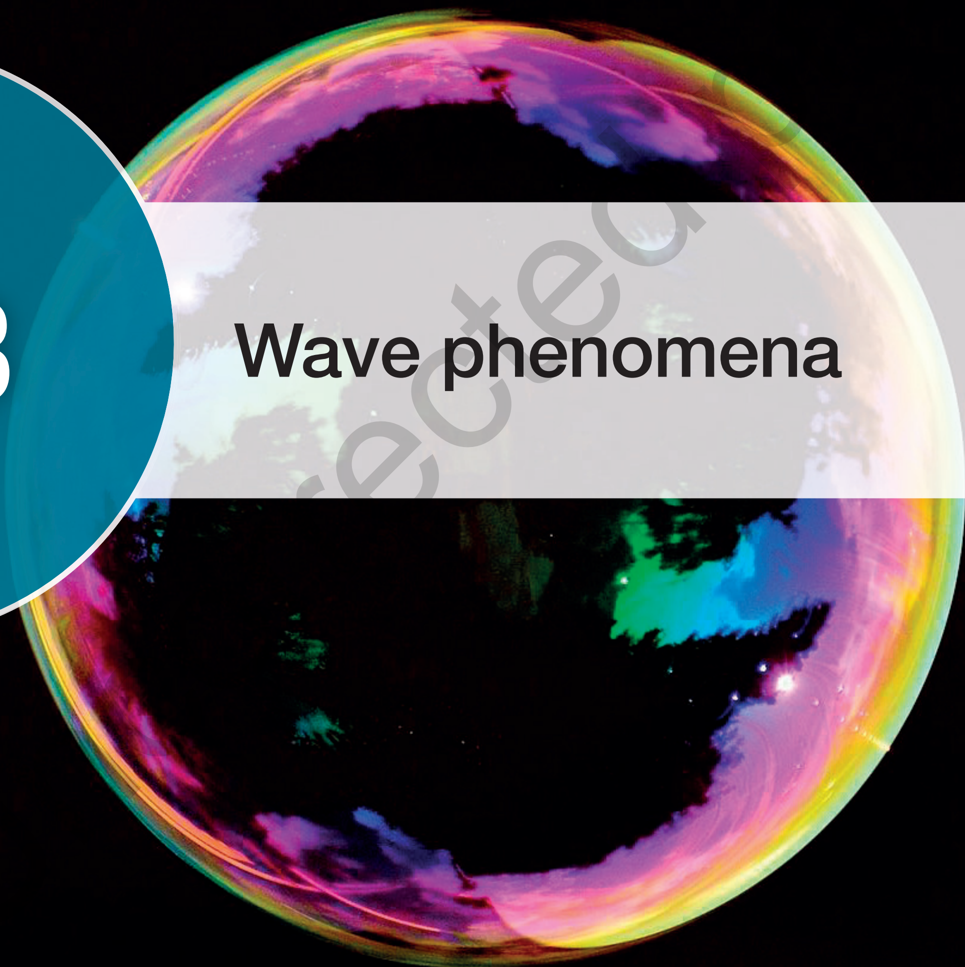


C.3 Wave phenomena

A bubble is a thin, spherical soap and water film. When white light from the Sun shines on a bubble, some is reflected at the uppermost air–soap surface and some refracts inside the film, with additional reflection taking place at the inner soap–air surface. The distance traveled by the light within the film is greater than the distance traveled by the light reflecting on the outside, which means that interference occurs. The colors that are most prominent from a given viewpoint are the result of constructive interference. Those that are not seen have undergone destructive interference. These change with the observer's position in an effect called iridescence.

Guiding Questions

How are observations of wave behaviors at a boundary between different media explained?

How is the behavior of waves passing through apertures represented?

What happens when two waves meet at a point in space?

If C.1 is for the study of oscillations of individual particles and C.2 is for the study of how the particles in a medium can be disturbed to transmit energy, C.3 comprises what traveling waves *do*.

They reflect, refract, transmit, diffract and interfere, and we'll make use of ray and wavefront diagrams and graphs of displacements to visualize these behaviors.

As you read, try to notice not only *what* these behaviors consist of and emerge from but also *how* you as a physicist are being asked to understand them: qualitatively and/or quantitatively?

Students should understand:

waves traveling in two and three dimensions can be described through the concepts of wavefronts and rays
wave behavior at boundaries in terms of reflection, refraction and transmission
wave diffraction around a body and through an aperture
wavefront-ray diagrams showing refraction and diffraction
Snell's law, critical angle and total internal reflection
Snell's law as given by $\frac{n_1}{n_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_2}{v_1}$ where n is the refractive index and θ is the angle between the normal and the ray
superposition of waves and wave pulses
double-source interference requires coherent sources
the condition for constructive interference as given by path difference = $n\lambda$
the condition for destructive interference as given by path difference = $\left(n + \frac{1}{2}\right)\lambda$
Young's double-slit interference as given by $s = \frac{\lambda D}{d}$ where s is the separation of fringes, d is the separation of the slits, and D is the distance from the slits to the screen
HL single-slit diffraction including intensity patterns as given by $\theta = \frac{\lambda}{b}$ where b is the slit width
HL the single-slit pattern modulates the double-slit interference pattern
HL interference patterns from multiple slits and diffraction gratings as given by $n\lambda = d\sin\theta$

These narrow beams of light at a concert are like rays; they are parallel to the direction of the energy transfer.

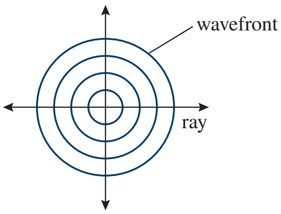
These 'ripples' in the sand are like wavefronts; they are perpendicular to the direction of energy transfer.

Wavefronts

In reality, not all waves are sinusoidal. The troughs between the peaks of water waves traveling towards a beach are much longer than the peaks

If a stone is thrown into a pond, then a pulse will be seen to spread out across the surface in two dimensions: energy has been transferred from the stone to the surface of the water. If the surface is disturbed continuously by an oscillating object (or the wind), a continuous wave will be formed whose profile resembles a sine wave. Viewed from directly above, the wave spreads out in circles. The circles that we see are actually the peaks and troughs of the wave; we call these lines **wavefronts**. A wavefront is any line joining points that are in phase. Wavefronts are perpendicular to the direction of energy transfer, which can be represented by an arrow called a **ray**.

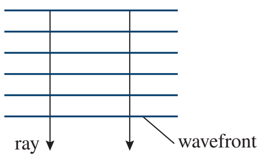
Ripples spreading out in circles after the surface is disturbed.



C.3 Figure 1 A circular wavefront spreading out from a point.

Point sources produce circular wavefronts, but if the source is far away, the waves will appear plane.

A plane wavefront moves toward the beach.

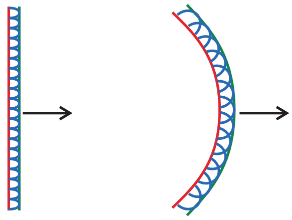


C.3 Figure 2 Parallel plane wavefronts.

Tidal bores, beloved of surfers, occur in rivers with large tidal ranges. These surfers take the path of rays respective to the wavefront crests.

Wave propagation (Huygens' construction)

We can think of a wavefront as being made up of an infinite number of new centers of disturbance. Each disturbance creates its own wavelet that progresses in the direction of the wave. The wavefront is made up of the sum of all these wavelets. Figure 3 shows how circular and plane wavefronts propagate according to this construction.



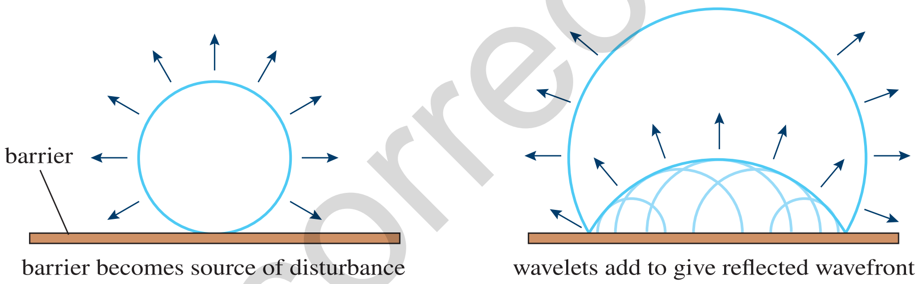
C.3 Figure 3 Huygens' construction used to find the new position of plane and circular wavefronts.

Nature of Science

The Huygens' construction treats a wavefront as if it is made of an infinite number of small point sources that only propagate forward. Huygens gave no explanation for the fact that propagation is only forward but the model correctly predicts the laws of reflection and Snell's law of refraction. Snell's law was the result of many experiments measuring the angles of light rays passing from one medium to another. The result gives the path with the shortest time, a result that is in agreement with Einstein's theory of relativity. There can be more than one theory to explain a phenomenon but they must give consistent predictions.

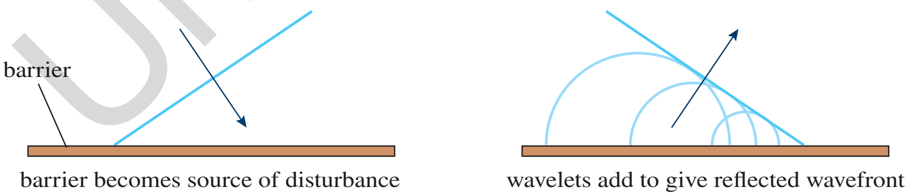
Reflection of water waves

When a wavefront hits a barrier, the barrier now behaves as a series of wavelet sources sending wavelets in the opposite direction. In this way, a circular wavefront is reflected as a circular wavefront that appears to originate from a point behind the barrier as in Figure 4.



C.3 Figure 4 Reflection of a circular wavefront.

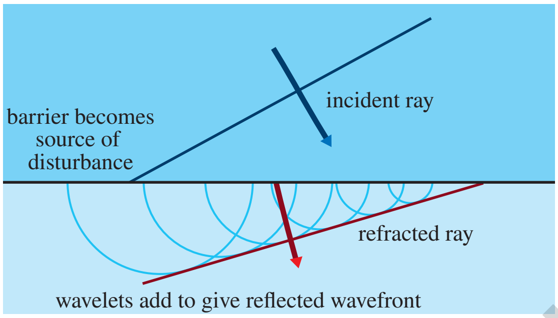
A plane wavefront reflects as a plane wavefront, making the same angle to the barrier as the incident wave, as shown in Figure 5.



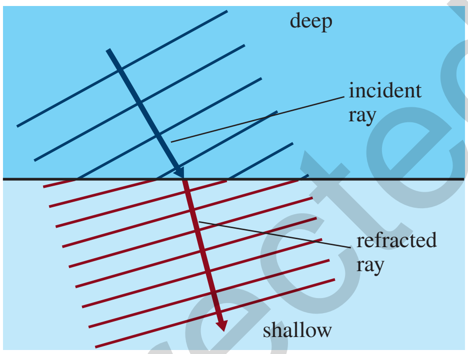
C.3 Figure 5 Reflection of a plane wavefront.

Refraction of water waves

Refraction is the change of direction of propagation when a wave passes from one medium to another. In the case of water waves, it is difficult to change the medium but we can change the depth. This changes the speed of the wave and causes the ray to change direction. This can again be explained using Huygens' construction as shown in Figure 6, where the wave is passing into shallower water, where it travels more slowly.



C.3 Figure 6 The wavefront and ray directions change.



The frequency of the wave does not change when the wave slows down so the wavelength must be shorter ($v = f\lambda$). Note that, although not drawn in Figure 7, when a wave meets a boundary such as this, it will be reflected as well as refracted.

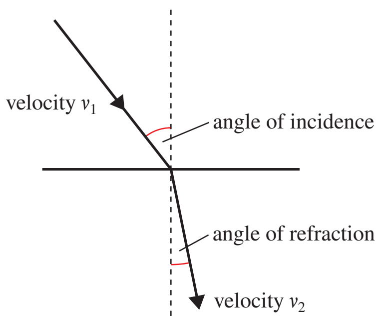
C.3 Figure 7 The optical density need not change for a change in direction to emerge.

Snell's law

Snell's law relates the angles of incidence and refraction to the ratio of the velocity of the wave in the different media. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is equal to the ratio of the velocities of the wave in the different media:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

Note that the angles are measured between the ray and the **normal**, or between the wavefront and the boundary.

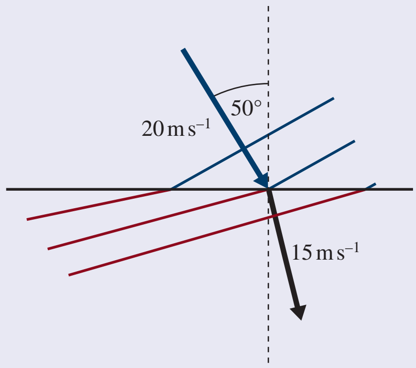


C.3 Figure 8 Angles of incidence and refraction.

We refer to the proportion of the energy of the wave that refracts at a boundary as being 'transmitted' to the new medium. The remainder is reflected.

Worked example

A water wave traveling at 20 m s^{-1} in deep water enters a shallow region where its velocity is 15 m s^{-1} (Figure 9). If the angle of incidence between the water wave and the normal of the boundary between regions is 50°, what is the angle of refraction?



Solution

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{20}{15}$$

Applying Snell's law:

$$\sin r = \frac{\sin 50°}{1.33} = 0.576$$

$$r = 35°$$

Exercise

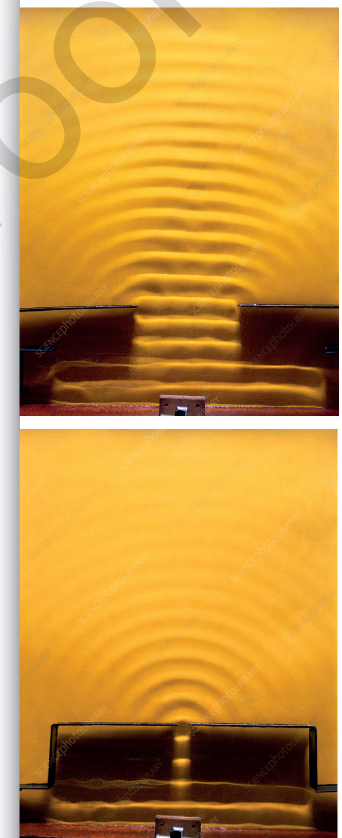
Q1. A water wave with wavelength 30 cm traveling with velocity 0.50 m s^{-1} meets the straight boundary to a shallower region at an angle of incidence 30°. If the velocity in the shallow region is 0.40 m s^{-1}, calculate:

(**a**) the frequency of the wave

(**b**) the wavelength of the wave in the shallow region

(**c**) the angle of refraction.

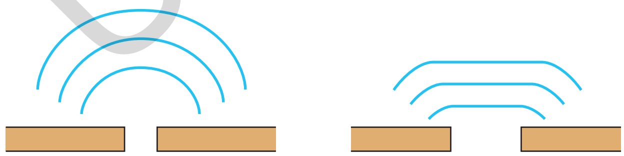
Q2. A water wave traveling in a shallow region at a velocity of 0.30 m s^{-1} meets the straight boundary to a deep region at angle of incidence 20°. If the velocity in the deep region is 0.50 m s^{-1}, calculate the angle of refraction.



Water waves diffracting through two different sized openings. The waves are diffracted more through the narrower opening.

Diffraction of water waves

Diffraction takes place when a wave passes through a small opening. If the opening is very small, then the wave behaves just like a point source as shown in Figure 9.



C.3 Figure 9 If the opening is a bit bigger then the effect is not so great.

Using Huygens' construction, we can explain why this happens. In the case of the very narrow slit, the wavefront is reduced to one wavelet that propagates as a circle.

Waves are also diffracted by objects and edges as shown in Figure 10. Notice how the wave seems to pass round the very small object.

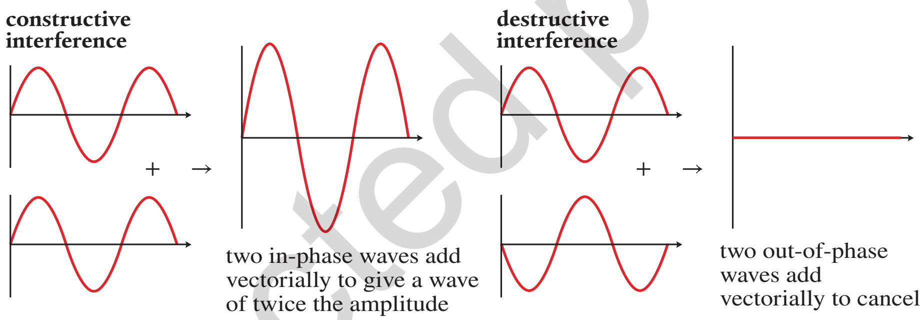
C.3 Figure 10 Diffraction around obstacles.

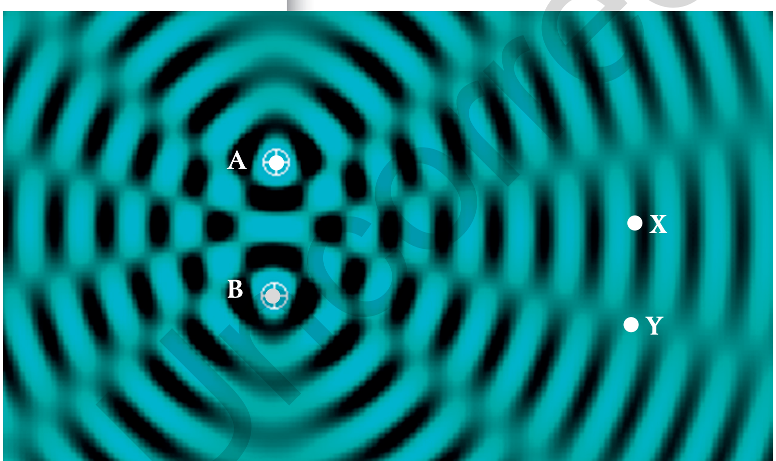
What evidence is there that particles possess wave-like properties such as wavelength? (NOS)

Interference of water waves

If two disturbances are made in a pool of water, two different waves will be formed. When these waves meet, the individual displacements will add vectorially. This is called **superposition**. If the frequency of the individual waves is equal, then the resulting amplitude will be constant and related to the **phase difference** between the two waves.

C.3 Figure 11





C.3 Figure 12 Ripple tank, a screenshot from www.paulfalstad.com.

When two identical point sources produce waves on the surface of a pool of water, a pattern like the one in Figure 12 is produced.

We can see that there are regions where the waves are interfering constructively (X) and regions where they are interfering destructively (Y). If we look carefully at the waves arriving at X and Y from A and B, we see that at X they are in phase and at Y they are out of phase (Figure 13). This is because the waves have traveled the same distance to get to X, but the wave from A has traveled $\frac{1}{2}\lambda$ extra to get to Y.

In general, constructive interference occurs if:

path difference = $n\lambda$

or:

phase difference = $2n\pi$

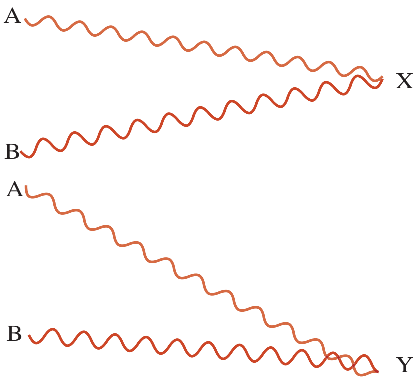
where n is a whole number.

Destructive interference occurs if:

path difference = $\left(n + \frac{1}{2}\right)\lambda$

or:

phase difference = $(2n + 1)\pi$



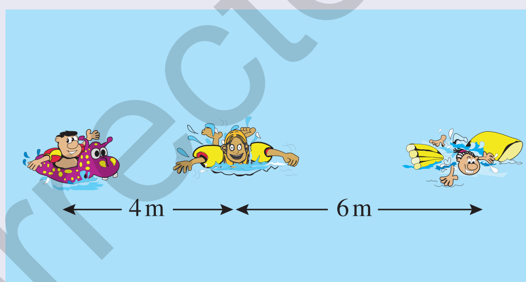
C.3 Figure 13 Path difference leads to phase difference.

Path difference and phase difference

We can see from the previous example that a path difference of $\frac{1}{2}\lambda$ introduces a phase difference of π, so if the path difference is d, then the phase difference, $\theta = \frac{2\pi d}{\lambda}$.

Worked example

Two boys playing in a pool make identical waves that travel toward each other. The boys are 10 m apart and the waves have a wavelength 2 m. Their little sister is swimming from one boy to the other. When she is 4 m from the first boy, will she be in a big wave or a small wave?



A diagram always helps, no matter how simple it is.

Solution

The waves from the boys will interfere when they meet. If the girl is 4 m from the first boy, then she must be 6 m from the other. This is a path difference of 2 m, one whole wavelength. The waves are therefore in phase and will interfere constructively.

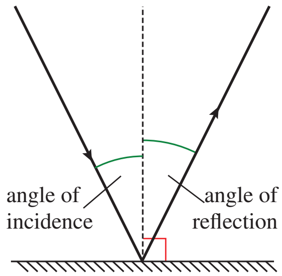
Exercise

Q3. Two wave sources, A and B, produce waves of wavelength 2.0 cm. What is the phase angle between the waves at:

(a) a point C, distance 6.0 cm from A and 6.2 cm from B?

(b) a point D, distance 8.0 cm from A and 7.0 cm from B?

(c) a point E, distance 10.0 cm from A and 11.5 cm from B?



C.3 **Figure 14**

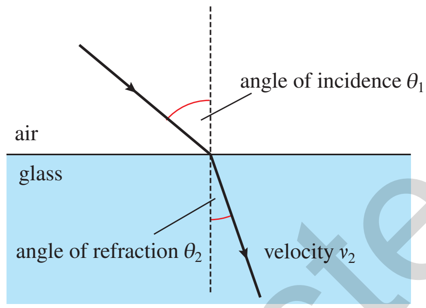
Reflection of light

When light hits an object, part of it is absorbed and part of it is reflected. It is the reflected light that enables us to see things. If the reflecting surface is uneven, the light is reflected in all directions, but if it is flat, the light is reflected uniformly so we can see that the angle of reflection equals the angle of incidence (Figure 14).

Refraction of light

The velocity of light is different for different transparent media, so when light passes from one medium to another, it changes direction. For example, the velocity of light is greater in air than it is in glass, so when light passes from air to glass, it refracts as in Figure 15.

The refractive index of a medium, n, is the ratio of the speed of light in a vacuum to the speed of light in the medium. This means that: $n_1 = \frac{c}{c_1}$ and $n_2 = \frac{c}{c_2}$



C.3 Figure 15 Light refracts from air to glass.

Rearranging: $c_1 = \frac{c}{n_1}$ and $c_2 = \frac{c}{n_2}$

Combining: $\frac{c_1}{c_2} = (\frac{c}{n_1})/(\frac{c}{n_2}) = \frac{n_2}{n_1}$

Applying Snell's law, we get: $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$

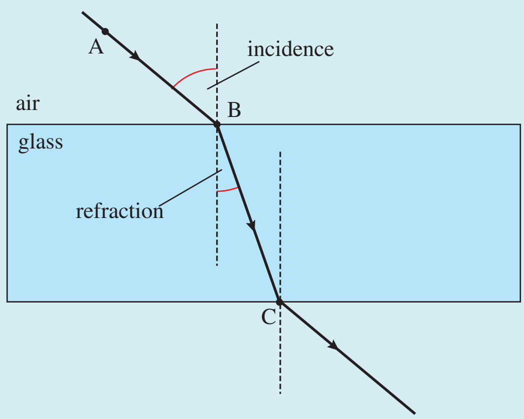
This can also be written $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$ where n_1 and n_2 are the **refractive indices** of the two media. The bigger the difference in refractive index, the more the light ray will be deviated. We say a medium with a high refractive index is 'optically dense'.

Light reflected off the straw is refracted as it comes out of the water, causing the straw

SKILLS

To measure the refractive index of a glass block, you can pass a narrow beam of light (e.g. from a laser) through it and measure the angles of incidence and refraction as the light passes from air into the glass. It is not possible to trace the ray as it passes through the block, but if you place the block on a sheet of paper and mark where the ray enters and leaves the block (at B and C in the figure on the right), then you can plot the path of the ray. If you do not have a light source, you can use an alternative method with pins. Place two pins on one side of the block in positions A and B then, looking through the block, place a third pin in line with the other two. Joining the dots will give the path of a ray from A through the block.

The angles can then be measured using a protractor and the refractive index calculated.



SKILLS

A worksheet with full details of how to carry out this experiment is available in your eBook.

Worked example

A ray of light traveling in air is incident on a glass block at an angle of 56°. Calculate the angle of refraction if the refractive index of glass is 1.5.

Solution

Applying Snell's law: $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$

Where: $\theta_1 = 56°$

$n_1 = 1$ (air)

$n_2 = 1.5$ (glass)

$$\frac{\sin 56°}{\sin\theta_2} = \frac{1.5}{1}$$

$$\sin\theta_2 = \sin\frac{\sin 56°}{1.5} = 0.55$$

$$\theta_2 = 34°$$

Exercise

Use the refractive indices in the table on the right to solve the following problems.

Q4. Light traveling through the air is incident on the surface of a pool of water at an angle of 40°. Calculate the angle of refraction.

Q5. Calculate the angle of refraction if a beam of light is incident on the surface of a diamond at an angle of 40°.

Q6. If the velocity of light in air is $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$, calculate its velocity in glass.

Q7. A fish tank made of glass contains water (and fish). Light travels from a fish at an angle of 30° to the side of the tank. Calculate the angle between the light and the normal to the glass surface as it emerges into the air.

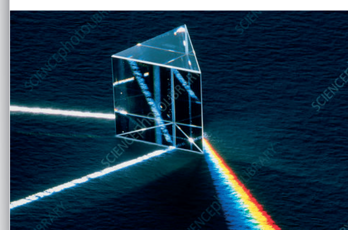
Q8. Light incident on a block of transparent plastic at an angle of 30° is refracted at an angle of 20°. Calculate the angle of refraction if the block is immersed in water and the ray is incident at the same angle.

Material	Refractive index
Air	1.0003
Water	1.33
Glass	1.50
Diamond	2.42

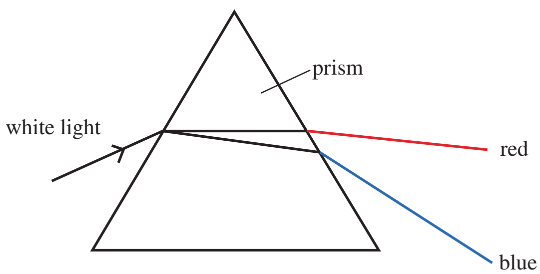
Some refractive indexes of different media.

Dispersion

The angle of refraction is dependent on the wavelength of the light. If red light and blue light pass into a block of glass, the blue light will be refracted more than the red, causing the colors to **disperse**. This is why you see a spectrum when light passes through a prism as in the photo. It is also the reason why rainbows are formed when light is refracted by raindrops.



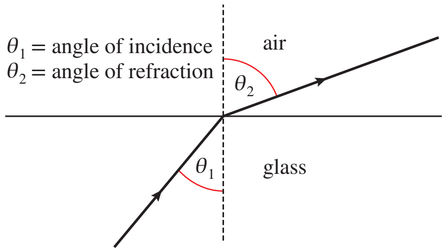
The white light is dispersed into the colors of the spectrum because the different colors of light travel at different speeds in glass.



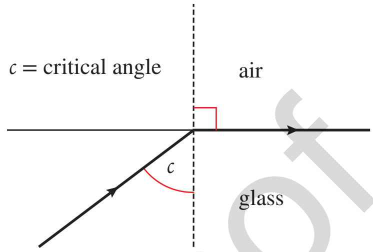
C.3 Figure 16 When white light is passed through a prism, blue light is refracted more than red.

The critical angle

If light passes into an optically less dense medium, e.g. from glass to air, then the ray will be refracted away from the normal as shown in Figure 17.



C.3 **Figure 17** Light refracted from glass to air.



C.3 **Figure 18** Light refracted at 90°.

If the angle of incidence increases, a point will be reached where the refracted ray is refracted along the boundary. The angle at which this happens is called the **critical angle**.

Applying Snell's law to this situation:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

Where: $\theta_1 = c$

$\theta_2 = 90°$

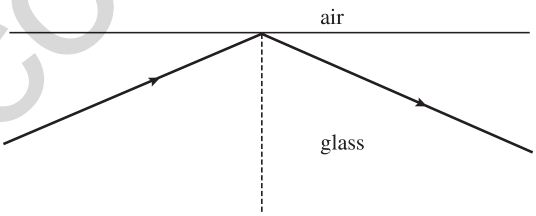
$n_1 = 1.5$ for glass

$n_2 = 1$

$c = 42°$

Total internal reflection

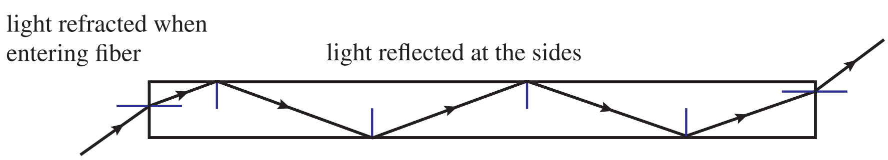
If the critical angle is exceeded, all of the light is reflected. This is known as **total internal reflection**. Since all the light is reflected, none is transmitted. This is not the case when light is reflected off a mirror when some is absorbed.



C.3 **Figure 19** Light totally internally reflected.

Optical fibers

An optical fiber is a thin strand of glass or clear plastic. If a ray of light enters its end at a small angle, the ray will be totally internally reflected when it meets the side. Since the sides are parallel, the ray will be reflected back and forth until it reaches the other end as in Figure 20. Optical fibers are used extensively in communication.



C.3 **Figure 20** Light reflected along a fiber.

Exercise

Q9. Light enters a glass block of refractive index 1.5 at an angle 70° as shown in Figure 21.

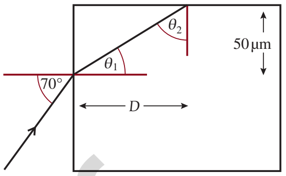
(**a**) Use Snell's law to calculate the angle of refraction θ_1.

(**b**) Use geometry to find the angle θ_2.

(**c**) Calculate the critical angle for glass.

(**d**) Will the ray be totally internally reflected?

(**e**) Calculate length D.

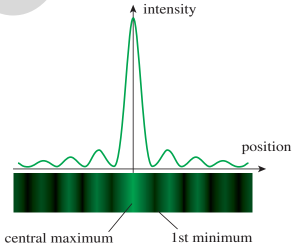


C.3 Figure 21

HL

Diffraction of light at a single slit

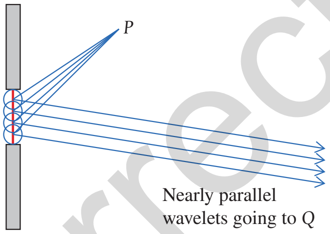
When light passes through a narrow slit, it diffracts, forming a series of bright and dark bands, as shown in Figure 22.



C.3 Figure 22 Single-slit diffraction.

We can derive an equation for the first minima in this pattern by applying Huygens' construction.

When a wavefront passes through a narrow slit, it will propagate as if there were a large number of wavelet sources across the slit, as in Figure 23.

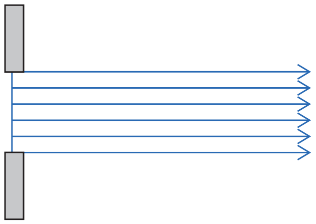


C.3 Figure 23 Wavelets add vectorially to give resultant intensity.

The resultant intensity at some point *P* in front of the slit is found by summing all the wavelets. This is not a simple matter since each wavelet has traveled a different distance so they will be out of phase when they arrive at *P*. To simplify the problem, we will consider a point Q a long way from the slits. Light traveling through the slit arriving at point Q is almost parallel, and if we say that it is parallel, then the geometry of the problem becomes much simpler.

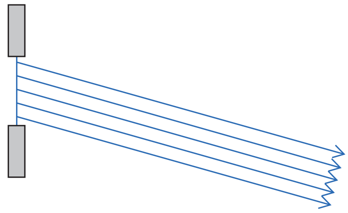
The central maximum

The central maximum occurs directly ahead of the slit. If we take a point a long way from the slit, all wavelets will be parallel and will have traveled the same distance, as shown in Figure 24. If all the wavelets have traveled the same distance, they will be in phase, so will interfere constructively to give a region of high intensity (bright).

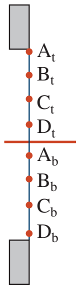


C.3 Figure 24 Wavelets traveling to the central maximum.

The first minimum



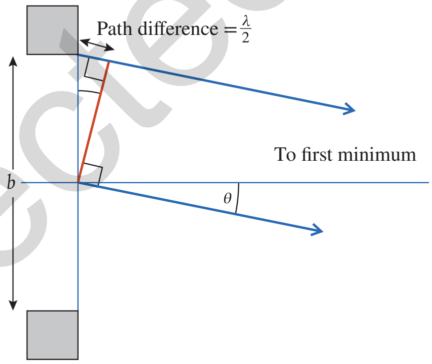
C.3 **Figure 25** Wavelets traveling to the first minimum.



C.3 **Figure 26** Wavelets in the top half cancel with wavelets in the bottom.

If we now consider wavelets traveling toward the first minimum, as in Figure 25, they are traveling at an angle so will not all travel the same distance. The wavelet at the top will travel further than the wavelet at the bottom. When these wavelets add together, they interfere destructively to form a region of low intensity (dark).

We can calculate the angle at which the first minimum is formed by splitting the slit into two halves, top and bottom, as in Figure 26. If all the wavelets from the top half cancel out all the wavelets from the bottom, the result will be a dark region. So if we have eight wavelet sources, four in the top half (A_t, B_t, C_t, D_t) and four in the bottom (A_b, B_b, C_b, D_b), and if A_t cancels with A_b and B_t cancels with B_b, etc., then all the wavelets will cancel with each other. For each pair to cancel, the path difference must be $\frac{1}{2}\lambda$. Figure 27 shows the situation for the top wavelet and the one in the middle.



C.3 **Figure 27** Geometric construction for the first minimum.

The orange line cuts across the two wavelets at 90° showing that the top one travels further than the bottom one. If the path difference shown is $\frac{\lambda}{2}$, then these wavelets will cancel and so will all the others. If the first minimum occurs at an angle θ as shown, then this will also be the angle of the triangle made by the orange line. We can therefore write:

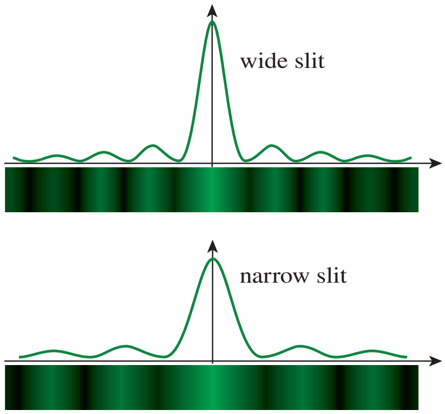
$$\sin\theta = \frac{\frac{\lambda}{2}}{\frac{b}{2}} = \frac{\lambda}{b}$$

But the angles are very small, so if θ is measured in radians: $\sin\theta = \theta$

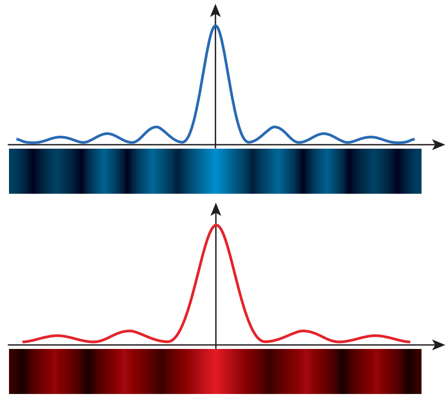
So: $$\theta = \frac{\lambda}{b}$$

Knowing the position of the first minimum tells us how spread out the diffraction pattern is. From the equation, we can see that if b is small, then θ is big, so the pattern is spread out as shown in Figure 28.

If white light is passed through the slit, light of different wavelengths forms peaks at different angles, resulting in colored fringes.



C.3 Figure 28 Notice that with a narrower slit the pattern is wider but less intense.



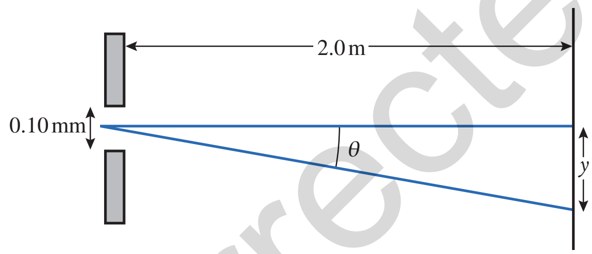
C.3 Figure 29 Same size slit but different wavelength light; longer wavelength gives a wider pattern.

> What are the similarities and differences between single-slit diffraction and diffraction to study atomic structures? (E.1)

Example

A diffraction pattern is formed on a wall 2.0 m from a 0.10 mm slit illuminated with light of wavelength 600 nm. How wide will the central maximum be?

First draw a diagram showing the relative positions of the slit and screen.



C.3 Figure 30 The angle between the center and the first minimum.

The angle θ can be calculated from:

$$\theta = \frac{\lambda}{b} = \frac{600 \times 10^{-9}}{0.10 \times 10^{-3}} = 0.0060\ \text{rad}$$

Since this angle is small, we can say that:

$$\theta = \frac{y}{2.0} \quad \text{so} \quad y = 0.006 \times 2.0 = 0.012\ \text{m} = 1.2\ \text{cm}$$

This is half the width of the maximum, so: width = 2.4 cm

> Note that the intensity of the diffraction maxima decreases as you move away from the central maximum. In reality, the first maximum would be smaller than shown in these diagrams (about $\frac{1}{20}$ the height of the central maximum).

Exercise

Q10. Light of wavelength 550 nm is passed through a slit of size 0.050 mm. Calculate the width of the central maximum formed on a screen that is 5.0 m away.

Q11. Calculate the size of the slit that would cause light of wavelength 550 nm to diffract, forming a diffraction pattern with a central maximum 5.0 cm wide on a screen 4.0 m from the slit.

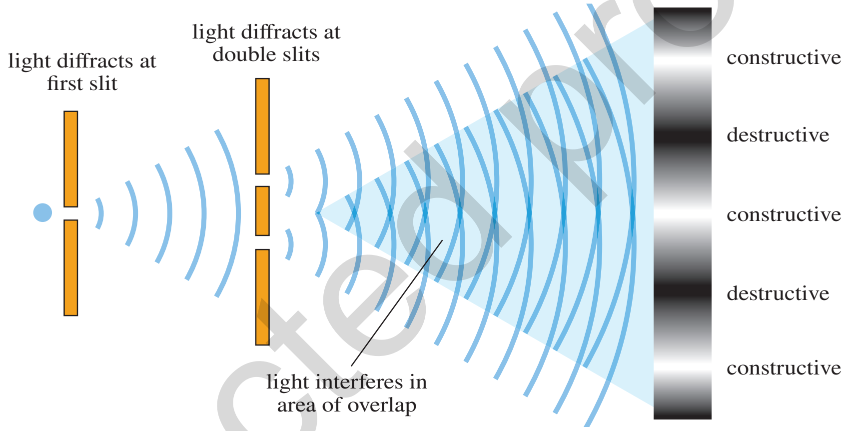
HL end

Interference of light

For the light from two sources to interfere, the light sources must be coherent. This means they have the same frequency, similar amplitude, and a constant phase difference. This can be achieved by taking one source and splitting it in two, using slits or thin films.

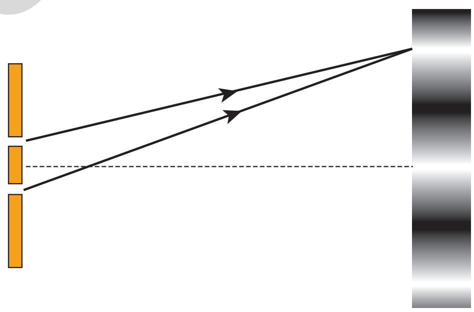
Two-slit interference

Light from a single source is split in two by parallel narrow slits. Since the slits are narrow, the light diffracts, creating an overlapping region where interference takes place. This results in a series of bright and dark parallel lines called fringes, as shown in Figure 31. This set-up is called Young's slits.

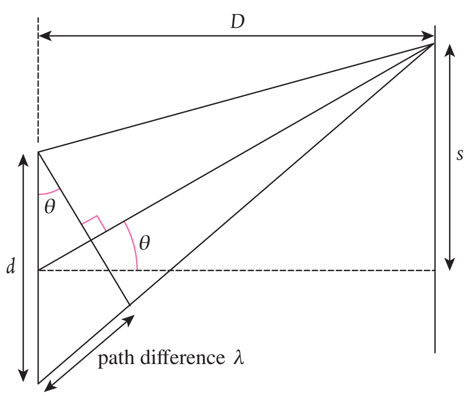


C.3 Figure 31 Double-slit interference.

To simplify matters, let us consider rays from each slit arriving at the first maximum as shown in Figure 32.



C.3 Figure 32 Paths taken by the rays from each slit to the first maximum



C.3 Figure 33 Light rays interfere at the first maximum.

Since this is the first interference maximum, the path difference between the waves must be λ. If we fill in some angles and lengths as in Figure 33 we can use trigonometry to derive an equation for the separation of the fringes.

Since the wavelength of light is very small, the angle θ will also be very small. This means that we can approximate θ in radians to $\frac{\lambda}{d} = \frac{s}{D}$ so the distance from the central bright fringe to the next one, $s = \frac{\lambda D}{d}$.

What can an understanding of the results of Young's double-slit experiment reveal about the nature of light? (C.2)

This equation is of great importance. This is the fringe spacing. We can therefore conclude that if the slits are made closer together, the fringes become more separated.

Exercise

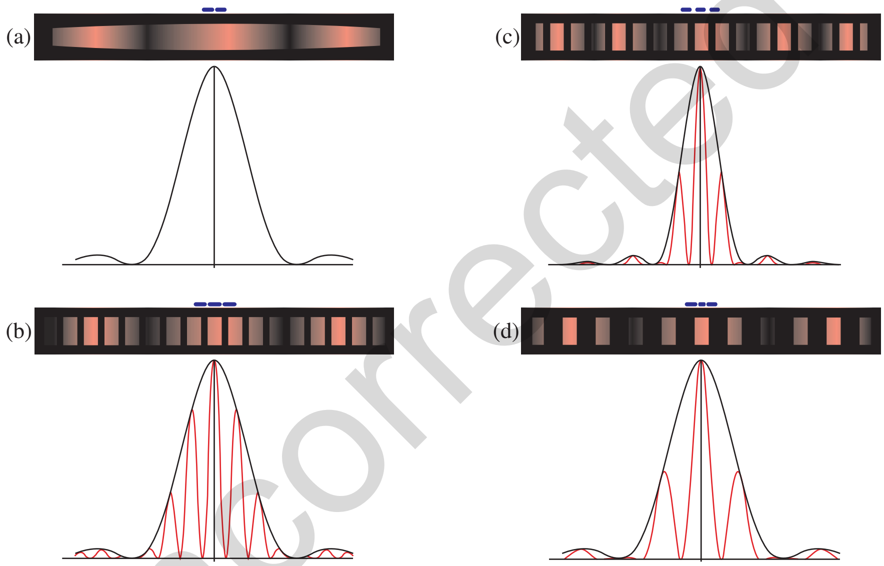
Q12. Two narrow slits, 0.01 mm apart (d), are illuminated by a laser of wavelength 600 nm. Calculate the fringe spacing (y) on a screen 1.5 m (D) from the slits.

Q13. Calculate the fringe spacing if the laser is replaced by one of wavelength 400 nm.

HL

Effect of diffraction

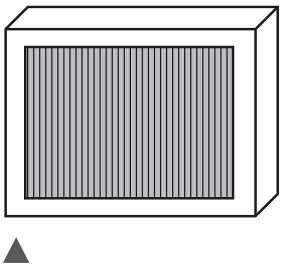
As we have seen, light is diffracted when it passes through each slit. This means that each slit will form a diffraction pattern on the screen, which causes the fringes to vary in brightness as shown in Figure 34. Here it can also be seen how the diffraction pattern has modulated the fringes.



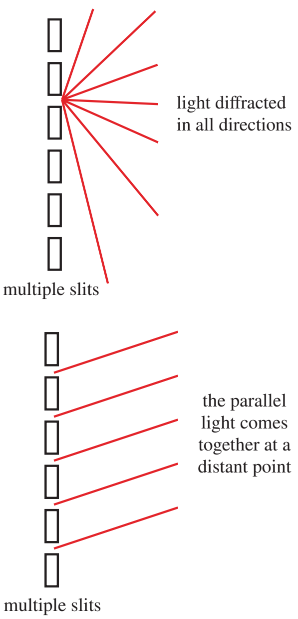
C.3 Figure 34 Fringes modulated by diffraction pattern.
(a) A single slit of width b.
(b) Double slits, each of width b and separation d.
(c) Double slits of width $>b$, giving a less spread diffraction pattern. The separation of the slits is d so the fringes are the same as in (b).
(d) Double slits of width b so the diffraction pattern is the same as (a). Separation of the slits $<d$ so the fringes are further apart.

Multiple-slit diffraction

The intensity of double-slit interference patterns is very low but can be increased by using more than two slits. A diffraction grating is a series of very fine parallel slits mounted on a glass plate.



C.3 Figure 35 Diffraction grating (the number of lines per millimeter can be very high: school versions usually have 600 lines per millimeter).



C.3 Figure 36 Light diffracted at each slit undergoes interference at a distant screen.

Diffraction at the slits

When light is incident on the grating, it is diffracted at each slit. The slits are very narrow so the diffraction causes the light to propagate as if coming from a point source.

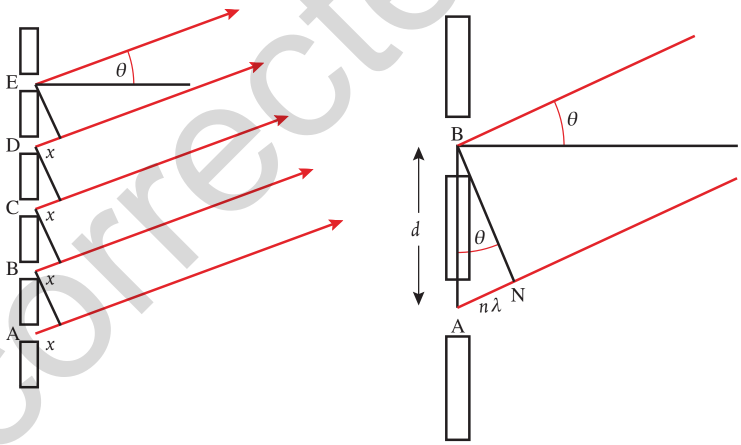
Interference between slits

To make the geometry simpler, we will consider what would happen if the light passing through the grating were observed from a long distance. This means that we can consider the light rays to be almost parallel. So the parallel light rays diffracted through each slit will come together at a distant point. When they come together, they will interfere.

Geometrical model

Let us consider waves that have been diffracted at an angle θ as shown in Figure 37 (remember, light is diffracted at all angles – this is just one angle that we have chosen to consider).

We can see that when these rays meet, the ray from A will have traveled a distance x further than the ray from B. The ray from D has traveled the same distance further than C, and so on. If the path difference between neighbors is λ, then they will interfere constructively; if $\frac{1}{2}\lambda$, then the interference will be destructive.



C.3 Figure 37 Parallel light travels through the grating and some is diffracted at an angle θ. The expansion shows just slits A and B. If the path difference is $n\lambda$, then constructive interference takes place.

The line BN is drawn perpendicular to both rays so angle N is 90°.

Therefore from triangle ABN, we see that: $\sin\theta = \frac{n\lambda}{d}$

Rearranging gives: $d \sin\theta = n\lambda$

If you look at a light source through a diffraction grating and move your head around, bright lines will be seen every time $\sin\theta = \frac{n\lambda}{d}$.

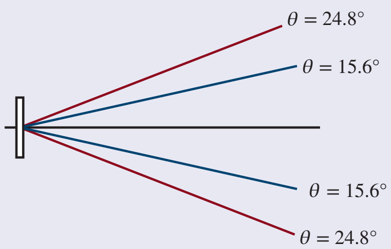
Producing spectra

If white light is viewed through a diffraction grating, each wavelength undergoes constructive interference at different angles. This results in a spectrum. The individual wavelengths can be calculated from the angle using the formula $d \sin\theta = n\lambda$.

Worked example

If blue light of wavelength 450 nm and red light of wavelength 700 nm are viewed through a grating with 600 lines mm^{-1}, at what angle will the first bright blue and red lines be seen?

Solution



If there are 600 lines/mm: $d = \frac{1}{600}$ mm = 0.001 67 mm

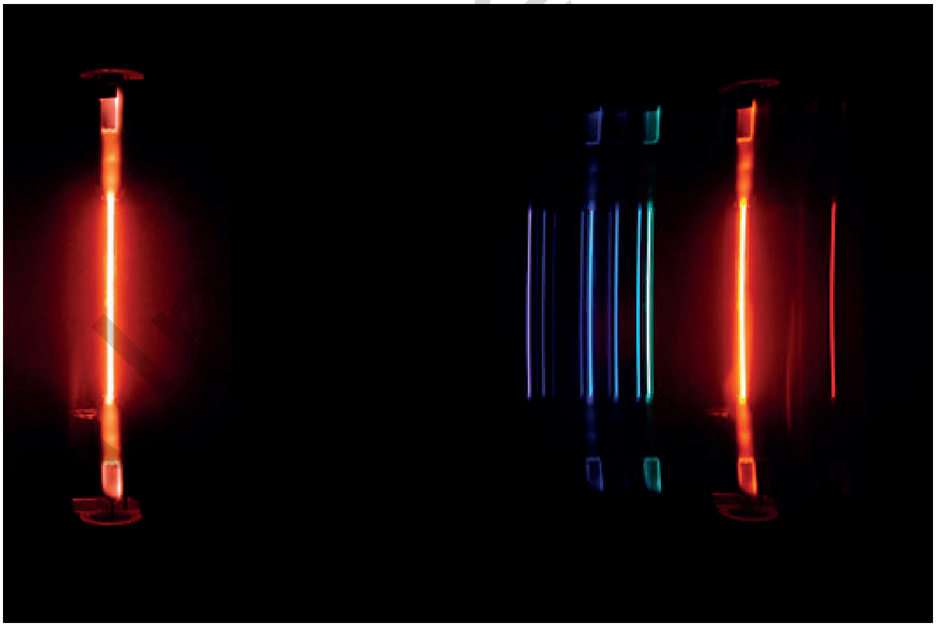
For the first line: $n = 1$

For blue light: $\sin\theta = \frac{450 \times 10^{-9}}{0.001\,67 \times 10^{-3}} = 0.269$

Therefore: $\theta_{blue} = 15.6°$

For red light: $\sin\theta = \frac{700 \times 10^{-9}}{0.001\,67 \times 10^{-3}} = 0.419$

Therefore: $\theta_{red} = 24.8°$



C.3 Figure 38 A hydrogen lamp viewed through a grating.

HL end

A compact disk (CD) can be used as a diffraction grating. Light reflecting off small silver lines on the CD diffract in all directions. Light from each line interferes constructively when the path difference = $n\lambda$. This takes place at different angles for different wavelengths, giving rise to the colors you see when viewing a CD in white light.

Guiding Questions revisited

How are observations of wave behaviors at a boundary between different media explained?

How is the behavior of waves passing through apertures represented?

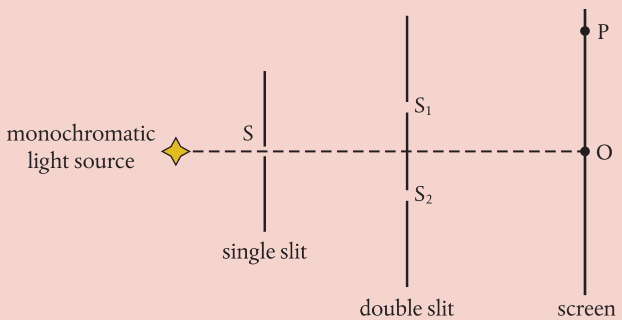
What happens when two waves meet at a point in space?

In this chapter, we have considered both wavefront and ray representations of the wave model to understand:

- That a boundary between two media causes a wave to separate into reflected and transmitted components.
- Refraction as the change in speed and direction of a wave when transmitted into a new medium.
- Snell's law as describing how the ratio of refractive indices of the two media is the inverse of the ratio of the speeds of the wave in the two media.
- Interference as a property of waves.
- Constructive interference as being associated with path differences of whole wavelengths and destructive interference as resulting from odd numbers of half-wavelengths.
- Superposition as the vector sum of the amplitudes of waves that interfere at a point.
- The characteristic interference pattern of a double slit, with the separation of fringes increasing with wavelength and distance from the slits to the screen and decreasing with the slits' separation.
- Diffraction as the spreading out of a wave's energy when passing around a body or through an aperture, with the effects most noticeable when the barrier or gap is approximately equal to the wavelength.
- HL The characteristic diffraction pattern of a single slit as a modulator to any double-slit interference pattern.
- HL The characteristic interference pattern of multiple slits and diffraction gratings.

Practice questions

1. The diagram shows an arrangement (not to scale) for observing the interference pattern produced by the superposition of two light waves. S_1 and S_2 are two very narrow slits. The single slit S ensures that the light leaving the slits S_1 and S_2 is coherent.



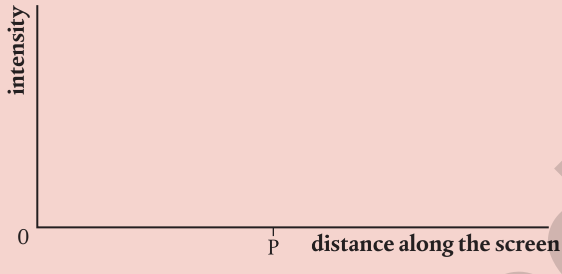
(a) (i) Define *coherent*. (1)

(ii) Explain why slits S_1 and S_2 need to be very narrow. (2)

The point O on the diagram is equidistant from S_1 and S_2 and there is maximum constructive interference at point P on the screen. There are no other points of maximum interference between O and P.

(b) (i) State the condition necessary for there to be maximum constructive interference at the point P. (1)

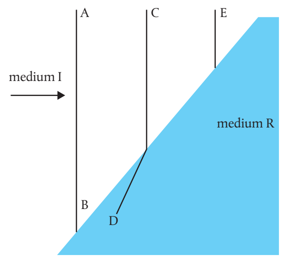
(ii) Copy the axes below and draw a graph to show the variation of intensity of light on the screen between the points O and P. (2)



(c) In this particular arrangement, the distance between the double slit and the screen is 1.50 m and the separation of S_1 and S_2 is 3.00×10^{-3} m. The distance OP is 0.25 mm. Determine the wavelength of the light. (2)

2. (a) By making reference to waves, distinguish between a *ray* and a *wavefront*. (3)

The diagram shows three wavefronts incident on a boundary between medium I and medium R. Wavefront CD is shown crossing the boundary. Wavefront EF is incomplete.

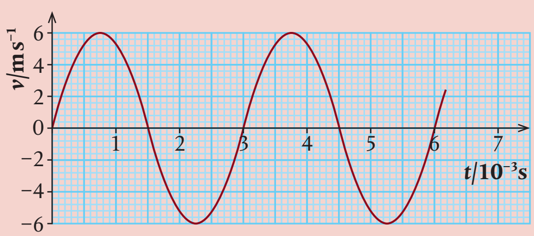


(b) (i) Copy the diagram and draw a line to complete the wavefront EF. (1)

(ii) Explain in which medium, I or R, the wave has the higher speed. (3)

(iii) By taking appropriate measurements from the diagram, determine the ratio of the speeds of the wave traveling from medium I to medium R. (2)

The diagram below shows the variation with time *t* of the velocity *v* of one particle of the medium through which the wave is traveling.



(c) (i) Explain how it can be deduced from the diagram that the particle is oscillating. (2)

(ii) Determine the frequency of oscillation of the particle. (2)

(iii) Copy the diagram and mark on the graph, with the letter M, one time at which the particle is at maximum displacement. (1)

(iv) Estimate the area between the curve and the *x*-axis from the time $t = 0$ to the time $t = 1.5$ ms. (2)

(v) Suggest what the area in (c)(iv) represents. (1)

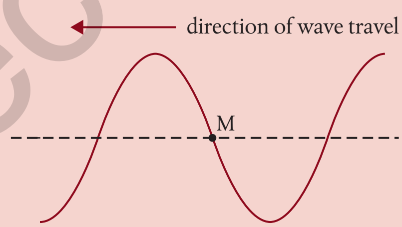
3. The diagram represents the direction of oscillation of a disturbance that gives rise to a wave.

(a) Copy the diagram and add arrows to show the direction of wave energy transfer to illustrate the difference between:

(i) a transverse wave (1)

(ii) a longitudinal wave. (1)

A wave travels along a stretched string. The diagram below shows the variation with distance along the string of the displacement of the string at a particular instant in time. A small marker is attached to the string at the point labeled M. The undisturbed position of the string is shown as a dotted line.



(b) Copy the diagram.

(i) Draw an arrow on the diagram to indicate the direction in which the marker is moving. (1)

(ii) Indicate, with the letter A, the amplitude of the wave. (1)

(iii) Indicate, with the letter λ, the wavelength of the wave. (1)

(iv) Draw the displacement of the string a time $\frac{T}{4}$ later, where T is the period of oscillation of the wave. Indicate, with the letter N, the new position of the marker. (2)

The wavelength of the wave is 5.0 cm and its speed is 10 cm s^{-1}.

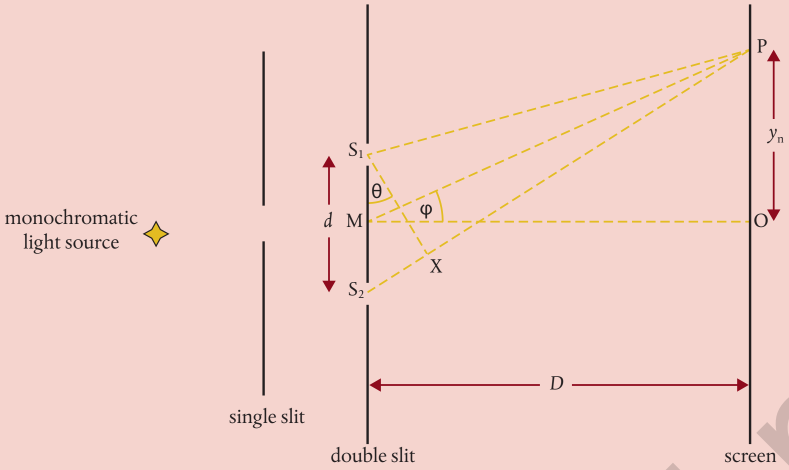
(c) Determine:

(i) the frequency of the wave (1)

(ii) how far the wave has moved in $\frac{T}{4}$ s. (2)

(d) By reference to the principle of superposition, explain what is meant by *constructive interference*. (4)

The diagram below (not drawn to scale) shows an arrangement for observing the interference pattern produced by the light from two narrow slits S_1 and S_2.



The distance S_1S_2 is d, the distance between the double slit and screen is D and $D >> d$ such that the angles θ and φ shown on the diagram are small. M is the midpoint of S_1S_2 and it is observed that there is a bright fringe at point P on the screen, a distance y_n from point O on the screen. Light from S_2 travels a distance S_2X further to point P than light from S_1.

(e) **(i)** State the condition in terms of the distance S_2X and the wavelength of the light λ, for there to be a bright fringe at P. (2)

(ii) Deduce an expression for θ in terms of S_2X and d. (2)

(iii) Deduce an expression for φ in terms of D and y_n. (1)

For a particular arrangement, the separation of the slits is 1.40 mm and the distance from the slits to the screen is 1.50 m. The distance y_n is the distance of the eighth bright fringe from O and the angle $\theta = 2.70 \times 10^{-3}$ rad.

(f) Using your answers to (e) to determine:

(i) the wavelength of the light (2)

(ii) the separation of the fringes on the screen. (3)

4. Three quantities used to describe a light wave are:

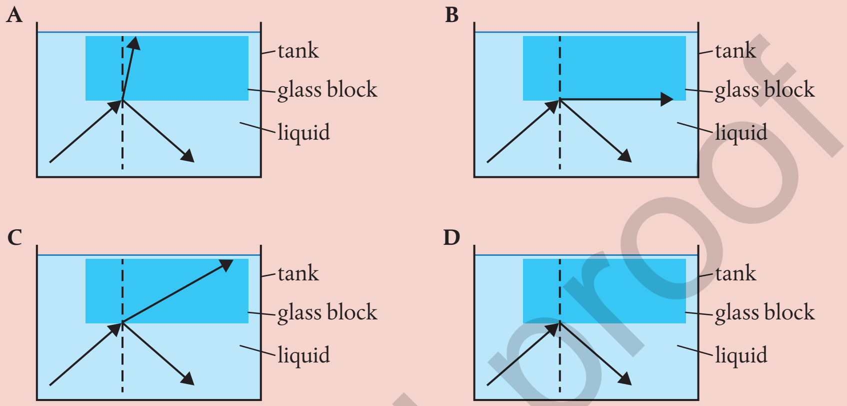
I. frequency **II.** wavelength **III.** speed

Which quantities increase when the light wave passes from water to air? (1)

A I and II only **C** II and III only

B I and III only **D** I, II and III

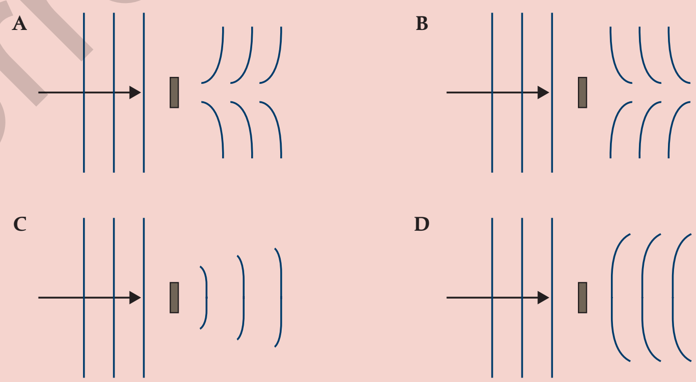
5. A glass block of refractive index 1.5 is immersed in a tank filled with a liquid of higher refractive index. Light is incident on the base of the glass block. Which is the correct diagram for rays incident on the glass block at an angle greater than the critical angle? (1)



6. A glass block has a refractive index in air of n_g. The glass block is placed in two different liquids: liquid X with a refractive index of n_X and liquid Y with a refractive index of n_Y. In liquid X, $\frac{n_g}{n_X} = 2$, and in liquid Y, $\frac{n_g}{n_Y} = 1.5$. What is $\frac{\text{speed of light in liquid X}}{\text{speed of light in liquid Y}}$? (1)

A $\frac{2}{4}$ **B** $\frac{3}{4}$ **C** $\frac{4}{3}$ **D** 3

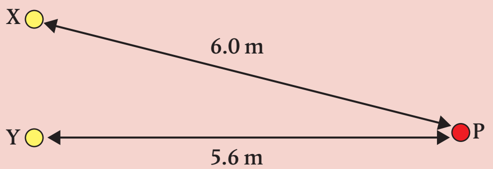
7. Which diagram shows the shape of the wavefront as a result of the diffraction of plane waves by an object? (1)



8. Two identical waves, each with amplitude X_0 and intensity I, interfere constructively. What are the amplitude and intensity of the resultant wave? (1)

	Amplitude of the resultant wave	Intensity of the resultant wave
A	X_0	$2I$
B	$2X_0$	$2I$
C	X_0	$4I$
D	$2X_0$	$4I$

9. X and Y are two coherent sources of waves. The phase difference between X and Y is zero. The intensity at P due to X and Y separately is I. The wavelength of each wave is 0.20 m. What is the resultant intensity at P? (1)



A 0 **B** I **C** $2I$ **D** $4I$

10. In a Young's double-slit experiment, the distance between fringes is too small to be observed. What change would increase the distance between fringes? (1)

A Increasing the frequency of light

B Increasing the distance between slits

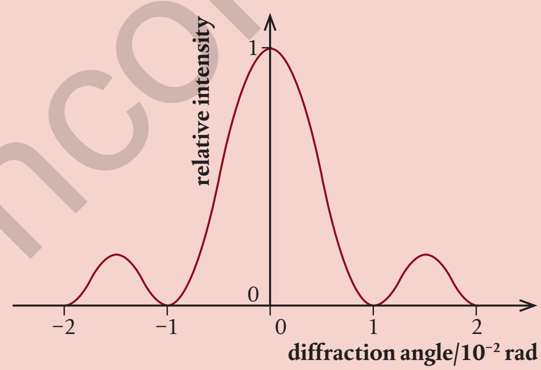
C Increasing the distance from the slits to the screen

D Increasing the distance between light source and slits

11. Monochromatic light of wavelength λ is incident on a double slit. The resulting interference pattern is observed on a screen a distance y from the slits. The distance between consecutive fringes in the pattern is 55 mm when the slit separation is a. λ, y and a are all doubled. What is the new distance between consecutive fringes? (1)

A 55 mm **B** 110 mm **C** 220 mm **D** 440 mm

12. HL The diagram shows the diffraction pattern for light passing through a single slit. What is $\frac{\text{wavelength of light}}{\text{width of slit}}$?



A 0.01 **B** 0.02 **C** 1 **D** 2

13. HL White light is incident normally on separate diffraction gratings, X and Y. Y has a greater number of lines per meter than X. Three statements about differences between X and Y are:

I. Adjacent slits in the gratings are further apart for X than for Y.

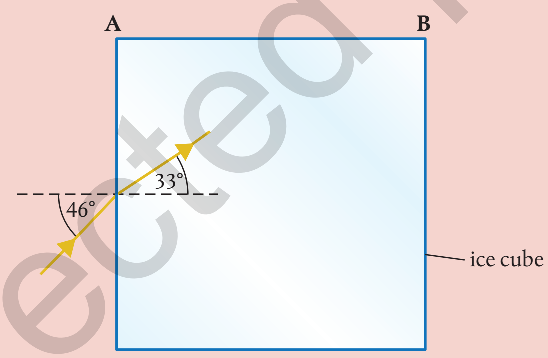
II. The angle between red and blue light in a spectral order is greater in X than in Y.

III. The total number of visible orders is greater for X than for Y.

Which statements are correct? (1)

A I and II only
B I and III only
C II and III only
D I, II and III

14. A large cube is formed from ice. A light ray is incident from a vacuum at an angle of 46° to the normal on one surface of the cube. The light ray is parallel to the plane of one of the sides of the cube. The angle of refraction inside the cube is 33°.

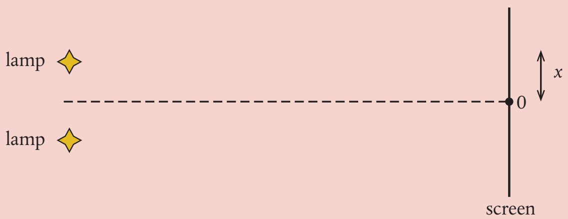


(a) Calculate the speed of light inside the ice cube. (2)

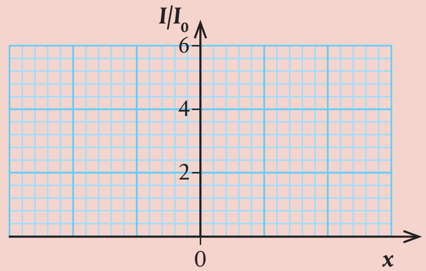
(b) Show that no light emerges from side AB. (3)

(c) Copy the diagram and sketch the subsequent path of the light ray. (2)

15. Monochromatic light from two identical lamps arrives on a screen. The intensity of light on the screen from each lamp separately is I_0.

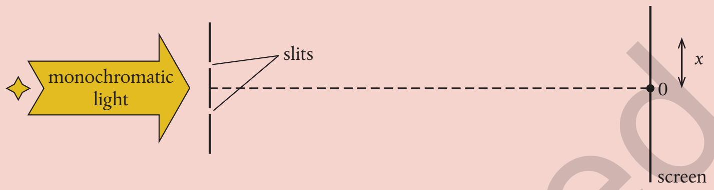


(a) Copy the axes below and sketch a graph to show the variation with distance x on the screen of the intensity I of light on the screen. (1)

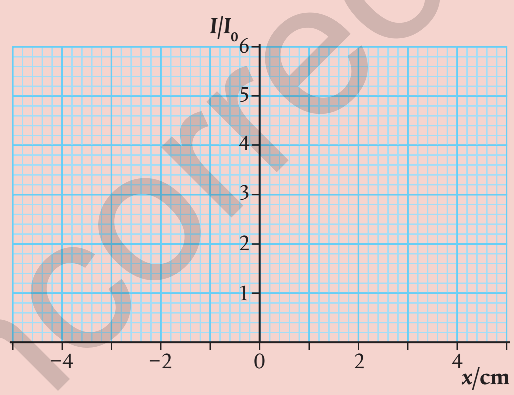


Monochromatic light from a single source is incident on two thin, parallel slits. The following data are available:

Slit separation = 0.12 mm   Wavelength = 680 nm   Distance to screen = 3.5 m

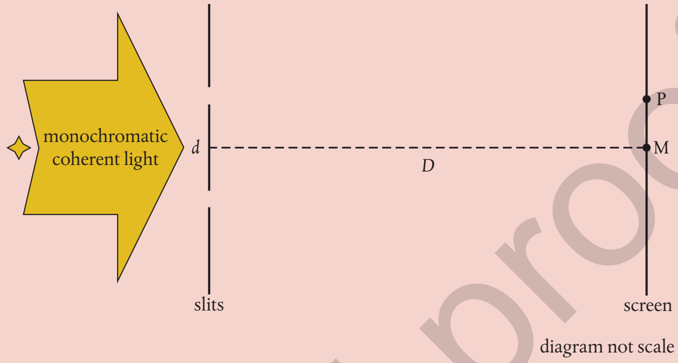


(b) The intensity I of light at the screen from each slit separately is I_0. Copy the axes below and sketch a graph to show the variation with distance x on the screen of the intensity of light on the screen for this arrangement. (3)



(c) The slit separation is increased. Outline **one** change observed on the screen. (1)

16. HL Monochromatic coherent light is incident on two parallel slits of negligible width, a distance d apart. A screen is placed a distance D from the slits. Point M is directly opposite the midpoint of the slits. Initially, the lower slit is covered and the intensity of light at M due to the upper slit alone is $22\ W\ m^{-2}$. The lower slit is now uncovered.



(**a**) Deduce, in $W\ m^{-2}$, the intensity at M. (3)

(**b**) P is the first maximum of intensity on one side of M. The following data are available:

$d = 0.12$ mm
$D = 1.5$ m
Distance MP = 7.0 mm

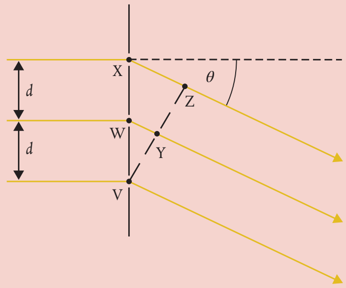
Calculate, in nm, the wavelength λ of the light. (2)

The width of each slit is increased to 0.030 mm. D, d and λ remain the same.

(**c**) Suggest why, after this change, the intensity at P will be less than that at M. (1)

(**d**) Show that, due to single slit diffraction, the intensity at a point on the screen a distance of 28 mm from M is zero. (2)

17. HL Monochromatic light of wavelength λ is normally incident on a diffraction grating. The diagram shows adjacent slits of the diffraction grating, labeled V, W and X. Light waves are diffracted through an angle θ to form a second-order diffraction maximum. Points Z and Y are labeled.

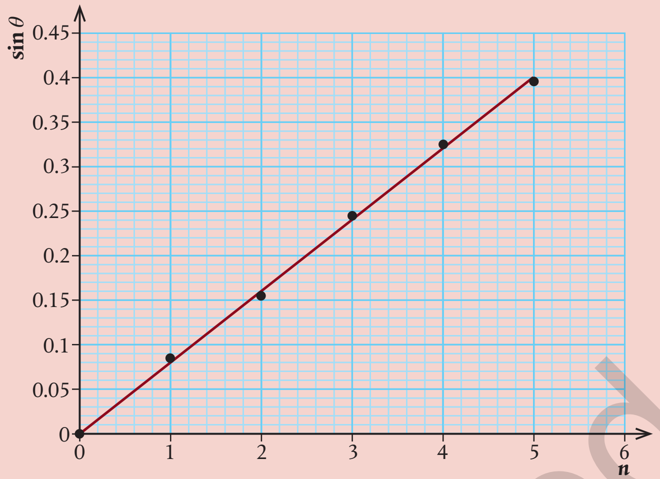


(**a**) State the phase difference between the waves at V and Y. (1)

(**b**) State, in terms of λ, the path length between points X and Z. (1)

(**c**) The separation of adjacent slits is d. Show that for the second-order diffraction maximum, $2\lambda = d\sin\theta$. (1)

Monochromatic light of wavelength 633 nm is normally incident on a diffraction grating. The diffraction maxima incident on a screen are detected and their angle θ to the central beam is determined. The graph shows the variation of $\sin\theta$ with the order n of the maximum. The central order corresponds to $n = 0$.



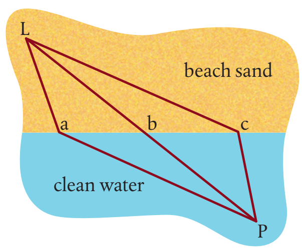
(**d**) Determine a mean value for the number of slits per millimeter of the grating. (4)

(**e**) State the effect on the graph of the variation of $\sin\theta$ with n of:

(**i**) using a light source with a smaller wavelength (1)

(**ii**) increasing the distance between the diffraction grating and the screen. (1)

18. A lifeguard at L on the beach must rescue a person in need in the water at P. Time is of the essence. Which path from L to P will take the least time and why? (2)



19. A coin is underwater. What does it appear to be? (1)

A Nearer the surface than it really is

B Farther from the surface than it really is

C As deep as it really is

20. The two wave forms in the diagram are traveling in opposite directions. Draw three diagrams to show the resultant wave forms when point O reaches A, B and C. (4)