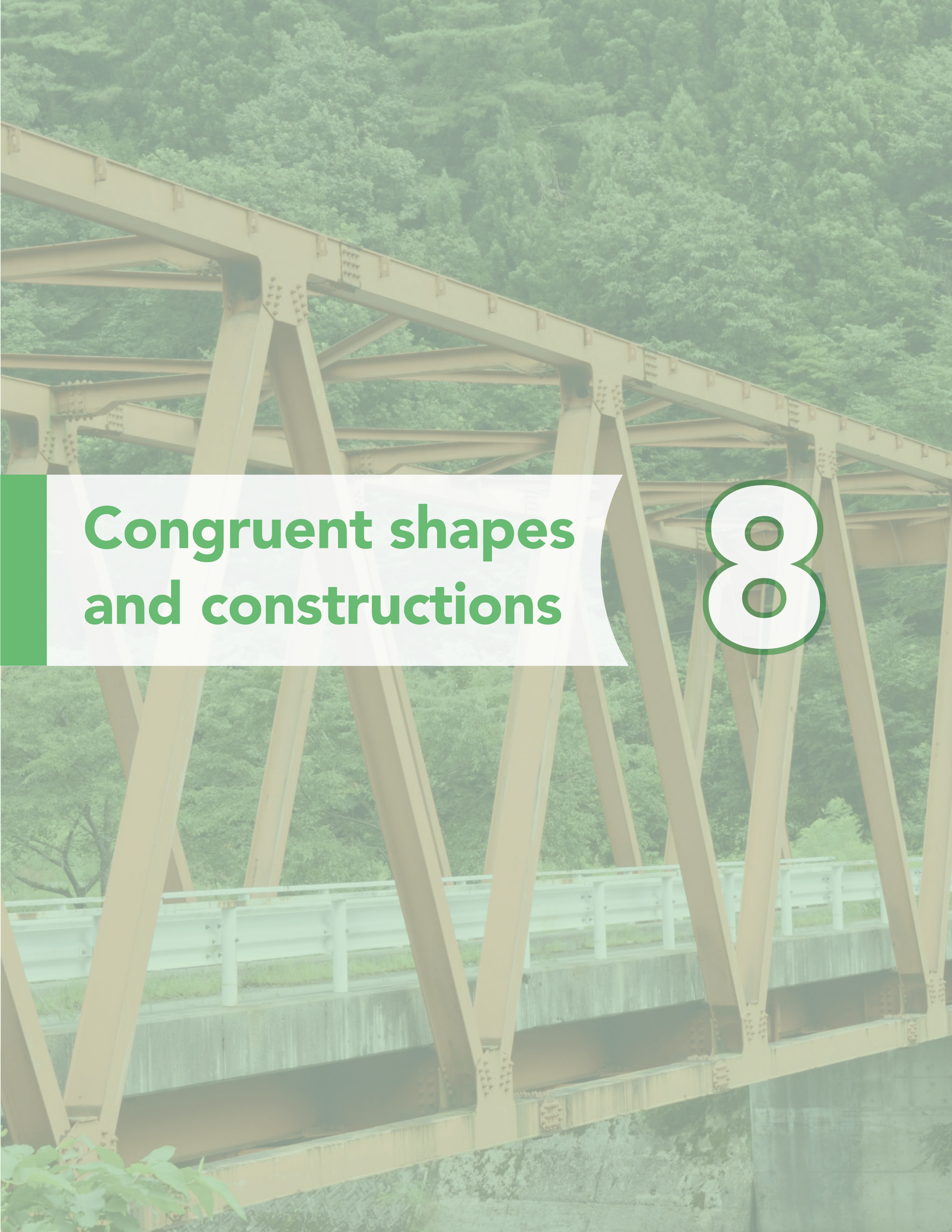


Congruent shapes and constructions

8

8 Congruent shapes and constructions

KEY CONCEPT

Logic

RELATED CONCEPTS

Equivalence, Patterns, Representation, Space, Systems

GLOBAL CONTEXT

Orientation in space and time

Statement of inquiry

Logical systems can be used to represent equivalent shapes in space.

Factual

- What are congruent shapes?

Conceptual

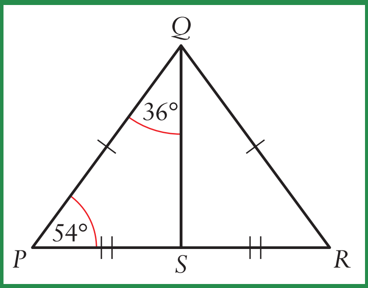
- How can you test if shapes are congruent?

Debatable

- Do congruent shapes make a structure stronger?
- Where can you find congruent shapes?

Do you recall?

1 Look at the diagram below and answer the questions.



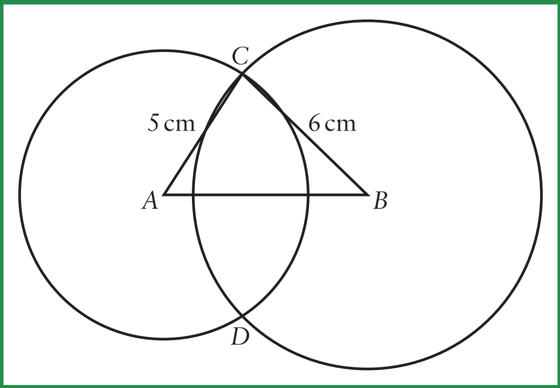
a Which side of triangle RQS is congruent to PQ?

b What length is equal to PS?

c Which angle is congruent to $\angle QPR$?

d What is the size of each of the angles $\angle QRP$, $\angle QSP$ and $\angle RQS$?

2 In the diagram below A and B are the centres of the circles.



a How far is C from A?

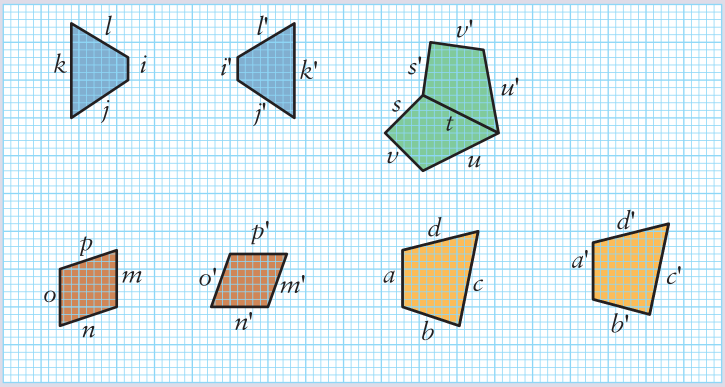
b How far is C from B?

c Which point or points are 5 cm from A and 6 cm from B?

d Describe how you could find a point 4 cm from A and 7 cm from B.

8.1 Congruent shapes

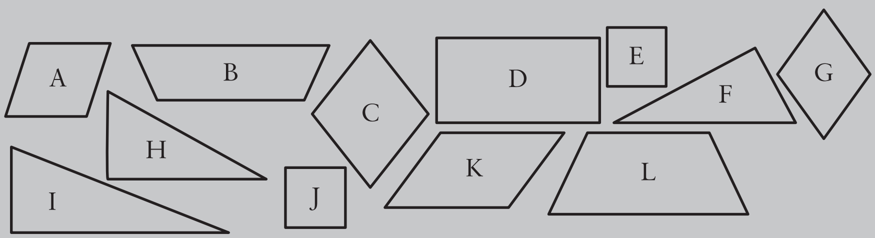
Explore 8.1



The diagram shows four pairs of shapes. Three of the pairs are congruent; one pair is not. Discuss with others what you think congruency might mean. Are you confident with your decision or would you like to see one or more pairs of shapes? Draw two shapes that you think are congruent and see if others agree. Research the word congruency (in its mathematical context) to see if you are correct.

Worked example 8.1

Which pairs of these figures are congruent?



Solution

Understand the problem

Pair the figures that are the same shape and size.

Make a plan

Measure the sides and angles of each figure and then compare them with each other to see which are the same.

Carry out the plan

A and G are congruent.

The angles and sides in figures A and G are equal.

E and J are congruent. E and J are squares of the same size.

F and H are congruent. The angles and sides in triangles F and H are equal.

Look back

Are the pairs of figures congruent and are there any other pairs of congruent figures? Yes, the pairs of figures are congruent as the sides and angles measure the same in each pair. There are not any other pairs of congruent figures as they are either different shapes or have different measurements.

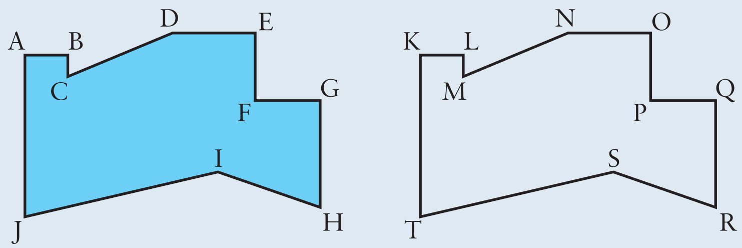
Reflect

Look at the Worked example. Can you explain why triangle I is not congruent to triangles F and H, or why figure C is not congruent to figure G?

Draw a rectangle on squared paper. Reflect your rectangle, translate your rectangle and rotate your rectangle. Can you explain why all of the rectangles you have drawn are congruent?

Practice questions 8.1

1 The two figures here are congruent.



a Use them to write down the angle that matches with:

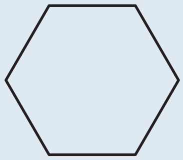
i $\angle CDE$ **ii** $\angle GHI$ **iii** $\angle HIJ$

b Use them to write down the side that matches with lines:

i AJ **ii** CD **iii** HI

c Gaia says, 'The area of the two shapes is the same'. Is she correct? Explain your answer.

2 Make four copies of this regular hexagon.



a Divide your first hexagon into two congruent figures.

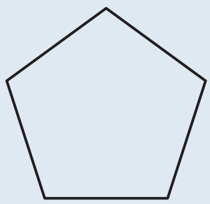
b Divide your second hexagon into three congruent figures.

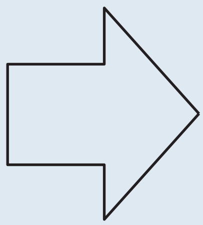
c Divide your third hexagon into six congruent figures.

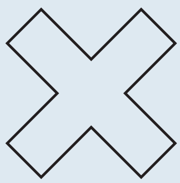
d Divide your fourth hexagon into four congruent figures.

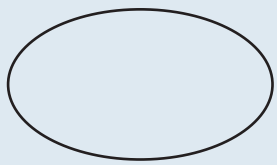
e Look back at parts **a** to **d**. Are there any other ways of dividing the hexagon into these numbers of congruent figures?

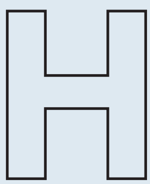
3 Copy each of these figures. For each figure, use one line to divide them into two congruent figures.

a


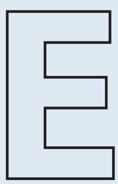
b


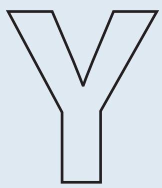
c


d


e


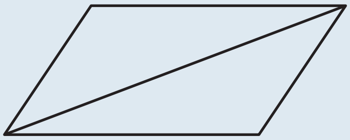
f


g


h


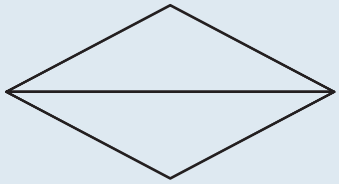
4 The following figures have each been divided into two triangles.
Write down whether or not the two triangles are congruent.

a



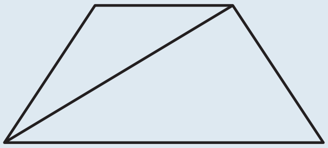
Parallelogram

b



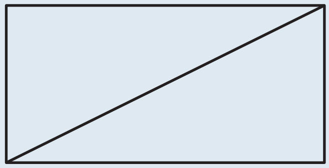
Rhombus

c



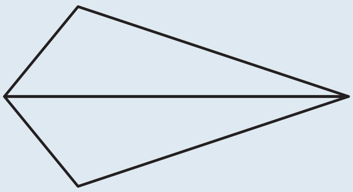
Trapezium

d



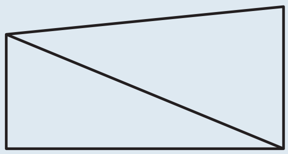
Rectangle

e



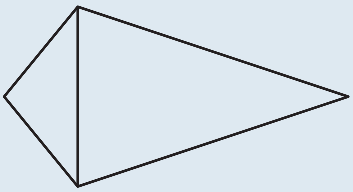
Kite

f



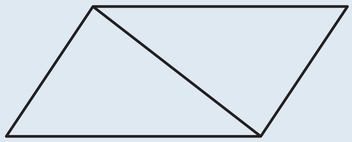
Trapezium

g



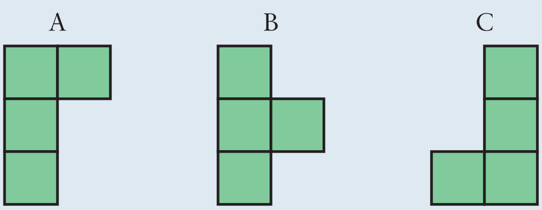
Kite

h



Parallelogram

5 The diagram shows three figures.



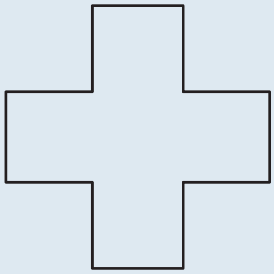
a Write down the area of each figure.

b Which two figures are congruent?

c Do figures A and B have the same area? The same perimeter?

d Are figures A and B congruent? Explain your answer.

6 Copy this diagram four times.



a Divide the figure into two congruent figures.

b Divide the figure into four congruent figures.

c Divide the figure into five congruent figures.

d Divide the figure into eight congruent figures.

e Look back at parts **a** to **d**. Are there any other ways of dividing the diagram into these numbers of congruent figures?

7 The diagram shows two congruent triangles ABC and CDE.

a Use the diagram to write down the angle that matches with:

i $\angle CAB$

ii $\angle ABC$

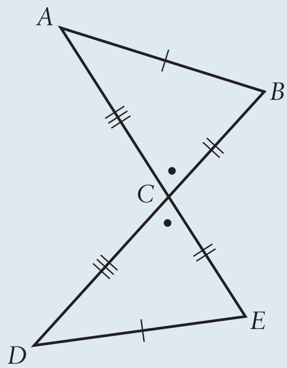
iii $\angle ACB$

b Use the diagram to write down the side that matches with lines:

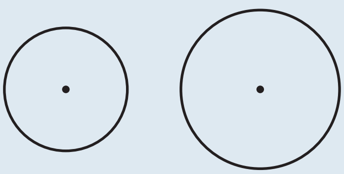
i AB

ii BC

iii AC



8 Here are two circles:



a Are these circles congruent? Explain your answer.

b What conditions must be satisfied for two circles to be congruent?

9 Idris looks at figure A and then draws figure B.

Figure A

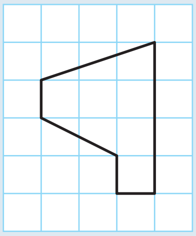
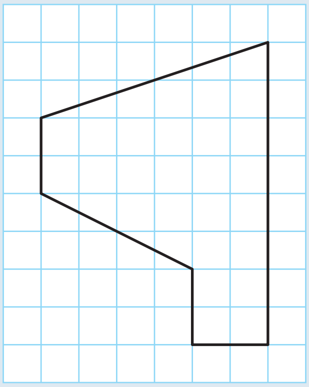


Figure B



Idris says, 'Figure B is the same as figure A, so it is congruent'.

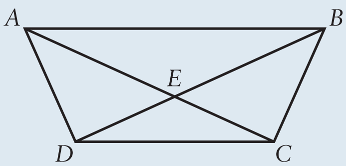
Kiara says, 'Figure B is not congruent to figure A'.

Who is correct? Explain your answer.

10 Opel says, 'All squares are congruent'. Is she correct? Explain your answer.

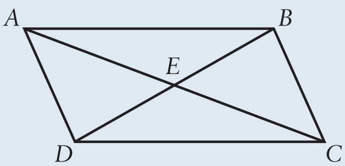
11 Identify the pairs of congruent figures in each of these diagrams.

a



Isosceles trapezium

b



Parallelogram

Explore 8.2

The pictures show two different rollercoaster cars.

Why do you think rollercoaster cars are congruent figures?

Thinking skills

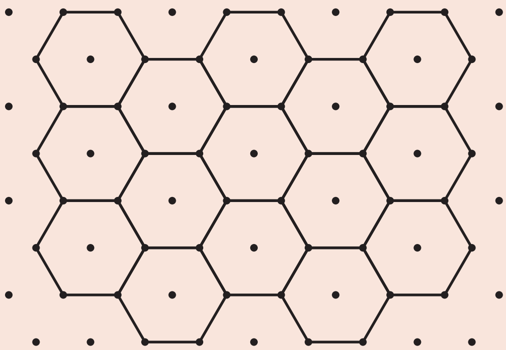
Communication skills

Research skills

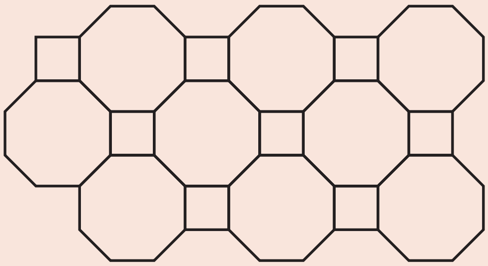
Investigation 8.1

A tessellation is a tiling pattern using one or more congruent shapes.

Here is an example of a tessellation using one shape:



Here is an example of a tessellation using two shapes:



Draw some patterns using triangles that tessellate on their own.

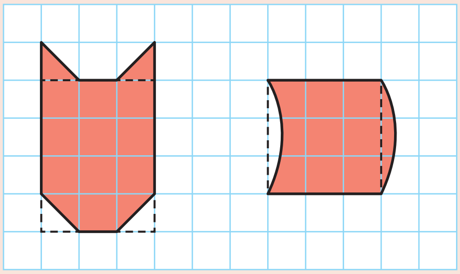
Do all types of triangle tessellate? Explain why.

Try investigating other shapes that tessellate on their own. What quadrilaterals tesselate?

What shapes do not tessellate?

Tessellations have been used in many art designs. One artist famous for using tessellations is M. C. Escher, born in the Netherlands in 1898. Research Escher to find out more about his artwork. Can you see how he used the idea of tessellations and developed this within some of his pieces of art?

Design your own tessellation artwork. First, start with a shape that tessellates on its own, for example, a rectangle. Translate small parts of the shape to its opposite side. Both the following examples start with a rectangle. The first example takes two triangles from each corner at the bottom of the rectangle and translates them to the top of the rectangle. The second example takes a segment from the left of the rectangle and translates it to the right of the rectangle.



Tessellate your shape and then turn it into art. The first example above could be developed into a cat tessellation artwork:

Here is more complicated cat tessellation using the style of graphic artist, M. C. Escher:

8.2 Basic constructions

8.2.1 Constructing triangles

Construction means to build something. Constructions in mathematics are linked to this as they are accurate drawings of shapes, angles and lines. When you are asked to construct you will need a ruler, a pencil and a pair of compasses.

Explore 8.3

Draw a triangle with sides 5 cm, 6 cm and 8 cm, as accurately as you can.

Swap your triangle with someone else and check each other's accuracy. Explain the steps you followed to draw your triangle.

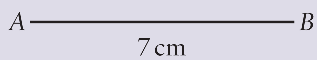
Explore 8.4

Diane was asked to construct the triangle ABC where:

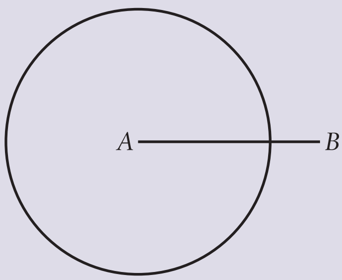
$AB = 7$ cm, $AC = 5$ cm and $BC = 4$ cm.

Here are the steps she took:

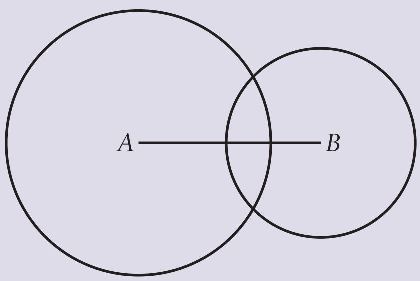
Step 1



Step 2 Draw an arc using compasses 5 cm from point A.

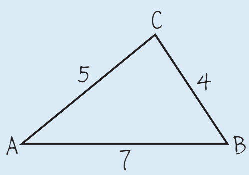


Step 3

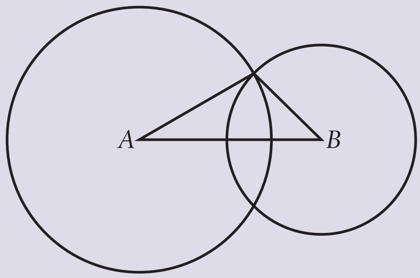


Hint

In construction, it is good practice to first make a hand drawn sketch of the figure you are about to construct. For example, for this Explore activity:



Step 4



Describe what you think she did. How many such triangles are you able to find?

Reflect

When constructing a triangle, does it matter which side you choose as your first side? Can you give a reason for your answer?

Practice questions 8.2.1

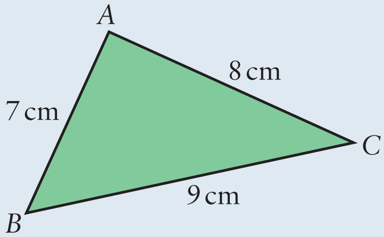
1 Construct a triangle with sides of length:

a 5 cm, 6 cm and 7 cm

b 8 cm, 5 cm and 8cm

c 9 cm, 5 cm and 7 cm.

2 **a** Use compasses to construct a triangle that is congruent to triangle *ABC*.

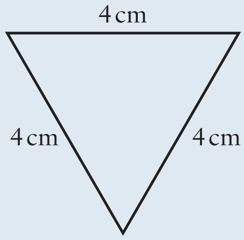


b Use compasses to construct a triangle with side lengths that are half of the side lengths of triangle *ABC*. Label your triangle *PQR*.

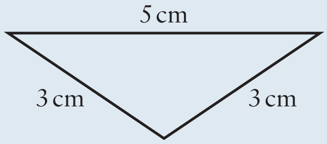
3 Using your triangles from Questions 2a and 2b, write down the matching pairs of angles and sides in these two congruent triangles.

4 Use a ruler and pair of compasses to accurately draw these triangles.

a



b

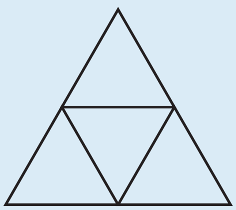


5 Construct an equilateral triangle with side length 6 cm.

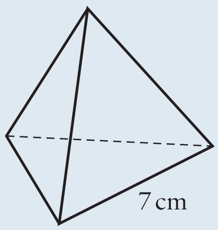
Challenge Q6

Hint Q6

A tetrahedron is a triangular-based pyramid. All four faces are equilateral triangles. The diagram shows a sketch of the net.



6 A tetrahedron is shown below.

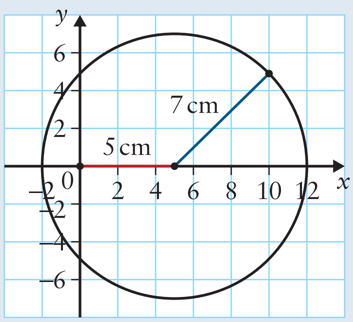


Construct an accurate drawing of a net of the tetrahedron.

7 A triangle cannot be constructed with sides 5 cm, 7 cm and 15 cm or sides 6 cm, 7 cm and 13 cm. Explain why and state the conditions that must be satisfied in order to construct a triangle.

Hint Q7

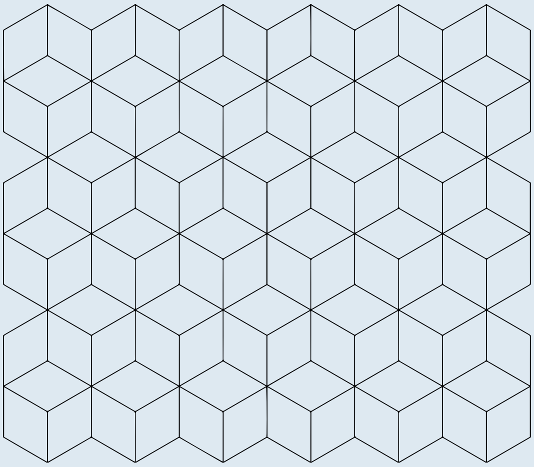
First draw a 5 cm line segment, then draw a circle of 7 cm with its centre at one end of the line segment and then contemplate what a third side, of 15 cm, would look like.



Challenge Q8

8 Construct all possible isosceles triangles with side lengths 5 cm and 7 cm.

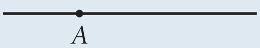
9 This tessellation uses a rhombus.



Each rhombus is constructed using two equilateral triangles with side of length 4 cm.

Using a ruler and compasses, construct one rhombus in the tessellation.

10 Draw a line with point A on the line like this:



Construct an angle of 60° with vertex at *A*.

Challenge Q9

Challenge Q10

Thinking skills

8.2.2 Bisecting angles

To bisect means to cut exactly into two equal parts.

Explore 8.5

Draw these angles using a protractor.

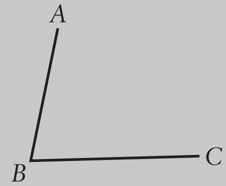
40°, 160°, 72°, 135°.

Can you draw a line to divide your angles exactly in half?

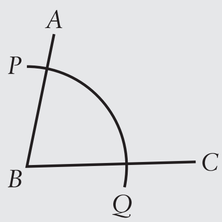
Were any of the angles easier to divide into half than others? Give a reason for your answer.

Worked example 8.2

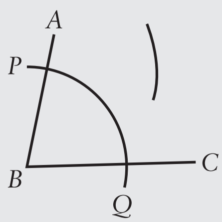
Bisect the angle ABC using only a pair of compasses and a straight edge.



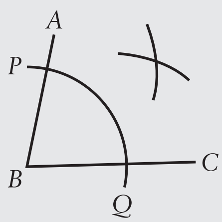
Solution



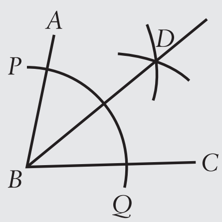
We put the tip of our compasses at B and draw an arc that crosses AB and BC.



With the point of intersection P as the centre, we draw an arc.



Without changing the setting of the compasses, we draw an arc with the point of intersection Q as the centre. The two arcs drawn from both points of intersection should cross. Call the point D.



We draw a straight line from point B through the two intersecting arcs.

This line bisects the angle ABC.

Reflect

Can you explain why the line drawn in the worked example above bisects the angle ABC?

Which is more accurate: Constructing the angle bisector using compasses or using a protractor to split the angle into two halves?

Hint

Connect PD and QD.

Practice questions 8.2.2

1 Draw two different acute angles and bisect each of them.

2 Draw two different obtuse angles and bisect each of them.

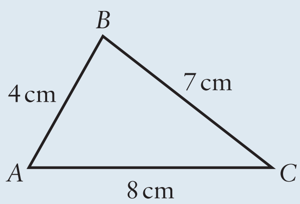
3 Draw each angle using a protractor and construct its angle bisector using compasses and straight edge.

a 50° **b** 85° **c** 110° **d** 162°

4 Draw a scalene triangle with one obtuse angle. Bisect the obtuse angle in your triangle.

5 **a** Construct the triangle *ABC*:

b Bisect angle *ABC*.



6 **a** Construct an equilateral triangle with side length 7 cm.

b Bisect an angle in your triangle.

c Without using a protractor, what is the size of the two angles you created in part **b**?

Challenge Q6

Investigation 8.2

Construct an equilateral triangle with side length 10 cm.

Construct the angle bisector of each angle in your triangle.

What do you notice with your three angle bisectors?

Try this for isosceles and scalene triangles.

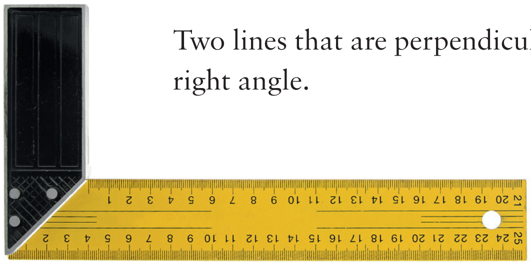
Does the same happen again?

Generalise your findings.

Thinking skills

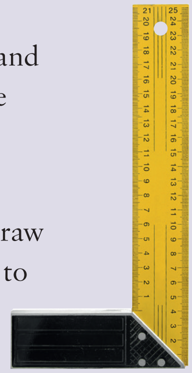
Communication skills

8.2.3 Perpendicular bisectors



Two lines that are perpendicular to each other form a right angle.

Explore 8.6



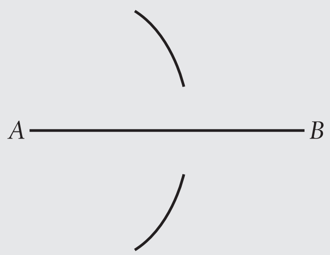
Draw a line *AB* that is 9 cm long. Can you use a ruler and protractor to draw a line that is perpendicular to line *AB* and cuts the line *AB* in half? Swap your diagram with someone else's and check how accurate theirs is.

As with bisecting angles, perpendicular bisectors can be constructed more accurately using a pair of compasses. Draw another line *AB* that is 9 cm long. Can you figure out how to use a pair of compasses to construct the perpendicular bisector of your line?

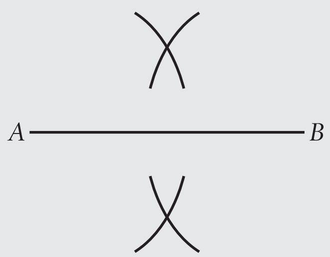
Worked example 8.3

Bisect the line segment *AB*.

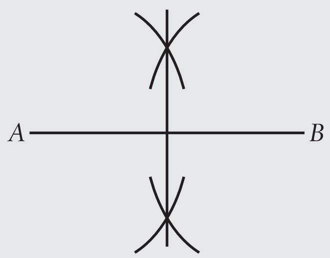
Solution



We open up our compasses to a radius of more than half of *AB*, and draw an arc above and below the line *AB* from point *A*.



Without changing the setting of our compasses, we draw an arc above and below the line *AB* from point *B*.



We draw a line passing through the intersecting arcs.

The line we have drawn bisects the line segment *AB* and forms a right angle with *AB*.

Reflect

Can you explain why the line constructed in Worked example 8.3 above bisects segment AB?

Can you explain why the compasses need to be set so the radius is set greater than half the length of the segment being bisected?

Practice questions 8.2.3

1 Draw a straight line segment 8 cm long.
Construct the perpendicular bisector of the line segment.

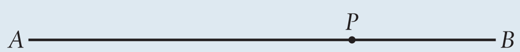
2 Draw each straight line segment and construct its perpendicular bisector.
a 10 cm **b** 7 cm **c** 12 cm **d** 6 cm

3 Draw a scalene triangle and construct the perpendicular bisector of one of its sides.

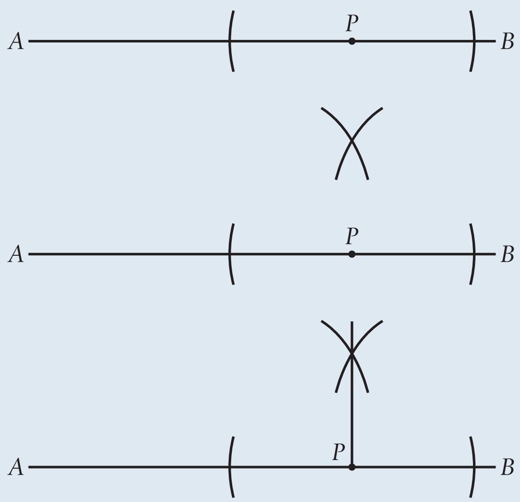
4 **a** Construct the triangle ABC, where $AB = 7$ cm, $BC = 6$ cm and $AC = 5$ cm.
b Construct the perpendicular bisector of the side AB.

5 **a** Draw a straight line segment 10 cm long and construct its perpendicular bisector.
b Construct the angle bisector of one of the right angles.
c Without using a protractor, what is the size of the two angles you created in part **b**?

6 Point P lies on the line AB.



Trace the diagram and construct a perpendicular line to the line AB from point P. Follow the steps in the diagrams below to help you.

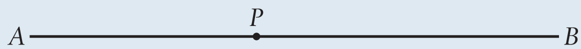


Hint Q6

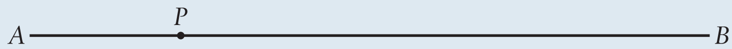
The steps for constructing a perpendicular line to the line AB from point P is the same as bisecting the angle APB.

7 Copy each diagram. Construct a perpendicular line to the line *AB* from point *P* for each one.

a

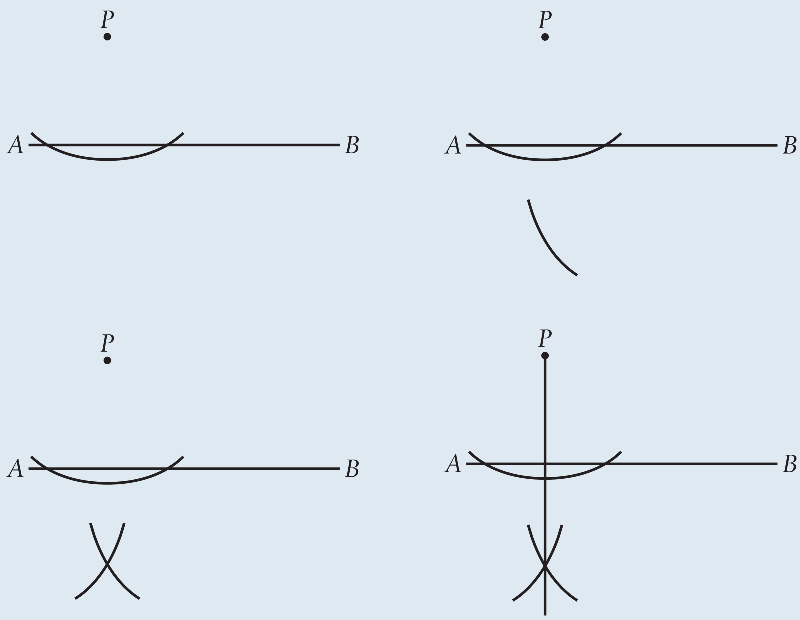


b



8 Point *P* is above the line *AB*. Trace the diagram and construct a line perpendicular to *AB* from point *P*.

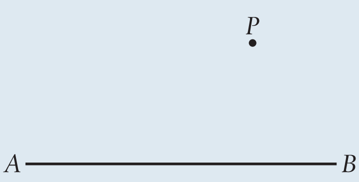
Follow the steps below to help you.



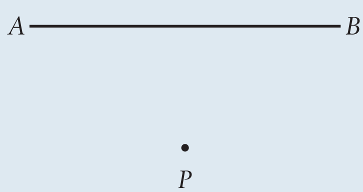
9 Copy each diagram.

Construct a perpendicular line to the line *AB* from point *P* for each one.

a



b



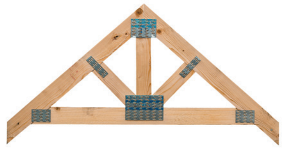
10 The picture on the right shows a roof truss.

For this design of roof truss, the triangular outline of the roof truss is an isosceles triangle. The vertical piece of wood bisects the angle at the top of the roof truss. The other two interior pieces of wood are at right angles to the two equal sides of the roof truss.

Construct a scale drawing of a roof truss of this design with sides 6 m, 6 m and 10 m. Use a scale of 1 cm for 10 m.

You may complete this question using GeoGebra geometry.

Challenge Q10



11 The picture on the right shows the steel frame of a commercial building. The outline of the windows are squares with length 2 metres.

Using ruler and compasses, construct an outline of the window using a scale of 1 cm for 0.25 m.

Challenge Q11

Investigation 8.3

Construct an equilateral triangle with side length 8 cm on paper or using GeoGebra.

Construct the perpendicular bisector of each side of your triangle.

What do you notice with your three perpendicular bisectors?

Try this for isosceles and scalene triangles.

Does the same happen again?

Generalise your findings.

Investigation 8.4

You are going to design and make your own bridge using popsicle (or lolly) sticks. You could use glue, elastic bands, sticky tape or Blu Tack® to join your popsicle sticks together.

Your bridge must span a gap of at least 30 centimetres and use congruent figures in its design.

First, sketch some designs. Choose which design you are going to construct.

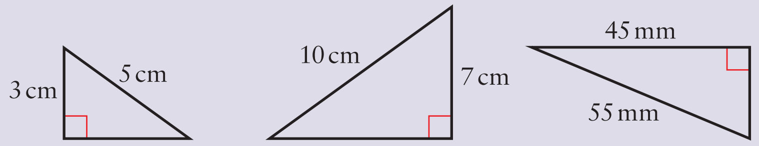
Draw an accurate construction of your bridge using a ruler and compasses or GeoGebra geometry before building it.

What problems did you come across whilst designing and building your bridge? How did you overcome these problems?

What is the greatest mass your bridge will hold? How could you make your bridge stronger?

Explore 8.7

Using your knowledge of constructing triangles and perpendicular bisectors, can you construct these triangles? (Do not use a protractor.)



Explain your method.

Thinking skills

Social skills

Communication skills

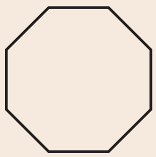
Self-assessment

- I can identify congruent figures.
- I can construct triangles using a ruler and compasses.
- I can bisect angles.
- I can construct the perpendicular bisector of a line segment.
- I can construct a perpendicular line to a line segment from a point on the line.
- I can construct a perpendicular line to a line segment from a point above or below the line.

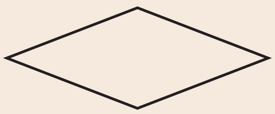
Check your knowledge questions

1 Copy each of these figures. For each figure, use one line to divide them into two congruent figures.

a

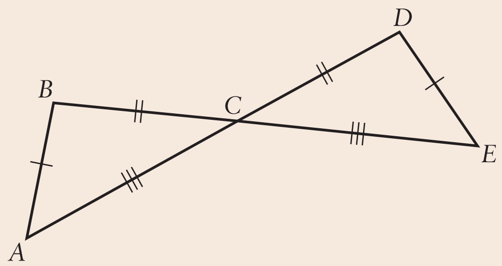


b



c

2 The diagram shows two congruent triangles ABC and CDE.



a Use the diagram to write down the angle that matches with

i $\angle CAB$ ii $\angle ABC$ iii $\angle ACB$

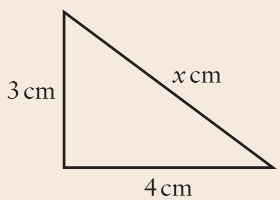
b Use the diagram to write down the side that matches with

i AB ii BC iii AC

3 A rectangle has area 20 cm². Using a ruler and compasses, construct a triangle with the same area as the rectangle.

4 Construct an equilateral triangle with side length 4 cm.

5 The diagram shows a triangle.



The side labelled x cm is 25% longer than the base of the triangle. Construct an accurate drawing of the triangle using a ruler and compasses.

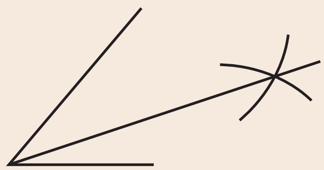
6 Construct all possible isosceles triangles with side lengths 6 cm and 9 cm.

Challenge Q6

7 Draw an obtuse angle and bisect it.

8 Construct an isosceles triangle with one obtuse angle. Bisect the obtuse angle in your triangle.

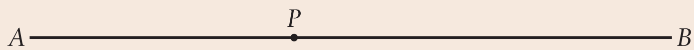
9 Jay uses his ruler and compasses to bisect an angle. The diagram shows their work. Explain what mistake Jay has made.



10 Use your triangle from Question 4 and construct the perpendicular bisector of one of the sides.

11 Copy each diagram. Construct a perpendicular line to the line AB from point P for each diagram.

a



b