CHAPTER 1

Measurement

What You'll Learn

- Convert measurements in imperial units.
- Convert measurements between SI units and imperial units.
- Find surface areas and volumes of:
  prisms
  pyramids
  cylinders
  cones
  spheres

Why It's Important

Imperial units are used by:

- construction workers, to order, measure, and cut building materials
- graphic designers, to label products and containers that are sold in both the United States and Canada

Surface area and volume are used by:

- interior designers, to find the amount of material needed to cover furniture, paint walls, or make drapes and blinds
- farmers, to find the storage space in a silo or bin

Key Words

imperial units
referent
SI units
slant height
lateral area
prism
pyramid
cylinder
cone
sphere

1.1 Skill Builder

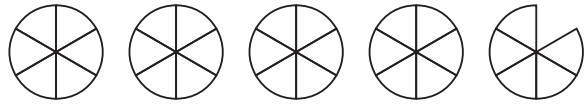
Converting between Mixed Numbers and Improper Fractions

To write the mixed number $4\frac{5}{6}$ as an improper fraction:

$4\frac{5}{6} = 4 + \frac{5}{6}$

$4\frac{5}{6} = \frac{24}{6} + \frac{5}{6}$

$4\frac{5}{6} = \frac{29}{6}$



A quick way is to think:

$4\frac{5}{6} = \frac{4 \times 6 + 5}{6} = \frac{29}{6}$

To write the improper fraction $\frac{29}{6}$ as a mixed number:

Think: How many times does 6 go into 29?

6 goes into 24 four times.

So, 6 goes into 29 four times, with remainder 5.

We write: $\frac{29}{6} = 4\frac{5}{6}$

Check

1. Write each mixed number as an improper fraction.

a) $4\frac{1}{2} = 4 + \_\_\_\_$

$4\frac{1}{2} = \frac{8}{2} + \_\_\_\_$

$4\frac{1}{2} = \_\_\_\_$

b) $7\frac{3}{4} = 7 + \_\_\_\_$

$7\frac{3}{4} = \frac{28}{4} + \_\_\_\_$

$7\frac{3}{4} = \_\_\_\_$

c) $5\frac{7}{12} = \_\_\_\_ + \_\_\_\_$

$5\frac{7}{12} = \_\_\_\_ + \_\_\_\_$

$5\frac{7}{12} = \_\_\_\_$

2. Write each improper fraction as a mixed number.

a) $\frac{13}{5} = \frac{10}{5} + \_\_\_\_$

$\frac{13}{5} = 2 + \_\_\_\_$

$\frac{13}{5} = \_\_\_\_$

b) $\frac{37}{8} = \_\_\_\_ + \frac{5}{8}$

$\frac{37}{8} = \_\_\_\_ + \_\_\_\_$

$\frac{37}{8} = \_\_\_\_$

c) $\frac{83}{12} = \_\_\_\_ + \_\_\_\_$

$\frac{83}{12} = \_\_\_\_ + \_\_\_\_$

$\frac{83}{12} = \_\_\_\_$

1.1 Imperial Measures of Length

FOCUS **Find referents for imperial units.**

Some imperial units are: the inch, the foot, the yard, and the mile
We can use a personal referent to estimate a length. For example:

Unit	Referent
Inch (in.)	Thumb length
Foot (ft.)	Foot length
Yard (yd.)	Arm span
Mile (mi.)	Distance walked in 20 min

The thumb length, foot length, and arm span are **referents**.
Each referent is an approximate measure for an imperial unit.

Example 1 Estimating Lengths Using Imperial Units

Describe how you would estimate the width of your locker.

Solution

The most appropriate imperial unit is the inch.
I use the width of my hand as a referent. It is about 4 in. across.

I line up one hand with one edge of the locker.
I count how many times I place my hands, one next to the other, to go from one edge of the locker to the other.
I multiply the number of hands by 4, to get the approximate width of my locker in inches.

Check

1. Describe how you would estimate the length of a driveway.
The most appropriate imperial unit is: _____
The referent is: _____

Here are some relationships between imperial units:

1 ft. = 12 in.

1 yd. = 3 ft. 1 yd. = 36 in.

1 mi. = 1760 yd. 1 mi. = 5280 ft.

To convert 4 ft. to inches, think:

1 ft. = 12 in.

So, 4 ft. = 4 × 12 in.

4 ft. = 48 in.

Example 2 Converting between Imperial Units

Convert:

a) 5 mi. to yards

b) 100 in. to feet and inches

Solution

a) 1 mi. = 1760 yd.

So, multiply by 1760 to convert miles to yards.

5 mi. = 5 × 1760 yd.

5 mi. = 8800 yd.

It takes more smaller units to measure the same length, so the number should be greater.

b) 12 in. = 1 ft.

So, divide by 12 to convert inches to feet.

100 in. = (100 ÷ 12) ft., or

$100 \text{ in.} = \frac{100}{12} \text{ ft.}$

Write the improper fraction as a mixed number.

$100 \text{ in.} = 8\frac{4}{12} \text{ ft.}$

Write $\frac{4}{12}$ ft. as 4 in.

100 in. = 8 ft. 4 in.

It takes fewer larger units to measure the same length, so the number should be less.

Check

1. Convert:

a) 14 ft. to inches

1 ft. = _____ in.

14 ft. = 14 × _____

14 ft. = _____

b) 14 ft. to yards and feet

_____ ft. = 1 yd.

14 ft. = (14 ÷ _____) yd.

14 ft. = _____ yd.

14 ft. = _____ yd.

14 ft. = _____ yd. _____ ft.

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Example 3 Solving a Problem Involving Conversions

Alex purchased 7 yd. of ribbon to trim some napkins. The ribbon is sewn around a napkin, which is 14 in. wide and 16 in. long. How many napkins can Alex trim with this ribbon?

Solution

Write the measurements in the same units.

It's often easier to change to the smaller unit.

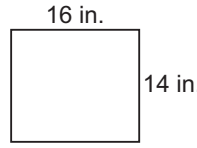
Convert 7 yd. to inches.

1 yd. = 36 in.

So, 7 yd. = 7 × 36 in.

7 yd. = 252 in.

Find the perimeter of each napkin.



Perimeter = 2 × 16 in. + 2 × 14 in.

Perimeter = 32 in. + 28 in.

Perimeter = 60 in.

The number of napkins that can be trimmed is:

$\frac{252 \text{ in.}}{60 \text{ in.}} = 4.2$

Alex can trim 4 napkins with this ribbon.

Check

1. One of Eric's steps is 18 in. long. How many steps would he take to walk 1 mi.?

Write the measurements in the same units.

Convert miles to yards, then inches:

1 mi. = _____ yd.

1 yd. = _____ in.

1760 yd. = _____ × _____

1760 yd. = _____

The number of steps is: _____

Eric would take _____ steps to walk 1 mi.

Practice

1. To estimate the width of the classroom:

a) Which is the most appropriate imperial unit? Why?

_____

b) Which referent could you use?

_____

2. To estimate the greatest width of a maple leaf:

a) Which is the most appropriate imperial unit? Why?

_____

b) Which referent could you use?

_____

3. To estimate the length of your friend's shadow:

a) Which is the most appropriate imperial unit? Why?

_____

b) Which referent could you use?

_____

4. a) Convert yards to feet.

1 yd. = _____ ft.
2 yd. = _____
3 yd. = _____
4 yd. = _____
5 yd. = _____

b) Convert feet to inches.

1 ft. = _____ in.
2 ft. = _____
3 ft. = _____
4 ft. = _____
5 ft. = _____

5. Convert:

a) 8 ft. to inches

1 ft. = _____
8 ft. = 8 × _____
8 ft. = _____

b) 235 yd. to feet

1 yd. = _____
235 yd. = _____ × _____
235 yd. = _____

c) 6 mi. to yards

1 mi. = _____
6 mi. = _____
6 mi. = _____

6. Convert:

a) 72 in. to feet

_____ in. = 1 ft.

72 in. = (72 ÷ _____) ft.

72 in. = _____

b) 87 ft. to yards

_____ ft. = 1 yd.

87 ft. = (_____ ÷ _____) yd.

87 ft. = _____

c) 288 in. to yards

_____ in. = 1 yd.

288 in. = (_____) yd.

288 in. = _____

7. Convert:

a) 67 in. to feet and inches

_____ in. = 1 ft.

67 in. = (67 ÷ _____) ft., or

67 in. = _____ ft.

67 in. = _____ ft.

67 in. = _____ ft. _____ in.

b) 418 ft. to yards and feet

_____ ft. = 1 yd.

418 ft. = (_____ ÷ _____) yd., or

418 ft. = _____ yd.

418 ft. = _____ yd.

418 ft. = _____ yd. _____ ft.

c) 2148 in. to yards and feet

12 in. = _____ ft.

2148 in. = (_____ ÷ _____) ft., or

2148 in. = _____ ft.

2148 in. = _____ ft.

_____ ft. = 1 yd.

_____ ft. = (_____ ÷ _____) yd., or

_____ ft. = _____ yd.

_____ ft. = _____ yd.

2148 in. = _____ yd. _____ ft.

8. Convert:

a) 7 ft. 5 in. to inches

1 ft. = _____ in.

7 ft. = _____ × _____

7 ft. = _____

Add _____

7 ft. 5 in. = _____ + _____

7 ft. 5 in. = _____

b) 9 yd. 1 ft. to feet

1 yd. = _____ ft.

9 yd. = _____ × _____

9 yd. = _____

Add _____

9 yd. 1 ft. = _____ + _____

9 yd. 1 ft. = _____

c) 11 yd. 2 ft. to inches

1 yd. = _____

11 yd. = _____ × _____

11 yd. = _____

1 ft. = _____ in.

2 ft. = _____ × _____

2 ft. = _____

Add the inches.

11 yd. 2 ft. = _____ + _____

11 yd. 2 ft. = _____

d) 3 mi. 128 yd. to feet

1 mi. = _____ ft.

3 mi. = _____ × _____

3 mi. = _____

1 yd. = _____ ft.

128 yd. = _____

128 yd. = _____

Add the feet.

3 mi. 128 yd. = _____ + _____

3 mi. 128 yd. = _____

9. Sue wants to fence part of her yard. She measures the perimeter as 44 yd. What is the perimeter in feet?
1 yd. = 3 ft.

The perimeter is _____

10. A dog trainer advises that, when walking a dog on a city street, the walker should allow the dog 42 in. of leash. Greg bought a retractable leash that extends $5\frac{1}{2}$ yd. Greg follows the trainer's advice. What length of the leash, in inches, is not used?

Write $5\frac{1}{2}$ as an improper fraction: _____

Convert _____ yd. to inches.

_____

Find the difference between the two measures.

_____ of the leash is not used.

11. A mural is 3 yd. long and 2 yd. wide. It has a border that is made with sections that are 48 in. long. How many sections were needed to make the border?

_____ were needed to make the border.

1.2 Math Lab: Measuring Length and Distance

FOCUS **Measure lengths and distances.**

Work with a partner.

The materials you need are listed on page 14 of the Student Text.

Part A

Choose one object and sketch it below.

Use referents. Estimate all linear measures, in imperial units and SI units.

Record these measures on the sketch.

Repeat for two more objects.

Object 1:	Referent for imperial unit: Referent for SI unit:
Object 2:	Referent for imperial unit: Referent for SI unit:
Object 3:	Referent for imperial unit: Referent for SI unit:

For each object, justify your choice of units.

Object 1: _____

_____

Object 2: _____

Object 3: _____

_____

Part B

Sketch each object in the spaces below.

Use imperial units and SI units to measure the lengths you estimated.

Record these measures on each sketch.

Object 1:	Measuring instrument for imperial units: Measuring instrument for SI units:
Object 2:	Measuring instrument for imperial units: Measuring instrument for SI units:
Object 3:	Measuring instrument for imperial units: Measuring instrument for SI units:

List the measures that were hard to find.

_____

Describe how you found these measures.

_____

_____

_____

Practice

1. Which of these items is best for measuring each object below: calipers, ruler, yard stick, measuring tape, or string?

a) the thickness of 50 sheets of paper: _____

b) the length of a frog: _____

c) the length of a bus : _____

d) the distance around a can of juice: _____

2. Which of these imperial units would you use to measure each item in question 1: inch, foot, yard, or mile?

a) the thickness of 50 sheets of paper: _____

b) the length of a frog: _____

c) the length of a bus : _____

d) the distance around a can of juice: _____

3. For each object below:

- Describe how you would measure it.
- Include the measuring device and the imperial unit.

a) the length of your foot

b) the width of a computer screen

c) the perimeter of the top of a large garbage can

d) the distance between your classroom door and the next classroom door

1.3 Skill Builder

Converting between Metric Units of Length

1 m = 100 cm
1 cm = 10 mm
1 mm = 0.1 cm

1 m = 1000 mm
1 cm = 0.01 m
1 mm = 0.001 m

To convert 12 m to centimetres:
Start with 1 m = 100 cm
Then, 12 m = 12 × 100 cm
So, 12 m = 1200 cm

Mark a decimal point after 12. To multiply by 100, move the decimal point 2 places to the right. Use zeros as place holders.

To convert 372 mm to centimetres:
Start with 1 mm = 0.1 cm
Then, 372 mm = 372 × 0.1 cm
So, 372 mm = 37.2 cm

Mark a decimal point after 372. To multiply by 0.1, move the decimal point 1 place to the left.

Check

1. Convert each measurement to centimetres.

a) 7 m = _____ × 100 cm
7 m = _____

b) 33 m = _____ × _____
33 m = _____

c) 45 mm = _____ × 0.1 cm
45 mm = _____

d) 6 mm = _____ × _____
6 mm = _____

2. Convert each measurement to millimetres.

a) 9 cm = _____ × 10 mm
9 cm = _____

b) 89 cm = _____ × _____
89 cm = _____

c) 3 m = _____ × 1000 mm
3 m = _____

d) 38 m = _____ × _____
38 m = _____

3. Convert each measurement to metres.

a) 800 cm = _____ × 0.01 m
800 cm = _____

b) 27 cm = _____ × _____
27 cm = _____

c) 9000 mm = _____ × 0.001 m
9000 mm = _____

d) 235 mm = _____ × _____
235 mm = _____

1.3 Relating SI and Imperial Units

FOCUS **Convert between SI units and imperial units.**

Use this table to convert a measure in imperial units to SI units.

Imperial Units to SI Units
1 in. ≐ 2.5 cm
1 ft. ≐ 30 cm 1 ft. ≐ 0.3 m
1 yd. ≐ 90 cm 1 yd. ≐ 0.9 m
1 mi. ≐ 1.6 km

"SI units" is another name for metric units.

Example 1 Converting from Imperial Units to SI Units

Convert:

a) 17 ft. to metres

b) 5 mi. to kilometres

Solution

a) From the table,
1 ft. ≐ 0.3 m
So, 17 ft. ≐ 17 × 0.3 m
17 ft. ≐ 5.1 m

b) From the table,
1 mi. ≐ 1.6 km
So, 5 mi. ≐ 5 × 1.6 km
5 mi. ≐ 8 km

Check

1. Convert:

a) 2 yd. to centimetres
1 yd. ≐ _____
2 yd. ≐ 2 × _____
2 yd. ≐ _____

b) 4 ft. to centimetres
1 ft. ≐ _____
4 ft. ≐ _____ × _____
4 ft. ≐ _____

c) 13 in. to centimetres
1 in. ≐ _____
13 in. ≐ _____ × _____
13 in. ≐ _____

d) 87 yd. to metres
1 yd. ≐ _____
87 yd. ≐ _____ × _____
87 yd. ≐ _____

Use this table to convert a measure in SI units to imperial units.

SI Units to Imperial Units
1 mm $\doteq \frac{4}{100}$ in.
1 cm $\doteq \frac{4}{10}$ in.
1 m $\doteq$ 39 in. 1 m $\doteq 3\frac{1}{4}$ ft.
1 km $\doteq \frac{6}{10}$ mi.

Remember that imperial units are written as fractions, not decimals.

Example 2 Converting from SI Units to Imperial Units

Convert:

a) 16 cm to inches

b) 58 m to feet

Solution

a) From the table,

$1 \text{ cm} \doteq \frac{4}{10}$ in.

So, $16 \text{ cm} \doteq 16 \times \frac{4}{10}$ in.

$16 \text{ cm} \doteq \frac{16}{1} \times \frac{4}{10}$ in. Multiply the numerators. Multiply the denominators.

$16 \text{ cm} \doteq \frac{64}{10}$, or $\frac{32}{5}$ in. Write as a mixed number.

$16 \text{ cm} \doteq 6\frac{2}{5}$ in.

b) From the table,

$1 \text{ m} \doteq 3\frac{1}{4}$ ft.

So, $58 \text{ m} \doteq 58 \times 3\frac{1}{4}$ ft. Write the mixed number as an improper fraction.

$58 \text{ m} \doteq \frac{58}{1} \times \frac{13}{4}$ ft.

$58 \text{ m} \doteq \frac{754}{4}$, or $\frac{377}{2}$ ft. Write as a mixed number.

$58 \text{ m} \doteq 188\frac{1}{2}$ ft.

Check

1. Convert:

a) 94 m to inches

1 m ≐ _____

So, 94 m ≐ _____ × _____

94 m ≐ _____

b) 183 km to miles

1 km ≐ _____

So, 183 km ≐ _____

183 km ≐ _____

183 km ≐ _____

You may need to do more than one conversion to solve a problem.

Example 3 Problem Solving with Conversions

The school librarian needs to reach a shelf that is 1.7 m above the floor. The librarian can only reach to 5 ft. 11 in. from the floor. Will the librarian be able to reach the shelf? Justify the answer.

Solution

First, find how high the librarian can reach in inches.

1 ft. = 12 in.

So, 5 ft. = 5 × 12 in.

5 ft. = 60 in.

And, 5 ft. 11 in. = 60 in. + 11 in.

= 71 in.

You can also solve the problem by converting 1.7 m to inches.

Convert the measurement in inches to centimetres.

1 in. ≐ 2.5 cm

So, 71 in. ≐ 71 × 2.5 cm

71 in. ≐ 177.5 cm

Convert the measurement in centimetres to metres.

100 cm = 1 m

So, $177.5 \text{ cm} = \frac{177.5}{100} \text{ m}$

177.5 cm = 1.775 m

The librarian can reach 1.775 m high.

The shelf is 1.7 m high.

So, the librarian can reach the shelf.

Check

1. A truck driver wants to park her 3.25-m high truck in a storage shed that is 11 ft. 6 in. high. Will the truck fit in the shed? Justify your answer.

Convert the height of the storage shed to inches.

1 ft. = _____ in.

11 ft. = 11 × _____ in.

11 ft. = _____

11 ft. 6 in. = _____ + 6 in.

= _____

Convert inches to centimetres.

1 in. ≐ _____ cm

_____ in. ≐ _____ × _____ cm

_____ ≐ _____

Convert centimetres to metres.

_____ cm = 1 m

$$\text{_____ cm} = \frac{\text{_____}}{100}\ \text{m}$$

_____ cm = _____ m

Since the height of the truck is 3.25 m and the height of the shed is _____, the truck will _____.

Practice

1. Convert:

a) 24 yd. to centimetres

1 yd. ≐ _____ cm

24 yd. ≐ 24 × _____

24 yd. ≐ _____

b) 5 in. to centimetres

1 in. ≐ _____ cm

5 in. ≐ _____ × _____

5 in. ≐ _____

c) 8 ft. to metres

1 ft. ≐ _____

8 ft. ≐ _____ × _____

8 ft. ≐ _____

d) 7 mi. to kilometres

1 mi. ≐ _____

7 mi. ≐ _____

7 mi. ≐ _____

2. Convert:

a) 12 m to inches

1 m ≐ _____ in.

12 m ≐ 12 × _____

12 m ≐ _____

Don't forget to write the fractions in simplest form.

b) 7 km to miles

1 km ≐ _____ mi.

7 km ≐ _____ × _____

7 km ≐ _____

7 km ≐ _____

c) 276 mm to inches

1 mm ≐ _____

276 mm ≐ 276 × _____

276 mm ≐ _____

276 mm ≐ _____

Use the table on page 14.

d) 86 cm to inches

1 cm ≐ _____

86 cm ≐ _____ × _____

86 cm ≐ _____

86 cm ≐ _____

3. Convert:

a) 4 ft. 9 in. to millimetres

Convert feet to inches first.

1 ft. = _____

4 ft. = _____ × _____

4 ft. = _____

4 ft. 9 in. = _____ + 9 in.

4 ft. 9 in. = _____

Convert inches to millimetres.

1 in. ≐ _____ cm

1 in. ≐ _____ mm

_____ in. ≐ _____ mm

_____ ≐ _____

4 ft. 9 in. ≐ _____

b) 3 yd. 2 ft. to centimetres

Convert yards to feet first.

1 yd. = _____

3 yd. = _____ × _____

3 yd. = _____

3 yd. 2 ft. = _____ + _____

3 yd. 2 ft. = _____

Convert feet to centimetres.

1 ft. ≐ _____

_____ ft. ≐ _____ cm

_____ ≐ _____

3 yd. 2 ft. ≐ _____

4. Tammy drove 85 km to a camp site.
Todd drove 52 mi. to meet Tammy at the camp site.
Who drove farther? Justify your answer.
Convert miles to _____.
1 mi. ≐ _____
52 mi. ≐ _____
52 mi. ≐ _____
Since _____ is _____ than _____, _____ drove farther.

5. To qualify for the school volleyball try-outs, Rick needs to be able to jump and touch a line on a wall that is 8 ft. 2 in. off the ground. In his workouts, Rick jumped 243 cm. Will he qualify for the try-outs? Justify your answer.
Convert feet to inches.

Convert inches to centimetres.

Since _____ is _____ than _____, Rick _____ qualify.

6. The cross-country running team is preparing for a meet. Katy ran 18 laps around the 400-yd. track and Ben ran 7.5 km. Who ran farther? Justify your answer.

Since Katy ran approximately _____ and Ben ran _____,
_____ ran farther.

Can you...

- estimate a length using a referent?
- convert between imperial units?
- convert between SI units and imperial units?

1.1 **1. a)** To estimate the length of a semi-truck and trailer

i) Which is the most appropriate imperial unit? Why?

_____

ii) Which referent could you use?

_____

b) To estimate the width of your text book:

i) Which is the most appropriate imperial unit? Why?

_____

ii) Which referent could you use?

_____

2. Describe how you would estimate:

a) the distance from your classroom to the school's office

The most appropriate imperial unit is: _____

The referent is: _____

b) the length of a pencil

The most appropriate imperial unit is: _____

The referent is: _____

3. Convert:

a) 99 in. to feet and inches

_____ in. = 1 ft.

99 in. = (99 ÷ _____) ft., or

99 in. = _____ ft.

99 in. = _____ ft.

99 in. = _____ ft. _____ in.

b) 4 yd. 2 ft. to inches

1 yd. = _____ in.

4 yd. = _____ × _____

4 yd. = _____

1 ft. = _____ in.

2 ft. = _____ × _____

2 ft. = _____

Add the inches.

4 yd. 2 ft. = _____ + _____

4 yd. 2 ft. = _____

4. Two students measured a set for a school play. One student said it was 3 yd. 2 ft. wide. The other student said it was 132 in. wide. Are the students' measurements the same? Show your work.

_____, the measurements _____ the same.

1.2 **5. a)** Which of these imperial units is best for measuring each item below: inch, foot, yard, or mile?

i) the width of your bedroom: _____

ii) the length of your longest finger: _____

b) Which of these SI units is best for measuring each item below: millimetre, centimetre, metre, kilometre?

i) the distance between your house and school: _____

ii) the width of a pencil: _____

6. Describe one method to measure the circumference of a marker pen. Identify the unit and measuring device you would use.

1.3 **7.** Convert:

a) 16 ft. to metres

1 ft. ≐ _____ m

16 ft. ≐ 16 × _____ m

16 ft. ≐ _____

b) 27 mi. to kilometres

1 mi. ≐ _____ km

27 mi. ≐ _____ × _____ km

27 mi. ≐ _____

c) 44 mm to inches

1 mm ≐ _____ in.

44 mm ≐ _____ × _____ in.

44 mm ≐ $\frac{\text{\_\_\_\_\_}}{\text{\_\_\_\_\_}}$ in.

44 mm ≐ _____

d) 840 cm to yards and feet

_____ cm ≐ 1 ft.

840 cm ≐ (_____ ÷ _____) ft., or

840 cm ≐ $\frac{\text{\_\_\_\_\_}}{\text{\_\_\_\_\_}}$ ft.

840 cm ≐ _____ ft.

_____ ft. = 1 yd.

_____ ft. = $\frac{\text{\_\_\_\_\_}}{\text{\_\_\_\_\_}}$ yd.

_____ ft. = $\frac{\text{\_\_\_\_\_}}{\text{\_\_\_\_\_}}$ yd.

840 cm ≐ _____ yd. _____ ft.

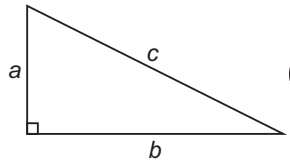
8. Nina ran an 8-km race for charity. What is this distance in miles and yards?

8 km is approximately _____.

1.4 Skill Builder

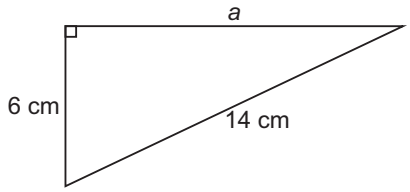
The Pythagorean Theorem

In a right triangle with hypotenuse length c and leg lengths a and b,
$c^2 = a^2 + b^2$



The hypotenuse is the side opposite the right angle.

In the right triangle below, to find the value of a to the nearest tenth of a centimetre:



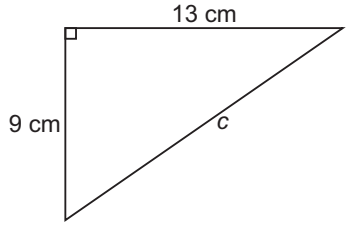
Use:

$c^2 = a^2 + b^2$	Substitute: $c = 14$ and $b = 6$
$14^2 = a^2 + 6^2$	Simplify.
$196 = a^2 + 36$	Solve for a^2.
$a^2 = 196 - 36$	
$a^2 = 160$	Solve for a.
$a = \sqrt{160}$	
$a = 12.6491\ldots$	

a is about 12.6 cm.

Check

1. Find the value of c, to the nearest tenth of a centimetre.



Use: $c^2 = a^2 + b^2$ Substitute: $a =$ _____ and $b =$ _____

$c^2 =$ _____ + _____

$c^2 =$ _____ + _____

$c^2 =$ _____

$c =$ _____

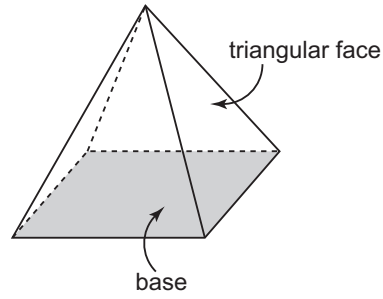
$c =$ _____

c is about: _____

1.4 Surface Areas of Right Pyramids and Right Cones

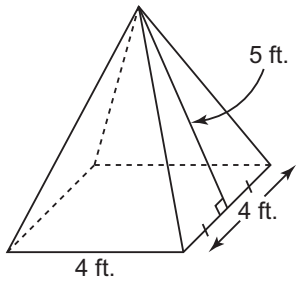
FOCUS **Find the surface areas of pyramids and cones.**

The surface area of a pyramid is:
Area of base + area of triangular faces



Example 1 Finding the Surface Area of a Square Pyramid

A square pyramid has a base with side length 4 ft. Each triangular face has height 5 ft. Find the surface area of the pyramid.



*The height of a triangular face is the **slant height** of a square pyramid.*

Solution

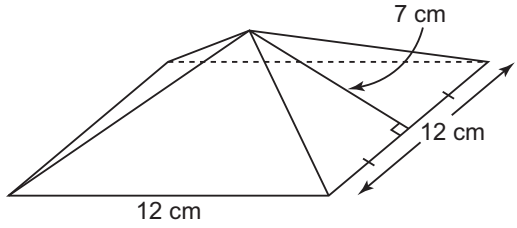
The base of the pyramid is a square, so the area of the base is:
$4 \times 4 = 16$
There are 4 triangular faces.
Each face has base 4 ft. and height 5 ft.
The area of each face is:

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 5$$
$$= 10$$

The area of all 4 faces is: $4 \times 10 = 40$
The surface area of the pyramid is: $16 + 40 = 56$
The surface area of the pyramid is 56 square feet.

Check

1. Find the surface area of this square pyramid.



Area of the base is: _____ × _____ = _____

Area of each triangular face is: $\frac{1}{2}$ × _____ × _____ = _____

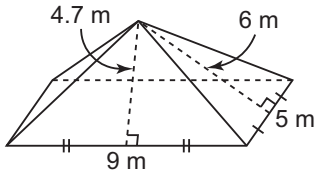
Area of all 4 faces is: 4 × _____ = _____

The surface area of the pyramid is: _____ + _____ = _____

The surface area of the pyramid is: _____ cm^2

Example 2 Finding the Surface Area of a Rectangular Pyramid

Find the surface area of this rectangular pyramid.



Solution

The base of the pyramid is a rectangle, so the area of the base is: $9 \times 5 = 45$

There are 2 triangular faces with base 9 m and height 4.7 m.

The area of each face is: $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 9 \times 4.7$

$= 21.15$

Opposite triangular faces are congruent.

There are 2 triangular faces with base 5 m and height 6 m.

The area of each face is: $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 5 \times 6$

$= 15$

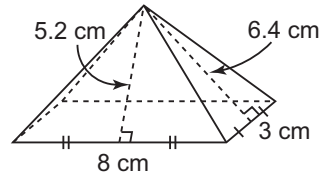
The area of all 4 faces is: $(2 \times 21.15) + (2 \times 15) = 72.3$

The surface area of the pyramid is: $45 + 72.3 = 117.3$

The surface area of the pyramid is 117.3 m^2.

Check

1. Find the surface area of this rectangular pyramid.



The base of the pyramid is a _____,
so the area of the base is: _____
There are 2 triangular faces with base _____
and height _____.
The area of each face is:

$\frac{1}{2} \times$ base $\times$ height = _____
= _____

There are 2 triangular faces with base _____ and height _____.
The area of each face is:

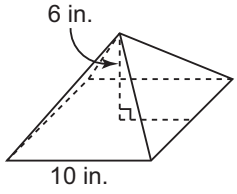
$\frac{1}{2} \times$ base $\times$ height = _____
= _____

The area of all 4 faces is: _____ + _____ = _____
The surface area of the pyramid is: _____ + _____ = _____
The surface area of the pyramid is: _____

Sometimes we must find the slant height of an object before we can calculate its surface area.

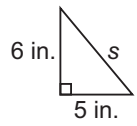
Example 3 Finding the Surface Area of a Pyramid Given Its Height

Find the surface area of this square pyramid, to the nearest square inch.



Solution

Sketch a triangle that shows the height of the pyramid and the slant height.



The base of this triangle is: $\frac{1}{2} \times 10$ in. $= 5$ in.

Let the slant height be s inches.

Use the Pythagorean Theorem to find the slant height, s.

$s^2 = 6^2 + 5^2$ Simplify.

$s^2 = 36 + 25$

$s^2 = 61$

$s = \sqrt{61}$

The slant height is $\sqrt{61}$ in.

Now find the surface area of the pyramid.

The area of the base is: $10 \times 10 = 100$

The area of the 4 faces is:

$4 \times \frac{1}{2} \times \text{base} \times \text{height} = 2 \times \text{base} \times \text{slant height}$

$= 2 \times 10 \times \sqrt{61}$

$= 20 \times \sqrt{61}$

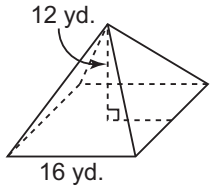
The height of a face is equal to the slant height of the pyramid.

The surface area of the pyramid is: $100 + 20 \times \sqrt{61} = 256.2049\ldots$

The surface area of the pyramid is about 256 square inches.

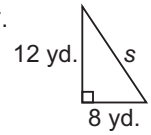
Check

1. Find the surface area of this square pyramid, to the nearest square yard.



This triangle shows the height of the pyramid and its slant height, s.

The base of the pyramid has length 16 yd.



So, the base of this triangle is: $\frac{1}{2} \times 16 \text{ yd.} = 8 \text{ yd.}$

Use the Pythagorean Theorem to find the slant height.

$s^2 =$ _____ + _____ Simplify.

$s^2 =$ _____

$s^2 =$ _____

$s =$ _____

The slant height is: _____

For the surface area of the pyramid:
Area of the base is: _____
Area of the 4 faces is:

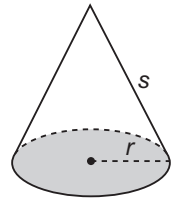
$4 \times \frac{1}{2} \times \text{base} \times \text{height} = 2 \times \text{base} \times \text{slant height}$
$= $ _____
$= $ _____

The surface area of the pyramid is: _____ = _____

The surface area of the pyramid is about: _____

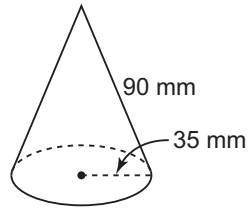
The surface area, *SA*, of a cone is: Lateral area + base area

For a cone with slant height *s* and base radius *r*:
$SA = \pi rs + \pi r^2$



Example 4 Finding the Surface Area of a Cone

Find the surface area of this cone, to the nearest square millimetre.

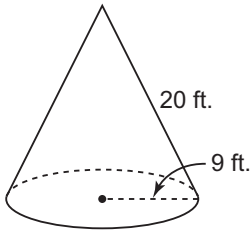


Solution

Use the formula:
$SA = \pi rs + \pi r^2$
Substitute: $r = 35$ and $s = 90$
$SA = \pi(35)(90) + \pi(35)^2$
$SA = 13\,744.4678\ldots$
The surface area is about 13 744 mm^2.

Check

1. Find the surface area of this cone, to the nearest square foot.

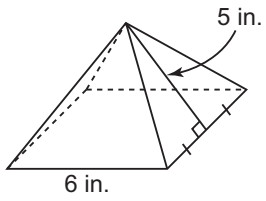


$SA = \pi rs + \pi r^2$ Substitute: _____
$SA = $ _____ + _____
$SA = $ _____
The surface area is about: _____

Practice

1. Find the surface area of each object, to the nearest square unit.

a) a square pyramid



Area of the base is: _____

Area of each triangular face is: $\frac{1}{2} \times$ _____ $\times$ _____ = _____

Area of all 4 faces is: $4 \times$ _____ = _____

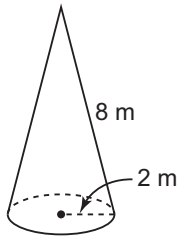
The surface area of the pyramid is:

_____ + _____ = _____

The surface area of the pyramid is:

_____ square inches

b) a cone



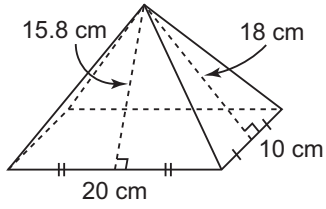
$SA = \pi rs + \pi r^2$

$SA =$ _____ + _____

$SA =$ _____

The surface area is about: _____

c) a rectangular pyramid



The base of the pyramid is a _____,
so the area of the base is: _____

Two triangular faces have:
base _____ and height _____

Area of the 2 faces is:

$2 \times \frac{1}{2} \times$ base $\times$ height = _____

= _____

Two triangular faces have:
base _____ and height _____

The area of the 2 faces is:

$2 \times \frac{1}{2} \times$ base $\times$ height = _____

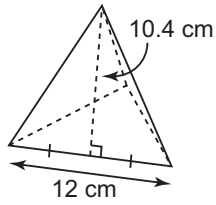
= _____

The surface area of the pyramid is:

_____ + _____ + _____ = _____

The surface area of the pyramid is: _____

2. A triangular pyramid has 4 congruent faces. Find its surface area.



Four triangular faces have: base _____ and height _____

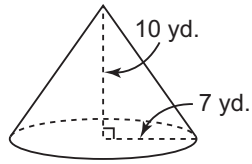
The area of the 4 faces is:

$4 \times \frac{1}{2} \times$ base $\times$ height = _____

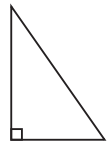
= _____

The surface area of the pyramid is: _____

3. Find the slant height of this cone, to the nearest tenth of a unit.



Label this triangle to show the height and slant height of the cone.
Let the slant height be s yards.



Use the Pythagorean Theorem to find the slant height.

s^2 = _____ + _____

s^2 = _____ + _____

s^2 = _____

s = _____

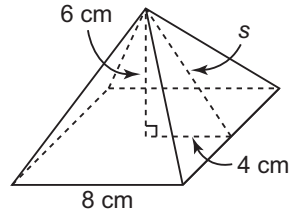
s = _____

Remember to write an imperial measure as a fraction.

The slant height is about: _____

4. A wooden square pyramid is to be painted. The side length of the base is 8 cm and the height of the pyramid is 6 cm. To the nearest square centimetre, what is the area that will be painted?

The painted area is the surface area of the pyramid.
The slant height is: s centimetres.
Use the Pythagorean Theorem to find the slant height.



s^2 = _____

s = _____

The slant height is: _____

The area of the base is: _____

The area of the 4 faces is:

$4 \times \frac{1}{2} \times$ base $\times$ height = $2 \times$ base $\times$ slant height

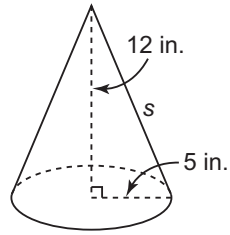
= _____

= _____

The surface area of the pyramid is: _____ + _____ = _____

The surface area of the pyramid is about: _____

5. A cone-shaped hat is to be made with radius 5 in. and height 12 in.
To the nearest square inch, how much material will be needed for the hat?



The amount of material is the lateral surface area of the cone.

Use the _____ to find the slant height, s.

The hat is a cone with no base.

s^2 = _____

s = _____

The slant height is: _____

Use the formula for the lateral surface area of a cone.

$SA = \pi rs$ Substitute: _____

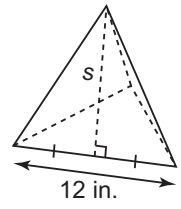
SA = _____

SA = _____

The surface area is about: _____

About _____ of material will be needed.

6. This triangular pyramid has 4 congruent faces.
The surface area of this pyramid is 250 square inches.
Find its slant height, to the nearest tenth of an inch.



The slant height is: s inches

Use the formula to write an expression for the surface area.

SA = _____

This expression is equal to: _____

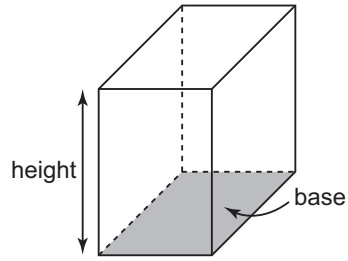
Write an equation, then solve for s.

The slant height is about: _____

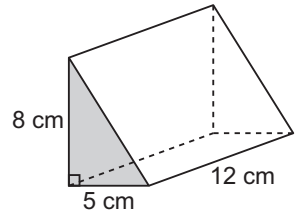
1.5 Skill Builder

Volume of a Prism

To find the volume, V, of a prism, use this formula:
V = base area × height

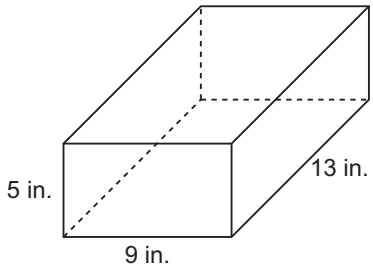


To find the volume, V, of this triangular prism:
The base is a triangle.
So, the area of the base is: $\frac{1}{2} \times 8 \times 5 = 20$
V = base area × height
V = 20 × 12
V = 240
The volume of the prism is 240 cm^3.



Check

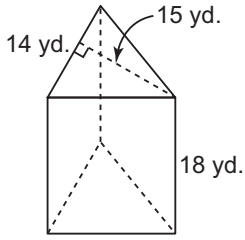
1. Find the volume of this rectangular prism.



The base is a _____.
So, the area of the base is: _____
V = base area × height
V = _____
V = _____
The volume of the prism is: _____

For a rectangular prism, V = length × width × height, or $V = lwh$

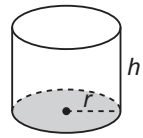
2. Find the volume of this triangular prism.



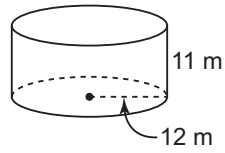
The base is a _____.
So, the area of the base is: _____
V = _____
V = _____
V = _____
The volume of the prism is: _____

Volume of a Cylinder

The volume, V, of a cylinder is: base area $\times$ height
The base of a cylinder is a circle.
So, for a cylinder with base radius r and height h:
$V = \pi r^2 h$



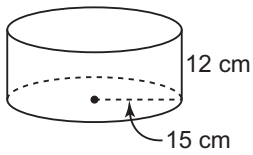
To find the volume, V, of this cylinder:
Use the formula:
$V = \pi r^2 h$ Substitute: $r = 12$ and $h = 11$
$V = \pi \times 12^2 \times 11$
$V = 4976.2827\ldots$
The volume of the cylinder is about 4976 m^3.



Check

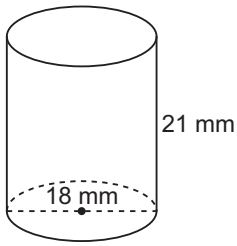
1. Find the volume of each cylinder, to the nearest tenth of a cubic unit.

a)



$V = \pi r^2 h$ Substitute: $r =$ _____ and $h =$ _____
$V =$ _____
$V =$ _____
The volume of the cylinder is about: _____

b)

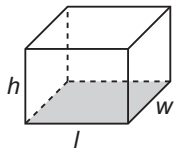


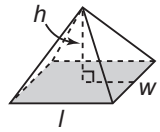
The diameter of the cylinder is: _____
So, its radius is: _____
$V =$ _____ Substitute: _____
$V =$ _____
$V =$ _____
The volume of the cylinder is about: _____

1.5 Volumes of Right Pyramids and Right Cones

FOCUS **Find the volumes of pyramids and cones.**

When a prism and a pyramid have the same base and the same height, the volume of the pyramid is $\frac{1}{3}$ the volume of the prism.



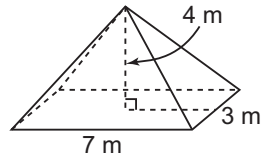


Volume, V, of a pyramid is: $V = \frac{1}{3} \times$ base area $\times$ height

For a rectangular pyramid, $V = \frac{1}{3}lwh$

Example 1 Finding the Volume of a Rectangular Pyramid

Find the volume of this rectangular pyramid.



Solution

Use the formula:

$V = \frac{1}{3}lwh$ Substitute: $l = 7$, $w = 3$, and $h = 4$

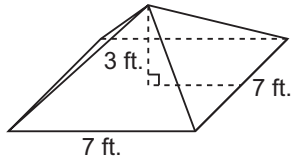
$V = \frac{1}{3} \times 7 \times 3 \times 4$

$V = 28$

The volume is 28 m^3.

Check

1. Find the volume of this square pyramid.



Use the formula:

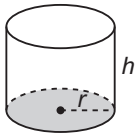
$V = \frac{1}{3}lwh$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is: _____

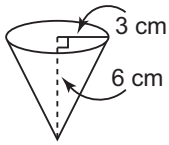
When a cylinder and a cone have the same base and the same height, the volume of the cone is $\frac{1}{3}$ the volume of the cylinder.



Volume, V, of a cone is: $V = \frac{1}{3}\pi r^2 h$

Example 2 Finding the Volume of a Cone

Find the volume of this cone, to the nearest tenth of a cubic centimetre.



Solution

Use the formula:

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 3$ and $h = 6$

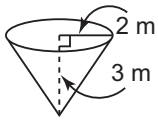
$V = \frac{1}{3} \times \pi \times 3^2 \times 6$

$V = 56.5486...$

The volume is about 56.5 cm^3.

Check

1. Find the volume of this cone, to the nearest tenth of a cubic metre.



Use the formula:

$V = \frac{1}{3}\pi r^2 h$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

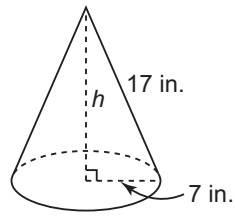
When the slant height of a pyramid or cone is given, we find the height before calculating the volume.

Example 3 Finding the Volume of a Cone Given Its Slant Height

A cone has radius 7 in. and slant height 17 in.
Find the volume of the cone, to the nearest cubic inch.

Solution

Sketch a diagram. Let the height be h inches.



Use the Pythagorean Theorem to find h.

$17^2 = h^2 + 7^2$ Simplify.

$289 = h^2 + 49$ Solve for h^2.

$h^2 = 289 - 49$

$h^2 = 240$ Solve for h.

$h = \sqrt{240}$

Use this value of h in the formula for the volume.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 7$ and $h = \sqrt{240}$

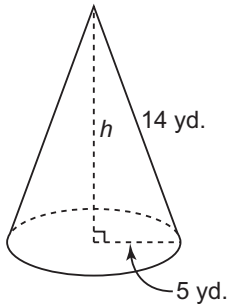
$V = \frac{1}{3} \times \pi \times 7^2 \times \sqrt{240}$

$V = 794.9326\ldots$

The volume is about 795 cubic inches.

Check

1. Find the volume of this cone, to the nearest cubic yard.



Use the Pythagorean Theorem to find h.

$14^2 =$ _____ + _____

$h =$ _____

Use this value of h in the formula for the volume.

$V = \frac{1}{3}\pi r^2 h$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

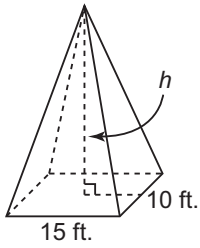
We can use a volume measure to find an unknown measure.

Example 4 Finding the Height of a Pyramid

A rectangular pyramid can hold 1250 cubic feet of water.
The base of the pyramid is 15 ft. by 10 ft.
What is the height of the pyramid?

Solution

Let the height of the pyramid be h feet.



Use the formula for the volume of a rectangular pyramid:

$V = \frac{1}{3}lwh$ Substitute: $V = 1250$, $l = 15$, and $w = 10$

$1250 = \frac{1}{3} \times 15 \times 10 \times h$ Simplify.

$1250 = 50h$ Divide each side by 50.

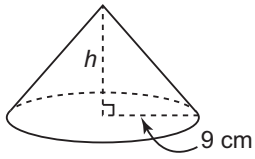
$h = \frac{1250}{50}$

$h = 25$

The pyramid is 25 ft. high.

Check

1. This cone can hold 870 cm^3 of sand. What is the height of the cone, to the nearest tenth of a centimetre?



Use the formula for the volume of a cone:

$V = \frac{1}{3}\pi r^2 h$ Substitute: _____

Remember to use brackets when you divide by the denominator.

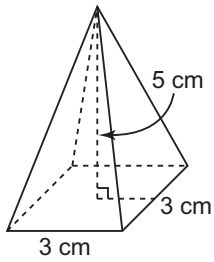
$h =$ _____

The height of the cone is about: _____

Practice

1. Find the volume of each pyramid.

a) a square pyramid



Use the formula:

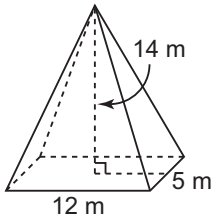
$V = \frac{1}{3}lwh$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is: _____

b) a rectangular pyramid



Use the formula:

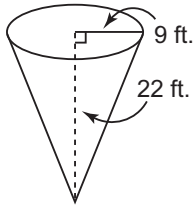
$V =$ _____ Substitute: _____

$V =$ _____

$V =$ _____

The volume is: _____

2. Find the volume of this cone, to the nearest cubic foot.



Use the formula:

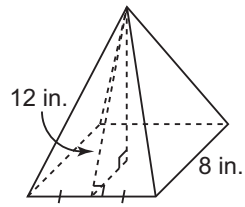
$V = \frac{1}{3}\pi r^2 h$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

3. Find the volume of this square pyramid, to the nearest cubic inch.



Label a triangle to show the height and slant height of the pyramid.

Let the height be h inches.

The base of the triangle is: $\frac{1}{2} \times$ _____ = _____

Use the Pythagorean Theorem to find h.

12^2 = _____ + _____

h = _____

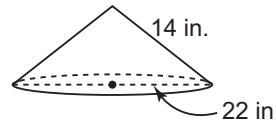
Use this value of h in the formula for the volume.

$V = \frac{1}{3}lwh$ Substitute: _____

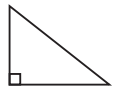
V = _____

The volume is about: _____

4. Find the volume of this cone, to the nearest cubic inch.



Label this triangle to show the height and slant height of the cone.



Let the height be h inches.

Use the Pythagorean Theorem to find h.

h = _____

Use this value of h in the formula for the volume.

$V = \frac{1}{3}\pi r^2 h$

V = _____

V = _____

The volume is about: _____

5. The base of a square prism has side length 8 cm; its height is 12 cm.
The base of a square pyramid has side length 12 cm; its height is 17 cm.
Which object has the greater volume?

The volume of the prism is:

The volume of the pyramid is:

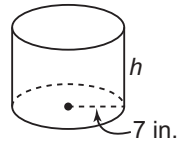
The volume of the prism is: _____

The volume of the pyramid is: _____

The _____ has the greater volume.

6. Both a cylinder and a cone have volume 1525 cubic inches and base radius 7 in.
Find the height of each object, to the nearest tenth of an inch.

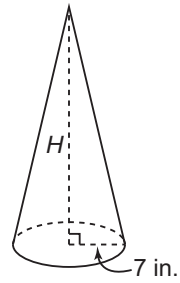
Let the height of the cylinder be h inches.
To find h, use the formula for the volume of a cylinder.



The height of the cylinder is about: _____

Let the height of the cone be H inches.

To find H, use the formula for the volume of a cone.

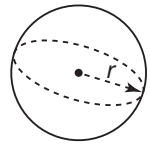


The height of the cone is about: _____

1.6 Surface Area and Volume of a Sphere

FOCUS **Find the surface area and volume of a sphere.**

To find the surface area of a sphere, use this formula:
$SA = 4\pi r^2$



To find the surface area of a hemisphere, use this formula:
$SA = 3\pi r^2$

Example 1 Finding the Surface Area of a Sphere

A glass sphere has radius 25 cm. What is the surface area of the sphere, to the nearest square centimetre?

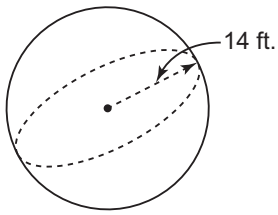
Solution

Use the formula:
$SA = 4\pi r^2$ Substitute: $r = 25$
$SA = 4 \times \pi \times 25^2$
$SA = 7853.9816\ldots$
The surface area of the sphere is about 7854 cm^2.

Check

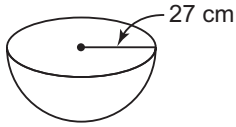
1. Find the surface area of each object, to the nearest square unit.

a)



Use the formula:
$SA = 4\pi r^2$ Substitute: _____
$SA =$ _____
$SA =$ _____
The surface area is about: _____

b)



Use the formula:
$SA = 3\pi r^2$ Substitute: _____
$SA =$ _____
$SA =$ _____
The surface area of the hemisphere is about: _____

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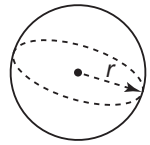
To find the volume of a sphere, use this formula:

$V = \frac{4}{3}\pi r^3$



To find the volume of a hemisphere, use this formula:

$V = \frac{2}{3}\pi r^3$

Example 2 Finding the Volume of a Sphere

A sphere has diameter 8 yd. What is the volume of the sphere, to the nearest cubic yard?

Solution

Radius is: $\frac{1}{2} \times 8$ yd. $= 4$ yd.

Use the formula:

$V = \frac{4}{3}\pi r^3$ Substitute: $r = 4$

The radius is one-half the diameter.

$V = \frac{4}{3} \times \pi \times 4^3$

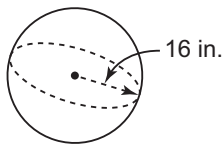
$V = 268.0825\ldots$

The volume of the sphere is about 268 cubic yards.

Check

1. Find the volume of each object, to the nearest cubic unit.

a)



Use the formula:

$V = \frac{4}{3}\pi r^3$ Substitute: _____

$V =$ _____

$V =$ _____

The volume of the sphere is about: _____

b)

Radius is: _____

Use the formula:

$V = \frac{2}{3}\pi r^3$ Substitute: _____

$V =$ _____

$V =$ _____

The volume of the hemisphere is about: _____

If the surface area of a sphere or hemisphere is known, we can find its radius.

Example 3 Finding the Radius of a Sphere

A globe has surface area 2735 cm^2. Find the radius of the globe, to the nearest tenth of a centimetre.

Solution

Use the formula:

$SA = 4\pi r^2$ Substitute: $SA = 2735$

$2735 = 4\pi r^2$ Solve for r^2. Divide each side by 4π.

$r^2 = \frac{2735}{4\pi}$ Solve for r. Take the square root of each side.

$r = \sqrt{\frac{2735}{4\pi}}$

$r = 14.7527\ldots$

The radius of the globe is about 14.8 cm.

Remember to use brackets in the denominator. Input: $\sqrt{\frac{2735}{(4\pi)}}$

Check

1. A sphere has surface area 3567 m^2. What is the radius of the sphere, to the nearest tenth of a metre?

Use the formula:

$SA = 4\pi r^2$ Substitute: _____

$r^2 =$ _____

$r =$ _____

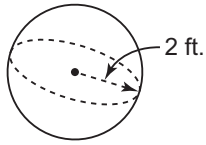
$r =$ _____

The radius of the sphere is about: _____

Practice

1. Find the surface area of each object, to the nearest square unit.

a)



Use the formula:

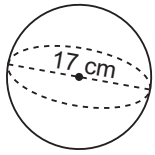
$SA = 4\pi r^2$ Substitute: _____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

b)



Radius: _____

Use the formula:

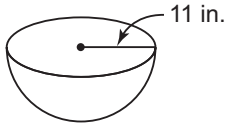
$SA =$ _____ Substitute: _____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

c)



Use the formula:

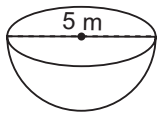
$SA = 3\pi r^2$ Substitute: _____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

d)



Radius: _____

Use the formula:

$SA =$ _____ Substitute: _____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

2. Find the volume of each object in question 1.
Give the answers to the nearest cubic unit.

a) Use the formula:

$V = \frac{4}{3}\pi r^3$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

b) Use the formula:

$V =$ _____ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

c) Use the formula:

$V = \frac{2}{3}\pi r^3$ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

d) Use the formula:

$V =$ _____ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

3. A solid cork ball is covered in gold plating. It has diameter 14 cm.

a) To the nearest tenth of a square centimetre, what is the area of gold plating?
Find the surface area of the ball.

Radius: _____

$SA = 4\pi r^2$

$SA =$ _____

$SA =$ _____

The area of gold plating is about: _____

b) To the nearest cubic centimetre, what is the volume of cork?
Find the volume of the ball.

Radius: _____

$V = \frac{4}{3}\pi r^3$

$V =$ _____

$V =$ _____

The volume of cork is about: _____

4. A ball has a surface area of 28 square inches.
Find the radius of the ball, to the nearest tenth of an inch.

Let the radius of the ball be r inches.
Use the formula:

$SA =$ _____ Substitute: $SA =$ _____
Solve for r.

$r^2 =$ _____

$r =$ _____

$r =$ _____

The radius of the ball is about: _____

5. A disco ball is covered in 9 square feet of silver foil. To the nearest inch, what is the diameter of the ball?

Assume a disco ball approximates a sphere.

Let the radius of the ball be r feet.
Use the formula:

$SA =$ _____ Substitute: $SA =$ _____
Solve for r.

$r^2 =$ _____

$r =$ _____

$r =$ _____

Diameter: 2 × _____ ft. = _____ ft.

Convert feet to inches:

1 ft. = 12 in.

_____ ft. = _____ × 12 in.
= _____

The diameter of the ball is about: _____

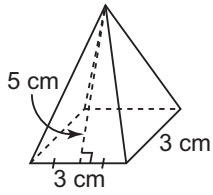
CHECKPOINT 2

Can you...

- find the height or slant height of an object using the Pythagorean Theorem?
- find the surface areas of pyramids, cones, and spheres?
- find the volumes of pyramids, cones, and spheres?

1.4 **1.** Find the surface area of each object.

a) a square pyramid



The base of the pyramid is a _____, so the base area is: _____ × _____ = _____

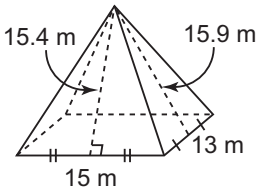
Area of each triangular face: $\frac{1}{2}$ × _____ × _____ = _____

Area of all 4 faces: 4 × _____ = _____

Surface area of the pyramid: _____ + _____ = _____

The surface area of the pyramid is: _____

b) a rectangular pyramid



The base of the pyramid is a _____, so the base area is: _____ = _____

Two triangular faces have base _____ and height _____.

Area of these 2 faces is:

$2 \times \frac{1}{2} \times$ base × height = _____

= _____

Two triangular faces have base _____ and height _____.

Area of these 2 faces is:

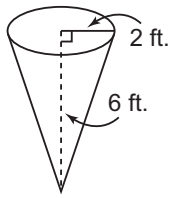
$2 \times \frac{1}{2} \times$ base × height = _____

= _____

Surface area of the pyramid: _____ + _____ + _____ = _____

The surface area of the pyramid is: _____

2. Find the surface area of this cone, to the nearest square foot.



Label this triangle to show the height and slant height of the cone.
Let the slant height be s feet.

Use the Pythagorean Theorem to find the slant height.

$s^2 =$ _____ + _____ Simplify.

$s^2 =$ _____ + _____

$s^2 =$ _____

$s =$ _____

The slant height is: _____

Use the formula for the surface area of a cone.

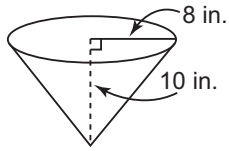
$SA =$ _____ Substitute: _____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

1.5 **3.** Find the volume of this cone, to the nearest cubic inch.



Use the formula for the volume of a cone.

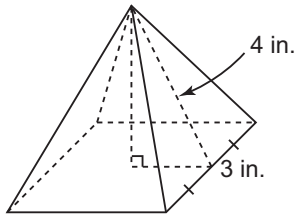
$V =$ _____ Substitute: _____

$V =$ _____

$V =$ _____

The volume is about: _____

4. A glass paperweight has the shape of a square pyramid. Alex measured the edge of the base as 3 in. and the slant height as 4 in. What is the volume of the paperweight, to the nearest cubic inch?



Label this triangle to show the height and slant height of the pyramid.
Let the height be h inches.

Use the Pythagorean Theorem to find h.

Solve for h.

$h =$ _____

Substitute this value of h in the formula for the volume.

$V = \frac{1}{3}lwh$ Substitute: _____

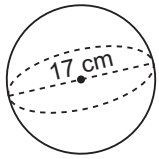
$V =$ _____

$V =$ _____

The volume is about: _____

1.6 **5.** Find the surface area of each object, to the nearest tenth of a square unit.

a) a sphere



Radius: _____

Use the formula:

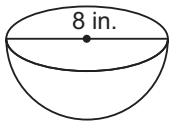
$SA =$ _____ Substitute: _____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

b) a hemisphere



Radius: _____

Use the formula:

SA = _____ Substitute: _____

SA = _____

SA = _____

The surface area is about: _____

6. To the nearest cubic unit, find the volume of each object in question 5.

a) Use the formula for the volume of a sphere:

V = _____ Substitute: _____

V = _____

V = _____

The volume is about: _____

b) Use the formula for the volume of a hemisphere:

V = _____ Substitute: _____

V = _____

V = _____

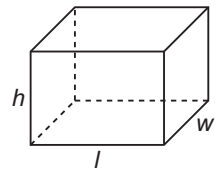
The volume is about: _____

1.7 Skill Builder

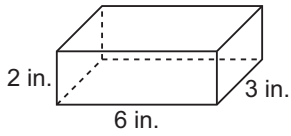
Surface Area of a Prism

The surface area, SA, of a prism, is:
SA = sum of the areas of all the faces

The surface area of a rectangular prism is:
$SA = 2(lw + lh + wh)$



To find the surface area of this rectangular prism:



Use:

$SA = 2(lw + lh + wh)$ Substitute: $l = 6$, $w = 3$, and $h = 2$

$SA = 2(6 \times 3 + 6 \times 2 + 3 \times 2)$ Use the order of operations.

$SA = 2(18 + 12 + 6)$

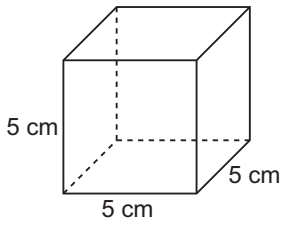
$SA = 2(36)$

$SA = 72$

The surface area of the prism is 72 square inches.

Check

1. Find the surface area of this cube.



Use:

$SA = 2(lw + lh + wh)$ Substitute: _____

$SA =$ _____

$SA =$ _____

$SA =$ _____

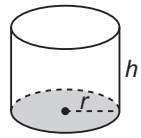
$SA =$ _____

The surface area of the cube is: _____

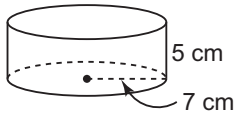
Surface Area of a Cylinder

The surface area, SA, of a cylinder, is:
SA = Area of the 2 bases + Area of curved surface

The surface area of this cylinder is:
$SA = 2\pi r^2 + 2\pi rh$



To find the surface area of this cylinder:

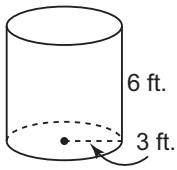


Use:
$SA = 2\pi r^2 + 2\pi rh$ Substitute: $r = 7$ and $h = 5$
$SA = 2\pi(7)^2 + 2\pi(7)(5)$
$SA = 527.7875\ldots$
The surface area of the cylinder is about 528 cm^2.

Check

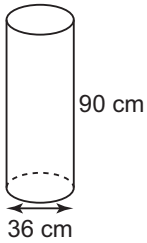
1. Find the surface area of each cylinder, to the nearest square unit.

a)



Use:
$SA = 2\pi r^2 + 2\pi rh$ Substitute: _____
SA = _____
SA = _____
The surface area is about: _____

b)



Radius: _____
Use:
SA = _____ Substitute: _____
SA = _____
SA = _____
The surface area is about: _____

1.7 Solving Problems Involving Objects

FOCUS **Find the surface areas and volumes of composite objects.**

The formulas for surface area and volume are summarized in this chart:

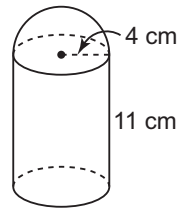
Object	Surface Area Formula	Volume Formula
Prism	SA = Area of two bases + area of faces	V = (Base area)(height)
Rectangular prism	$SA = 2(lw + lh + wh)$	$V = lwh$
Cube	$SA = 6s^2$	$V = s^3$
Pyramid	SA = Area of base + area of faces	$V = \frac{1}{3}$(Base area)(height)
Rectangular pyramid	SA = Area of base + 2(Area of one triangular face) + 2(Area of different triangular face)	$V = \frac{1}{3}lwh$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$	$V = \pi r^2 h$
Cone	$SA = \pi rs + \pi r^2$	$V = \frac{1}{3}\pi r^2 h$
Sphere	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Hemisphere	$SA = 3\pi r^2$	$V = \frac{2}{3}\pi r^3$

To find the volume of a composite object, add the volumes of the objects that make up the composite object.

A composite object is made from 2 or more objects.

Example 1 Finding the Volume of a Composite Object

Find the volume of this composite object, to the nearest tenth of a cubic centimetre.



Solution

The composite object is a hemisphere on a cylinder.
Find the volume of each separate object.

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Volume of the hemisphere:

$V = \frac{2}{3}\pi r^3$

Substitute: $r = 4$

$V = \frac{2}{3}\pi(4)^3$

$V = 134.0412\ldots$

Volume of cylinder:

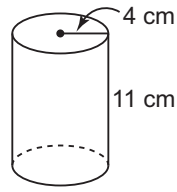
$V = \pi r^2 h$

Substitute:

$r = 4$ and $h = 11$

$V = \pi(4)^2(11)$

$V = 552.9203\ldots$



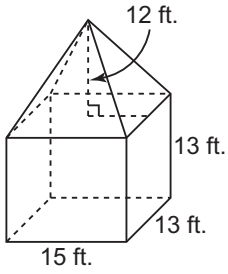
Add the volumes of the hemisphere and the cylinder.

Total volume is: $134.0412\ldots + 552.9203\ldots = 686.9615\ldots$

The volume of the composite object is about 687.0 cm^3.

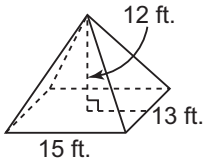
Check

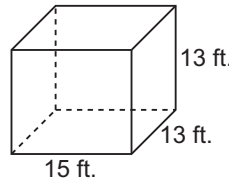
1. Find the volume of this composite object.



The composite object is a rectangular pyramid on a rectangular prism.

Find the volume of each separate object.





Volume of the rectangular pyramid:

$V = \frac{1}{3}lwh$

Substitute: $l =$ _____, $w =$ _____, $h =$ _____

$V =$ _____

$V =$ _____

Volume of the rectangular prism:

$V = lwh$

Substitute: $l =$ _____, $w =$ _____, $h =$ _____

$V =$ _____

$V =$ _____

Add the volumes.

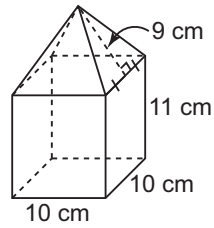
Total volume is: _____ + _____ = _____

The volume of the composite object is: _____

To find the surface area of a composite object, add the areas of all exposed surfaces.

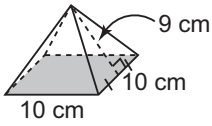
Example 2 Finding the Surface Area of a Composite Object

Find the surface area of this composite object.

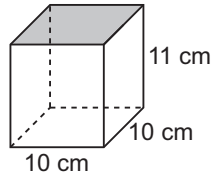


Solution

The composite object is a square pyramid on a square prism.
Find the area of the exposed surfaces of each separate object.



*The shaded regions are **not** included in the surface area.*



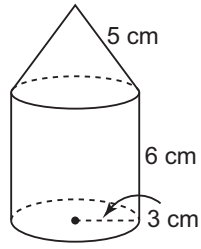
Surface area of the square pyramid
is: area of 4 triangular faces
Area of 1 face is: $\frac{1}{2} \times 10 \times 9 = 45$
Area of 4 faces is: $4 \times 45 = 180$

Surface area of the square prism is:
area of 4 rectangular faces + area of base
Area of 1 face is: $10 \times 11 = 110$
Area of 4 faces is: $4 \times 110 = 440$
Area of base is: $10 \times 10 = 100$
Surface area is: $440 + 100 = 540$

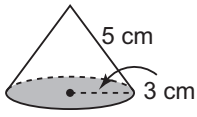
Add the surface areas.
Total surface area is: $180 + 540 = 720$
The surface area of the composite object is 720 cm^2.

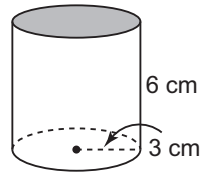
Check

1. Find the surface area of this composite object, to the nearest square centimetre.



The composite object is: _____
Find the surface area of each separate object.





Surface area of the: _____

$SA = \pi rs$

Substitute: $r =$ _____ and $s =$ _____

$SA =$ _____

$SA =$ _____

Surface area of the: _____

$SA = 2\pi rh + \pi r^2$

Substitute: $r =$ _____ and $h =$ _____

$SA =$ _____

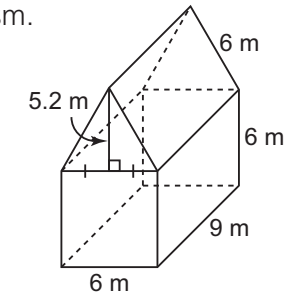
$SA =$ _____

Total surface area is: _____ + _____ = _____

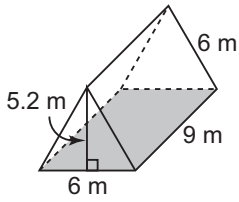
The surface area is about: _____

Practice

1. This composite object is a triangular prism on a rectangular prism. The base of the triangular prism is an equilateral triangle. Find the surface area of this object.



Find the area of the exposed surfaces of each separate object.



Surface area of: _____

is the area of _____ triangular faces + the area of _____ rectangular faces

Area of 1 triangular face is:

_____ = _____

Area of _____ triangular faces is:

_____ × _____ = _____

Area of 1 rectangular face is:

_____ × _____ = _____

Area of _____ rectangular faces is:

_____ = _____

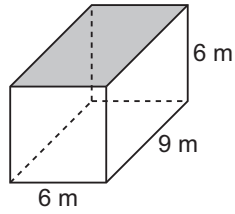
Surface area of: _____

is: _____ + _____ = _____

Add the surface areas.

Total surface area is: _____ + _____ = _____

The surface area is: _____



Surface area of: _____

is the area of _____ square faces + the area of _____ rectangular faces

Area of 1 square face is:

_____ = _____

Area of _____ square faces is:

_____ × _____ = _____

Area of 1 rectangular face is:

_____ × _____ = _____

Area of _____ rectangular faces is:

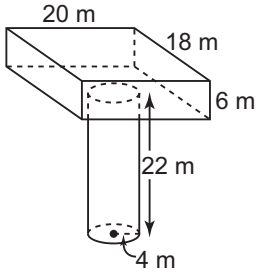
_____ = _____

Surface area of: _____

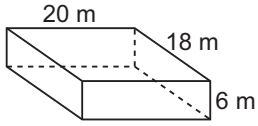
is: _____ + _____ = _____

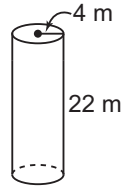
2. Determine the volume of each composite object, to the nearest cubic unit.

a) a rectangular prism on a cylinder



Find the volume of each separate object.





Volume of the rectangular prism:

$V =$ _____

Substitute: _____

$V =$ _____

$V =$ _____

Volume of the cylinder:

$V =$ _____

Substitute: _____

$V =$ _____

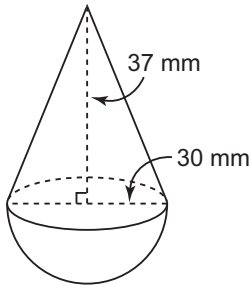
$V =$ _____

Add the volumes.

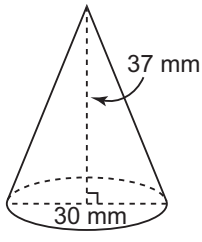
Total volume is: _____ + _____ = _____

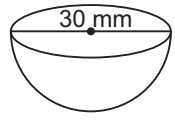
The volume is about: _____

b) a cone on a hemisphere



Find the volume of each separate object.





Radius: _____

Volume of the cone:

$V =$ _____

Substitute: _____

$V =$ _____

$V =$ _____

Volume of the hemisphere:

$V =$ _____

Substitute: _____

$V =$ _____

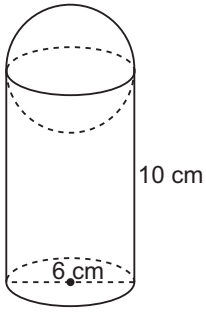
$V =$ _____

Add the volumes.

Total volume is: _____ + _____ = _____

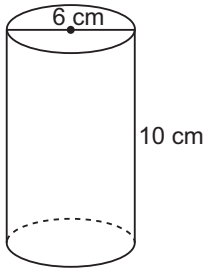
The volume is about: _____

3. A sphere of flavoured ice is served in a cylinder-shaped paper cup. The cup has diameter 6 cm and height 10 cm. The sphere has the same diameter as the cup. To the nearest cubic centimetre, how much space is left inside the cup? (Hint: One-half of the sphere is below the rim of the cup.)



Volume of space = Volume of _____ − volume of _____

Radius: _____



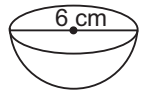
Volume of: _____

$V = \pi r^2 h$

Substitute: _____

$V =$ _____

$V =$ _____



Volume of: _____

$V = \frac{2}{3}\pi r^3$

Substitute: _____

$V =$ _____

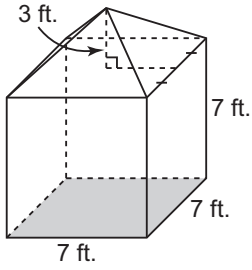
$V =$ _____

Subtract the volumes.

Volume of space is: _____ − _____ = _____

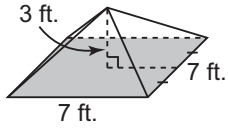
There is about _____ of space in the cup.

4. A tent has the shape of a square pyramid on top of a cube, as shown. To the nearest square foot, find the amount of material needed to make the tent.



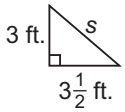
Do not include the floor.

Find the area of the exposed surfaces of each separate object.



Surface area of square pyramid
is the area of _____ triangular faces.

Find the slant height of the pyramid.
Let s represent the slant height.



Use the Pythagorean Theorem.

s^2 = _____ + _____

s^2 = _____

s = _____

Area of 1 triangular face is: _____

Area of _____ triangular faces is:

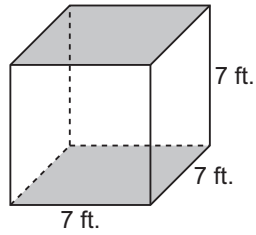
_____ = _____

Add the surface areas.

Total surface area is:

_____ + _____ = _____

The amount of material needed is about: _____



Surface area of cube
is the area of _____ square faces.

Area of 1 square face is:

_____ = _____

Area of _____ square faces is:

_____ = _____

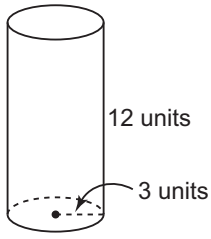
Chapter 1 Puzzle

Figure It Out

To simplify this expression: $[(AB) \div E]^C + D$, you need to find the value of each letter.

Calculate the surface area and volume of each object below on another sheet of paper. Write each measure to the nearest whole unit. Record your answers below, then answer the questions that follow.

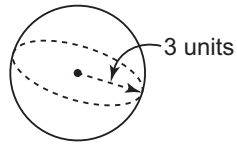
A.



$SA \doteq$ _____

$V \doteq$ _____

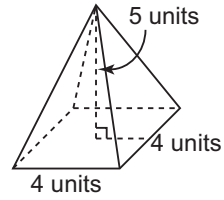
B.



$SA \doteq$ _____

$V \doteq$ _____

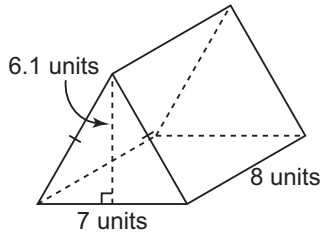
C.



$SA \doteq$ _____

$V \doteq$ _____

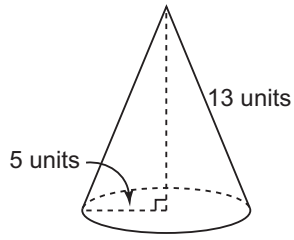
D.



$SA \doteq$ _____

$V \doteq$ _____

E.



$SA \doteq$ _____

$V \doteq$ _____

Answer these questions with the letter that represents the object.

Which object has its volume and surface area differ by 40? _____ = 1

Which object has a volume of 314? _____ = 2

Which object has the least volume? _____ = 3

Which object has the second to least volume? _____ = 4

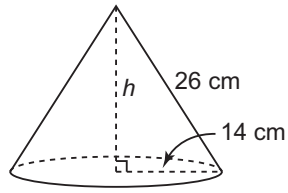
Which object has the greatest difference in its volume and surface area? _____ = 5

Substitute the number that corresponds to each letter in the expression below, then simplify.

$[(AB) \div E]^C + D =$

The solution is: _____

Chapter 1 Study Guide

Skill	Description	Example
Convert between units in the imperial system.	Use the relationships between the units.	Convert: 19 yd. to feet 1 yd. = 3 ft. 19 yd. = 19 × 3 ft. 19 yd. = 57 ft.
Convert between units in the imperial system and the SI system.	Use the relationships between the systems.	Convert: 37 mi. to kilometres 1 mi. ≐ 1.6 km 37 mi. ≐ 37 × 1.6 km 37 mi. ≐ 59.2 km
Find the slant height or height of a cone.	Use the Pythagorean Theorem.	To the nearest centimetre, find the height of this cone.  $26^2 = 14^2 + h^2$ $h^2 = 26^2 - 14^2$ $h = \sqrt{26^2 - 14^2}$ $h = 21.9089\ldots$ The height is about 22 cm.
Find the surface areas of pyramids, prisms, cones, cylinders, and spheres.	For a pyramid or a prism, add the area of the base or bases to the area of the faces. For a cone or a cylinder, add the area of the base or bases to the curved surface area. Surface area of a sphere is: $4\pi(\text{radius})^2$	To the nearest square foot, find the surface area of a cylinder with radius 3 ft. and height 9 ft. $SA = 2\pi r^2 + 2\pi rh$ $SA = 2\pi(3)^2 + 2\pi(3)(9)$ $SA = 226.1946\ldots$ The surface area is about 226 square feet.
Find the volumes of prisms, cylinders, pyramids, cones, and spheres.	Volume of a prism and a cylinder is: (base area)(height) Volume of a pyramid and a cone is: $\frac{1}{3}$(base area)(height) Volume of a sphere is: $\frac{4}{3}\pi(\text{radius})^3$	To the nearest cubic inch, find the volume of a cylinder with radius 2 in. and height 8 in. $V = \pi r^2 h$ $V = \pi(2)^2(8)$ $V = 100.5309\ldots$ The volume is about 101 cubic inches.

Chapter 1 Review

1.1 **1.** Which imperial unit is best to measure each item below: mile, yard, foot, or inch?

a) the distance between your locker and the front door of the school: _____

b) the width of a house: _____

c) the distance around a pop can: _____

2. Convert:

a) 84 ft. to yards

_____ ft. = 1 yd.

84 ft. = (_____ ÷ _____) yd., or

84 ft. = _____ yd.

_____

84 ft. = _____

b) 9 ft. 7 in. to inches

1 ft. = _____ in.

9 ft. = _____ × _____ in.

9 ft. = _____

Add the inches.

9 ft. 7 in. = _____ + _____

9 ft. 7 in. = _____

See page 6 of the Student Text for a conversion chart.

1.2 **3.** For each object below:

- Describe how you measure it.
- Include the measuring device.
- State the imperial unit.
- State the SI unit.

a) the greatest distance around a fish bowl

Imperial unit: _____

SI unit: _____

Measuring device: _____

b) the width of your kitchen floor

Imperial unit: _____

SI unit: _____

Measuring device: _____

1.3 **4.** Convert:

a) 17 yd. to metres

1 yd. ≐ _____ m

17 yd. ≐ 17 × _____

17 yd. ≐ _____

b) 68 mi. to kilometres

1 mi. ≐ _____ km

68 mi. ≐ _____ × _____

68 mi. ≐ _____

See page 13 for a conversion chart.

5. Ryan drove 19 km to watch a lacrosse game. Julie drove 11 mi. to see the same game. Who drove farther?

Ryan drove: _____

Convert Julie's distance to kilometres.

1 mi. ≐ _____

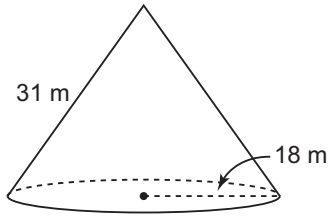
11 mi. ≐ _____ × _____

11 mi. ≐ _____

Since _____ is greater than _____, _____ drove farther.

1.4 **6.** Find the surface area of each object, to the nearest square unit.

a) a right cone



For a cone:

$SA = \pi$_____ + π_____

Substitute: $r =$ _____ and $s =$ _____

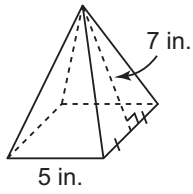
$SA = \pi$_____ + π_____

$SA =$ _____

The surface area is about: _____

See page 27 for the formula for the surface area of a cone.

b) a square pyramid



For a square pyramid:

Area of base is: _____

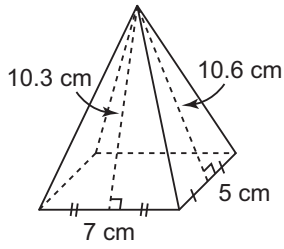
Area of each triangular face is: $\frac{1}{2} \times$ _____ × _____ = _____

Area of all 4 faces is: 4 × _____ = _____

The surface area of the pyramid is: _____ + _____ = _____

The surface area is: _____

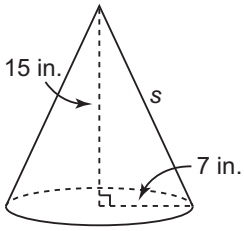
7. Find the surface area of this rectangular pyramid.



For a rectangular pyramid:
SA = area of rectangular base + area of each triangular face

The surface area of the pyramid is: _____.

8. Is the surface area of this cone less than 500 square inches?

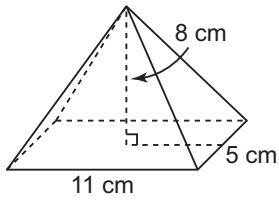


Find the slant height, *s*.
Use the Pythagorean Theorem.
s^2 = _____ + _____
s^2 = _____
s = _____

Find the surface area:
SA = _____ + _____
Substitute: _____ = _____ and _____ = _____

The surface area of the cone is about: _____
The surface area of the cone is _____ than 500 square inches.

1.5 **9.** Find the volume of this rectangular pyramid, to the nearest cubic centimetre.



See page 33 for the formula for the volume of a pyramid.

For a rectangular pyramid:

$V =$ _____

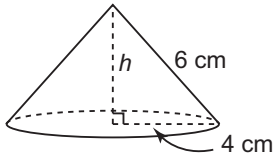
Substitute: _____ = _____, _____ = _____, _____ = _____

$V =$ _____

$V =$ _____

The volume is about: _____

10. A bowl of sugar was knocked over. The spilled sugar formed this cone.
How much sugar was in the pile?



Use the Pythagorean Theorem to find h.

_____ $= h^2 +$ _____

$h^2 =$ _____

$h^2 =$ _____

$h =$ _____

Find the volume:

$V =$ _____

Substitute: _____ = _____ and _____ = _____

$V =$ _____

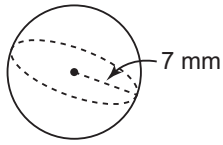
$V =$ _____

See page 34 for the formula for the volume of a cone.

There was about _____ of sugar in the pile.

1.6 **11.** Find the surface area and volume of each object. Give the answers to the nearest whole number of units.

a) a sphere



See pages 40 and 41 for the formulas for a sphere.

For a sphere:

$SA =$ _____ Substitute: ____ = ____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

$V =$ _____

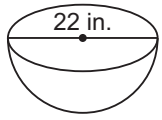
Substitute: _____ = _____

$V =$ _____

$V =$ _____

The volume is about: _____

b) a hemisphere



Radius is: _____

For a hemisphere:

$SA =$ _____ Substitute: ____ = ____

$SA =$ _____

$SA =$ _____

The surface area is about: _____

$V =$ _____

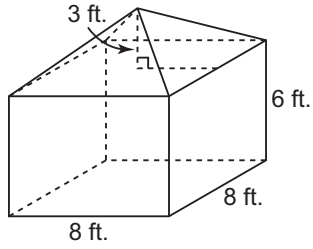
Substitute: ____ = ____

$V =$ _____

$V =$ _____

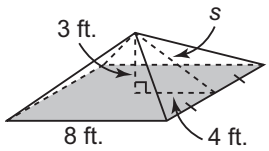
The volume is about: _____

1.7 **12.** A garden shed has the shape of a square pyramid on top of a square prism. Both the pyramid and the prism have base side length 8 ft. The prism is 6 ft. high and the pyramid is 3 ft. high.



a) Find the surface area of the shed. Do not include the base of the shed.

Let the slant height of the pyramid be s feet.



Surface area of square pyramid is the area of _____ triangular faces.

Find the slant height of the pyramid.
Use the Pythagorean Theorem.
s^2 = _____

s = _____

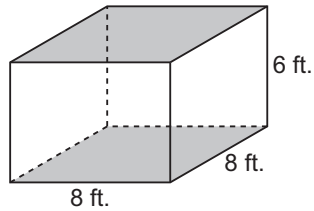
Area of 1 triangular face is: _____ = _____

Area of _____ triangular faces is: _____ = _____

Add the surface areas.

Total surface area is: _____ + _____ = _____

The surface area of the shed is: _____



Surface area of square prism is the area of _____ rectangular faces.

Area of 1 rectangular face is:
_____ = _____

Area of _____ rectangular faces is:
_____ = _____

b) How much space is inside the shed?

Find the volume of the shed.

Volume of square pyramid:

V = _____

Substitute: _____

V = _____

V = _____

Volume of square prism:

V = _____

Substitute: _____

V = _____

V = _____

Total volume is: _____ + _____ = _____

There is _____ of space in the shed.

