

Section E

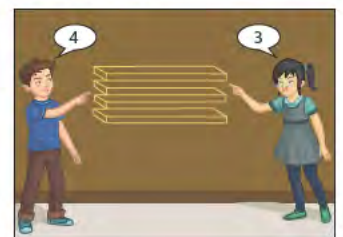
Perspective Taking



Perspective taking refers to the ability to view and understand an object or visual scene from a different point of view; it involves the ability to picture something—in your mind—from a perspective other than your own. It can also involve physically taking a different view of an object or visual scene. Perspective taking has long been an area of interest for psychologists who have studied its various aspects, notably visual, cognitive, and affective (as in empathy) (Dawson & Fernald, 1987). Until recently, however, mathematics education researchers have given relatively limited attention to *spatial* perspective taking, or the ability to imagine ourselves in another place or to physically take another perspective in order to see a different view.

Many math curricula do include some expectations related to perspective taking, most often in the form of sketching or replicating the isolated faces of three-dimensional (3D) structures from the top, front, and side views, or building models when given these three views. These expectations are usually listed for students older than the children we are focusing on in this book (students in Grade 6, for example). However, the researchers and educators in the Math for Young Children (M4YC) project have observed time and again that young children can develop their perspective-taking abilities and that they

FIGURE E.1

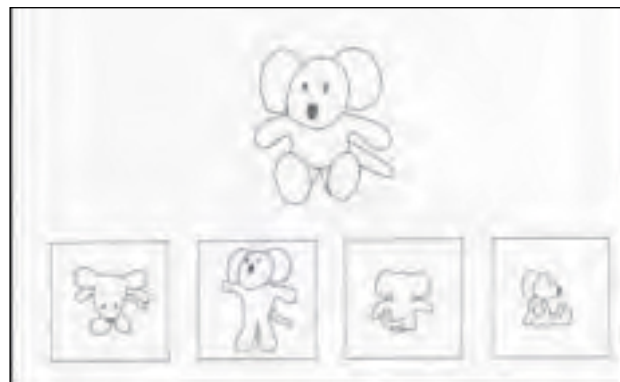


find the tasks engaging. The M4YC research has made it clear that young children possess the ability to develop their perspective-taking skills and that they should be given opportunities to do so. Essentially, perspective taking is a cognitive skill we use in our everyday lives to make sense of the physical world. Recent studies also show that strong abilities in perspective taking relate significantly to overall mathematical ability (van den Heuvel-Panhuizen, Elia, & Robitzsch, 2015).

There are two main types of perspective taking. One involves moving the body or an object to experience a different view; this is sometimes called “embodied perspective taking.” This kind of perspective taking might take the form of a person walking around a car to see it from different angles, or hovering over a desk to see its top view, or rotating a vase of flowers to see the blooms from different perspectives. In these examples, the body moves to take a new perspective, or the object moves while the body stays fixed. The other main type of perspective taking is called “imaginary perspective taking,” where we try to *imagine* what something looks like from a different perspective. Imagine flying over and around a building to see it from different angles. Doing so is a form of imaginary perspective taking.

We know that perspective taking is an important skill, but how do we assess or measure it? How do we know whether children are perspective taking when much of the activity is taking place in the mind? Researchers have developed a number of ways to determine this. For example, van den Heuvel-Panhuizen, Elia, & Robitzsch (2015) have published a paper-and-pencil test involving imaginary perspective taking, which asks children to identify what objects would look like from different views.

FIGURE E.2 “How do you see Mouse if you look at him from above like a bird?” Children answer the question by choosing the picture that shows the correct image.



In this example, children are asked what the mouse would look like from above. The children then point to the picture that they think best matches the top view. In their research with more than 600 Kindergarten students, van den Heuvel-Panhuizen, Elia, & Robitzsch (2015)

have demonstrated that there is significant interaction (a positive relationship) between the ability to answer questions like the one presented in Figure E.2 and overall mathematical ability.

Why Engage Children in These Tasks?

Perspective taking has been typically reserved for older children and adults in the belief that young children are not capable of engaging in and making sense of these activities. Recent research, including the M4YC project, has found this to be inaccurate. Young children can learn to take different perspectives when presented with appropriate tasks and tools. Emerging research, such as the study described above, is beginning to reveal a correlation between perspective taking, spatial reasoning ability, and mathematics achievement.

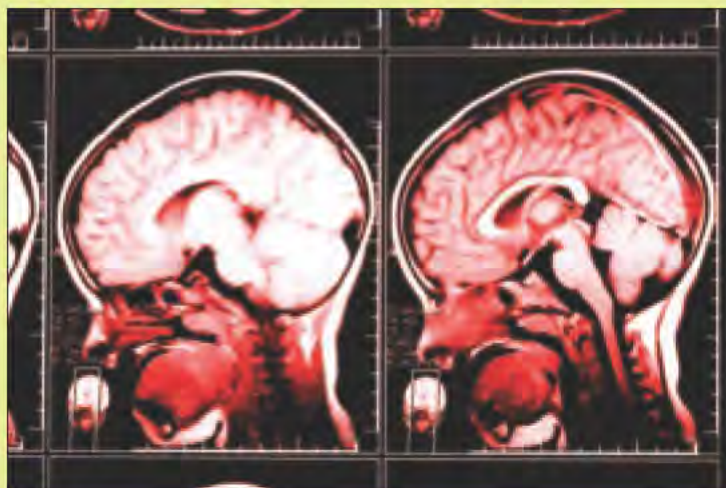
In mathematics classrooms, we know that perspective taking is very important, and the National Council of Teachers of Mathematics (NCTM) recognizes this. The Council's standards of 2000 for Kindergarten to Grade 2 urge educators to pay attention to important spatial skills related to perspective taking, such as the ability to specify locations and interpret relative positions in space, the ability to visualize, and the ability to recognize and represent shapes from different perspectives. Similar statements can be found in curricula and standards around the world, including England, Australia, the Netherlands, and Cyprus. In Ontario, the 2005 mathematics curriculum expects Grade 1 students to “describe the relative locations of objects or people using positional language (e.g., *over, under, above, below, in front of, behind, inside, outside, beside, between, along*)” (p. 37), and by Grade 2, children should “describe the relative locations (e.g., beside, two steps to the right of) and the movements of objects on a map (e.g., ‘The path shows that he walked around the desk, down the aisle, and over to the window.’)” (p. 47). Any time we are describing relative movement or locations (see Section D of this resource), we are almost always engaging in perspective taking, as well. Consider, for example, a child who is looking at some playground equipment. She is lying on the grass under a slide. What does she see? Can she explain her relative location to the slide (even though she is not outside of herself to view this)—that she is under the slide or looking up at the underside of the slide and that the slide is above her? Can she imagine what the slide looks like from above? Now consider a child looking at an apple placed on an acrylic surface. The educator asks, “If you were below the apple looking up, what would you see?” In this example, the child needs to understand location words and ideas as well as imagine himself under the acrylic surface looking up at the base of the apple.

Perspective taking is important to daily human activity, such as giving directions to another person (where one has to imagine where that person is located and is going to), and assembling furniture and toys. Perspective taking is also involved in topographical mapping, orienteering, and almost all way-finding tasks that involve maps. (Most maps typically represent the perspective of a “top view” of countries, cities, and towns.) Furthermore, perspective taking helps with decision-making in careers such as chemistry (when considering chemical compound structures), geography and cartography, engineering, design work and architecture, the arts, and medicine.

Did You Know?

Doctors use perspective taking when viewing and making a diagnosis from X-rays and MRIs (magnetic resonance imaging tests). These types of important visual data must be well understood and interpreted by professionals. There is a standard approach to all imaging: doctors view an image as if they were standing at the patient’s feet, facing the patient. The viewer, in fact, needs to note that the cracked rib on the right side of the digital image is a left rib. Being able to distinguish a visual depiction of “normal” lungs compared to abnormal lungs is crucial. Doctors use the viewing principle that two X-ray or image views are better than one for accuracy of diagnoses. They compare the two views to determine the location of a lesion or anomaly. Views are generally perpendicular to each other. For CT and MRI scans, the same is done by multiple images at different levels. Symmetry and comparison of one side to the other is done for almost all images. Assessing for symmetry of the pelvis or any other area of the body is similarly important in diagnosing fractures and misalignments. All of these examples demonstrate the importance of perspective taking.

Figure E.3 Two images from an MRI series of the head



Section Overview

This section presents lessons and activities to help build children’s perspective-taking ability, most often using playful and imaginative contexts. The section includes one detailed guided lesson that uses a playful narrative about a mother bird flying over a sculpture. The mother bird comes back to her nest and tells her babies what she sees; the children must then consider what the sculpture might look like from different perspectives. The children are given an image of the top view that the mother bird sees and are asked to try to build the sculpture using interlocking cubes with the information that they have. This section also includes a range of shorter tasks that the children can do in small-group settings at the carpet or a centre to further build their perspective-taking skills. In many of these shorter tasks, the children use grids, images, interlocking cubes, or LEGO® bricks to help integrate and isolate different views. The tasks in this section offer multiple entry points and engaging opportunities for children to play with the concept of, and develop the spatial skills of, perspective taking.

Section Content

Lesson Title	Details	Overview
1: Mother Bird Finds a Sculpture page 217	Type: Guided lesson Time: 60 minutes Age Range: JK–Grade 2 Grouping: Whole group and pairs Math and Spatial Reasoning Focus: 2D representations of a 3D structure; perspective taking	Children are introduced to the idea of a “top view” with a story about a mother bird who flies over a sculpture. Children work in pairs to create an object from interlocking cubes based on information about its top view. The class regroups to compare the structures created.
2: Shoebox Goggles page 228	Type: Quick challenge Time: 20–30 minutes Age Range: JK–Grade 2 Grouping: Groups of 3–7 children Math and Spatial Reasoning Focus: perspective taking; spatial language	Children look at a structure from different views looking through a small window, name those views, and discuss what they see.
3: Build It, Photograph It, Make It page 231	Type: Exploratory activity Time: 30 minutes Age Range: JK–Grade 2 Grouping: Whole-class introduction, then a centre activity Math and Spatial Reasoning Focus: perspective taking; connecting 3D objects with 2D representations (photographs)	Children make a structure using interlocking cubes, then photograph it from different perspectives. The printed photographs can be labelled and used by another child as a set of blueprints to build a copy of the structure.

(Continued)

Lesson Title	Details	Overview
4: Crazy Creatures page 235	Type: Quick challenge Time: 15 minutes (may decrease in time with practice) Age Range: JK–Grade 2 Grouping: Individual; alternatively, with the whole-class, or at a centre with educator guidance Math and Spatial Reasoning Focus: perspective taking; composition of 3D structures; translations between 2D representations and 3D objects	Children build a crazy creature based on a photograph. They justify why their creature is the same as the one in the photo, and physically label the top, front, and side of their object to match the views in the photo.
5: What Do You See? page 238	Type: Quick challenge Time: 10 minutes Age Range: JK–Grade 2 Grouping: Whole class to start, then repeated as a quick task at any time Math and Spatial Reasoning Focus: perspective taking/imaginary perspective taking; visualization	Children are asked to imagine what an object would look like when viewed from a particular perspective, and then pick from a selection the drawing that best represents that perspective.
6: Secret LEGO® Buildings page 241	Type: Quick challenge Time: 10 minutes Age Range: JK–Grade 2 Grouping: Whole-class introduction (may take 20 minutes if there is a discussion), then in small groups, as a centre, or at the carpet Math and Spatial Reasoning Focus: perspective taking; composition of 3D objects, visual-spatial working memory	Children build a LEGO® sculpture from memory after viewing it for 5 to 10 seconds.

Mother Bird Finds a Sculpture

Lesson Type	Time	Age Range	Groupings
Guided lesson	60 minutes	JK–Grade 2	Whole group and pairs

Math and Spatial Reasoning Focus

2D representations of a 3D structure; perspective taking

Materials

- Card with top view of a sculpture for each pair of students (see LM E01)
- Bowl of 30 same-coloured interlocking cubes for each pair of students. *Note:* It is important that the cubes are the same size as the squares on the card for later comparison.
- Bird puppet or other toy bird or flying creature
- Flat boards or surfaces for students to work on
- Barriers (made from a cardboard box or other material that can stand alone and block the work of one group from the others). Alternatively, have the children scatter in their pairs around the room to remove the need for barriers.

Overview

This lesson begins with a playful activity and discussion about what a “top view” looks like. The children then listen to a story at the carpet about a mother bird who flies over a sculpture made of cubes and notices the view of the structure from directly overhead—the top view. Back at the nest, she tells her babies what she saw; the children are asked to use their imagination to envision the sculpture just as the baby birds do. The children then work in pairs to create an object from interlocking cubes based on information about the top view of the sculpture. They regroup at the carpet to discuss their mathematical thinking. The children’s attention is then drawn to how the structures created by their peers are the same and different, even though the top view of each is the same.



FIGURE E.4 Identical cube structures placed on the classroom carpet (left); children “flying” over the objects to get a top view (right)



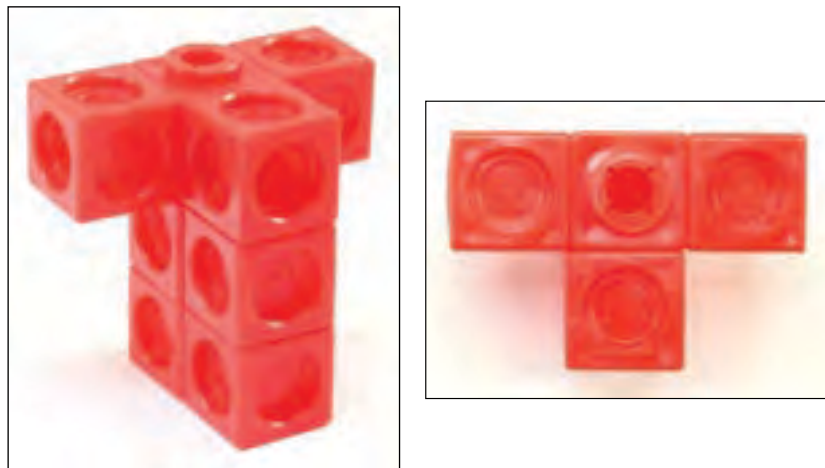
Math Language in Focus

top view, side view, front view, bird's-eye view, over top, snap cubes or interlocking cubes, storeys tall, high, height, wide, numbers 1 through 30, up, down, on the side, perspective, angle, turn, flip, rotate, upside down, right-side up, 2D, 3D

LESSON SEQUENCE

1. Place several identical interlocking cube structures on the floor or carpet. Be sure to use a simple structure such as the one shown in Figure E.5. Invite students to pretend that they are little birds flying over the carpet: *Pretend you are a little bird and the carpet is a park you are flying over. You see the tops of these objects. Let's try that...* [Children get up and "fly" above the objects.] *We will need to be careful not to fly into one another.* Then, ask children to land on the ground (sit on the carpet) and tell a classmate beside them what they saw. Once children have talked about what they saw, regroup them and ask them to tell you what they saw: *Let's talk about the blocks together. When we are looking at an object from above, it is called the "top view." What did you see from that top view?* As children are describing what they saw, you might draw that top view, show it with the blocks, or share a previously taken photograph of the top view with children.

FIGURE E.5 A simple cube structure (left) and the top view of this same structure (right)

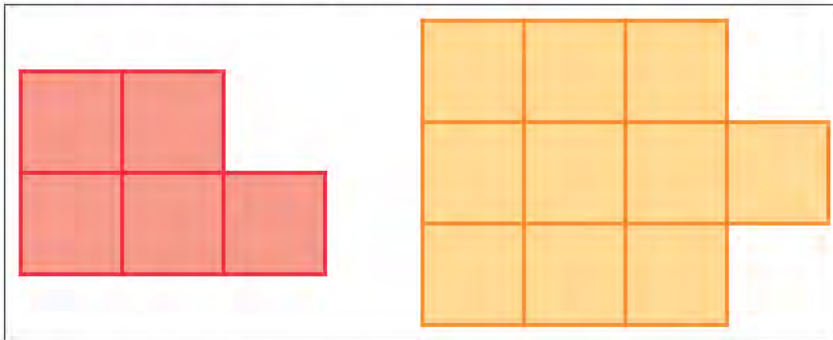


2. Tell children a story along these lines: *A mother bird is carrying some worms for her babies. The babies are hungry and waiting for her back in the nest. Along the way, the mother bird flies over a sculpture and looks down at it from where she is, high up in the sky.* [Using a puppet or other toy bird, show what it was

like for the bird to be flying.] *When the mother bird gets back to her nest, she tells the babies about the sculpture she saw. The babies want to know more. How big was the structure? What did the front and sides look like? Well, she says, I know it was tall and made with cubes, but I'm not sure what it looks like from the front or the sides because I only flew over the top of it. She gives them a picture of the top view of the structure. Here is what she shows them.*

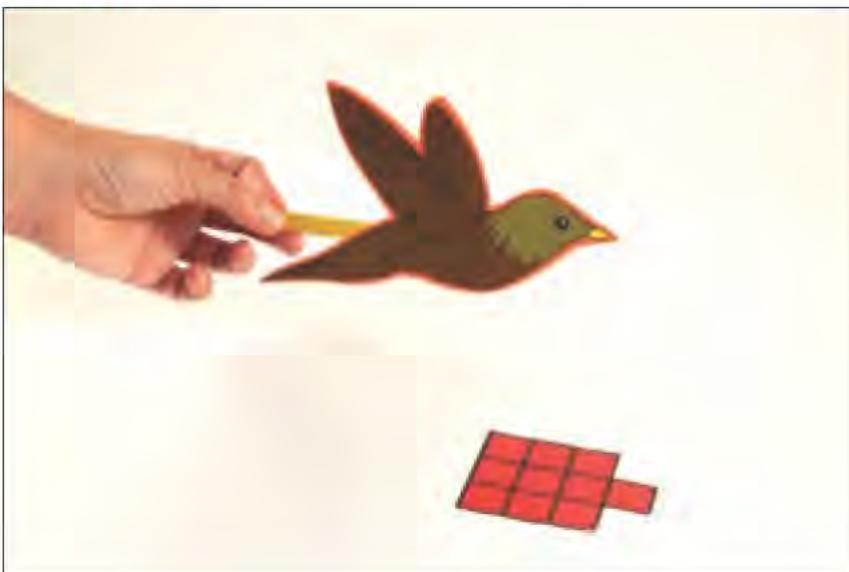
- Place one of the top-view cards down so that the students can see it.

FIGURE E.6 The image on the left shows a top view suitable for use with Junior and Senior Kindergarten classes. The image on the right is suitable for use with students in Grades 1 and 2.



- Say to the class: *I wonder what the sculpture looks like. Working with a partner, can you build something that looks like this when viewed from the top? Think about what it might look like from the front and sides.*

FIGURE E.7



5. Pair up children, and distribute to each pair one top view card, a bowl of 30 same-coloured interlocking cubes, and a barrier. Tell them: *Here is a bowl of cubes you can use to make your sculpture. You don't have to use all of the cubes, but the top view of your sculpture needs to look like the top view in the mother bird's picture. You and your partner also get a barrier. That's so you can keep your ideas a surprise to share with others at the end. Let's spend the next 10 minutes building.* (If you do not want children to use barriers, pairs of students can go to different areas in the classroom so that they can focus on their sculpture without making comparisons to what their peers are doing.)
6. Provide 10–15 minutes for the pairs to build. Circulate, reminding children to make sure that the top view of their sculpture matches the design on their card. You can also remind children that they do not need to use all of the cubes in their bowl. Children will have many questions. Help them focus on the key information: *You have the top view, and you have some cubes to build. Remember: you are trying to make a cube sculpture that you think will have the same top view that the mother bird saw.* Children may ask, for example, about the height of the sculpture. Explain to the students that the mother bird does not know how tall the sculpture is because she could see it only from the top.
7. Once children have a sculpture constructed, be sure to ask them to check their top view one more time.
8. Invite children to take a “gallery walk” around the sculptures that their classmates have constructed and then return to the carpet for sharing with the whole group. *Now let's talk about the sculptures that you made.*

Key Questions

- What were some things you noticed about the other sculptures?
- How are these structures the same?
- Do the top views match?
- How are they different?
- Are all the front [side] views the same?
- How is it possible that we have different structures?

Students will realize that there can be many different structures having the same top view—the structures could be different heights, for example. Students may observe that the top view alone does not provide enough information for everyone to be able to make exactly the same structure. This is an excellent

conclusion to draw. The main parameter of this task—that children use the information from the top view to make a sculpture—is what is known as an “enabling constraint,” a challenge that pushes children’s thinking while also allowing them to be creative and explorative.

In response to the Key Question *What were some things you noticed about the other sculptures?* children in a Grade 1 class gave answers such as these:

- “Ours was very different.”
- “Some people did some things that other people didn’t really notice.”
- “Some people had a different plan for their sculpture.”
- “We all had different sculptures in our head, but from the top we all had the same images.”

What to Look For, What to Listen For: Ongoing Assessment of Student Learning

Watch for gestures and body movement. Do the children make full body movements to take different perspectives, such as hovering over the top of the sculpture, or do they use head tilting as they imagine a different view? Perspective taking is challenging for children, and watching their strategies for doing so is interesting and informative. Pointing gestures may mean that children are referring to a specific feature of the sculpture, while whole-hand gestures mean that they are referring to the object as a whole. All of these may be helping the children to imagine different views of the sculpture.

How do children deal with creating the structure, knowing only the top view? While working, are children reluctant to change any of the cubes because they think it will change the top view? Do some children build the entire structure so that each storey (or layer) is exactly the same, or do they vary the organization of the cubes from layer to layer while maintaining an accurate top view? Do others build the top view like a tabletop and then add columns so that it is not a solid structure? Using the same configuration on each level is a chunking or stacking strategy, which is a way for young children to make sense of a complex structure. If students do not seem to need to do this, they may have a better sense of the overall structure without having to replicate the same configuration on each level.

FIGURE E.8 Each of these three structures has the same top view. As children become more comfortable with this kind of activity, they will understand that structures can be configured differently while still sharing the same top view.



Notice whether some children relate their sculpture to something familiar, such as a dog or table. When children relate a whole shape to another familiar object (sometimes called the “gestalt” of the design), it can reduce the task’s cognitive demand. Relating the shape to a familiar object is a way of summarizing the contents of the design, helping students to remember and replicate a design and reducing the burden of having to pay attention to all the bits of information that make up the figure.

Note whether both members of each pair are contributing equally to building the sculpture. Is one child leading the building while the other watches? This task requires clear communication between the partners to build the sculpture. If the partners are building without discussing, it is possible that one partner will dominate during the building of the sculpture. Mathematical language and communication will also help the children be more precise about their building; encourage the children to talk with their partner as much as possible about what they are doing, and to co-construct the sculpture.

Supporting Learners

Children would benefit from prior experiences with building structures using interlocking cubes. These experiences may include building structures with interlocking cubes during free play. The children may also want to have a “perspective buddy” such as a small, handheld horse or bird that has “special seeing powers.” The children can manipulate the perspective buddy to navigate around a structure to play with the idea of different perspectives.

After the children have developed familiarity with the materials, prompt them to try to replicate pre-built structures. This exercise is a good preparation for the introduction of enabling constraints which put parameters on what children create. For example, you might ask children to use 3 cubes to make a three-storey building; after they have done so, you can then ask them to use 10 cubes to make a three-storey building, adding greater dimension to the structures, as shown in Figure E.9.

FIGURE E.9



Children will also find the task more accessible if there are some prior activities or discussions about the concept of a top view. One literature connection is the picture book *As the Crow Flies: A First Book of Maps* by Gail Hartman. In this story, the main character is a crow that flies over countryside and towns; illustrations in the book show what the crow sees from her perspective (and also explores what useful maps for other creatures might look like). The fact that not all of the images are top views in this story can foster interesting discussions. Other training and exploratory tasks in this perspective-taking section might also support children in accessing the challenges of this guided lesson.

Some students might need to make a direct comparison of their structure against the top view. To do so, they may place the top-view card on top of the structure to see whether the outline fits exactly. Similarly, we have observed students who pick up their sculpture and place it top down on the top-view card to verify that the outline is an exact match. Be sure that the top-view outline uses the same dimensions as your interlocking cubes. Making a copy of Line Master E01 at 100 percent will match the most common 1-inch interlocking cubes.

Do the children have difficulty building a sculpture with only the top-view information? This task may take some time for children to grasp, and they will need problem-solving time in their pairs. If a child, or children, cannot make sense of a bird's-eye view, try the Shoebox Goggles task (page 228).

If children are reluctant to change any of their cubes because they worry that doing so will change the top view, you might tape the top view so that it seems fixed and then encourage the students to modify the lower components.

Finally, if students are not contributing equally and one student is dominating the building exercise, encourage both partners to build together and to discuss as they go. If the dynamic fails to shift, consider providing another bowl of 30 interlocking cubes so that each individual can try building a structure. Encourage the students to continue the discussion in partners, even if they are each working on a sculpture of their own.

Assessment for Reporting

This fairly detailed lesson requires children to consider information about the top view of a cube sculpture, and from that create a full 3D object using cubes. When assessing children during this task, you may notice three stages of understanding that increase in depth and breadth from a beginning stage through to a mastering stage. These observations will help you to assess the understandings of the individual children in the classroom, and to plan for extensions of the activities presented in this lesson. Some of the statements below can be used for reporting purposes.

Beginning Stage

The child is able to compose 3D structures using interlocking cubes but may be confused by the idea of the top view, or, the child is able to recognize and even replicate the top view but has difficulty relating this idea to the whole structure. The child is just beginning to understand that there are different perspectives (such as top, front, and side views) of the same structure. The child is also beginning to recognize the overall design of a structure and can replicate a given structure, but focuses mostly on a structure's individual components or pieces. When presented with different views of the same structure, this child might duplicate each view and then try to merge these smaller structures into one structure. The child is not yet able to meet multiple criteria simultaneously for one structure. (As introduced in the extension tasks, there might be additional parameters to the given top view, such as making the structure a certain number of storeys high or using a set number of blocks.)

Developing Stage

The child is able to compose 3D structures using interlocking cubes and understands that there are different perspectives of the same structure—top, front, and side views among them. The child is able

to describe and match at least one perspective of a structure successfully. The child can combine information about different views to make one structure, but there may be omissions in the design. The child uses the overall look of a structure and connects this to familiar objects, thus making the design more manageable to work with. For example, she might note: “It looks like a table.” The child is able to meet several criteria simultaneously for one structure. (As introduced in the extension tasks, there are additional parameters to the given top view, such as making the structure a certain number of storeys high or using a set number of blocks.)

Mastering Stage

The child is able to compose 3D structures using interlocking cubes and clearly understands and articulates that the same structure has different perspectives (such as top, front, and side views). The child is able to describe and match several views of a structure successfully. The child can combine information about different views to make one structure and manipulates the structure and her or his perspective taking with ease. While still paying attention to detail, the child uses the overall design (the look, or gestalt) of a structure and connects this to familiar objects, making the design more manageable to work with. The child is easily able to meet multiple criteria simultaneously for one structure. (As introduced in the extension tasks, there are additional parameters to the given top view, such as making the structure a certain number of storeys high and using a set number of blocks.)

Extensions and Variations

One Additional Constraint

Once the discussion has occurred and if the children are still engaged, continue on with the story as follows: *The mother bird was curious to see more of the sculpture, and so once all the baby birds were fed and resting, she flew out of the nest and visited the sculpture again. This time she flew a bit lower and circled around the sculpture. What she saw was a side view of the sculpture.*

Use the puppet to show how the mother bird might fly along the side of a building or structure, as shown in Figure E.10 (below). *The mother bird looked at the side of the sculpture and was able to count how tall the sculpture was. She discovered that it was four cubes tall. Now, turn to your partner and look at your sculpture. Without changing your top view, can you make your sculpture four cubes tall?*

Provide children with time to determine how they can make their sculpture 4 cubes tall without changing the top view. Discuss the results.

Key Questions

- Were you able to keep the same top view and make the sculpture four cubes tall?
- Do we all have the same sculptures now? [It is most likely that there will still be many variations to the sculpture that meet the two criteria of height and top view.]
- What did you have to do to your sculpture to make it four cubes tall?

FIGURE E.10



FIGURE E.11 Here is an example of a structure that meets all three key constraints.



Two Additional Constraints

If the children are still engaged, and there is time, another extension to the task is to limit the number of cubes that can be used to still meet the top view and 4-cube-high criteria. Or, this can be done at a later time if the sculptures are kept intact and labelled with the children's names. To introduce this challenge, explain: *Now the mother bird returned to look at the sculpture because the baby birds kept asking more questions. The baby birds really wanted to know how many cubes there were in the sculpture altogether. This time, the mother bird flew out to the sculpture, and she counted all the cubes. There were only 18 cubes in the sculpture! Look again at your sculpture, and try to make it more like the sculpture that the mother bird saw. Can you make it 18 cubes in total—and keep the top view the same, and keep it 4 cubes high?* Children (and adults) find this a very challenging task because they usually have to remove cubes strategically to arrive at only 18, while maintaining the parameters of the same top view and the 4-cube-height rule, which becomes increasingly difficult.

Discuss the different solutions in the children's sculptures. There will likely still be several correct variations. See Figure E.11 (above) for a sample solution.

Key Questions

- Does your sculpture match all the things that the mother bird saw? Does the top view match? Is the sculpture 4 cubes tall? Are there 18 cubes?
- Is everyone's sculpture the same now?
- What parts of this activity did you find challenging?

FROM THE CLASSROOM

In the M4YC research, the following comments have been made by children engaging in this lesson:

- "This makes an awkward sculpture."
- "That was awesome!"
- "If we were birds, we would see a strange building!"
- "They [the views] are all blending together to make one sculpture."

One child talked about what the "back view" might look like. She compared her sculpture to an elephant, noticing the overall gestalt of the sculpture.

One pair of children took a very long time (15 minutes) working on the top view of their sculpture. They wanted to be sure that it was right. Eventually, they decided that they could flip their sculpture onto the top-view card to match the structure and the image.

On observing the variety of structures created by students, one educator commented: "It was nice to see that they understood that the sculptures could be very different and still match the criteria of the task."

Students in one Grade 1 classroom attempted drawings with different views of a car. First, they decided to try drawing the top view of a car. They then attempted a front view. The educator asked, "What is the front view of the car?" The educator remarked at how quickly the children began to draw different views. She stated: "Normally, it is really hard for Grade 1 students to represent something like that, but it transferred so easily from what we've been doing in math. It's exciting to see that it does affect their thinking in other areas." One child in this class decided to draw a front-view picture of her house, family car, and herself.

FIGURE E.12

