

PEARSON

Foundations and Pre-calculus
Mathematics 10

SASKATCHEWAN
CURRICULUM COMPANION

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Saskatchewan Curriculum Companion for Pearson Foundations and Pre-calculus Mathematics 10

Saskatchewan Curriculum Planning Charts

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Lessons to Address Saskatchewan-Specific Content

Teaching Notes

| | |
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| Lesson 1.3b | page 6 |
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Masters

| | | |
|------|--|---------|
| 1.20 | Lesson 1.3b: SI and Imperial Measures of Area and Volume | page 10 |
| 1.21 | Answers for Master 1.20 | page 17 |
| 1.22 | Lesson 1.3b Check Your Understanding Solutions | page 18 |

About This Module

The Saskatchewan Curriculum Framework departs from the WNCP Common Curriculum Framework in a few specific areas, often reflected in the Indicators. Use the planning charts to get general background on such curricular discrepancies. Use the Teaching Notes and Masters for support as you implement the material unique to Saskatchewan.

Chapter 1 Measurement

| Lesson | Curriculum Note |
|---|--|
| 1.1 Imperial Measures of Length | Include an additional probing question to elicit the observation that changing the unit does not change the length. |
| 1.2 Math Lab: Measuring Length and Distance | Include an additional probing question to elicit the observation that changing the unit does not change the length. |
| 1.3 Relating SI and Imperial Units | Include an additional probing question to elicit the observation that changing the unit does not change the length. |
| 1.3b SI and Imperial Measures of Area and Volume | Use Master 1.20, Lesson 1.3b, to cover content required in Saskatchewan only, for FP10.3: Demonstrate understanding of SI and imperial units of measurement... Indicator f: Develop, generalize, explain, and apply strategies and/or formulas for converting between units within the imperial or SI system of measurements [for] ... surface area and volume units Indicator g: Develop, generalize, explain, and apply strategies and/or formulas for converting between: SI and imperial units of ... surface area and volume measure, imperial and SI units of ... surface area and volume measure Also use Master 1.20, Lesson 1.3b, to provide content background that may be required for Saskatchewan outcome FP10.3: Analyze a treaty for its inclusion of measurements ... and create and solve situational questions... |
| Checkpoint 1 | |
| 1.4 Surface Areas of Right Pyramids and Right Cones | |
| 1.5 Volumes of Right Pyramids and Right Cones | |
| 1.6 Surface Area and Volume of a Sphere | |
| Checkpoint 2 | |
| 1.7 Solving Problems Involving Objects | |
| Study Guide and Review | |
| Practice Test | |

Note: Allow students to use calculators as they work through the Exercise questions in this chapter.

Chapter 3 Factors and Products

| Lesson | Curriculum Note |
|---|---|
| 3.1 Factors and Multiples of Whole Numbers | |
| 3.2 Perfect Squares, Perfect Cubes, and Their Roots | The Saskatchewan Grade 10 curriculum also requires students to investigate and report about the numbers 0 and 1 with respect to factors, multiples, square roots, and cube roots. Include this topic as an additional probing question. |
| Checkpoint 1 | |
| 3.3 Common Factors of a Polynomial | |
| 3.4 Math Lab: Modelling Trinomials as Binomial Products | |
| 3.5 Polynomials of the form $ax^2 + bx + c$ | |
| 3.6 Polynomials of the form $x^2 + bx + c$ | |
| Checkpoint 2 | |
| 3.7 Multiplying Polynomials | |
| 3.8 Factoring Special Polynomials | |
| Study Guide and Review | |
| Practice Test | |

Chapter 4 Roots and Powers

| Lesson | Curriculum Note |
|---|--|
| 4.1 Math Lab: Estimating Roots | |
| 4.2 Irrational Numbers | The Saskatchewan Grade 10 curriculum also requires students to create and explain a pattern that describes the decimal form of an irrational number. Present an additional example and include another question to cover this topic. |
| 4.3 Mixed and Entire Radicals | |
| Checkpoint 1 | |
| 4.4 Fractional Exponents and Radicals | |
| 4.5 Negative Exponents and Reciprocals | |
| Checkpoint 2 | |
| 4.6 Applying the Exponent Laws | |
| Study Guide and Review | |
| Practice Test | |
| Project: Algebra and Number <i>Human Calculators</i> | |
| Cumulative Review: Chapters 1–4 | |

1.3b

SI and Imperial Measures of Area and Volume

Lesson Organizer

60 - 75 min

Key Math Learnings

Linear conversion factors can be squared to give conversion factors for area, and cubed to give conversion factors for volume.

Curriculum Focus

Convert area and volume measurements within and between SI units and imperial units.

| SO | AI (SK) |
|--------|---------|
| FP10.3 | f, g |

Processes

C, CN, ME, PS, R, V

Student Materials

- Master 1.20
- linking cubes
- grid paper (optional)
- scientific calculator

Vocabulary

hectare, acre, section

Make Connections, page 10

Every year forest fires destroy a significant area of Saskatchewan's forest. Often, the area destroyed is reported in hectares.

For other units that are used to measure area, students may suggest square units, such as square kilometres, or square feet. Those who live in rural areas may suggest acres, sections, or quarter sections.

To compare the sizes of the units, students may suggest comparing the side length of squares with each area.

Construct Understanding, page 10

If students use centimetre cubes, explain that each cube represents a cube with edge length 1 ft. Remind students of these linear conversion factors: 1 yd. = 3 ft., 1 m = 100 cm, and 1 in. \doteq 2.5 cm. Provide grid paper for students who wish to use it in *Step E*.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- If each linking cube represents a cube with edge length 1 ft., then what is the edge length of the large cube in feet? (3 ft.)
- How can you use the number of cubes that form a face of the cube to determine the area of the face in square feet? (*The area of 1 face of a linking cube is 1 square foot, so the number of cubes that form the face of the large cube equals the area of the face of the cube in square feet.*)
- How can you tell how many square feet are equal to 1 square yard? (*Nine linking cubes form 1 face of the large cube, so the area of the face is 9 square feet. The area of this face is also 1 square yard. So, 9 square feet are equal to 1 square yard.*)
- How can you determine the volume of the large cube in cubic feet? (*The large cube is made of 3 layers with 9 linking cubes in each layer, so it is made up of 3×9 , or 27 linking cubes. Each linking cube has volume 1 cubic foot, so the volume of the large cube is 27 cubic feet.*)

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty determining the volume of the large cube in *Step C*.

- Remind the student that each linking cube represents a volume of 1 cubic foot, so the number of linking cubes in the large cube represents the volume of the cube in cubic yards.

Assessment for Learning

What to look and listen for:

- Students can convert an area or volume from an SI unit to another SI unit, or from an imperial unit to another imperial unit.
- Students can convert an area or volume from an SI unit to an imperial unit, and vice versa.
- Students can convert a measurement of land area from a square unit to hectares or acres, and vice versa.
- Students can use diagrams to explain and support their approach for converting area and volume measurements.
- Students can verify a unit conversion using unit analysis.
- Students can solve problems that involve the conversion between and within SI and imperial units.

DI Extending Thinking

Ask students to assume each linking cube has edge length 1 in. Have them determine the number of linking cubes that would be needed to represent 1 cubic yard. Students should see that they would need $(36)^3$, or 46 656 cubes since $36 \text{ in.} = 1 \text{ yd.}$

Debrief Try This: Assessment for Learning

Have students compare their results with those of other pairs of students. Encourage students to generalize their results for converting area or volume measurements.

Ask questions such as:

- What pattern did you see in *Step D*? Why do you think this is? (*The numbers increased by a factor of 3 each time. Area is a measure in 2 dimensions and volume is a measure in 3 dimensions. For each new dimension, I have to multiply by 3 since 1 yard equals 3 feet.*)
- What strategy did you use to determine the number of square millimetres that were equal to 1 cm^2 in *Step E*? (*I used grid paper. I assumed each grid square had side length 1 mm. Since $10 \text{ mm} = 1 \text{ cm}$, I sketched a square with side length 10 mm and counted the number of grid squares inside the large square: 100. So, $1 \text{ cm}^2 = 100 \text{ mm}^2$.)*)
- What pattern did you see in *Step E*? (*The numbers increased by a factor of 10 each time.*)
- What pattern do you expect to see in *Step F*? Why? (*I know $1 \text{ in.} \doteq 2.5 \text{ cm}$, so I expect the numbers to increase by a factor of approximately 2.5 each time.*)
- How many square feet are in 2 square yards? How can you tell? (*Two square yards is the area of 2 faces of the large cube from Step B. So, $2 \text{ square yards} = 2 \times 9$, or 18 square feet.*)
- How many cubic feet are in 3 square yards? How can you tell? (*Three cubic yards is the volume of 3 large cubes from Step B. So, $3 \text{ cubic yards} = 3 \times 27$, or 81 cubic feet.*)

After *Example 1*, the surface area of the box is calculated in square metres to determine a conversion factor for square metres and square centimetres. You may also wish to calculate the volume of sand in the box in cubic metres to determine a conversion factor for cubic metres and cubic centimetres.

DI Extending Thinking

- Have students use the exact linear conversion ratio $1 \text{ in.} = 2.54 \text{ cm}$ to determine exact conversion ratios for square inches and square centimetres, and cubic inches and cubic centimetres.
- Have students investigate prefixes used with the SI system of measures, including: milli-, centi-, deci-, hecta-, and kilo-

DI Identifying Common Difficulties: Possible Intervention

The student uses a linear conversion ratio to convert an area measure or a volume measure, without squaring or cubing the ratio.

- Have the student calculate the surface area or volume of an object by first converting the units. Then, have the student convert the units after calculating its surface area or volume to see that they must square or cube the conversion ratio to get the same answer using this strategy.

Curriculum Note

Before *Example 4*, new units of area, the hectare (ha) and the acre, are introduced. You may also wish to develop and/or present the approximate conversion factor for hectares and acres: $1 \text{ ha} \approx 2.471 \text{ acres}$. This section prepares students for Saskatchewan curriculum outcome FP10.3, indicator m, where they analyze a treaty for its inclusion of area measurements, and create and solve situational questions.

DI Cultural Perspectives

- The hectare is based on the “are”, a unit of area equal to 100 m^2 . It is not officially used in the SI system, but it is still used in some other countries to measure areas of land.
- In the land survey system of Saskatchewan, the land is divided into squares of various sizes. The basic unit is a section, which is a square with area 1 square mile, or 640 acres. Thirty-six sections form a township. One-quarter of a section, or 160 acres, is a quarter section. Legal subdivisions (LSDs) typically have an area of one-sixteenth of a section, or 40 acres.
- The government of Saskatchewan provides resources related to treaty education on their web site. See: <http://k-12treatyeducation.gov.sk.ca/Support-Resources/> (accessed Dec. 2010).

Assessing Understanding: Discuss the Ideas, page 15

Sample Responses

- 1. An advantage of converting the dimensions of an object before calculating its surface area and volume is that it is easy to convert linear units because we have conversion factors for many of them. Once the dimensions are converted, they can be used for both the surface area and volume calculation. A disadvantage is that you may have to perform more conversions.*
- 2. I can use linear conversion factors to create conversion factors for area or volume measures. I square the linear conversion factors to create conversion factors for area measures, and I cube the linear conversion factors to create conversion factors for volume measures.*
- 3. I might need to convert units of area to compare areas of land, or to determine the number of cans of paint I need to buy to cover an area measured in square feet when the paint coverage is given in square metres. I might need to convert units of volume to determine how much concrete I need for the foundation of a building when the building measurements are given in SI units, and the volume of concrete I need to order is given in imperial units.*

Exercises, page 15

Solutions for conversions between SI and imperial units may vary depending on the conversion ratios used. For this exercise set, the answers are given using the exact conversion ratio: 1 in = 2.54 cm.

Assessment of Learning – selected questions

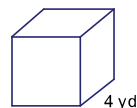
| AI (SK) | f | g |
|-----------------|---|----|
| Question number | 9 | 10 |

Sample Response: Reflect, page 16

For a cube with edge length 4 yd.:

$$\text{Surface Area: } 6(4 \text{ yd.})^2 = 96 \text{ yd.}^2$$

$$\text{Volume: } (4 \text{ yd.})^3 = 64 \text{ yd.}^3$$



A conversion ratio for yards and feet is: 1 yd. = 3 ft.

To convert 96 yd.^2 to square feet, use the square of the conversion ratio for yards and feet:

$$96 \text{ yd.}^2 = 96(1 \text{ yd.})^2$$

$$96 \text{ yd.}^2 = 96(3 \text{ ft.})^2$$

$$96 \text{ yd.}^2 = 96(9 \text{ ft.}^2)$$

$$96 \text{ yd.}^2 = 864 \text{ ft.}^2$$

The surface area of the cube is 864 square feet.

To convert 64 yd.^3 to cubic feet, use the cube of the conversion ratio for yards and feet:

$$64 \text{ yd.}^3 = 64(1 \text{ yd.})^3$$

$$64 \text{ yd.}^3 = 64(3 \text{ ft.})^3$$

$$64 \text{ yd.}^3 = 64(27 \text{ ft.}^3)$$

$$64 \text{ yd.}^3 = 1728 \text{ ft.}^3$$

The volume of the cube is 1728 cubic feet.

1.3b SI and Imperial Measures of Area and Volume

Lesson Focus: Convert measurements of area and volume within and between SI units and imperial units.

Make Connections

In June 2009, a forest fire destroyed 2000 hectares of forest land near Canwood, Saskatchewan. One hectare is a unit of area equal to the area of a square with side length 100 m. What other units are used to measure area? How could you compare the sizes of the units?

Construct Understanding

Try This

Work with a partner. You will need about 30 linking cubes.

Assume that each cube has edge length 1 ft.

- A.** What is the volume of a cube in cubic feet?
What is the area of one face of the cube in square feet?
- B.** Use linking cubes to build a cube with edge length 1 yd.
What is the area of a square face of the cube in square yards?
How many linking cubes form a square face?
What is the area of the square face in square feet?
- C.** Use the cube you built in *Step B*.
How many linking cubes form the cube?
What is the volume of the cube in cubic feet?
- D.** Complete each statement below:
 $1 \text{ yd.} = \underline{\quad} \text{ ft.}$
 $1 \text{ yd.}^2 = \underline{\quad} \text{ ft.}^2$
 $1 \text{ yd.}^3 = \underline{\quad} \text{ ft.}^3$
 Describe the pattern you see.
 Explain why this relationship exists.
- E.** Complete each statement below.
Describe the strategy you used.
 $1 \text{ cm} = \underline{\quad} \text{ mm}$
 $1 \text{ cm}^2 = \underline{\quad} \text{ mm}^2$
 $1 \text{ cm}^3 = \underline{\quad} \text{ mm}^3$
- F.** What number do you think completes each statement below?
Justify your answers.
 $1 \text{ in.}^2 = \underline{\quad} \text{ cm}^2$
 $1 \text{ in.}^3 = \underline{\quad} \text{ cm}^3$

Workers must often use the system of measurement that is the standard in their industry. For example, a contractor planning a residential project may decide to use imperial units because most construction materials are sold in imperial units.

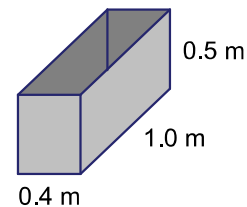
In some situations, a person may be required to use measures from both SI and imperial systems of measurement. For example, in Canada, a contractor planning a commercial project must refer to building codes in SI units. However, she will likely purchase construction materials sold in imperial units.

In both of these situations, it is necessary to have strategies for converting area, surface area, or volume units. One strategy is to convert the given dimensions before performing any calculations.

EXAMPLE 1 Converting Units Before Calculating Area and Volume

This open rectangular box is filled with sand.

- What is the surface area of the exterior of the box in square centimetres?
- What is the volume of sand in the box in cubic centimetres?



Solution

- Convert each dimension to centimetres.

$$\begin{aligned} 0.4 \text{ m} &= 0.4(100 \text{ cm}) \\ &= 40 \text{ cm} \end{aligned}$$

$$\begin{aligned} 1.0 \text{ m} &= 1.0(100 \text{ cm}) \\ &= 100 \text{ cm} \end{aligned}$$

$$\begin{aligned} 0.5 \text{ m} &= 0.5(100 \text{ cm}) \\ &= 50 \text{ cm} \end{aligned}$$

The surface area, SA , of the exterior of the box is:

$$SA = 2(40 \text{ cm})(50 \text{ cm}) + 2(100 \text{ cm})(50 \text{ cm}) + (40 \text{ cm})(100 \text{ cm})$$

$$SA = 4000 \text{ cm}^2 + 10\,000 \text{ cm}^2 + 4000 \text{ cm}^2$$

$$SA = 18\,000 \text{ cm}^2$$

The exterior of the box has surface area $18\,000 \text{ cm}^2$.

- Use the dimensions of the box in centimetres from part a.

The volume, V , of the box is:

$$V = (\text{length})(\text{width})(\text{height})$$

$$V = (100 \text{ cm})(40 \text{ cm})(50 \text{ cm})$$

$$V = 200\,000 \text{ cm}^3$$

The volume of sand in the box is $200\,000 \text{ cm}^3$.

CHECK YOUR UNDERSTANDING

A cube has edge length 0.5 m .

- What is the surface area of the cube in square centimetres?
- What is the volume of the cube in cubic centimetres?

[Answers: a) $15\,000 \text{ cm}^2$ b) $125\,000 \text{ cm}^3$]

Determine the surface area of the box in *Example 1* in square metres.

The surface area, SA , of the exterior of the box is:

$$SA = 2(0.4 \text{ m})(0.5 \text{ m}) + 2(1.0 \text{ m})(0.5 \text{ m}) + (0.4 \text{ m})(1.0 \text{ m})$$

$$SA = 0.4 \text{ m}^2 + 1.0 \text{ m}^2 + 0.4 \text{ m}^2$$

$$SA = 1.8 \text{ m}^2$$

The surface area of the box is 1.8 m^2

Since the box is the same, the surface areas must be equal.

$$1.8 \text{ m}^2 = 18\,000 \text{ cm}^2$$

Divide each side of this equation by 1.8 to determine a conversion factor for square metres and square centimetres.

$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$

We can determine this conversion factor directly.

We know: $1 \text{ m} = 100 \text{ cm}$

$$\text{So, } (1 \text{ m})^2 = (100 \text{ cm})^2$$

$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$

In general, any conversion factor for linear units of measure can be squared to give a conversion factor for square units and cubed to give a conversion factor for cubic units.

EXAMPLE 2 Calculating Area and Volume Before Converting Units

A cylindrical block is 15 cm high, with base radius 5 cm.

- What is its surface area, to the nearest square inch?
- What is its volume, to the nearest cubic inch?

Solution

- Calculate the surface area of the block in square centimetres.

Use the formula for the surface area, SA , of a cylinder with radius r and height h .

$$SA = 2\pi r^2 + 2\pi rh$$

$$SA = 2\pi(5 \text{ cm})^2 + 2\pi(5 \text{ cm})(15 \text{ cm})$$

$$SA = 50\pi \text{ cm}^2 + 150\pi \text{ cm}^2$$

$$SA = 200\pi \text{ cm}^2$$

Convert $200\pi \text{ cm}^2$ to square inches.

$$\text{We know: } 1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$$

$$\text{So, } (1 \text{ cm})^2 \doteq \left(\frac{4}{10} \text{ in.}\right)^2$$

$$1 \text{ cm}^2 \doteq \frac{16}{100} \text{ in.}^2$$

$$\text{And, } 200\pi \text{ cm}^2 \doteq 200\pi \left(\frac{16}{100}\right) \text{ in.}^2$$

$$200\pi \text{ cm}^2 \doteq 100.5309\dots \text{ in.}^2$$

The surface area of the block is approximately 101 square inches.

- b) Calculate the volume of the block in cubic centimetres.

Use the formula for the volume, V , of a cylinder with radius r and height h .

$$V = \pi r^2 h$$

$$V = \pi(5 \text{ cm})^2(15 \text{ cm})$$

$$V = 375\pi \text{ cm}^3$$

Convert $375\pi \text{ cm}^3$ to cubic inches.

$$\text{We know: } 1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$$

$$\text{So, } (1 \text{ cm})^3 \doteq \left(\frac{4}{10} \text{ in.}\right)^3$$

$$1 \text{ cm}^3 \doteq \frac{64}{1000} \text{ in.}^3$$

$$\text{And, } 375\pi \text{ cm}^3 \doteq 375\pi \left(\frac{64}{1000}\right) \text{ in.}^3$$

$$375\pi \text{ cm}^3 \doteq 75.3982\dots \text{ in.}^3$$

The volume of the block is approximately 75 cubic inches.

CHECK YOUR UNDERSTANDING

A rectangular block has dimensions 20 cm by 25 cm by 40 cm.

- a) What is its surface area, to the nearest square inch?
 b) What is its volume, to the nearest cubic inch?

[Answers: a) 736 square inches b) 1280 cubic inches]

When only the area or volume of an object is given, we must use a conversion factor for units of area or volume.

EXAMPLE 3 Converting Volume Measurements

The volume of air in a 4-person hot air balloon is 2200 m^3 .

- a) Determine this volume to the nearest thousand cubic feet.
 b) Use unit analysis to verify the conversion.

Solution

- a) Convert 2200 m^3 to cubic feet.

$$\text{We know: } 1 \text{ m} \doteq 3\frac{1}{4} \text{ ft.}$$

$$1 \text{ m} \doteq \frac{13}{4} \text{ ft.}$$

$$\text{So, } (1 \text{ m})^3 \doteq \left(\frac{13}{4} \text{ ft.}\right)^3$$

$$1 \text{ m}^3 \doteq \left(\frac{13}{4}\right)^3 \text{ ft.}^3$$

$$\text{And, } 2200 \text{ m}^3 \doteq 2200 \left(\frac{13}{4}\right)^3 \text{ ft.}^3$$

$$2200 \text{ m}^3 \doteq 75\,521.9 \text{ ft.}^3$$

The volume of the hot air balloon is approximately 76 000 cubic feet.

We could also have written the conversion factor as:

$$1 \text{ m}^3 \doteq \frac{2197}{64} \text{ ft.}^3$$

- b) Use unit analysis.

Write a conversion factor for feet and metres, with feet in the numerator: $\frac{3\frac{1}{4} \text{ ft.}}{1 \text{ m}}$

$$\begin{aligned} \text{Then, } 2200 \text{ m}^3 \times \left(\frac{3.25 \text{ ft.}}{1 \text{ m}}\right)^3 &\doteq 2200 \text{ m}^3 \times \frac{34.328125 \text{ ft.}^3}{1 \text{ m}^3} \\ &\doteq 75\,521.9 \text{ ft.}^3 \end{aligned}$$

Since this measurement is equal to the measurement in part a, the conversion is verified.

CHECK YOUR UNDERSTANDING

The volume of air in a 12-person hot air balloon is 4000 m^3 .

- a) Determine this volume to the nearest ten thousand cubic feet.
b) Use unit analysis to verify the conversion.

[Answer: a) Approximately 140 000 cubic feet]

Land area is often measured in hectares, acres, and sections.

The hectare is a unit of area used with the SI system. It is represented by the symbol ha.

$$1 \text{ ha} = 10\,000 \text{ m}^2$$

$$1 \text{ km}^2 = 100 \text{ ha}$$

Acres and sections are units of area used in the Imperial system.

$$1 \text{ acre} = 4840 \text{ square yards}$$

$$1 \text{ square mile} = 640 \text{ acres}$$

$$1 \text{ section} = 1 \text{ square mile}$$

EXAMPLE 4 Converting Area Measurements

The total area of Saskatchewan is $651\,900 \text{ km}^2$.

- a) Saskatchewan's forests cover approximately 33.9 million hectares.
What percent of Saskatchewan's area is covered by forest?
b) In 2006, Saskatchewan had approximately 64.3 million acres of farmland.
What percent of Saskatchewan was farmland?

Solution

To compare the areas, convert one measurement so the units are the same.

- a) To convert 33 900 000 ha to square kilometres, divide by 100.

$$\begin{aligned} 33\,900\,000 \text{ ha} &= \frac{33\,900\,000}{100} \text{ km}^2 \\ &= 339\,000 \text{ km}^2 \end{aligned}$$

The area covered by forest, as a fraction of Saskatchewan's total area, is:

$$\frac{339\,000 \text{ km}^2}{651\,900 \text{ km}^2} = 0.5200\dots$$

Write this decimal as a percent: $0.5200\dots \times 100\% \doteq 52\%$

Approximately 52% of Saskatchewan is covered by forest.

- b) First, convert 64 300 000 acres to square miles.

$$64\,300\,000 \text{ acres} = \frac{64\,300\,000}{640} \text{ mi.}^2$$

$$= 100\,468.75 \text{ mi.}^2$$

Convert 100 468.75 square miles to square kilometres.

We know: $1 \text{ mi.} \doteq 1.6 \text{ km}$
 So, $(1 \text{ mi.})^2 \doteq (1.6 \text{ km})^2$
 $1 \text{ mi.}^2 \doteq 2.56 \text{ km}^2$

And, $100\,468.75 \text{ mi.}^2 \doteq 100\,468.75(2.56 \text{ km}^2)$
 $\doteq 257\,200 \text{ km}^2$

The area of farmland in 2006, as a fraction of Saskatchewan's total area, was:

$$\frac{257\,200 \text{ km}^2}{651\,900 \text{ km}^2} = 0.3945\dots$$

Write this decimal as a percent: $0.3945\dots \times 100\% \doteq 39\%$

In 2006, approximately 39% of Saskatchewan was farmland.

CHECK YOUR UNDERSTANDING

The total area of Alberta is 661 848 km².

- a) Alberta's forests cover approximately 38 million hectares.
 What percent of Alberta's area is covered by forest?
- b) In 2006, Alberta had 52.1 million acres of farmland.
 What percent of Alberta was farmland?

[Answers: a) Approximately 57% b) Approximately 31%]

Discuss the Ideas

1. What are some advantages and disadvantages of converting the dimensions of an object before calculating its surface area and volume?
2. How do linear conversion factors help you convert area and volume measures?
3. When might you need to convert units of area or volume?

Exercises

A

4. Convert each measurement.
 - a) 1 mm² to square centimetres
 - b) 1 square mile to square yards
 - c) 1 km³ to cubic metres
 - d) 1 cubic inch to cubic feet
5. Convert each measurement.
 - a) 1 square foot to the nearest hundred square centimetres
 - b) 1 m² to the nearest hundred square inches
 - c) 1 cubic inch to the nearest cubic centimetre
 - d) 1 km³ to the nearest tenth of a cubic mile

B

6. Convert each measurement.
 - a) 2 km² to square metres
 - b) 3 square feet to square inches
 - c) 1.5 cm³ to cubic millimetres
 - d) $\frac{1}{2}$ cubic mile to cubic yards
7. Convert each measurement.
 - a) 11 square yards to the nearest square metre
 - b) 19 km² to the nearest square mile
 - c) 4 cubic feet to the nearest ten thousand cubic centimetres
 - d) 6 m³ to the nearest cubic yard

8. Use a diagram to explain why 1 m^3 does not equal 100 cm^3 .
9. A rectangular box has dimensions 3 ft. by 1 ft. by 2 ft.
- What is the surface area of the box in square inches?
 - What is the volume of air inside the box in cubic inches?
10. A pillar candle is a cylinder with height 14 cm and base radius 2 cm.
- Determine the surface area of the candle to the nearest square inch.
 - What is the volume of the candle to the nearest cubic inch?
11. The area of an NHL hockey rink is approximately 16 327 square feet. The area of an international hockey rink is approximately 1814.9 m^2 .
- Determine the area of an NHL rink to the nearest hundred square metres.
 - Which surface is larger? Explain how you know.
12. The Great Sand Hills of Saskatchewan are a region of arid plains and sand dunes that cover 1900 km^2 . The Cypress Hills Dark-Sky Preserve in Saskatchewan covers 36 600 ha.
- Which region is larger?
 - Use unit analysis to verify the conversion.
13. In 1995, Saskatchewan's production of natural gas peaked at 8.7 billion cubic metres. What is this volume to the nearest hundred billion cubic feet?
14. In the late 1800s, Bell Farm covered approximately 53 000 acres and was the largest farm of continuous land in the world. In 2006, the average size of a Saskatchewan farm was 587 ha. How many times larger was Bell Farm than an average farm in 2006? Give your answer to the nearest whole number.
15. The Addison Sod House near Kindersley, Saskatchewan, was built between 1909 and 1911, and was occupied until 2006. The walls of a sod house were constructed by overlapping pieces of sod. Each piece of sod was approximately 4 in. deep, 3 ft. wide, and 5 ft. long.
- What is the volume of a typical piece of sod in cubic feet?
 - Suppose 200 pieces of sod were used to construct the walls of a house. Determine the volume of sod that was used, to the nearest cubic metre.
16. A portable storage container is a rectangular prism with dimensions 16 ft. 5 in. by 7 ft. 3 in. by 7 ft. 3 in.
- Determine the volume of the container in cubic inches.
 - Moving companies estimate that approximately 200 cubic feet of storage are needed per room. How many rooms could be moved using one of these storage containers?
- C**
17. Holly would like to change the flooring in a bedroom. The floor is rectangular and measures 5.6 m by 4 m.
- Carpet costs $\$59/\text{yd}^2$. Determine the cost of carpeting the floor.
 - Laminate costs $\$6/\text{ft}^2$. Determine the cost of using laminate to cover the floor.
18. Mulch is sold in increments of one-half of a cubic yard.
- The recommended mulch depth is 3 in. To the nearest square foot, what area can be covered by 1 cubic yard of mulch at this depth?
 - Jon needs to cover an area of 12 m^2 in mulch. How much mulch should he buy?

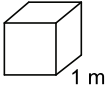
Reflect

Using an example, explain how to convert surface area units and volume units.

Answers for Master 1.20

Master 1.20, for Saskatchewan Lesson 1.3b

Answers may vary based on conversions ratios used.

4. **a)** 0.01 cm^2 **b)** 3 097 600 square yards
c) 1 000 000 000 m^3 **d)** $\frac{1}{1728}$ cubic foot
5. **a)** Approximately 900 cm^2 **b)** Approximately 1500 square inches
c) Approximately 16 cm^3 **d)** Approximately $\frac{2}{10}$ cubic mile
6. **a)** $2\,000\,000 \text{ m}^2$ **b)** 432 square inches
c) 1500 mm^2 **d)** 2 725 888 000 cubic yards
7. **a)** Approximately 9 m^2 **b)** Approximately 7 square miles
c) Approximately $110\,000 \text{ cm}^2$ **d)** Approximately 8 cubic yards
8. Consider a cube with volume 1 m^3 .
The cube has edge length 1 m, or 100 cm.
So, the volume of the cube in centimetres is:
 $(100 \text{ cm})^3$, or $1\,000\,000 \text{ cm}^3$.
- 
9. **a)** 3168 square inches **b)** 10 368 cubic inches
10. **a)** Approximately 32 square inches
b) Approximately 11 cubic inches
11. **a)** Approximately 1500 m^2
b) The international hockey rink has the larger surface.
 $1814.9 \text{ m}^2 > 1500 \text{ m}^2$
12. **a)** The Great Sand Hills are the larger region.
13. Approximately 300 billion cubic feet
14. Approximately 37 times larger
15. **a)** 5 cubic feet of sod **b)** Approximately 28 m^3
16. **a)** 1 491 093 cubic inches **b)** 4 rooms
17. **a)** Approximately \$1593 **b)** Approximately \$1452
18. **a)** 108 square feet **b)** $1\frac{1}{2}$ cubic yards of mulch

1. A cube has edge length 0.5 m.
- What is the surface area of the cube in square centimetres?
 - What is the volume of the cube in cubic centimetres?

Solution

- a) Convert the edge length to centimetres.

$$\begin{aligned}0.5 \text{ m} &= 0.5(100 \text{ cm}) \\ &= 50 \text{ cm}\end{aligned}$$

The surface area, SA , of the exterior of the box is:

$$SA = 6(50 \text{ cm})(50 \text{ cm})$$

$$SA = 15\,000 \text{ cm}^2$$

The exterior of the box has surface area $15\,000 \text{ cm}^2$.

- b) Use the edge length of the cube in centimetres from part a.

The volume, V , of the box is:

$$V = (\text{edge length})^3$$

$$V = (50 \text{ cm})^3$$

$$V = 125\,000 \text{ cm}^3$$

The volume of sand in the box is $125\,000 \text{ cm}^3$.

2. A rectangular block has dimensions 20 cm by 25 cm by 40 cm.
- What is its surface area, to the nearest square inch?
 - What is its volume, to the nearest cubic inch?

Solution

- a) Calculate the surface area of the block in square centimetres.

The surface area, SA , of the block is:

$$SA = 2(20 \text{ cm})(25 \text{ cm}) + 2(20 \text{ cm})(40 \text{ cm}) + 2(25 \text{ cm})(40 \text{ cm})$$

$$SA = 1000 \text{ cm}^2 + 1600 \text{ cm}^2 + 2000 \text{ cm}^2$$

$$SA = 4600 \text{ cm}^2$$

Convert 4600 cm^2 to square inches.

$$\text{We know: } 1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$$

$$\text{So, } (1 \text{ cm})^2 \doteq \left(\frac{4}{10} \text{ in.}\right)^2$$

$$1 \text{ cm}^2 \doteq \frac{16}{100} \text{ in.}^2$$

$$\text{And, } 4600 \text{ cm}^2 \doteq 4600 \left(\frac{16}{100}\right) \text{ in.}^2$$

$$4600 \text{ cm}^2 \doteq 736 \text{ in.}^2$$

The surface area of the block is approximately 736 square inches.

- b) Calculate the volume of the block in cubic centimetres.

The volume, V , of the block is:

$$V = (\text{length})(\text{width})(\text{height})$$

$$V = (20 \text{ cm})(25 \text{ cm})(40 \text{ cm})$$

$$V = 20\,000 \text{ cm}^3$$

Convert $20\,000 \text{ cm}^3$ to cubic inches.

$$\text{We know: } 1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$$

$$\text{So, } (1 \text{ cm})^3 \doteq \left(\frac{4}{10} \text{ in.}\right)^3$$

$$1 \text{ cm}^3 \doteq \frac{64}{1000} \text{ in.}^3$$

$$\text{And, } 20\,000 \text{ cm}^3 \doteq 20\,000 \left(\frac{64}{1000}\right) \text{ in.}^3$$

$$20\,000 \text{ cm}^3 \doteq 1280 \text{ in.}^3$$

The volume of the block is approximately 1280 cubic inches.

3. The volume of air in a 12-person hot air balloon is 4000 m^3 .
- Determine this volume to the nearest ten thousand cubic feet.
 - Use unit analysis to verify the conversion.

Solution

- a) Convert 4000 m^3 to cubic feet.

$$\text{We know: } 1 \text{ m} \doteq 3\frac{1}{4} \text{ ft.}$$

$$\text{So, } (1 \text{ m})^3 \doteq \left(3\frac{1}{4} \text{ ft.}\right)^3$$

$$1 \text{ m}^3 \doteq \left(3\frac{1}{4}\right)^3 \text{ ft.}^3$$

$$\text{And, } 4000 \text{ m}^3 \doteq 4000 \left(3\frac{1}{4}\right)^3 \text{ ft.}^3$$

$$4000 \text{ m}^3 \doteq 137\,313 \text{ ft.}^3$$

The volume of the hot air balloon is approximately 140 000 cubic feet.

- b) Use unit analysis.

Write a conversion factor for feet and metres, with feet in the numerator: $\frac{3\frac{1}{4} \text{ ft.}}{1 \text{ m}}$

$$\begin{aligned} \text{Then, } 4000 \text{ m}^3 \times \left(\frac{3.25 \text{ ft.}}{1 \text{ m}}\right)^3 &\doteq 4000 \text{ m}^3 \times \frac{34.328125 \text{ ft.}^3}{1 \text{ m}^3} \\ &\doteq 137\,313 \text{ ft.}^3 \end{aligned}$$

Since this measurement is equal to the measurement in part a, the conversion is verified.

4. The total area of Alberta is $661\,848\text{ km}^2$.
- Alberta's forests cover approximately 38 million hectares.
What percent of Alberta's area is covered by forest?
 - In 2006, Alberta had 52.1 million acres of farmland.
What percent of Alberta's area was farmland?

Solution

To compare the areas, convert one measurement so the units are the same.

- a) To convert 38 000 000 ha to square kilometres, divide by 100.

$$\begin{aligned} 38\,000\,000\text{ ha} &= \frac{38\,000\,000}{100}\text{ km}^2 \\ &= 380\,000\text{ km}^2 \end{aligned}$$

The area covered by forest, as a fraction of Alberta's total area, is:

$$\frac{380\,000\text{ km}^2}{661\,848\text{ km}^2} = 0.574\,15$$

Write this decimal as a percent: $0.574\,15 \times 100\% \doteq 57\%$

Approximately 57% of Alberta is covered by forest.

- b) First, convert 52 100 000 acres to square miles.

$$\begin{aligned} 52\,100\,000\text{ acres} &= \frac{52\,100\,000}{640}\text{ mi.}^2 \\ &= 81\,406.3\text{ mi.}^2 \end{aligned}$$

Convert 81 406.3 square miles to square kilometres.

We know: $1\text{ mi.} \doteq 1.6\text{ km}$

So, $(1\text{ mi.})^2 \doteq (1.6\text{ km})^2$
 $1\text{ mi.}^2 \doteq 2.56\text{ km}^2$

And, $81\,406.3\text{ mi.}^2 \doteq 81\,406.3(2.56\text{ km}^2)$

$$\doteq 208\,400\text{ km}^2$$

The area of farmland in 2006, as a fraction of Alberta's total area, was:

$$\frac{208\,400\text{ km}^2}{661\,848\text{ km}^2} = 0.314\,876$$

Write this decimal as a percent: $0.314\,876 \times 100\% \doteq 31\%$

In 2006, approximately 31% of Alberta was farmland.


PEARSON

Foundations and Pre-calculus Mathematics 10

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