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Second Edition

Children's Mathematics

Cognitively Guided Instruction

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DEDICATION

Cognitively Guided Instruction (CGI) teachers frequently talk about how they learn from their students. We have been fortunate to be able to learn from both teachers and their students. The teachers we have worked with have truly been partners in our research. They have helped to extend our understandings of children's mathematical thinking, shown us what is possible as they support student learning in classrooms, and inspired us to continue our research and professional development.

We dedicate this book to all the teachers, children, and teacher leaders who have helped to make CGI what it is today. This really is their story, and we are grateful to them for helping us tell it.

The continuing popularity of this book is testimony to the importance of its central message: Young children *can* think mathematically and instruction that builds on their thinking takes them further and deeper into the core concepts of arithmetic than teachers might imagine.

—James Hiebert,
Robert J. Barkley Professor of Education,
University of Delaware

The training we have had in Cognitively Guided Instruction and through *Children's Mathematics* has shown us that understanding "how" children think mathematically is so much more valuable to their ongoing development than trying to "give" them the way to think. Our students have flourished!

—Kathy Goecke, principal

Cognitively Guided Instruction not only changed the way I teach and restored my passion for teaching, it has had a positive impact on thousands of children.

—Debbie Gates, elementary mathematics specialist

This exemplary resource is essential for teachers, professional developers, and researchers who are interested in understanding, supporting, and extending children's ways of reasoning.

—Vicki Jacobs,
Yopp Distinguished Professor of Mathematics Education,
UNC Greensboro

Children's Mathematics is by far the most powerful and practical book that I have used in my work with teachers. The potential it has to not only deepen students' conceptual understanding of mathematics but their teachers' as well is unparalleled . . . it is essential reading for every teacher who teaches mathematics.

—Andrew Jenkins, principal

I am far more engaged and excited about teaching math myself because *Children's Mathematics* helped me understand what questions to ask and what student comments to listen for that will lead to student discovery.

—Lesley Wagner, teacher

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9

Developing Classroom Practice: Posing Problems and Eliciting Thinking

Teachers regularly ask, "What does a CGI classroom look like?" "What do I do to make my classroom a CGI classroom?" There is no "one way" to organize your classroom for CGI. However, we have observed that instruction in CGI classrooms can be characterized by a set of classroom practices that are powerful for engaging children's mathematical thinking. In this chapter and the next, we describe some research-based classroom practices and the principled ideas that inform them. We draw on examples from a number of CGI teachers working in a variety of contexts, at different points in their CGI journey. This chapter elaborates the practices of posing problems and eliciting student thinking.

When you see CGI teachers engaging children in problem solving, the work may seem effortless. However, those teachers have prepared the way for students' engagement by choosing problem contexts that are accessible, ensuring students have tools available to support their thinking, encouraging students to do what makes sense to them, and making sure all students have a way to get started. Thus, CGI teachers work at creating an environment where students see that a range of strategies are expected and celebrated. This work begins with posing problems.

POSING PROBLEMS

Prior to posing a problem, CGI teachers select a problem type and numbers that allow students to engage with the mathematical goals they have in mind. Choosing which problem to pose depends on understanding the details of your students' mathematical thinking; understanding your students' thinking requires posing problems and listening to and watching the strategies your students use. In getting started, posing a problem that can easily be directly modeled offers the greatest possibility that students will be successful. The more you learn about your students' thinking, the easier it will be for you to choose problems and numbers to meet students' learning needs.

Problem posing starts with the teacher reading the problem to students, the class reading it together, or students reading it on their own. During this initial reading, check for understanding of the context and watch and listen to see if students can get started. As students begin to engage in problem solving, pay special attention to making sure the students know what the story is about, because students' understanding of the story drives their strategies. Engaging the class in discussion of the problem context before reading the problem, while reading it, or after the students have heard the problem can support students in making sense of the problem.

> If you are looking for support in deciding what problem to pose, try some of the questions for reflection in Chapters 3 (question 2), 4 (questions 3 and 4), 6 (question 4), and 7 (question 3).

If the problem context makes sense to students and they know what they might do to start on a solution, they will be able to engage in problem solving. When posing the problem, it can be helpful to unpack the problem with students. We use the term *unpack* to refer to engaging students in making sense of the context and the action or situation of the problem to ensure that all students understand the problem. When or how much to unpack depends on the context, the students' experiences, and what the students understand about the mathematics. With younger students, it may be necessary to spend more time making sure that they understand the context of the problem, particularly the consequences of the action on the number of objects in the story. Teachers in dual language classes or classes where English is not some students' primary language may choose to unpack in ways that ensure students have the opportunity to use their native language as a resource in making sense of the context. When posing a particular problem type for the first time, you might spend more time making sure students understand what is happening in the story as it relates to the mathematics involved. Or you might choose not to unpack as a way to assess students' abilities to make sense of the context on their own. Most important, when considering when and how to unpack a problem, remember that the goal of unpacking is to provide opportunity for students to make sense of the problem context, not to walk them through how to solve the problem.

CLIP 9.1 Unpacking the problem context in Ms. Scott's class http://smarturl.it/CM9.1

Unpacking the Problem

In the following episode, Ms. Scott posed a problem to her second-graders by unpacking the problem context and having a discussion with students about what was happening:

> *Estabán has 71 marbles. On his way to school 39 marbles fell out of his backpack. How many marbles does Estabán have left?*
>
> Ms. Scott asked if her students were all ready for math today, and they responded with a "Yes!" She then asked, "I wonder who is in the story today?" and looking at her clipboard, said, "Oh, it's Estabán."

Ms. S: *Estabán, do you have marbles at home?*

Estabán: Yes.

Ms. S: *I had a feeling. How many do you have—a bunch, a little bit?*

Estabán: I have 100.

Ms. S: *[with eyes wide] Really? Do you ever bring your marbles to school?*

Estabán: My mom told me if I take them to school, my teacher might take them away.

Ms. S: *Well, you have a smart mom. Well, today we are going to be pretending that you brought your marbles to school. Can you all pretend with Estabán about that?*

Students: Yes.

Ms. S: *I am going to tell you the story and then we are going to say it together. Estabán had 71 marbles, and on his way to school, 39 marbles fell out of his backpack.*

Estabán: Oh, man.

Ms. S: *You had a hole in your backpack.*

Estabán: No, I don't.

Ms. S: *Well, let's all pretend you did. We want to know how many marbles Estabán had left. So I am going to say it again. [Rereads entire problem.]*

The students repeated the problem, along with Ms. Scott, two more times. Ms. Scott then asked the students if they had a way they might figure it out. When the students responded that they did, Ms. Scott sent them to their tables to solve the problem.

The Separate (Result Unknown) problem Ms. Scott created for her students to solve involved a student in the class and a situation she thought would make sense to her students. The power of the context in a word problem is that it draws on relationships that students can reason about in ways that enable them to understand the mathematics of the problem. The context of Ms. Scott's problem told a story about marbles getting lost, a situation that was familiar to students and one that they could see as involving a separating action.

The key to unpacking a problem with students is supporting students to understand the context in relation to the mathematics of the problem. In unpacking the problem, Ms. Scott did not focus on "key words," the question at the end of the word problem, or what operation students should use. She also did not discuss how the problem could be solved. Instead, as she posed the problem, she focused on what the story was about. She checked with Estabán about his marble collection and highlighted the hole in Estabán's backpack to explain how he lost the marbles. Similar to what a teacher might do during a literacy activity, Ms. Scott engaged the students in comprehending the story.

CLIP 9.2 Unpacking the problem context to personalize the problem for a student http://smarturl.it/CM9.2

CLIP 9.3 Unpacking the problem context to help a student get started http://smarturl.it/CM9.3

Students Share Ideas About a Familiar Context

In the preceding example, Ms. Scott unpacked the problem as she posed it to the class by asking students questions along the way. Some teachers unpack a problem by having students talk to each other about the context. As we see in the next example, Mr. Torres has his first-graders pair share around what the story is about and then engages the students together in a whole-group conversation. The pair share allows more students the chance to verbalize their understanding of the context, and it can uncover aspects of students' personal experiences that the teacher needs to address, as the following example shows.

Before reading the problem, Mr. Torres asked his students about the breakfast items served in the cafeteria. He called on a number of different students to share their ideas. Mr. Torres then told the class they were going to solve a problem about breakfast in the cafeteria. On the board he had written a Compare (Difference Unknown) problem:

> *At breakfast in the cafeteria, 12 children ate cereal. 7 children ate eggs. How many more children ate cereal than eggs?*

He read the problem to the students and had the students read the problem to him. He asked the students to turn to their partner and discuss what the problem is about. Mr. Torres moved around listening to the students' discussions. He then called them back together.

Mr. T: *So who can tell me one thing about the story?*

Senica: It is about cereal.

Mr. T: *Okay, what do we know about the cereal? Jesus?*

Jesus: 12 children ate it.

Mr. T: *Yes. [to the class] How many children were eating cereal for breakfast at the cafeteria?*

Students: 12!

Mr. T: *What else can you tell me, someone tell me one other thing about the story. Yvette?*

Yvette: 7 had eggs.

Aaron: But Mister, I had eggs and there were more people having eggs.

Mr. T: *Yvette, you are correct, the problem says 7 children ate eggs. Aaron is saying that today he had eggs and he thinks more than 7 people had eggs. That is good noticing. But today the problem we are solving is about 7 children eating eggs so we want to pay attention to those 7 people having eggs. So, Aaron, let's make sure we know what is going on in this problem. Jesus and Yvette told us we know that in this story, 12 children had cereal and 7 children had eggs. Do you agree with that Aaron? [Aaron agrees.] So what are we trying to find out? Arleta?*

Arleta: How many.

Mr. T: *How many what?*

Arleta: How many more people had cereal.

Mr. T: *How many more people ate cereal than ate eggs. Do you agree with Arleta?*

Students: Yes.

Mr. T: *So are there more children eating cereal or eggs? [Students call out, "Cereal."] If we are trying to find out how many more people ate cereal than eggs, do you think it could be that 20 more people ate cereal than eggs? [Students call out, "No."]*

Brian: No, 'cause that would be too many.

Mr. T: *Okay, Caro?*

Caro: Only 12 have cereal. So that is too many.

Mr. T: *Okay, raise your hand if you have a way to get started. If you still want to talk more about it, stay here with me for a minute.*

Mr. Torres not only asked his students to pair share about the context; he also asked them to take the lead in describing the context. He supported Aaron to see that while he had particular knowledge of egg eating in the cafeteria, the problem being posed was about a different situation. Sometimes students draw on their experiences in ways that can confuse them; at other times students do not understand the context well enough to make sense of what is going on. In closing his problem posing, Mr. Torres posed a question to support students to think about the relationship between the action and the quantities: Could it be that 20 more people ate cereal than eggs? Students' responses to this question provided insight into their understanding of the mathematical relationships in the problem.

Making Sense of Problems

One of the biggest challenges in unpacking a problem with students is that you can end up doing all of the mathematical work. It is easy to lead students through the problem in a way that provides the students a strategy to use so that students already have the answer before you are finished unpacking the problem. The goal of unpacking the problem is to support sense making around the context, not to help students come up with a strategy or an answer. By focusing unpacking on having the students explain what the story is about, you can avoid doing too much of the work for the students.

CLIP 9.4 Ms. Byron, Ms. Hassay, and Ms. Grace begin by posing the problem http://smarturl.it/CM9.4

CGI teachers have found that students gradually learn to make sense of the context on their own. Students learn that whether the problem is about monkeys or squids or about a student in the class or about a stranger does not make a difference in the strategy they choose. Rather, students learn to look for the mathematical relationships that are a part of the story and use them to get started on a solution.

While the details of unpacking problems can vary, a set of principles underlies the unpacking in all of these instances (see Figure 9.1).

Principles for Unpacking Problems
Focus on story comprehension.
Attend to the connection between the story and the mathematics.
Support each student's participation and access.
Support students to learn how to unpack problems on their own.
Avoid doing the mathematical work of solving the problem.

FIGURE 9.1

ELICITING STUDENT THINKING

Asking students, "Can you tell me how you solved that?" is a central feature of CGI classrooms. The wording of the question may vary, but the underlying idea that students make explicit their mathematical thinking is common to all CGI classrooms. This general question, along with variations such as "What did you do?" or "Tell me about your strategy," is a productive way to start engaging students in explaining the strategies they used to solve a problem. More specific follow-up questions support students to elaborate the details of their strategy. Having a student share more details of her thinking engages the student in articulating, explaining, and justifying her thinking and enables the teacher and other students to understand the strategy the student used. Research shows that when students are expected to describe their strategies in detail with the teacher and with each other, they demonstrate higher mathematical achievement (see Webb et al. 2008 and 2009). Both the general initial question and the specific follow-up questions that teachers ask support students in explaining their strategies.

You can elicit student thinking throughout problem solving in ways that seek out how students solved problems or support students to complete or correct a strategy or extend students' mathematical thinking. There is no perfect sequence of questions nor can questions all be preplanned because supporting students to explain the details of their ideas requires listening and responding to what students share. The first examples that follow focus on eliciting student thinking after the students have solved the problem; the next section examines eliciting thinking when students' solutions are incomplete or incorrect and when teachers draw from student written work. The final section looks at eliciting where teachers work to extend students' mathematical thinking.

> *Your best learning comes from your students so really listen to them and be open to the many ways they will teach you.*
>
> **—Shernice Lazare, teacher**

Eliciting Student Thinking After Students Have Solved the Problem

Teachers often search for the "best" question to ask in eliciting a student's thinking. However, it turns out there is no best question. Rather, the most productive questions following the initial general "Tell me what you did" are those questions that refer specifically to what a student shared in talk or writing. Your job is to listen and observe the details of the student's strategy, and then ask him a question about something you noticed.

For example, consider the interaction between Liz and her teacher about the following problem:

> *Kevin has 7 dollars. He wants to save up 11 dollars to buy a toy. How much more money does Kevin need?*

Liz: I got 4.
Ms. T: *How did you figure that out, Liz?*
Liz: I put 4 with 7 and got 11.
Ms. T: *Tell me about the 4. What do you mean that you put it with the 7?*
Liz: I knew that 7 and 3 is 10 and 1 more is 11 and 3 and 1 is 4.

Liz's initial explanation let Ms. Thompson see that Liz had arrived at a correct answer and she was thinking about putting the 4 with the 7. Ms. Thompson's follow-up question prompted Liz to focus specifically on the 4 and describe the remainder of her strategy. The question supported Liz to detail her strategy so the teacher and class could see the specific derived fact she used.

Asking a Series of Follow-up Questions

Often one follow-up question is not enough to support the student to describe her entire strategy. Sometimes you need to ask a series of questions as illustrated by the following example. Kirta is solving the problem:

> *Joshua had 8 buckets of sand for his sand castle. His brother brought him 11 more buckets of sand. How many buckets of sand does Joshua have now?*

Ms. N: *Kirta, how did you solve the problem?*
Kirta: I made 8 and 11.
Ms. N: *Okay, you made 8 and 11. Can you tell me how you made the 8?*
Kirta: I used my cubes and counted 1, 2, 3, 4, 5, 6, 7, 8 and then I counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and I got 19.
Ms. N: *So you counted out 8 cubes and then 11 cubes. How did you use that to get 19?*
Kirta: I counted 8 [sweeping her hand over the set of 8 cubes], 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 [pointing to each cube in the set of 11].

With each follow-up question, Ms. Navarro picked up a detail from Kirta's explanation and used that to ask her another question. Ms. Navarro asked about the 8, and how Kirta counted to get to 19. Kirta could have counted her cubes in several different ways. Kirta might have counted all 19 cubes in the combined set starting with 1. Or she might have taken advantage of the fact that she already knew that there were 8 cubes in one of the collections and started counting on at 9 to count the remaining cubes. Ms. Navarro learned that Kirta started counting from the 8 in the initial group and counted the second group. The follow-up questions allowed Kirta to work through explaining the details of her strategy and let Ms. Navarro and the class hear Kirta describe each step of her Direct Modeling strategy, which showed that she was beginning to transition to a Counting strategy.

Following up on student explanations not only helps the teacher and other students learn more about a student's thinking, it also helps the student doing the sharing in two important ways. First, sharing helps the student enhance his understanding because as he verbalizes he can make and deepen connections among mathematical ideas. Second, sharing allows the student and the class to learn what counts as a complete mathematical explanation.

Eliciting Student Thinking When Student Solutions Are Incomplete or Incorrect

As you observe students solving problems, you will see students who are unsure about how to proceed. Sometimes these students are on the right track, and sometimes they are not. In both cases, eliciting the student's thinking is an important way to support the student to engage in problem solving. How then do you apply the same principles about attending to the details of the student's strategy? You can start with what the student *has done* or on *what the student is thinking*. Christi Byron encountered this issue as her students solved a Join (Result Unknown) problem:

CLIP 9.5 Ms. Byron works with a student who has a partially correct strategy http://smarturl.it/CM9.5

> *Natalie has 30 jelly beans. Her mom gives her 23 more jelly beans. How many jelly beans does Natalie have now?*
>
> Ms. Byron was checking in with students while they solved the problem. Samuel had finished solving the problem and arrived at the answer 33 (circled on his paper). Ms. Byron asked him to tell her about his thinking. (See Figure 9.2.)

Ms. B: *Will you tell me your thinking on here, Samuel?*
Samuel: [points to the circled tens on his paper] 10.

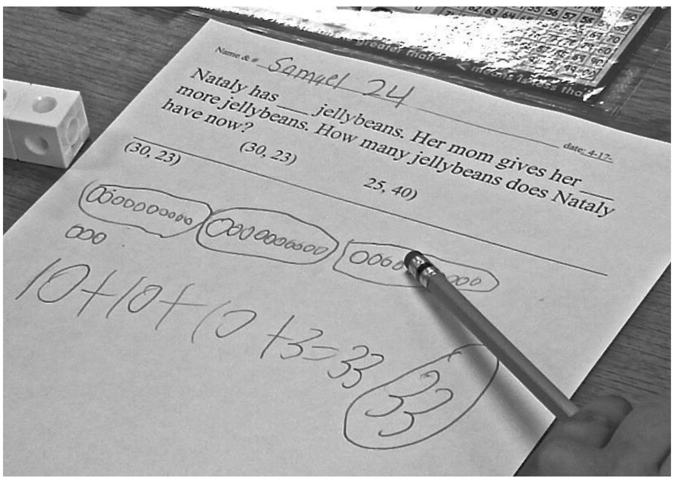

FIGURE 9.2 Samuel's solution

Ms. B: *10, okay.*

Samuel: Plus 10, plus 10, plus 3, equals 33.

Ms. B: *Equals 33, all right, let's read the story together.*

Ms. B and Samuel together: *Natalie has 30 jelly beans, her mom gives her 23 more jelly beans. How many jelly beans does Natalie have now?*

Ms. B: *All right. So can you show me the 30 jelly beans that Natalie had?*

Samuel: [points to his paper with his pencil where he has 3 groups of 10].

Ms. B: *Let's count them.*

Samuel: [with Ms. B] 10, 20, 30.

Ms. B: *Oh, so how many more jelly beans did her mom give her?*

Samuel: These [points to the 3 smaller circles under the 3 tens on his paper].

Ms. B: *Let's read that number. How many jellybeans did her mom give her?*

Samuel: 23.

Ms. B: *Where is the 23 in your picture?*

Samuel: [points to the 3 circles].

Ms. B: *Can you count those for me?*

Samuel: 10, 20, 21 [Counts 2 circles as tens and 1one as a one. It is not clear how Samuel solved the problem but Ms. B did notice that he was sometimes counting his circles as tens and sometimes as ones.]

Ms. B: *[points to his 3 circles] Right here is 21? Or is that, how much is this?*

[Ms. Byron picks up some base-ten blocks and asks Samuel to make 23 with the tens and ones. She sees he is having trouble thinking about the tens in 23. So she then goes on to ask him to build the 23 using just ones blocks. She leaves Samuel as he is making the 23 and checks in later to see if he counted all of his initial 30 and the 23 he counted out with the ones blocks.]

Ms. Byron followed the same principles as Ms. Navarro in eliciting the details of Samuel's thinking. She asked Samuel to show her what he was doing. She listened to him and watched him explain his strategy, asking very specific questions and asking him to be specific in his responses. When Ms. Byron was unsure about what Samuel had on his paper and how he was thinking about the problem, she asked him to reread the problem with her. She asked him to be specific about where he had shown the 30 from the problem and then the 23. She saw he was having trouble making his 23 using tens and ones, and recognized that when he made the 30, he made all 30 circles in groups of 10, so she gave him ones blocks and left him to see if he could finish. In summary, she asked specific questions about what he did and what he was thinking, and she referred back to the problem and provided appropriate tools to support him to move forward in his solution.

> *I ask questions of all students (right or wrong) and encourage students to give more than one explanation. I also ensure (through record keeping and checklists) that over time all students have an opportunity to share their thinking and their strategies.*
>
> **—Darlene Fish, teacher**

Eliciting Student Thinking from Student Written Work

This same principle of asking students questions about the details of their explanations also applies to students' written work. In Ms. Reid's second-grade class, the students had solved a Join (Change Unknown) problem. Ms. Reid noticed that Katie had finished and so she asked her about her work (see Figure 9.3.).

Ms. R: *Katie what did you find out about the Pokémon cards?*
Katie: Her mom got her 33 more.
Ms. R: *I see you have 28, 38, 48, 58 here [points to paper]. Can you tell me about that?*
Katie: I counted up from the 28.
Ms. R: *Okay, what about these tens that you wrote?*
Katie: [points to the 28] 28 to 38 is 10 and 38 to 48 is 10 and 48 to 58 is 10 and then I went 59, 60, 61 and that is 3.

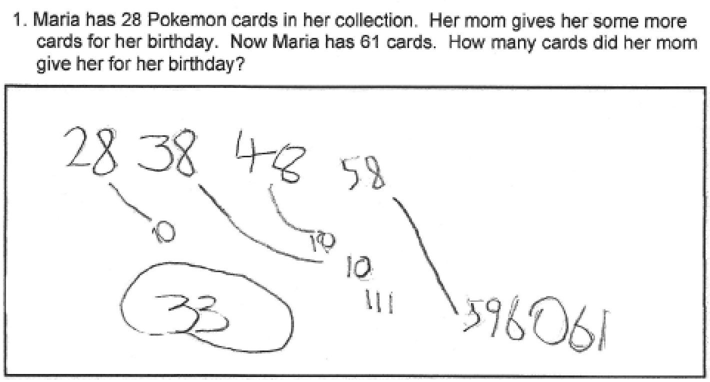

FIGURE 9.3 Katie's written work

Ms. R: So I see how you got the 3. Where did the 30 come from?
Katie: 10 and 10 and 10 is 30.
Ms. R: Nice.

Alex solved the same problem, but his answer was incorrect (Figure 9.4). Ms. Reid stops by Alex's desk to check in with him.

Ms. R: [points to the 28 on Alex's paper] I see you have a 28 written here. Can you tell me about what you were doing with the 28?
Alex: I counted from 28.
Ms. R: Okay, you counted from 28. Can you tell me what numbers you were counting?
Alex: I did 29, 30, 31 . . . 61 [pointing to the tallies on his paper].
Ms. R: I see. How did you get 34?
Alex: [points to his tallies and counts] 5, 10, 15, 20, 25, 30, 31, 32, 33, oh, 33.
Ms. R: So her mom gave her 33. I see you made tallies, how many groups of 5 did you have?
Alex: [points to the tallies of 5 one at a time] 1, 2, 3, 4, 5, 6. 6.
Ms. R: 6 groups of 5. How many groups of 10 are there?
Alex: [points to 2 groups of 5] 1, [points to 2 groups of 5] 2, [points to 2 groups of 5] 3.

As Ms. Reid questioned Alex, he was able to fix his miscount. He also worked through the details of his strategy and explained in words what he had done on paper. Alex had counted on by ones, but he had grouped the ones in groups of 5, which made them easier to count. He could count by fives, but when Ms. Reid asked a specific question about

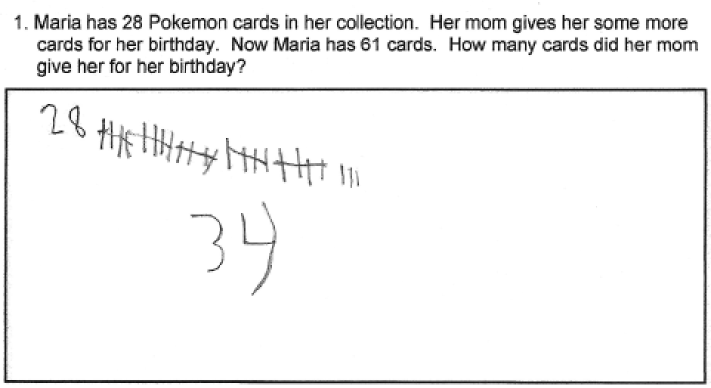

FIGURE 9.4 Alex's written work

groups of 10, he showed that he was able to count by tens. In asking him about tens, Ms. Reid was able to gauge his understanding of tens and get Alex to think about the use of tens in his solution. The use of tens is a goal Ms. Reid has for Alex. Ms. Reid's questioning gave Alex a chance to reflect on his strategy and his representation and begin to extend his ideas.

In each of these examples, the teachers listened to students' mathematical explanations and used what they heard or saw to ask a specific follow-up question to elicit more of the details of the student's strategy. Sometimes the additional student explanation that resulted from the teacher's questioning allowed the teacher to know what strategy the student used, and other times it provided more detail about a mathematical idea central to the strategy. Working through the explanations also gave students the opportunity to work through their mathematical ideas verbally and in written form, allowing them to synthesize and connect ideas and make sense of new mathematical ideas.

While the students in these examples responded to the teachers' questions, there can be times when students do not respond. When you first begin to ask students to share their strategies, your questions may be met with silence. A student's lack of response does not necessarily mean that the student does not have a strategy to share or that the student is unable to share. Sometimes students do not know what you want to hear. Other times, students are worried about saying something wrong.

CLIP 9.6 Ms. Dominguez works with a student who has an error http://smarturl.it/CM9.6

CLIP 9.7 Ms. Hassay works with a student who does not know how to get started http://smarturl.it/CM9.7

If students do not answer your questions at first, consider a different approach to eliciting their thinking. If the student has written something or used some tools, one option is to ask him if he can show you what he did. Another option is to ask the student to show you what she is doing while she is still solving the problem. As students show you how they solved the problem, you can talk about what you notice them doing to help them learn what it means to explain their strategy. For example, if a student starts solving a Join (Result Unknown) problem with 40 and 25 by placing 4 ten-rods in front of her, you could say, "I see that you started with 4 tens. Did you start with 4 tens?" Depending on the student, you could ask, "Why did you start with 4 tens?" You can also ask more specific questions, such as, "What did you do first when you were solving the problem?" "Did you use any tools, your fingers, the cubes, the pencil and paper? Can you tell me what you did with those?"

Some students are able to use strategies that they can not yet explain. This may be because a student's verbal abilities are not yet developed in a way that supports their mathematical explanation or because their understanding of the strategy is not solid enough for them to produce an explanation. In supporting students to explain their strategies, it is wise to be particularly careful not to encourage students to use less sophisticated strategies just because they are easier to explain. You can support students to develop their abilities to explain using the approaches shown above, and in the beginning, allow students to explain a piece of their strategy and to use language that makes sense to them.

Eliciting Student Thinking to Extend Mathematical Ideas

In eliciting students' thinking, CGI teachers also support students to extend their mathematical ideas by inviting them to go beyond what they have shared in some way. To extend or build on a student's mathematical thinking, you can ask questions, provide tools, and suggest different representations to encourage the student to try a new strategy or explore a new idea. Extending student thinking requires that the teacher know what strategies could come next in the development of students' mathematical thinking. In the earlier example, Ms. Reid asked Alex about tens in his solution. His written work showed that he could count on by ones and group them by fives to count the number of tallies he made. Ms. Reid knew that developing tens would advance his strategy use and that he had the beginnings of the idea in his written solution to this problem.

Although it is productive to support students to extend their thinking, it is not all up to you as the teacher. Students extend their ideas on their own, as they explain, use different tools, talk to other students, engage with other students as they share, and so on. It is also important to realize that supporting students to extend their ideas is not the goal of each interaction. You want to support students to extend their ideas when you see evidence that they clearly understand the strategy they are using and are starting to move toward using strategies at the next level. Supporting students to extend their thinking works best when students see themselves in charge of deciding if and when to take up a new idea. This also enables them to develop productive skills and identities as learners of mathematics.

CLIP 9.8 Ms. Grace works with a student to extend a mathematical idea http://smarturl.it/CM9.8

CLIP 9.9 Ms. Barron works with students to extend a mathematical idea http://smarturl.it/CM9.9

Eliciting student thinking starts with regularly asking students to explain their thinking. Consistently asking students to share their thinking, expecting each student to do so, and accepting whatever ideas students can provide will help students come to understand how to do math in your class, what it means to explain their mathematical ideas, and that everyone is capable of explaining mathematics.

As you work on developing your ability to elicit student thinking, consider the principles in Figure 9.5.

Principles for Eliciting Student Thinking
Consistently ask students to share their thinking.
Find ways for each student to explain his thinking to you or other students.
Follow up with specific questions drawing from what the student shared or did.
Support students to work all the way through the details of their strategies.
Ask about correct, incorrect, and incomplete strategies.
Watch for students to tell or show you that they are ready to be supported to adapt their strategy or try a new one.
Listen, observe. Try to not impose your ideas on students.

FIGURE 9.5

Several years ago, I went through my first summer session of CGI. I can vividly remember being slightly overwhelmed and wondering how long it would take for me to be able to slip this to the side and reinstitute "real teaching" in my classroom. While teaching for understanding and not memorization made sense to me, I didn't understand how an eight-year-old's thinking was supposed to guide the direction of an entire lesson. I remember the first day of school that year. It was about 2:30 in the afternoon and amazingly enough, I had run out of all those cutesy, first-of-the-year, "get to know you"–type activities. I had a room full of second-graders ready to pounce and I had to act fast. Out of desperation, I gathered all my little darlings in a circle on the floor, dusted off the Unifix cubes, and passed them around. While I don't remember the problems I gave the students, nor the strategies they used, I do remember their engagement—and I was surprised. Surprised that they had so much to say about exactly how they got their correct or incorrect *answers and shocked that they were so eager to share their ways of solving with their peers. Even after this experience, CGI was by no means cemented in my mind as one of the greatest teaching philosophies of all time, like it is now. I was busy "teaching" so CGI wasn't practiced on a daily or even weekly basis at first. In the beginning, I only "did CGI" when our [math] specialist was coming for a visit or when we were supposed to take student work to our next CGI meeting. However, over time, the more and more I practiced CGI with my students, the less generic and more authentic and genuinely probing my questions became. Because of this, slowly, my students' thinking became deeper and more advanced. While powerful, the change was so subtle, I'm not sure I realized it until the end of the year.*

[The] math specialist asked me to give a specific problem to my students to see what they would do. When I saw the problem, I was more than a little worried.

> *Rachel and her brother are making gift bags for 6 friends. Each bag has 3 pencils and 2 erasers in it. How many total items are there in 1 bag? How many total items are in 6 bags?*

"Oh, come ON!" I remember thinking. This problem involves two different operations, one being multiplication, and is a two-step problem at that. All I could think about was how embarrassed I was going to be when my students bombed this question. The numbers weren't all that large, but I thought that the overall complexity of the problem would surely do them in. Well, you know what? It didn't. In fact, they did amazing. When I got back together with the specialist to look at the student strategies, we were thrilled with the results. That's when she informed me that my students had just solved a fifth-grade problem. A fifth-grade benchmark released item.

It was then that I realized how powerful the work we had been doing with CGI had been. The countless questions and strategies and the hours of understanding the problems and listening to student thinking had paid off, and this was only a piece of the evidence. I then began reflecting

> *on the work we had done that year and thinking about those first days in a circle on the carpet with the Unifix cubes. I began to realize that because the progress my students had made was so subtle and slow, that it was twice as meaningful as all the fast and furious "learning" and intensive remediation we had done in years passed. Since then, and the years that have passed, I find myself thinking every day of how thankful I am for CGI. It not only changed my students, but it changed me, as a teacher, as well.*
>
> **—Tara Sanders, teacher**

USING KNOWLEDGE OF CHILDREN'S THINKING TO GUIDE INSTRUCTION

All of the teaching practices described in this chapter and throughout the book are based on knowledge of children's mathematical thinking, including both knowledge of the research frameworks for how children typically progress and knowledge of the thinking of individual children in your classroom. Decisions about what problem to pose, what numbers to use, what questions to ask, who to ask, whose idea to share, whose idea could be connected to the strategy shared all can be supported by your knowledge of the development of children's mathematical thinking. Each of the teaching examples in this chapter involved teachers considering what they knew about students' mathematical thinking in general and what they knew about their own students' thinking in relation to those general ideas. Ms. Scott described why she posed the problem about Estebán and his marbles. Her rationale was linked to the two kinds of Invented Algorithms she had seen her students use for joining problems and wanting to support the use of similar strategies in a separating context (shown in the following video). Ms. Byron purposefully stopped to check on Samuel as she had noticed that he was still working to make sense of tens and use them to solve problems even though he might not have developed enough understanding to do so. The framework detailed in earlier chapters can help you know what to listen for and how to think about where to go next. We know from our research that even teachers new to CGI can readily listen to their students' mathematical thinking and, as they make sense of this thinking in relation to the framework, they get better and better at using what they hear from their students to make instructional decisions.

CLIP 9.10 Ms. Byron's lesson
http://smarturl.it/CM9.10

I need a framework. I definitely think there's a framework with CGI that's made a big difference for me. Strategies have been identified, there's definitely a hierarchy. That's helped. . . . I mean, a lot of curriculum materials have problem solving. A lot of them do. But you don't know what to do with it. I mean, how do you decide why problems would be more difficult than others for children to solve? You know, what makes this problem difficult? And with CGI, that has been researched, and I think accurately researched, and it enables me to know why certain kids are struggling, what I can do to facilitate that. So I think a framework is critical. For a person like me, especially. I want to know where I'm going and where I'm taking them. So I need that framework.

—Sue Berthouex, teacher