are taught according to their needs and strengths, learning is maximized.	Resources should provide suggestions on how instruction and assessment can be differentiated to support the different learning needs and strengths of students in a diverse classroom.

The Math Learning Community/Environment

Encouraging Growth Mindsets

When students and teachers embrace a growth mindset, they believe that success is determined not by innate ability, but by persistence through challenges. From this perspective, mistakes are viewed as opportunities for learning and growth. Encouraging a growth mindset increases a student's motivation, confidence, and academic performance.

What the Research Says

> This growth mindset is based on the belief that your basic qualities are things you can cultivate through your efforts. Although people may differ in every which way – in their initial talent and aptitudes, interests, or temperaments – everyone can change and grow through application and experience.
>
> (Dweck, 2006)

"It is never too early to nurture the growth of a positive disposition towards mathematics, and improve the relationship that children have with mathematics content and processes."
(Colgan, 2014)

"The new evidence from brain research tells us that everyone, with the right teaching and messages, can be successful in math, and everyone can achieve at the highest levels in school … Although I am not saying that everyone is born with the same brain, I am saying that there is no such thing as a "math brain" or a "math gift," as many believe. No one is born knowing math, and no one is born lacking the ability to learn math."
(Boaler, 2016b)

"When anxiety is regulated or reframed, students often see a marked increase in their math performance."
(Maloney & Beilock, 2012)

"Young children have a natural inquisitiveness about mathematics, and teachers can build on this inquisitiveness to help students develop the positive attitudes that often occur when one understands and makes sense of a topic."
(Ontario Ministry of Education, 2003)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need to build on their students' natural inquisitiveness to motivate and enhance student learning. When students have positive and engaging mathematics experiences, they develop positive attitudes toward mathematics and gain confidence in their ability to do mathematics.	Resources should be challenging and engaging, supporting student success in mathematics and leading to a positive attitude. Resources should have multiple entry points to meet learners where they are, building confidence in their ability to learn mathematics.

Student Talk

Meaningful learning can occur when students are engaged in purposeful discussions about mathematics. These discussions provide students with an exposure to a variety of strategies, and with opportunities to consolidate their thinking as they communicate about mathematical ideas. Math talk provides students with opportunities to think critically, collaborate with others, and acquire meaningful mathematical learning.

What the Research Says

> "Productive talk promotes … conceptual understanding: It can be useful in helping students build individual mental connections, and it is the core activity of a community of learners who together are trying to make sense of mathematical truths."
>
> (Chapin, O'Connor, & Anderson, 2009)

"Mathematics educators nationwide agree that student engagement in meaningful mathematical discourse has a positive effect on their mathematical understanding as they increase the connections between ideas and representations."
(Garcia, 2011)

"Research tells us that student interaction – through classroom discussion and other forms of interactive participation – is foundational to deep understanding and related student achievement."
(Bruce, 2007)

"Social interaction provides us with the opportunity to use others as resources, to share our ideas with others, and to participate in the joint construction of knowledge."
(Smith & Stein, 2011)

"Participating in a mathematical community through discourse is as much a part of learning mathematics as the conceptual understanding of the mathematics itself."
(Stein, 2007)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources that provide suggestions on how to establish a learning community where "talking mathematics" is valued and routine. Teachers need support in understanding how talk can reflect and extend students' understanding of mathematical concepts. When students are part of a community where "talking mathematics" is valued, they have opportunities to share ideas, critique others' reasoning, and increase their own mathematical understanding.	Resources should offer frequent opportunities for teachers and students to engage in meaningful mathematical discourse.

Building a Community of Math Learners

Building a community of Math learners involves creating an inclusive learning environment where all students feel safe to explore and take mathematical risks. Teachers and students work collaboratively and learn from each other, thus creating a supportive learning environment.

What the Research Says

> "Teachers can foster mathematical understanding by providing a safe space for taking mathematical risks, allowing for exploration, and promoting collaborative learning."
>
> (Suurtamm, Quigley, & Lazarus, 2015)

"An engaging and encouraging climate for children's early encounters with mathematics develops their confidence in their ability to understand and use mathematics."
(National Council of Teachers of Mathematics, 2013)

"Knowledge about early mathematics learners and the mathematics for teaching can help educators create a rich environment and guide students to attain strong conceptual understandings, positive attitudes and self-efficacy."
(Ontario Ministry of Education, 2011)

"Learning mathematics in a community of practice … foster[s] mindful, strategic learning by engaging students in collaborative forms of inquiry."
(Goos, 2004)

"The classroom culture we create … needs to be one where questioning and deep thinking are valued, mistakes are seen as useful, all students contribute and their suggestions are valued, being stuck is seen as honourable and students learn from shared discussion with the teacher … and peers."
(Pennant, 2013)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources that support the creation of a classroom culture where respectful questioning and valuing of one another's mistakes and misconceptions are part of the learning process. When students are part of an engaging and encouraging classroom environment, they feel safe to explore their ideas and they develop confidence in their ability to make sense of mathematics.	Resources should support the creation of a classroom environment in which curiosity, risk-taking, cooperative learning, and self-confidence are fostered and celebrated.

First Nations, Métis, and Inuit Learners

Today's classroom should foster a mathematics community where First Nations, Métis, and Inuit learners feel meaningfully engaged and respected and where the rich history and cultures of its learners are honoured.

What the Research Says

> "An educational environment that honours the culture, language and world view of the Aboriginal student is critical."
>
> (Toulouse, 2008)

"There is a need in contemporary education to understand how to provide Indigenous students with a meaningful connection to their learning."
(Beatty & Blair, 2015)

"All children have the right to be taught in an inclusive environment that is safe and caring, respects diversity and the rights of persons, and provides equitable opportunities for success."
(Alberta Teachers' Association, 2010)

"One of the most readily implemented ways to begin teaching Mathematics in a First Peoples context is to establish meaningful connections for students between mathematics skills and 'content' and First Peoples themes and topics."
(First Nations Education Steering Committee, 2011)

"Culturally relevant ethnomathematical and ethnoscientific curricula connect students with their heritage."
(Barta, Jette', & Wiseman, 2003)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need access to resources with content that establishes meaningful connections to Indigenous students' heritage and honours their culture. When students are given opportunities to engage in culturally responsive activities involving Indigenous themes and topics, they feel more connected to their heritage.	Resources should provide support for teachers to develop meaningful and culturally responsive content that connects students with their heritage, and allow the development of meaningful relationships in the classroom. Resources should be developed in collaboration with elders and other Indigenous community members.

Positive Personal and Cultural Identity

Today's classroom should foster a mathematics community where students of different backgrounds and cultures are respected and appreciated. It is important that all students are treated equally, and that they feel safe and included in the learning environment.

What the Research Says

> "Exposing students to the contributions of members of their own and other cultures can help them gain confidence, self-esteem, and a sense of belonging, as well as respect for the mathematical thinking of all cultures."
>
> (Wiest, 2001)

"The learning of mathematics is important for all children and this is especially true in Canada's First Nation communities as they begin to re-establish their self-government and self-determination."
(Borden, 2010)

"Educators who are from different cultural perspectives than those present in the families and communities of the children they teach may render it difficult to "see" the cultural identities shaping the behaviors and achievement of their students."
(Gilliard & Moore, 2006)

"Family activities such as gardening, sewing, scheduling, cooking, and playing games are mathematical resources available to students and teachers to support mathematics learning of children."
(Civil, 2007)

"Students learn that numerical systems can be very different yet possess many strikingly similar, vital mathematical principles."
(Zaslavsky, 2001)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources that support the creation of an educational environment that honours students' cultures, fosters cross-cultural understanding, and respects all learners. When mathematics instruction is approached in a culturally meaningful way, teachers see themselves as co-learners and students can connect mathematics to their daily lives, and feel respected, included, and equal.	Resources should strive to approach mathematics instruction in a culturally meaningful and enriching way so that all learners feel included and respected.

Classroom Management

Classroom management is foundational to teaching and ensures that all students have an equal opportunity to learn. The implementation of these classroom management strategies keeps students meaningfully occupied, allowing teachers to work with those who require additional support.

What the Research Says

> The teacher involves her students in understanding the nature of the classroom and in making it work for everyone …
>
> (Tomlinson & Eidson, 2003)

"The teacher needs to imbue in students a sense of responsibility for their own learning … the advanced social skills hence developed by students would help to make each class, single grade or combined grade, a real 'learning community.'"
(Lataille-Démoré, 2007)

"By allowing our students to learn math content through choice, writing, peer activities, and one-on-one conferences with the teacher, we have seen rapid growth and an increase in student achievement."
(Ktytor & Waechter, 2014)

"… individual classroom teachers can have a major impact on student achievement. Of the three roles of the classroom teacher … classroom management is arguably the foundation."
(Marzano, Marzano, & Pickering, 2008)

"Teachers work daily to find ways to reach out to individual learners at their varied points of readiness, interest, and preferred approaches to learning."
(Tomlinson, 2014)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
When teachers are able to effectively manage their classrooms and meet the needs of all learners, students experience greater success. Teachers need resources that provide suggestions for ways to encourage students to take responsibility for their own learning. Teachers need resources that provide strategies and suggestions that allow them to meet the needs of all learners and to manage their classrooms so that effective learning and teaching can take place.	Resources should be developed with full consideration of practical challenges, and include suggestions for teachers regarding management of the classroom environment as well as management of materials.

Teacher Learning

Professional Learning

Educators are lifelong learners. Professional learning can help improve teaching practices and provide new perspectives and insights. In mathematics, it is essential that professional learning focuses on mathematical content as well as on pedagogical strategies, and that resources support teachers in naming and noticing children's mathematical thinking. When new knowledge and strategies are shared, both teaching and student learning are enriched.

What the Research Says

> "What teachers know, do and believe has a major influence on what students learn. If we are to improve the quality of teaching and learning in our schools we must invest in the learning of teachers."
>
> (OSE Office of School Education, n.d.)

"… gains in teachers' mathematical knowledge predicted changes in the quality of their lesson design, their mathematical agenda, and the classroom climate."
(Copur-Gencturk, 2015)

"Teachers want better insight into children's mathematical development. They want to know: 'What does this child's thinking tell me about what he or she understands?'"
(Lawson, 2015)

"… mathematics teachers recognize that their own learning is never finished and continually seek to improve and enhance their mathematical knowledge …"
(National Council of Teachers of Mathematics, 2014)

"There is no system in the world or any school in the country that is better than its teachers. Teachers are the lifeblood of the success of schools."
(Robinson, 2013)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need to have access to effective professional learning that meets their specific needs and that supports their students' understanding and engagement in mathematics. When professional learning opportunities are provided for teachers, students have the best chance at success.	Professional learning should be foundational to the resource, and it should be concrete, targeted, sustainable, and scalable. Professional learning needs to be available to teachers through a range of delivery methods and design, including at point of use, in school-based communities of learning, and in focused programs that explore particular areas of mathematics teaching and learning.

Early Math

How Young Children Best Learn Math

Young children construct mathematics concepts differently at different ages and developmental stages. Classroom activities need to be engaging and appealing, and build on students' knowledge and experience to make learning relevant and meaningful.

What the Research Says

> "… promoting play with mathematical objects and mathematical ideas is pedagogically powerful."
>
> (Clements & Sarama, 2014)

"Knowledgeable educators begin planning by carefully observing children at play or engaged in other activities in order to identify everyday mathematics."
(Ginsburg & Ertle, 2008)

"In this playful pedagogy approach, educators are seen as collaborative learning partners who create flexible, interest-driven experiences that encourage children's natural curiosity and "sense-making" processes."
(Moss, Bruce, Caswell, Flynn, & Hawes, 2016)

"Young children come to school with … some informal, yet powerful, problem-solving skills. These children have already constructed intuitive mathematical notions. We are challenged to make use of the child's physical and social environment as motivating arenas for further quantitative reasoning and problem-solving."
(Ontario Ministry of Education, 2003)

"Mathematics for young children should be an integrated whole. Connections—between topics, between mathematics and other subjects, and between mathematics and everyday life—should permeate children's mathematical experiences."
(Clements, Sarama, & DiBiase, 2004)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need to be able to combine play and engaging activities with intentional mathematics teaching and sound mathematics pedagogy. Teachers need to be able to recognize mathematical thinking as it emerges through play and rich tasks. When students are able to connect and apply their mathematics learning to their previous knowledge and experiences, and to communicate and explore mathematical concepts through engaging activities, they experience greater success.	Resources should recognize that when learning mathematics, there are multiple pathways that foster conceptual and procedural understanding and problem-solving skills; these varied approaches must be supported. Resources should foster student learning of mathematical concepts and the development of mathematical thinking. Resources should honour the fact that students come to class with prior knowledge.

Math Processes for Learning

21st-Century Competencies in Math

The challenges and opportunities of the 21st century require critical thinking, creativity, and problem solving; these are considered key competencies for success in the modern era.

What the Research Says

> “What’s new in the 21st century is the call for education systems to emphasize and develop these competencies in explicit and intentional ways through deliberate changes in curriculum design and pedagogical practice. The goal of these changes is to prepare students to solve messy, complex problems – including problems we don’t yet know about – associated with living in a competitive, globally connected, and technologically intensive world.”
>
> (Ontario Ministry of Education, 2016)

“What kind of learning experiences will prepare students for the demands of the twenty-first century? Research tells us that complex knowledge and skills are learned through social interaction (Vygotsky 1978; Lave and Wenger 1991).”
(Smith & Stein, 2011)

“The general mathematical processes of reasoning, representing, problem solving, connecting, and communicating are … a means both for making sense of abstract mathematics and for formulating real situations in mathematical terms – that is, for mathematizing the situations they [children] encounter.”
(National Research Council, 2009)

“In preparing our students to become mathematically aware, consideration needs to be given to how we might select contexts that … foster the appreciation of learning through classroom problem solving.”
(English & Gainsburg, 2016)

“Schools must be transformed in ways that will enable students to acquire the creative thinking, flexible problem solving, collaboration and innovative skills they will need to be successful in work and life.”
(Pacific Policy Research Center, 2010)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources to help them build mathematical processes into their teaching and to help their students develop 21st-century learning skills, such as collaboration and critical thinking. Opportunities to learn and apply mathematical processes and 21st-century learning competencies allow students to develop mathematical knowledge and skills, and make sense of the world around them.	Resources should provide embedded opportunities for students to acquire and apply mathematical processes and 21st-century competencies – the knowledge, skills, and attitudes that they will need to thrive in today’s world.

Home-School Connection

Home Connection

A strong connection between school and home can help teachers and parents work together to advance the mathematical development of children. The more parents are intentionally involved in their children's learning, the greater the opportunity for children to extend their knowledge outside the classroom.

What the Research Says

> Meaningful relationships that enhance parents' opportunities to make important contributions to student learning are vital to the work of teachers.
>
> (Pushor, 2010)

"When families and educators join forces, students of all ages can experience greater success in their learning."
(Council of Ontario Directors of Education, 2015)

"… parents, who are neither mathematics educators nor teachers can engage their children in activities that lead to actions, utterances, gestures, and other communicative acts that have something to do with logic and mathematics."
(Anderson, 1997)

"One of the best things parents can do to improve their children's math literacy is to regularly expose them to practical applications of math at home."
(Kormanik, 2012)

"Parent involvement that focuses on student learning in mathematics has a tremendously positive effect."
(Bruce, 2013)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need support for involving parents, caregivers, elders, and the community in students' mathematics learning. When parents, caregivers, elders, and the community are meaningfully involved in their children's mathematics education, children achieve more.	Resources should suggest opportunities for parents, caregivers, elders, and the community to become involved in their children's mathematics education. Resources should support clear and meaningful communication with parents and caregivers.

Learning Tools

Games and Manipulatives as Mathematical Thinking Tools

The intentional use of games and manipulatives helps support learning of mathematics by engaging students in explorations and concrete applications of mathematical concepts.

What the Research Says

> “Research from both learning theory and classroom studies shows that using manipulatives to help teach math can positively affect student learning. This is true for students at all levels and of all abilities.”
>
> (Research on the benefits of manipulatives, n.d.)

“… non-traditional activities and attention-grabbing resources can spark curiosity about mathematics, improve appreciation for and interest in mathematics and contribute to understanding the relevance of mathematics in everyday life.”
(Colgan, 2014)

“… students were more engaged in discussing strategies and sharing their thinking as a result of implementing game playing. They were also more enthusiastic about doing mathematics.”
(Lach & Sakshaug, 2005)

“If we are to make math experiential, we must present children with tactile tools with which they can learn …”
(Murray, 2001)

“… discussions with students about their thinking as they play games serve as useful assessment and instructional tools.”
(MacDonald & Shumway, 2016)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need access to manipulatives to allow students to explore concepts and demonstrate their thinking. Teachers need access to games that support the development and reinforcement of specific mathematical concepts. When students have opportunities to use manipulatives and play games, they can deepen their understanding through hands-on engaging activities.	Resources should provide manipulative-based tasks and games to engage students and to help them develop their own conceptual understanding.

Affordances of Technology

The integration of technology into the classroom setting can help motivate and engage students, allowing them to explore and deepen their understanding of mathematical concepts. Many technologies also have features that allow students to interact with and explore abstract mathematical concepts in ways that cannot be replicated with hands-on materials.

What the Research Says

> “These studies provide compelling evidence that computer use can have a major, positive impact on children’s social, emotional, language, and cognitive development.”
>
> (Murphy, DePasquale, & McNamara, 2003)

“… educators are positioned to improve program quality by intentionally leveraging the potential of technology and media for the benefit of every child.”
(NAEYC and Fred Rogers Center, 2012)

“Data analyses from our recent studies (Bruce, 2012; Bruce et al., 2011) suggest that pedagogical-technological interactivity involves 3-way interaction between teachers, students and the IWB [Interactive whiteboard], in a flexible web of exchanges.”
(Bruce & Flynn, 2012)

“… benefits from adopting mobile technology for student instruction included their potential to be engaging for students and to support personalization of instruction to meet the needs of different students.”
(Interactive Educational Systems Design, 2013)

“Integrating technology and mathematics instruction enables all students to engage in more complex activities at earlier levels.”
(Thach & Norman, 2008)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need access to technology to enrich instruction, to support students’ explorations of mathematical concepts, to engage students, and to teach diversity in different learning styles when possible. When students have access to technology, they can investigate and test mathematical ideas, engage in mathematical discourse with their peers and teacher, and make connections to previous mathematics learning.	Technology must be accessible, user-friendly, and student-friendly, it must support student learning, and it must be used in meaningful and enriching ways.

Math Content Areas

Algebraic Thinking, Spatial Reasoning, & Proportional Reasoning

In mathematics, several powerful processes span specific topics or content areas. Emphasizing these processes – and providing high-quality educational experiences that support their development – supports students' long-term success in mathematics. These processes include algebraic thinking, spatial reasoning, and proportional reasoning.

What the Research Says

> ... spatial reasoning and visual affordances in low and high tech environments are key to unlocking mathematics ideas for students.
>
> (Bruce, 2013)

"When teaching is based on students' mathematical ideas and promotes their mathematical curiosity, students tend to show algebraic ways of thinking in arithmetic, geometric or measuring contexts."
(Bastable & Schifter, 2008)

"The ability to think and reason proportionally is one essential factor in the development of an individual's ability to understand and apply mathematics."
(Ontario Ministry of Education, 2012)

"... the very act of learning about spatial reasoning and coming to understand its importance, and collaboratively designing tasks has had positive outcomes for students, teachers and researchers."
(Bruce, Sinclair, Moss, Hawes, & Caswell, 2015)

"A rich program of quantitative reasoning spurs the development of students' conceptual and representational capacities as it connects mathematics to the world of objects and situations, measurement, and change."
(Smith & Thompson, 2007)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources and activities that provide students with opportunities to apply proportional reasoning, algebraic thinking, and spatial reasoning, allowing students to develop a deeper understanding across all areas of mathematics. When students develop a strong sense of these processes, they are better mathematical thinkers and have a better conceptual understanding of the content they explore.	Resources should provide frequent opportunities for students to solve problems using deep algebraic thinking, starting in the early years. Resources should view spatial reasoning as an additional opportunity for students to develop coding and number skills. Resources should provide students with ongoing opportunities to apply proportional reasoning and make connections across mathematical concepts.

Building Computational Fluency by Developing Number Sense

Number sense involves the deep understanding of numbers, number relationships, magnitudes, and relative values. Strong number sense leads to flexibility in comparing and working with numbers. Students who develop this flexibility and computational fluency can use efficient strategies and calculate in multiple ways with ease to obtain accurate results.

What the Research Says

> “Whether or not we believe that fluency requires more than the recall of math facts, research evidence points in one direction: The best way to develop fluency with numbers is to develop number sense and to work with numbers in different ways, not to blindly memorize without number sense.”
>
> (Boaler, 2015)

“Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently.”
(National Council of Teachers of Mathematics, 2000)

“… students who are encouraged to use efficient mental computational strategies develop deeper understanding of number relationships.”
(Victoria State Government, Department of Education and Early Childhood Development, 2009)

“Developing number sense takes time; algorithms taught too early work against the development of good number sense.”
(Fosnot & Dolk, 2001)

“Fluency rests on a well-built mathematical foundation with three parts: an understanding of the meaning of the operations and their relationships to each other; the knowledge of a large repertoire of number relationships; and a thorough understanding of the base ten number system …”
(Russel, 2013)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need resources that provide opportunities for students to work with numbers in different ways and to develop deep understandings of numbers, quantities, and relationships.	Resources should provide students with opportunities to develop understandings of the meaning of number quantities and relationships, the base ten number system, and operations and their relationships, to develop flexibility and fluency in working with numbers.

Mental Math

Students who practise mental math are able to make calculations in their heads. Mental math encourages fluency and automaticity, fosters deeper understanding of concepts, builds confidence, improves problem-solving skills, and helps students to reason about the mathematics they encounter in daily situations.

What the Research Says

> “What does it mean to know mathematics? This is a complex question, but there is strong agreement that facility with numbers and skill in problem solving play important roles. Principles and Standards for School Mathematics [2000] calls for students to be proficient with tools that include pencil and paper and technology, as well as mental techniques. I would like to make a case for raising the importance of mental math as a major component in students’ tool kits of mathematical knowledge. Mental math is often associated with the ability to do computations quickly, but in its broadest sense, mental math also involves conceptual understanding and problem solving.”
>
> (Seeley, 2005)

“Students learn better if mental computation is taught and performed before written algorithms (and practice throughout education), along with appropriate work with concrete materials and drawings.”
(Clements & Sarama, 2009)

“Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.”
(Alberta Education, 2006)

“It is also significant that the integration of authentic mental math activity into an existing curriculum supported students’ development of number sense without any of the course content being sacrificed.”
(Whitacre & Nickerson, 2006)

“As students develop mental math proficiencies, they complete homework assignments faster, their confidence increases, and their overall understanding of numbers, operations, and algebraic thinking improves.”
(Olsen, 2015)

How This Informs Mathology

Implications for Teachers and Students	Implications for Resource Development
Teachers need opportunities to incorporate mental math strategies and activities into their lessons to enhance computation fluency and student confidence. When students develop and apply their mental math skills, a strong foundation of computational fluency is formed and they are set up for future success.	Resources should provide students with ongoing opportunities to establish their mental math skills to support fluency, which builds flexibility and confidence across many areas of mathematics.

Bibliography

PLANNING AND ASSESSMENT

Balanced Math

Devlin, K. (2010). The mathematical brain. In D. Sousa, (Ed.), *Mind, brain and education* (pp. 163–178). Bloomington, IN: Solution Tree Press. Available at https://books.google.ca/books?id=smYXBwAAQBAJ

Gangi, K. (2014). *Modeling what matters in K–12 Math: Lessons from literacy.* Teaching and Learning Alliance. Available at http://www.teachinglearningalliance.org/blog/modeling-what-matters-in-k-12-math-lessons-from-literacy

Hattie, J. (2015). *What doesn't work in education: The politics of distraction.* London, England: Pearson. Available at https://www.pearson.com/content/dam/corporate/global/pearson-dot-com/files/hattie/150602_DistractionWEB_V2.pdf

National Research Council; Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Executive summary. In *Adding it up: Helping children learn mathematics* (p. 11). Washington, DC: National Academies Press. Available at http://www.nap.edu/read/9822/chapter/2#11

Ottawa-Carleton District School Board. (n.d.). *Balanced mathematics instruction, K–12.* Available at http://www.ocdsb.ca/sta/bmi/Documents/Balanced%20Math.pdf

Protheroe, N. (2007). What does good math instruction look like? *Principal, 87* (1), 51–54. Available at https://www.naesp.org/resources/2/Principal/2007/S-Op51.pdf

Seeley, C. L. (2009). Balance is basic: What it takes to be mathematically prepared today. In *Faster isn't smarter: Messages about math, teaching, and learning in the 21st century* (pp. 93–100). Math Solutions. Available at http://mathsolutions.com/documents/FIS2E_9781935099727_Message14.pdf

Why Early Math Matters

Baroody, A., Lai, M., & Mix, K. (2006).The development of young children's early number and operation sense and its implications for early childhood education. In B. Spodek & O. Saracho (Eds.), *Handbook of research on the education of young children* (2nd ed.) (pp. 187–221). Mahwah, NJ: Erlbaum.

Bruce, C., Flynn, T., Moss, J. (2012). *Literature review: Mathematics for young children (M4YC).* Available at http://www.tmerc.ca/pub/M4YC_LiteratureReview_25june12.pdf

Colgan, L. (2014). *Making math children will love: Building positive mathitudes to improve student achievement in mathematics.* What Works: Research into Practice, Research Monograph #56. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_MakingMath.pdf

Foundations for numeracy: An evidence-based toolkit for early learning practitioners. (2010). Canadian Child Care Federation and Canadian Language and Literacy Research Network. Available at http://eyeonkids.ca/docs/files/foundations_for_numeracy.pdf

Jordan, N. C., Kaplan, D., Ramineni, C., Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*(3), 850–867. doi:10.1037/a0014939. Available at http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2782699/

Literacy and Numeracy Secretariat, Ontario Ministry of Education. (2008). *High-yield strategies to improve student learning: Viewer's guide.* Webcast Professional Learning Series. Toronto, ON: Queen's Printer for Ontario. Available at http://www.curriculum.org/secretariat/files/May2Guide.pdf

Moss, J., Bruce, C., Caswell, B., Flynn, T., & Hawes, Z. (2016). *Taking shape: Activities to develop geometric and spatial thinking, grades K–2.* Toronto, ON: Pearson Canada.

National Research Council, Committee on Early Childhood Mathematics; Cross, C. T., Woods, T. A., & Schweingruber, H. (Eds.). (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

Ritchie S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. P*sychological Science 24*(7), 1301–1308. doi:10.1177/0956797612466268. Epub May 2, 2013.

Vanbist, K., Ansari, D., Ghesquiere, P., & De Smedt, B. (2016). *Symbolic numerical magnitude processing is as important to arithmetic as phonological awareness is to reading. PLoS ONE 11*(3), e0151045. doi:10.1371/journal.pone.0151045

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher 43*(7), 352–60. doi:10.3102/0013189X14553660. Available at http://www.ncbi.nlm.nih.gov/pmc/articles/PMC4719158/

Wright, R. J., Ellemor-Collins, D., & Tabor, P. D. (2012). *Developing number knowledge: Assessment, teaching and intervention with 7–11 year olds.* Thousand Oaks, CA: Sage Publishing.

 |

Assessment

Barneveld, C. *Using data to improve student achievement.* (2008). What Works: Research into Practice, Research Monograph #15. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Using_Data.pdf

Clark, P., Owens, T., & Sutton, R. (2006). *Creating independent student learners, N–3: A practical guide to assessment for learning.* Winnipeg, MB: Portage & Main Press.

Darling-Hammond, L. (2008). Knowledge for teaching: What do we know? In M. Cochran-Smith, S. Feiman-Nemser, & D. J. McIntyre (Eds.), *Handbook of research on teacher education* (3rd ed.) (pp. 1316–1323). New York, NY: Routledge.

Ford, D. (2010). Culturally responsive classrooms: Affirming culturally different gifted students. *Gifted Child Today, 33*(1), 50–53. Available at http://files.eric.ed.gov/fulltext/EJ874024.pdf

Fosnot, C., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division.* Westport, CT: Heinemann.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 4*(2).

Literacy and Numeracy Secretariat, Ontario Ministry of Education. (2010). *Primary assessment: Lessons learned from kindergarten/grade 1 collaborative inquiry.* Capacity Building Series, Secretariat Special Edition 15. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_primaryassessment.pdf

Marzano, R., Pickering, D., & Pollock, J. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement.* Virginia: Association for Supervision & Curriculum (ASCD).

McMillan, J., & Hearn, J. (2009). Student self-assessment: The key to stronger student motivation and higher achievement. *Education Digest, 74*(8), 39–44.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM Press.

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools.* Toronto, ON: Queen's Printer for Ontario. Available at http://www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf

Ontario Ministry of Education. (2013). *Learning for all: A guide to effective assessment for all students, kindergarten to grade 12.* Available at http://www.edu.gov.on.ca/eng/general/elemsec/speced/LearningforAll2013.pdf

Ontario Ministry of Education. (2015). *Pedagogical documentation revisited: Looking at assessment and learning in new ways.* Capacity Building Series. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_PedagogicalDocument.pdf

Rogers, W. T. (1996). Principles for fair student assessment practices for education in Canada. *Canadian Journal of School Psychology 3*(3), 397–400. doi:10.1080/0969594960030308. Available at http://cdn.aac.ab.ca/wp-content/uploads/2015/10/eng_principles.pdf

Suurtamm, C. (2010, November). *Formative assessment to support student learning.* ETFO/FEEO Research for Teachers. Available at http://oere.oise.utoronto.ca/wp-content/uploads/2012/05/Research-for-Teachers-Number-2-Formative-Assessment.pdf

Stiggens, R., & Chappuis, J. (2005). Using student-involved classroom assessment to close achievement gaps. *Theory into Practice, 44*(1), 11–18.

Storeygard, J., Hamm, J., & Fosnot, C. T. (2010). Determining what children know: Dynamic versus static assessment. In C. T. Fosnot (Ed.), *Reweaving the tapestry: Models of intervention in mathematics.* Reston, VA: NCTM Press.

Selection and Use of Quality Resources

Cohen, D. K., & Ball, D. L. (2001, September). Making change: Instruction and its improvement. A Kappan Special Section on School Reform. *Phi Delta Kappan,* 73–77.

Harvey, R., & Averill, R. (2012). A lesson based on the use of contexts: An example of effective practice in secondary school mathematics. *Mathematics Teacher Education and Development, 14*(1), 41–59.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

National Research Council; Kilpatrick, J., Swafford, J., & Findell B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press.

National Council of Teachers of Mathematics. (2013). *Mathematics in early childhood learning: A position of the National Council of Teachers of Mathematics.* Available at http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/

Newton, D. P., & Newton, L. D. (2007). Could elementary mathematics textbook help give attention to reasons in the classroom. *Educational Studies in Mathematics, 64*(1), 69–84.

Nicol, C., & Crespo, S. (2005). Exploring math in imaginative places: Rethinking what counts as meaningful context for learning mathematics. *School Science and Mathematics, 105*(5), 240–251. Available at https://msu.edu/~crespo/SSMMathImagination.pdf

Stanovich, P., Stanovich, K. (2003). *Using Research and Reason in Education: How Teachers Can Use Scientifically Based Research to Make Curricular & Instructional Decisions.* Available at https://www.nichd.nih.gov/publications/pubs/Pages/using_research_stanovich.aspx

Tarr, J. E., Reys, B. J., Barker, D. D., & Billstein, R. (2006). Selecting high-quality mathematics textbooks. *Mathematics in the Middle School, 12*(1), 50–54.

Effective Lesson Model

Ackles, K., Fuson, K., & Sherin, M. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics.* Educational Practices 19. Geneva: International Bureau of Education. Available at http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_19.pdf

Askew, M., Brown, M., Rhodes, V., Johnson, D., & William, D. (1997). *Effective teachers of numeracy: Final report.* London: King's College. As cited in Doig, B., Groves, S., & Fujii, T. (2011). The critical role of task development in lesson study. In L. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 181–97). New York: Springer Science + Business Media B.V.

Chapin, S., O'Connor, C., & Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn.* Sausalito, California: Scholastics.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Oliviee, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Institute for Public School Initiatives, University of Texas at Austin. (n.d.). Algebra readiness, cycle 1. *The effective mathematics classroom.* Available at https://www.andrews.edu/sed/leadership_dept/webinars/presentationdocuments/the_effective_mathematics_classrroom.pdf

Lawson, A. (2007). *Learning mathematics vs following "rules": The value of student-generated methods.* What Works: Research into Practice, Research Monograph #2. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Lawson.pdf

Literacy and Numeracy Secretariat, Ontario Ministry of Education. (2010). *Communication in the mathematics classroom: Gallery walk, math congress and bansho.* Capacity Building Series, Special Edition 13. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_Communication_Mathematics.pdf

MacMath, S., Wallace, J., & Chi, X. (2009). *Problem-based learning in mathematics: A tool for developing students' conceptual knowledge.* What Works: Research into Practice, Research Monograph #22. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_problem_based_math.pdf

National Center on Universal Design for Learning at CAST. (2012). *Three principles of UDL.* Available at http://www.udlcenter.org/aboutudl/whatisudl/3principles

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all: Executive summary.* Available at http://www.nctm.org/PtA/

Richards, J. C. (1998). What's the use of lesson plans? In J. C. Richards (Ed.), *Beyond training* (pp. 103–121). New York: Cambridge University Press. Cited in Farrell, T. S. C. (2012). Lesson planning. In Jack C. Richards & Willy A. Renandya (Eds.), *Methodology in language teaching: An anthology of current practice* (pp. 30–39). Cambridge University Press. Available at https://books.google.ca/books?id=VxnGXusQlI8C&pg=PA30&lpg=PA30&dq#v=onepage&q&f=false

Small, M. (2013). *Teaching elementary mathematics.* ETFO/FEEO Research for Teachers.

Takahashi, A. (2006). Characteristics of Japanese mathematics lessons. *Tsukuba Journal of Educational Study in Mathematics, 25.* Available at http://www.human.tsukuba.ac.jp/~mathedu/2504.pdf

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., McGarvey, L. M., & Folk, S. (2015). *Elementary and middle school mathematics: Teaching developmentally* (4th Canadian ed.). Toronto, ON: Pearson.

Curriculum and Big Ideas

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38*(2), 115–131. Cited in National Research Council, Committee on Early Childhood Mathematics; C. T. Cross, T. A. Woods, & H. Schweingruber (Eds.). (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

Charles, R. I. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *NCSM Journal, 7*(3), 9–24. Available at http://www.authenticeducation.org/bigideas/sample_units/math_samples/BigIdeas_NCSM_Spr05v7.pdf

Clements, D., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach.* New York, NY: Routledge.

Dougherty, B. J., Flores, A., Louis, E., & Sophian, C. (2010). *Developing essential understanding of number and numeration for teaching mathematics in prekindergarten–Grade 2.* Reston, VA: National Council of Teachers of Mathematics.

Education Scotland. (2013). *The purpose of the curriculum.* Available at http://www.educationscotland.gov.uk/thecurriculum/whatiscurriculumforexcellence/thepurposeofthecurriculum/index.asp

Hurst, C., & Hurrell, D. (2014). Developing the big ideas of number. (2014). *International Journal of Educational Studies in Mathematics, 1*(2), 1–18. Available at http://www.ijesim.com/upload/dosya/1-2-1-eng_hurst_developing-the-big-ideas-of-number_15492af7d16370.pdf

Ontario Ministry of Education. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario.* Toronto, ON: Queen's Printer for Ontario. Available at http://edu.gov.on.ca/eng/document/reports/math/index.html

Small, M. (2009). *Big ideas from Dr. Small, grade K–3: Creating a comfort zone for teaching mathematics.* Toronto, ON: Nelson.

Differentiated Instruction

Benjamin, A. (2006). Valuing differentiated instruction. *Education Digest, 72*(1), 57–59.

Ford, D. (2010). Culturally responsive classrooms: Affirming culturally different gifted students. *Gifted Child Today, 33*(1), 50–53. Available at http://files.eric.ed.gov/fulltext/EJ874024.pdf

Lawrence-Brown, D. (2004). Differentiated instruction: Inclusive strategies for standards-based learning that benefit the whole class. *American Secondary Education, 32*(3), 34–62.

Literacy and Numeracy Secretariat, Ontario Ministry of Education. (2008). *High-yield strategies to improve student learning: Viewer's guide.* Webcast Professional Learning Series. Toronto, ON: Queen's Printer for Ontario. Available at http://www.curriculum.org/secretariat/files/May2Guide.pdf

Ontario Ministry of Education. (2013). *Learning for all: A guide to effective assessment for all students, kindergarten to grade 12.* Available at http://www.edu.gov.on.ca/eng/general/elemsec/speced/LearningforAll2013.pdf

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., McGarvey, L. M., & Folk, S. (2015). *Elementary and middle school mathematics: Teaching developmentally* (4th Canadian ed.). Toronto, ON: Pearson.

THE MATH LEARNING COMMUNITY/ENVIRONMENT

Encouraging Growth Mindsets

Ansari, D., & Matejko, A. (2015). The development of the numerate brain. *Principal Connections, 18,* (Catholic Principal's Council Ontario), 16–20. Available at https://cpco.on.ca/files/5214/1329/3912/NumerateBrain.pdf

Boaler, J. (2013). Ability and mathematics: The mindset revolution that is reshaping education. *FORUM, 55*(1). Available at http://www.youcubed.org/wp-content/uploads/14_Boaler_FORUM_55_1_web.pdf

Boaler, J. (2015). *What's math got to do with it? How teachers and parents can transform mathematics learning and inspire success.* New York, NY: Penguin.

Boaler, J. (2016a). *Advice for parents, from Professor Jo Boaler.* Youcubed at Stanford University [blog]. Available at https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2016/03/Parent-Night-Handout-vF.pdf

Boaler, J. (2016b). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* San Francisco, CA: Jossey-Bass.

Colgan, L. (2014). *Making math children will love.* What Works: Research into Practice, Research Monograph #56. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_MakingMath.pdf

Dweck, C. (2006). *Mindset: The new psychology of success.* New York, NY: Random House.

Dweck, C. (2007). *Mindset: The new psychology of success.* New York, NY: Ballantine Books.

Dweck, C. (2008). *Mindsets and math/science achievements.* Carnegie Corporation of New York, Institute for Advanced Study Commission on Mathematics and Science Education. Available at http://www.growthmindsetmaths.com/uploads/2/3/7/7/23776169/mindset_and_math_science_achievement_-_nov_2013.pdf

Eggen, P., & Kauchak, D. (2013). *Educational psychology: Windows on classrooms* (9th ed.). Upper Saddle River, NJ: Pearson Education.

Hinton, C., Fischer, K., & Glennon, C. (2012). *Mind, brain and education.* Students at the Center Series: Teaching and Learning in the Era of the Common Core. Available at http://www.studentsatthecenter.org/sites/scl.dl-dev.com/files/Mind%20Brain%20Education.pdf

Lyons, I., Price, G., Vaessen, A., Blomert L., & Ansari D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science 17*(5), 714–726.

Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences, 16*(8), 404–406. doi:10.1016/j.tics.2012.06.008. Available at https://hpl.uchicago.edu/sites/hpl.uchicago.edu/files/uploads/TiCS%20Final_Maloney%26Beilock_2012.pdf

Mighton, J. (2013, October 28). Kids can't teach themselves math. *Globe and Mail.* Available at http://www.theglobeandmail.com/news/national/education/kids-cant-figure-out-math-by-themselves/article15087557/

Moser, J. S., Schroder, H. S., Heeter, C., Moran, T. P., & Lee, Y. H. (2011). Mind your errors: Evidence for a neural mechanism linking growth mind-set to adaptive posterror adjustments. *Psychological Science, 22*(12), 1484–1489. Cited in Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* San Francisco, CA: Jossey-Bass.

Ontario Ministry of Education. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario.* Toronto, ON: Queen's Printer for Ontario. Available at http://edu.gov.on.ca/eng/document/reports/math/index.html

Price, G., & Ansari, D. (2013). Dyscalculia: Characteristics, causes, and treatments. *Numeracy, 6*(1). doi:10.5038/1936-4660.6.1.2

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development, 14*(2), 187–202. Available at http://www.tandfonline.com/doi/abs/10.1080/15248372.2012.664593

Small, M. (2013). *Teaching elementary mathematics.* ETFO/FEEO Research for teachers. Available at http://www.etfo.ca/Resources/ForTeachers/Documents/Research%20for%20Teachers%20-%20Number%2015%20-%20Teaching%20Elementary%20Mathematics.pdf

Student Talk

Bruce, C. (2007). *Student interaction in the math classroom: Stealing ideas or building understanding?* Research into Practice: Ontario Association of Deans of Education. Research Monograph #1 (premier edition), 1–4.

Burns, M. (2005). Looking at how students reason. *Educational Leadership, 63*(3), 26–31. Available at http://www.ascd.org/ASCD/pdf/journals/ed_lead/el200511_burns.pdf

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, grades K–6.* Sausalito, CA: Math Solutions.

Garcia, L. A. D. (2011). How to get students talking! Generating math talk that supports math learning. *Math Solutions, 39* [online newsletter]. Available at http://www.mathsolutions.com/documents/how_to_get_students_talking.pdf

Ontario Ministry of Education. (n.d.). *Math-talk learning community: Research synopsis.* Connecting Practice and Research in Mathematics Education. Available at http://www.edugains.ca/resources/LeadingChange/KeyDirectionsandFrameworks/MathTalkLearningCommunityResearchSynopsis.pdf

Smith, M., & Stein, M.K. (2011). 5 practices for orchestrating productive mathematics discussions. NCTM. Available at http://camsemsgeo.weebly.com/uploads/3/1/0/0/31008293/introch1.pdf

Stein, C. C. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher, 101*(4), 285–289. Available at http://jwilson.coe.uga.edu/EMAT7050/Students/Dwyer/mt2007-11-285a.pdf

Wagganer, E. L. (2015). Creating math talk communities. *Teaching Children Mathematics, 22*(4), 248–254. Available at http://www.nctm.org/Publications/Teaching-Children-Mathematics/2015/Vol22/Issue4/Creating-Math-Talk-Communities/

Building a Community of Math Learners

Clements, D. H., Sarama, J., & DiBiase, A. M. (Eds.). (2004). *Engaging young children in mathematics: Findings of the 2000 National Conference on Standards for Preschool and Kindergarten Mathematics Education.* Mahwah, NJ: Lawrence Erlbaum.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258–291. Available at https://espace.library.uq.edu.au/view/UQ:74224/SBE10UQ74224.pdf

Hattie, J. (2008). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* Florence, KY: Routledge.

National Council of Teachers of Mathematics. (2013). *Mathematics in early childhood learning.* A position statement. Available at http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/

Ontario Ministry of Education. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario.* Toronto, ON: Queen's Printer for Ontario. Available at http://edu.gov.on.ca/eng/document/reports/math/index.html

Ontario Ministry of Education. (2011). *Maximizing student mathematical learning in the early years.* Capacity Building Series, 22. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_Maximize_Math_Learning.pdf

Pennant, J. (2013). *Developing a classroom culture that supports a problem solving approach to mathematics.* NRICH, University of Cambridge. Available at https://nrich.maths.org/10341

Protheroe, N. (2007). What does good math instruction look like? *Principal, 87*(1), 51–54. Available at https://www.naesp.org/resources/2/Principal/2007/S-Op51.pdf

Sinclair, N. (2008). Attending to the aesthetic in the mathematics classroom. *For the Learning of Mathematics, 28*(1), 29–35.

Suurtamm, C., Quigley, B., & Lazarus, J. (2015). *Making space for students to think mathematically.* What Works: Research into Practice, Research Monograph #59. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_SpaceThinkMath.pdf

First Nations, Métis, and Inuit Learners

Alberta Teachers' Association. (2010). *Here comes everyone: Teaching in the intercultural classroom.* Available at http://www.teachers.ab.ca/SiteCollectionDocuments/ATA/Publications/Human-Rights-Issues/MON-3%20Here%20comes%20everyone.pdf

Alberta Education. (2005). *Our words, our ways: Teaching First Nations, Métis and Inuit learners.* Available at https://education.alberta.ca/media/563982/our-words-our-ways.pdf

Barta, J., Jette', C., & Wiseman, D. (2003). Dancing numbers: Cultural, cognitive, and technical instructional perspectives on the development of Native American mathematical and scientific pedagogy. *Educational Technology Research and Development, 51*(2), 87–97.

Beatty, R., & Blair, D. (2015). Indigenous pedagogy for early mathematics: Algonquin looming in a grade 2 math classroom. *International Journal of Holistic Early Learning and Development, 1.*

Borden, L. L. (2010). Transforming Mathematics Education for Mi'kmaw Students Through Mawikinutimatimk. Available at http://showmeyourmath.ca/wp-content/uploads/2016/10/LunneyBordenDissertation.pdf

Borden, L. L. (2011). The "verbification" of mathematics: Using the grammatical structures of Mi'kmaq to support student learning. *For the Learning of Mathematics, 31*(3), 8–13.

Borden, L. L. (2013). What's the word for…? Is there a word for…? How understanding Mi'kmaw language can help support Mi'kmaw learners in mathematics. *Mathematics Education Research Journal, 25*(1), 5–22.

First Nations Education Steering Committee. (2011). *Math First Peoples.* Available at http://www.fnesc.ca/resources/math-first-peoples/

Russel, G. L, & Chernoff, E. F. (2013). The marginalization of Indigenous students within school mathematics and the math wars: Resolution within ethical spaces. *Mathematics Education Research Journal, 25*(1), 109–227.

Toulouse, P. R. (2008). *Integrating Aboriginal teaching and values into the classroom.* What Works: Research into Practice, Research Monograph # 11. Available at https://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Toulouse.pdf

Positive Personal and Cultural Identity

Aguirre, J. M., & del Rosario Zavala, M. (2013). Making culturally responsive mathematics teaching explicit: a lesson analysis tool. *Pedagogies: An International Journal, 8*(2), 163–190.

Alberta Teachers' Association. (2010). *Here comes everyone: Teaching in the intercultural classroom.* Available at http://www.teachers.ab.ca/SiteCollectionDocuments/ATA/Publications/Human-Rights-Issues/MON-3%20Here%20comes%20everyone.pdf

Borden, L. L. (2010). Transforming Mathematics Education for Mi'kmaw Students Through Mawikinutimatimk. Available at http://showmeyourmath.ca/wp-content/uploads/2016/10/LunneyBordenDissertation.pdf

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nassir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105–117). New York, NY: Teachers College Press.

Civil, M., & Kahn, L. H. (2001). Mathematics instruction developed from a garden theme. *Teaching Children Mathematics, 7*(7), 400–405.

Gilliard, J. L., & Moore, R. A. (2006). An investigation of how culture shapes curriculum in early care and education programs on a Native American Indian reservation. *Early Childhood and Education Journal, 34*(4), 251–258.

Nicol, C., Archibald, J., & Baker, J. (2013). Designing a model of culturally responsive mathematics education: Place, relationships and storywork. *Mathematics Education Research Journal, 25*, 73–89.

Shirley, L. (1995). In P. A. House and A. F. Coxford (Eds.), *Connecting mathematics across the curriculum* (pp. 34–43). Reston, VA: National Council of Teachers of Mathematics.

Turner, E. E., Drake, C., Roth McDuffie, A., Aguirre, J. M., Bartell, T. G., & Foote, M. Q. (2012). Promoting equity in mathematics teacher preparation: A framework for advancing teacher learning of children's multiple mathematics knowledge bases. *Journal of Mathematics Teacher Education, 15*(1), 67–82.

Wiest, L. R. (2001). Teaching mathematics from a multicultural perspective. *Equity and Excellence in Education, 34*(1), 16–25.

Zaslavsky, C. (2001). Developing number: What can other cultures tell us? *Teaching Children Mathematics*, February, 312–319.

Classroom Management

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science, 27,* 1049–1053.

Ktytor, L., & Waechter, K. (2014). *The effects of the math daily 3 structure on student achievement and growth in an elementary school setting.* Masters of Arts in Education Action Research Papers, Paper 82. Available at http://sophia.stkate.edu/cgi/viewcontent.cgi?article=1077&context=maed

Lataille-Démoré, D. (2007). *Combined grade classroom.* What Works: Research into Practice, Research Monograph #9. Available at http://oere.oise.utoronto.ca/wp-content/uploads/2012/08/+WW_CombinedGrades.pdf

Literacy and Numeracy Secretariat, Ontario Ministry of Education. (2008). *Differentiating mathematics instruction.* Capacity Building Series, Secretariat Special Edition 7. Available at https://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/different_math.pdf

Marzano, R. J., Marzano, J. S., & Pickering, D. (2008). *Classroom management that works: Research-based strategies for every teacher.* Alexandria, VA: Association for Supervision and Curriculum Development.

Pierce, R. L, & Adams, C. M. (2004). Tiered lessons: One way to differentiate mathematics instruction. *Gifted Child Today, 27*(2), 58–66.

Price, G., & Ansari, D. (2013). Dyscalculia: Characteristics, causes, and treatments. *Numeracy, 6*(1). doi:10.5038/1936-4660.6.1.2

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners* (2nd ed.). Association for Supervision & Curriculum (ASCD). Ch. 1 available at http://www.ascd.org/publications/books/108029/chapters/What-Is-a-Differentiated-Classroom%C2%A2.aspx

Tomlinson, C., & Eidson, C. (2003). *Differentiation in practice: A resource guide for differentiating curriculum, grades K–5.* ASCD.

Tomlinson, C. A., & McTighe, J. (2006). *Integrating differentiated instruction and understanding by design.* ASCD.

TEACHER LEARNING

Professional Learning

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education, 51,* 241–247.

Ball, D. L, & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis and E. Simmt (Eds.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3–14). Edmonton, AB: CMESG/GCEMD.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, Fall,* 14–46.

Bruce, C. D., Esmonde, I., Ross, J., Gookie, L., & Beatty, R. (2010). The effects of sustained classroom-embedded teacher professional learning on teacher efficacy and related student achievement. *Teaching and Teacher Education, 26*(8), 1598–1608. Available at http://www.sciencedirect.com/science/article/pii/S0742051X10000922

Bruce, C. D, & Flynn, T. C. (2013). Assessing the effects of collaborative professional learning: Efficacy shifts in a three-year mathematics study. *Alberta Journal of Educational Research, 58*(4), 691–709.

Bruce, C. D., Flynn, T. C., & Bennett, S. (2015). A focus on exploratory tasks in lesson study: The Canadian "Math for Young Children" project. *ZDM Mathematics Education,* 1–14.

Copur-Gencturk, Y. (2015). The effects of changes in mathematical knowledge on teaching: A longitudinal study of teachers' knowledge and instruction. *University of Houston Journal for Research in Mathematics, 46*(3). Available at http://www.jstor.org/stable/10.5951/jresematheduc.46.3.0280?seq=1#page_scan_tab_contents

Darling-Hammond, L., & Ball, D. L. (2000). *Teaching for high standards: What policymakers need to know and be able to do.* CPRE paper, No. JRE-04. Philadelphia, PA: University of Pennsylvania, Consortium for Policy Research in Education.

Davis, B., & Simmt, E. (2016). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*(3), 293–319. Available at http://link.springer.com/article/10.1007%2Fs10649-006-2372-4

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*(4), 915–945.

Kajander, A. (2010). *Teachers' evolving mathematical understandings.* OERE Research Summary. Available at http://oere.oise.utoronto.ca/document/teachers-evolving-mathematical-understandings/

Lawson, A. (2015). *What to look for: Understanding and developing student thinking in early numeracy.* Toronto, ON: Pearson Education Canada.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (2nd ed.). New York, NY: Routledge.

Mason, J. (2003). *Researching your own practice: The discipline of noticing.* New York, NY: RoutledgeFalmer.

National Council of Teachers of Mathematics. (2013). *Mathematics in early childhood learning: A position of the National Council of Teachers of Mathematics.* Available at http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Available at www.nctm.org/PtA/

Nicol, C. (1999). Learning to teach mathematics: Questioning, listening, and responding. *Educational Studies in Mathematics,* 37, 47–66.

Ontario Ministry of Education. (2004). *Teaching and learning mathematics: The report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario.* Available at http://www.edu.gov.on.ca/eng/document/reports/numeracy/panel/numeracy.pdf

Ontario Ministry of Education. (2011). *Maximizing student mathematical learning in the early years.* Capacity Building Series, 22. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_Maximize_Math_Learning.pdf

Ontario Ministry of Education. (2013). *Leaders in educational thought: Special edition on mathematics.* Webcasts for Educators, Cathy Bruce Professional Learning Series. Available at http://www.curriculum.org/k-12/en/projects/leaders-in-educational-thought-special-edition-on-mathematics

OSE Office of School Education. (n.d.). T*he professional learning of teachers.* Available at https://lmrtriads.wikispaces.com/file/view/Professional_Learning_of_Teachers.pdf

Robinson, K. (2013, April). Ken Robinson: How to escape education's Death Valley [Video file]. Available at https://www.ted.com/talks/ken_robinson_how_to_escape_education_s_death_valley/

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: Seeing through teachers' eyes.* New York, NY: Routledge.

Sinclair, N., Zazkis, R., & Liljedahl, P. (2003). Number worlds: Visual and experimental access to elementary number theory concepts. *International Journal of Computers for Mathematical Learning, 8,* 235–263.

Towers, J. (2010). Learning to teach mathematics through inquiry: A focus on the relationship between describing and enacting inquiry-oriented teaching. *Journal of Mathematics Teacher Education, 13*(3), 243–263.

EARLY MATH

How Young Children Best Learn Math

Bailey, D. H. (2014). *What's the point of teaching math in preschool?* Brown Center Chalkboard Series Archive 87 [online only]. Available at http://www.brookings.edu/research/papers/2014/11/13-chalkboard-preschool-math-bailey

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38*, 115–131.

Bennett, N., Wood, L., & Rogers, S. (1997). *Teaching through play: Teacher's thinking and classroom practice.* Philadelphia, PA: Open University Press.

Boaler, J., Chen, L., Williams, C., & Cordero, M. (2016). *Seeing as understanding: The importance of visual mathematics for our brain and learning.* Youcubed at Stanford University [blog]. Available at https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2016/04/Visual-Math-Paper-vF.pdf

Clements, D. H. (2001). Mathematics in the preschool. In National Council of Teachers of Mathematics, *Teaching children mathematics* (pp. 270–275). Reston, VA: National Council of Teachers of Mathematics. Available at http://gse.buffalo.edu/fas/clements/files/Preschool_Math_in_TCM.pdf

Clements, D. H., & Sarama, J. (2014). *Math, and false dichotomies.* In Preschool Matters Today: A Blog of the National Institute for Early Education Research [blog]. Available at http://preschoolmatters.org/2014/03/03/play-mathematics-and-false-dichotomies/

Clements, D. H., Sarama, J., & DiBiase, A. M. (Eds.). (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education.* Mahwah, NJ: Lawrence Erlbaum.

Cornelle, B. 1997. Recognizing the mathematics. *Teaching Children Mathematics, 4*(2), 112–115.

Darling-Hammond, L. (2008). Knowledge for teaching: What do we know? In Marilyn Cochran-Smith, Sharon Feiman-Nemser, & D. John McIntyre (Eds.), *Handbook of Research on Teacher Education* (3rd ed.), pp. 1316–1323. New York, NY: Routledge, 2008.

Davis, B. (2001). Why teach mathematics to all students? *For the Learning of Mathematics, 21*(1), 17–24.

Education Commission of the States. (2013, October). *Math in the early years: A strong predictor of success.* The Progress of Education Reform, 14(5). Available at http://www.du.edu/kennedyinstitute/media/documents/math-in-the-early-years.pdf

Featherstone, H. (2000). "-Pat + Pat = 0": Intellectual play in elementary mathematics. *For the Learning of Mathematics, 20*(2), 14–23.

Fosnot, C. & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division.* Westport, CT: Heinemann, 5–7

Ginsburg, H., & Ertle, B. (2008). Knowing the mathematics in early childhood mathematics. In O.H. Saracho & B. Spodek (Eds.), *Contemporary perspectives in mathematics in early childhood education.* Charlotte, NC: Information Age Publishing

Hattie, J. (2012). Foreword. In C. B. Dean, E. R. Hubbell, H. Pitler, & B. J. Stone, *Classroom instruction that works: Research-based strategies for increasing student achievement* (2nd ed.). Virginia: Association for Supervision & Curriculum (ASCD). Available at http://www.ascd.org/publications/books/111001/chapters/Introduction@-Instruction-That-Makes-a-Difference.aspx

Hattie, J. (2015). *What doesn't work in education: The politics of distraction.* London: Pearson. Available at https://www.pearson.com/content/dam/corporate/global/pearson-dot-com/files/hattie/150602_DistractionWEB_V2.pdf

Hiebert, J. (Ed.). (1986). *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

Mason, J. (2002). *Researching your own practice: The discipline of noticing.* London & New York: Routledge and Falmer.

Moss, J., Bruce, C., Caswell, B., Flynn, T., Hawes, Z. (2016). *Taking shape: Activities to develop geometric and spatial thinking, grades K–2.* Toronto, ON: Pearson Canada.

National Research Council; Kilpatrick, J., Swafford, J., & Findell B. (Eds.). (2001a). Executive summary. In *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press. Available at http://www.nap.edu/read/9822/chapter/2pp. 1-14

National Research Council; Kilpatrick, J., Swafford, J., & Findell B. (Eds.). (2001b). The strands of mathematical proficiency. In *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press. Available at http://www.nap.edu/read/9822/chapter/6

Ontario Ministry of Education. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario.* Toronto: Queen's Printer for Ontario. Available at http://edu.gov.on.ca/eng/document/reports/math/index.html

Purpura, D. J., Baroody, A. J., & Lonigan, C. J. (2013). The transition from informal to formal mathematical knowledge: Mediation by numeral knowledge. *Journal of Educational Psychology, 105*, 453–464.

Rittle-Johnson, B., & Schneider, M. (n.d.). Developing conceptual and procedural knowledge of mathematics. In R. Cohen Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition.* Oxford, England: Oxford University Press. doi:10.1093/oxfordhb/9780199642342.013.014. Available at https://www.uni-trier.de/fileadmin/fb1/prof/PSY/PAE/Team/Schneider/RittleJohnsonSchneiderInPress.pdf

Rittle-Johnson, B., Schneider, M., & Star, J. (2015). Not a one-way street: Bi-directional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review, 27,* 587–597. doi:10.1007/s10648-015-9302-x

Seeley, C. L. (2009). *Faster isn't smarter: Messages about math, teaching, and learning in the 21st century.* Math Solutions. Available at https://cathyseeley.files.wordpress.com/2009/03/fasterisntsmarter.pdf

Seo, K. H. (2003). What children's play tells use about teaching mathematics. *YC Young Children, 56*(1), 28–34.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36,* 404–411.

Stigler, J. W., & Hiebert, J. (2009). Closing the teaching gap. *Phi Delta Kappan, 91*(3), 32–37.

U.S. Department of Education. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.*

MATH PROCESSES FOR LEARNING

21st-Century Competencies in Math

Ananiadou, K., & Claro, M. (2009). *21st century skills and competences for new millennium learners in OECD countries.* OECD Education Working Papers, 41. Available at http://dx.doi.org/10.1787/218525261154

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics.* Educational Practices 19. Geneva: International Academy of Education and International Bureau of Education. Available at http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_19.pdf

Bruce, C. (2007). *Student interaction in the math classroom: Stealing ideas or building understanding.* What Works: Research into Practice, Research Monograph #1. Available at http://www.curriculum.org/k-12/en/wp-content/uploads/2013/09/Student-Interaction-in-the-Math-Classroom-Stealing-Ideas-or-Building-Understanding.pdf

C21: Canadians for 21st Century Learning and Innovation. (2013). *21st century Canadian learning, innovation and skills agenda: Issues and considerations.* Available at http://www.c21canada.org/2013/12/09/21st-century-canadian-learning-innovation-skills-agenda-issues-considerations/

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education, 24*(5), 428–441. Cited in Ontario Ministry of Education. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario* (p. 10). Toronto: Queen's Printer for Ontario. Available at http://edu.gov.on.ca/eng/document/reports/math/index.html

Davis, M. G. (2010, November 7). *Thinking like mathematicians.* Opal School Blog [blog]. Available at http://opalschoolblog.typepad.com/opal-school-blog/2010/11/thinking-like-mathematicians.html

English, L., & Gainsburg, J. (2016). Problem solving in a 21st-century mathematics curriculum. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 313–335). New York: Routledge.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics, 28*(2).

Grouws, D. A., & Cebulla, K. J. (n.d.). *Improving student achievement in mathematics.* International Academy of Education and International Bureau of Education Educational Practices Series-4, UNESCO. Available at http://www.ibe.unesco.org/fileadmin/user_upload/archive/Publications/educationalpracticesseriespdf/prac04e.pdf

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4*(7), 410–414.

Krpan, C. M. (2013). *Math expressions: Developing student thinking and problem solving through communication.* Toronto, ON: Prentice.

Lai, E. R., Viering, M. (2012). *Assessing 21st century skills: Integrating research findings.* National Council on Measurement in Education. Vancouver, B.C.: Pearson Canada. Available at http://researchnetwork.pearson.com/wp-content/uploads/Assessing_21st_Century_Skills_NCME.pdf

Liljedahl, P., Chernoff, E., & Zazkis, R. (2007). Interweaving mathematics and pedagogy in task design: A tale of one task. *Journal of Mathematics Teacher Education, 10,* 239–249.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics. Available at www.nctm.org/PtA/

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Research Council; Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Executive summary. In *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press.

National Research Council, Committee on Early Childhood Mathematics; Cross, C. T., Woods, T. A., & Schweingruber, H. (Eds.). (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

Ontario Ministry of Education. (2016). *21st century competencies: Foundation document for discussion* (Winter 2016 ed.). Available at http://www.edugains.ca/resources21CL/About21stCentury/21CL_21stCenturyCompetencies.pdf

Pacific Policy Research Center. (2010). *21st century skills for students and teachers.* Honolulu, HA: Kamehameha Schools, Research & Evaluation Division. Available at http://www.ksbe.edu/_assets/spi/pdfs/21_century_skills_full.pdf

Papic, M. P., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3).

Raphael, T. E., & Hiebert, E. H. (1996). *Creating an integrated approach to literacy instruction.* Fort Worth, TX: Harcourt Brace. Available at http://textproject.org/assets/library/resources/Raphael-Hiebert-1996-Creating-an-Integrated-Approach-to-Literacy-Instruction.pdf

Reeves, N. (1990). The mathematics language connection. In J. Birkmore-Brand, *Language in mathematics* (pp. 90–99). Portsmouth, NH: Heinemann.

Rumsey, C., & Langrall, C. W. (2016). Promoting mathematical argumentation. *Teaching Children Mathematics, 22*(7), 412–419.

Sherin, M. (2002). A balancing act: Developing a discourse community in a mathematics classrooms. *Journal of Mathematics Teacher Education, 5,* 205–233.

Sherin, M., Mendez, E., & Louis, D. (2000). Talking about math talk. In M. C. Burke & F. Curico (Eds.), *Learning mathematics for a new century* (pp. 188–196). Reston, VA: National Council of Teachers of Mathematics.

Smith, M. & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* National Council of Teachers of Mathematics. Available at http://camsemsgeo.weebly.com/uploads/3/1/0/0/31008293/introch1.pdf

Stein, C. (2007). Let's talk: Promoting mathematical discourse in the classroom. *Mathematics Teacher, 101*(4), 285–289.

HOME-SCHOOL CONNECTION

Home Connection

Anderson, A. (1997). Families and mathematics: A study of parent-child interactions. *Journal for Research in Mathematics Education, 28*(4), 484–511.

Bruce, C. (2013, November 1). High expectations will raise math scores. *Globe and Mail.* Available at http://www.theglobeandmail.com/news/national/education/students-need-high-expectations-in-math/article15216621/

Council of Ontario Directors of Education. (2015). *Inspiring your child to learn and love math.* Available at http://www.ontariodirectors.ca/parent_engagement-math/en/

Kormanik, K. (2012). *A greater role in math education for parents: mathematical reasoning at home.* EdSource: Highlighting Strategies for Student Success. Available at http://edsource.org/2012/a-greater-role-in-math-education-for-parents-mathematical-reasoning-at-home/23918

Pushor, D. (2010, September). *Parent engagement.* ETFO/FEEO Research for Teachers. Available at http://oere.oise.utoronto.ca/wp-content/uploads/2012/05/Research-for-Teachers-Number-1-Parent-Engagement.pdf

Sheldon, S. B., & Epstein, J. L. (2005). Involvement counts: Family and community partnerships and mathematics achievement. *The Journal of Educational Research, 98*(4), 196–206.

LEARNING TOOLS

Games and Manipulatives as Mathematical Thinking Tools

Colgan, L. (2014). *Making math children will love: Building positive mathitudes to improve student achievement in mathematics.* What Works: Research into Practice, Research Monograph #56. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_MakingMath.pdf

Kamii, C. (2015). The importance of thinking: Direct vs indirect teaching of number concepts. *YC Young Children, 70*(1), 74–77. Available at https://sites.google.com/site/constancekamii/articles-available-for-printing

Lach, T. M., & Sakshaug, L. E. (2005). Let's do math: Wanna play? *Mathematics Teaching in the Middle School, 11*(4), 172–176.

MacDonald, B. L., & Shumway, J. F. (2016). Subitizing games: Assessing preschoolers' number understanding. *Teaching Children Mathematics, 22*(6), 340–348. Available at https://teal.usu.edu/plugins/work/blogger/178/files/Journal_Article.pdf

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175–197.

Murray, A. (2001). Ideas on manipulative math for young children. *YC Young Children, 56*(4), 28–29.

National Research Council; Kilpatrick, J., Swafford, J., & Findell B. (Eds.). (2001a). Executive summary. In *Adding it up: Helping children learn mathematics* (p. 9). Washington, DC: National Academies Press.

National Research Council; Kilpatrick, J., Swafford, J., & Findell B. (Eds.). (2001b). Developing proficiency with whole numbers. In *Adding it up: Helping children learn mathematics* (p. 198). Washington, DC: National Academies Press.

National Research Council; Kilpatrick, J., Swafford, J., & Findell B. (Eds.). (2001c). The strands of mathematical proficiency with whole numbers. In *Adding it up: Helping children learn mathematics* (pp. 118–119). Washington, DC: National Academies Press.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Available at www.nctm.org/PtA/

Ontario Ministry of Education. (2003). *Early math strategy: The report of the Expert Panel on Early Math in Ontario.* Toronto, ON: Queen's Printer for Ontario.

Research on the benefits of manipulatives. (n.d.). Hand2Mind, p. 4. Available at https://www.hand2mind.com/pdf/learning_place/research_math_manips.pdf

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science, 11*(5), 655–661.

Van de Walle, J. A., & Folk, S. (2010). *Elementary and middle school mathematics: Teaching developmentally* (2nd Canadian ed.). Toronto, ON: Pearson Canada.

Wickett, M., & Burns, M. (2003). Introduction. *In Lessons for extending division: Grades 4–5* (p. xvii.). Sausalito, CA: Math Solutions. Available at http://mathsolutions.com/-%20Number%2015%20-%20Teaching%20Elementary%20Mathematics.pdf

Affordances of Technology

Allsopp, D. H., McHatton, P. A., & Farmer, J. L. (2010). Technology, mathematics PS/RTI, and students with LD: What do we know, what have we tried, and what can we do to improve outcomes now and in the future? *Learning Disability Quarterly, 33*(4), 273–288.

Bruce, C. D. (2012). *Technology in the mathematics classroom: Harnessing the learning potential of interactive whiteboards.* What Works: Research into Practice, Research Monograph #38. Available at http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/WW_technology.pdf

Bruce, C. D., & Flynn, T. (2012). Exploring interactivity between students, teachers and the interactive whiteboard: Video analysis of pedagogical-technological interactivity during mathematics lessons. *RicercAzione,* (4)2, 183–197.

Couros, G. (2015). *The innovator's mindset: Empower learning, unleash talent, and lead a culture of creativity.* San Diego, CA: Dave Burgess Consulting.

Flores, A. (2002). By way of introduction: Learning and teaching mathematics with technology. *Teaching Children Mathematics, 8*(6), 308, 310.

Goodwind, K., & Highfield, K. (2012). *iTouch and iLearn: An examination of "educational" apps.* Early Education and Technology for Children. Available at http://www.eetcconference.org/wp-content/uploads/Examination_of_educational_apps.pdf

Interactive Educational Systems Design. (2013). *National survey on mobile technology for K–12 education: Research report* (Educator edition). Available at http://dnwssx4l7gl7s.cloudfront.net/amplifylive/default/page/-/Amplify_Mobile_Technology_Research_Report.pdf%20

Linder, S. M. (2012). Interactive whiteboards in early childhood mathematics: Strategies for effective implementation in pre-K-grade 3. *YC Young Children, 67*(3), 26–32.

McKewen, R. N., & Dubé, A. K. (2015). Engaging or distracting: Children's tablet computer use in education. *Journal of Educational Technology and Society, 18*(4), 9–23.

McManis, L. D., & Gunnewig, S. B. (2012). Find the education in educational technology with early learners. *YC Young Children, 67*(3), 14–24. Available at http://www.naeyc.org/yc/article/finding-education-in-educational-technology

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 372–377.

Murphy, K. L., DePasquale, R., & McNamara, E. (2003, November). Meaningful connections: Using technology in primary classrooms. *Beyond the Journal: Young Children on the Web.* Available at https://www.naeyc.org/files/yc/file/200311/TechInPrimaryClassrooms.pdf

National Association for the Education of Young Children (NAEYC) & Fred Rogers Center for Early Learning and Children's Media at Saint Vincent College. (2012). *Technology and interactive media as tools in early childhood programs serving children from birth through age 8.* A joint position statement. Available at http://www.eetcconference.org/wp-content/uploads/Use_of_Tech_Tools_and_Interactive_Media.pdf

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C., Özgün-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education , 9*(1) [Online serial]. Available at http://www.citejournal.org/vol9/iss1/mathematics/article1.cfm

Samara, J. & Clements, D. H. (2006). Young students, and computers: Software, teaching strategies and professional development. *Mathematics Educator, 9*(2), 112–134.

Schliemann, A. D. (2002). Analyzing tools: Perspectives on the role of designed artifacts in mathematics learning. *Journal of the Learning Sciences, 11*(2/3), 301–317.

Suh, J. (2010). Tech-knowledgy & diverse learners. *Mathematics Teaching in the Middle School, 15*(8), 440–447.

Thach, K. J, & Norman, K. A. (2008). Technology-rich mathematics instruction. *Teaching Children Mathematics, 15*(3), 152–158.

MATH CONTENT AREAS

Algebraic Thinking, Spatial Reasoning, & Proportional Reasoning

Bastable, V. & Schifter, D. (2008). Classroom stories: Examples of elementary students engaged in early algebra. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 165–184). New York, NY: Lawrence Erlbaum Associates.

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Lsler, I., & Kim, J. (2005). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics, 46*(1).

Bruce, C. (2013). *Efficacy, professional learning, content precision, and future directions.* Trent Mathematics Education Research Collaborative [online]. Available at https://mathforum1314.files.wordpress.com/2014/02/dr-cathy-bruce-research-paper-math-forum-2013.pdf

Bruce, C. D., Sinclair, N., Moss, J., Hawes, Z., & Caswell, B. (2015). Spatializing the curriculum. In B. Davis & Spatial Reasoning Study Group, *Spatial reasoning in the early years* (pp. 85–106). New York, NY: Routledge.

Gardiner, A. M., & Sawrey, K. (2016). *First graders and functional thinking, part 1.* National Council of Teachers of Mathematics, Teaching Children Mathematics [blog]. Available at http://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/First-Graders-and-Functional-Thinking,-Part-1/

Isler, I., Stephens, A., & Kang, H. (2016). *Even and odd numbers: A journey into the algebraic thinking practice of justification.* National Council of Teachers of Mathematics, Teaching Children Mathematics [blog]. Available at http://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Even-and-Odd-Numbers_-A-Journey-into-The-Algebraic-Thinking-Practice-of-Justification/

Kris, D. F. (2016, February 22). Can teaching spatial skills bridge the STEM gender gap? *KQED News.* Available at http://ww2.kqed.org/mindshift/2016/02/22/can-teaching-spatial-skills-help-bridge-the-stem-gender-gap/

Moss, J., & McNab, S. L. (2011). An approach to geometric and numeric patterning that fosters second grade students' reasoning and generalizing about functions and co-variation. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives.* Advances in Mathematics Education. New York: Springer.

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator, Summer.* Available at https://www.aft.org/sites/default/files/periodicals/Newcombe_1.pdf

Newcombe, N. S. (2013). Seeing relationships: Using spatial thinking to teach science, mathematics, and social studies. *American Educator, Spring,* 26–31, 40. Available at http://www.aft.org/sites/default/files/periodicals/Newcombe_0.pdf

Ontario Ministry of Education. (2012). *Paying attention to proportional reasoning.* Available at http://www.edu.gov.on.ca/eng/teachers/studentsuccess/ProportionReason.pdf

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237–269.

Small, M. (2015). *Building proportional reasoning across grades and math strands, K–8.* New York, NY: Teachers College Press.

Smith, J., & Thompson, P. W. (2007). Quantitative reasoning and the development of algebraic reasoning. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 95–132). New York, NY: Erlbaum.

Spatial Reasoning Think-Tank Group. (2013). *Early years spatial reasoning: Learning, teaching and research implications.* Research symposium. Available at http://www.ucalgary.ca/IOSTEM/files/IOSTEM/nctm15april2013.pdf

Stephens, A., Blanton, M., Knuth, E., Isler, I., & Gardiner, A. M. (2015). Just say yes to early algebra! *Teaching Children Mathematics, 22*(2). Available at http://www.nctm.org/Publications/Teaching-Children-Mathematics/2015/Vol22/Issue2/Just-Say-Yes-to-Early-Algebra!/

Warren, E., & Cooper, T. J. (2008). Generalizing the pattern rule for visual growth patterns: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics, 67,* 171–185.

Warren, E., & Cooper, T. J. (2009). Developing mathematics understanding and abstraction: The case of equivalence in the elementary years. *Mathematics Education Research Journal, 21*(2), 76–95.

Building Computational Fluency by Developing Number Sense

Boaler, J. (2015). F*luency without fear: Research evidence on the best ways to learn math facts.* Youcubed at Stanford University [blog]. Available at https://www.youcubed.org/fluency-without-fear/

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics, 29*(1).

Dyson, N., Jordan, N. C., Beliakoff, A., & Hassinger-Da, B. (2013). A kindergarten number-sense intervention with contrasting practice conditions for low-achieving children. *Journal of Learning Disabilities, 46*(2), 166–181.

Fosnot, C., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division.* Westport, CT: Heinemann.

Jordan, N. C., Glutting, J., Dyson, N., Hassinger-Das, B., & Irwin, C. (2012). Building kindergartners' number sense: A randomized controlled study. *Journal of Educational Psychology, 104*(3), 647–660.

Jordan, N. C., Kaplan, D., Oláh, L. N., Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*(1), 153–175.

Lyons, I., Price, G., Vaessen, A., Blomert L., & Ansari D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science, 17*(5), 714–726.

MacDonald, B. L., & Shumway, J. F. (2016). Subitizing games: Assessing preschoolers' number understanding. *Teaching Children Mathematics, 22*(6), 340–348.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Parrish, S. (2010). *Number talks: Helping children build mental math and computational strategies.* Sausalito, CA: Math Solutions.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346–362. Available at http://dx.doi.org/10.1037/0022-0663.93.2.346

Russel, S. J. (2013). *Developing computational fluency with whole numbers in elementary grades.* TERC, Investigations in Number, Data, and Space. Available at https://investigations.terc.edu/library/bookpapers/comp_fluency.cfm

Starkey, G. S., & McCandliss, B. D. (2014). The emergence of "groupitizing" in children's numerical cognition. *Journal of Experimental Child Psychology, 126*, 120–137.

Victoria State Government, Department of Education and Early Childhood Development. (2009). *Mental computation and estimation.* Melbourne, Australia: Available at https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/readmentalcompest.pdf

Mental Math

Alberta Education. (2006). *Western and Northern Canadian protocol common curriculum framework for K–9 mathematics.* Edmonton, AB: Alberta Education.

Clements, D., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach.* New York, NY: Routledge.

Olsen, J. (2015). Five keys for teaching mental math. *Mathematics Teacher, 108* (7), 543–547.

Seeley, C. (2005, December). Do the math in your head! President's message. *NCTM News Bulletin.* Reston, VA: National Council of Teachers of Mathematics. Available at http://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Cathy-Seeley/Do-the-Math-in-Your-Head!/

Whitacre, I., & Nickerson, S. (2006). Pedagogy that makes (number) sense: A classroom teaching experiment around mental math. *North American Chapter of the International Group for the Psychology of Mathematics Education, 2,* 736–743. Available at http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.97.4349&rep=rep1&type=pdf

Acknowledgements

The Pearson Canada Research Overview document was created, researched, and reviewed by the educator community across Canada.

Agawa, Pamala	Curriculum Coordinator, First Nation, Métis and Inuit Education, ON
Altenberg, Jennifer	Métis Educator, SK
Ansari, Daniel	Professor, Department of Psychology, Western University, ON
Auger, Marianne	Curriculum Consultant, ON
Ball, Sandra	Surrey School District #36, BC
Beatty, Ruth	Faculty of Education, Lakehead University, ON
Bernard, Kyla	Mi'kmaw Kina'matnewey, NS
Brooks, Towana	Curriculum Advisor, First Nation, Métis and Inuit Education, ON
Burden, Lois	Newfoundland and Labrador English School District, NL
Burdess, Alicia	Grande Prairie and District Catholic Schools, AB
Campbell, Karen	Mathematics Consultant, SK
Carter, Jennifer	School District #22, Vernon, BC
Cave, Phyllis	Early Years Consultant, ON
Costello, David	Elementary Numeracy Leader, PE
Darroch, Amber	Horizon School Division #67, AB
Davis, Garry	Mathematics Consultant, SK
Decoste, David	Mathematics Consultant, AB
Flynn, Tara	Trent University School of Education and Professional Learning, ON
Hantelmann, Lori Jane	Math Coordinator, SK
Haworth, Michael	School District #19, Revelstoke, BC
Irwin, Laura	Edmonton Catholic School District, AB
Jackson, Helen	Greater St. Albert Catholic School Board, AB
Johnston, Edwin	Elementary Math, Science, and Innovation Leader, PE
Kay, Laura	Halton District School Board, ON
Laliberte, Cornelia	Coordinator, First Nations, Métis and Inuit Education, SK
Leslie, Tamara	Edmonton Catholic Schools, AB
Luciw, Larysa	Edmonton Catholic Schools, AB
Malo, Sylvia	Elementary Math Consultant, AB
McGarvey, Lynn	PhD, University of Alberta, AB
McKillop, David	Mathematics Consultant, NS
McMurtry, Shannon	Winnipeg School Division, MB
Moher, Laurie	Numeracy Consultant, ON
Norton, Amanda	Regina Catholic School Division, SK
Novakowski, Janice	Richmond School District, BC
Oldridge, Matthew	Peel District School Board, ON

Acknowledgements

Pigeon, Ann	Thames Valley District School Board, ON
Postnikoff, Maureen	School District #20, Kootenay-Columbia, BC
Sinclair, Nathalie	Simon Fraser University, BC
Schmidt, Julie	Edmonton Catholic Schools, AB
So, Jonathan	Peel District School Board, ON
Soletsky, Ulana	Numeracy/Math Consultant, AB
Sproat, Diana	Greater Saskatoon Catholic Schools, SK
Strangway, Lynn	Toronto District School Board, ON
Torney, Jennipher	Toronto Catholic District School Board, ON
Tyssen, Kimberley	School District #22, BC
Vandesande, Deborah	School District #22, BC
Varty, Gerry	Education Consultant, AB
Vause, Lyn	Education Consultant, ON
Weight, Wendy	Education Consultant, MB
Wiebe, Jerrold	Pembina Trails School Division, MB
Zorzetto, Amy	Hamilton-Wentworth District School Board, ON

Pearson

Pearson Canada Research Overview

S17_E096