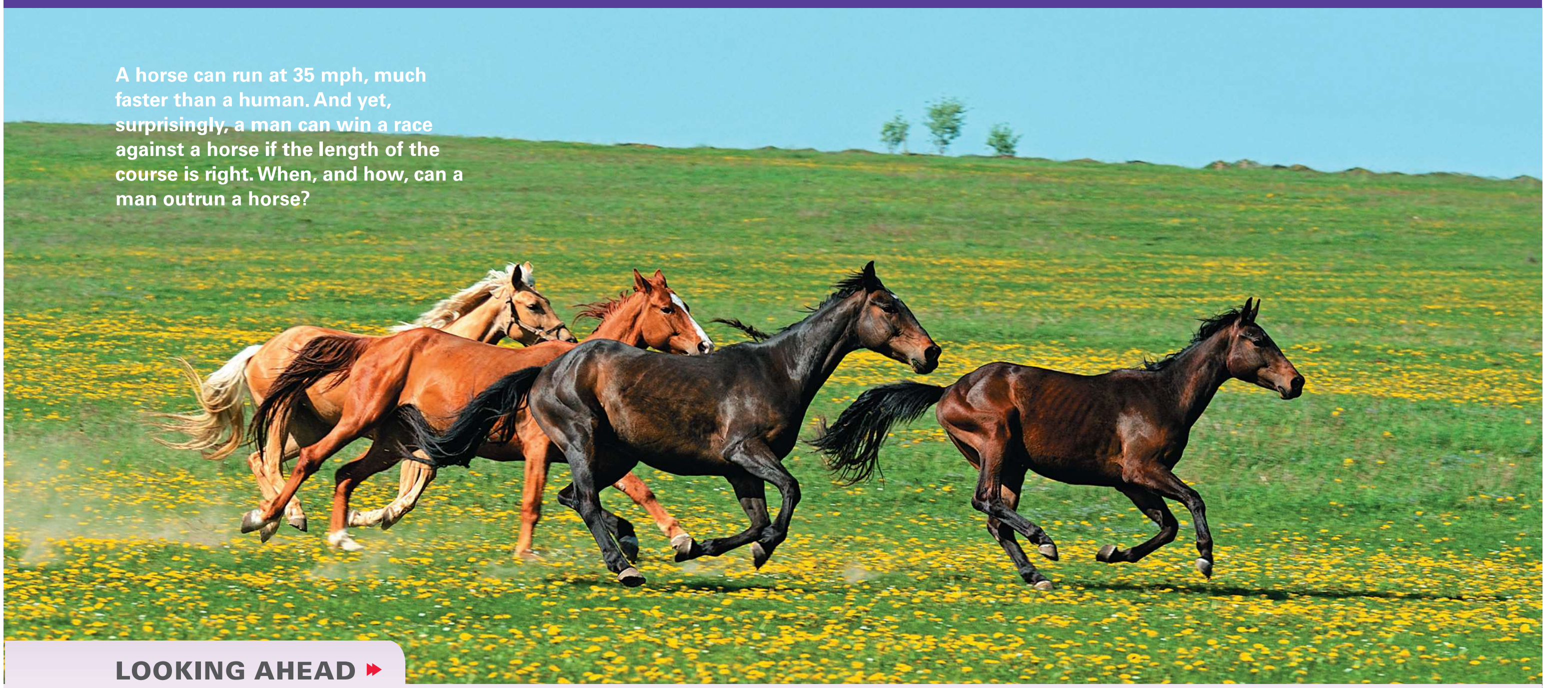
2 Motion in One Dimension



A horse can run at 35 mph, much faster than a human. And yet, surprisingly, a man can win a race against a horse if the length of the course is right. When, and how, can a man outrun a horse?

LOOKING AHEAD

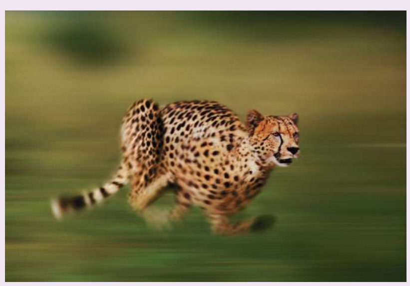
Uniform Motion

Successive images of the Segway rider are the same distance apart, so his velocity is constant. This is **uniform motion**.

You'll learn to describe motion in terms of quantities such as distance and velocity, an important first step in analyzing motion.

Acceleration

A cheetah is capable of running at very high speeds but, more important, it is capable of a rapid *change* in speed—a large **acceleration.**



You'll use the concept of acceleration to solve problems of changing velocity, such as races or predators chasing prey.

Free Fall

When the diver jumps, his motion—both going up and coming down—is determined by gravity alone. We call this **free fall.**

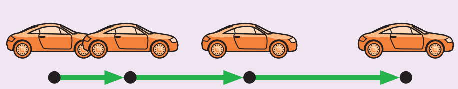
How long will it take this diver to reach the water? This is the type of free-fall problem you'll learn to solve.

GOAL To describe and analyze motion along a line.

LOOKING BACK

Motion Diagrams

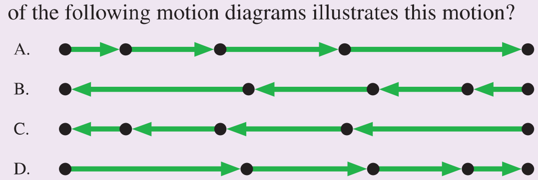
As you saw in Section 1.6, a good first step in analyzing motion is to draw a motion diagram, marking the position of an object at successive times.



In this chapter, you'll learn to create motion diagrams for different types of motion along a line. Drawing pictures like this is a good starting point for solving problems.

STOP TO THINK

A bicycle is moving to the left with increasing speed. Which of the following motion diagrams illustrates this motion?



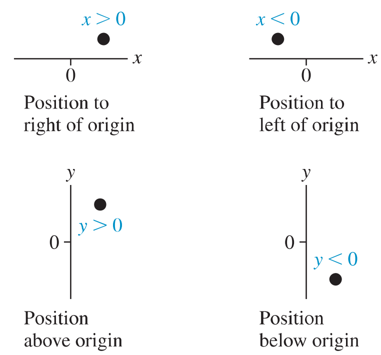
2.1 Describing Motion

In this chapter, we'll focus on how to *describe* motion using several different representations, including motion diagrams, graphs, and mathematical equations. We will defer a treatment of *why* objects move as they do until Chapter 4. The branch of physics that deals with the description of motion is **kinematics,** from the Greek word *kinema,* meaning "movement." You know this word through its English variation *cinema*—motion pictures!

Representing Position

As we saw in Chapter 1, *kinematic variables* such as position and velocity are measured with respect to a coordinate system, an axis that *you* impose on a system. We will use an x-axis to analyze both horizontal motion and motion on a ramp; a y-axis will be used for vertical motion. We will adopt the convention that the positive end of an x-axis is to the right and the positive end of a y-axis is up. This convention is illustrated in FIGURE 2.1.

FIGURE 2.1 Sign conventions for position.



NOTE ▸ The conventions illustrated in Figure 2.1 aren't absolute. In most cases, we are free to define the coordinate system, and doing so in this standardized way makes sense. In some cases, though, we'll want to make a different choice. ◂

Now, let's look at a practical problem. FIGURE 2.2 is a motion diagram of a straightforward situation, a student walking to school. She is moving horizontally, so we use the variable x to describe her motion. We have set the origin of the coordinate system, $x = 0$, at her starting position, and we measure her position in meters. We have included velocity vectors connecting successive positions on the motion diagram, as we saw we could do in Chapter 1.

FIGURE 2.2 The motion diagram of a student walking to school and a coordinate axis for making measurements.

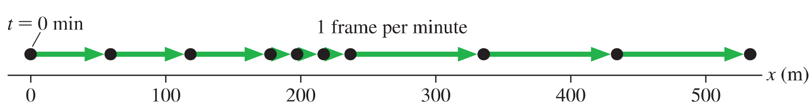


TABLE 2.1 Measured positions of a student walking to school

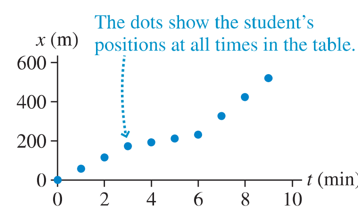
Time t (min)	Position x (m)	Time t (min)	Position x (m)
0	0	5	220
1	60	6	240
2	120	7	340
3	180	8	440
4	200	9	540

The motion diagram shows that she leaves home at a time we choose to call $t = 0$ min, and then makes steady progress for a while. Beginning at $t = 3$ min there is a period in which the distance traveled during each time interval becomes shorter—perhaps she slowed down to speak with a friend. Then, at $t = 6$ min, the distances traveled within each interval are longer—perhaps, realizing she is running late, she begins walking more quickly.

Every dot in the motion diagram of Figure 2.2 represents the student's position at a particular time. For example, the student is at position $x = 120$ m at $t = 2$ min. TABLE 2.1 lists her position for every point in the motion diagram.

The motion diagram of Figure 2.2 is one way to represent the student's motion. Presenting the data as in Table 2.1 is a second way to represent this motion. A third way to represent the motion is to use the data to make a graph. FIGURE 2.3 is a graph of the positions of the student at different times; we say it is a graph of x versus t for the student.

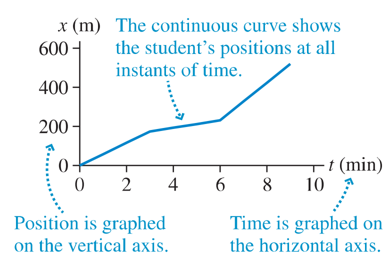
FIGURE 2.3 A graph of the student's motion.



NOTE ▸ A graph of "a versus b" means that a is graphed on the vertical axis and b on the horizontal axis. We say that such a graph represents a "as a function of" b. ◂

We can flesh out the graph of Figure 2.3, though. We can assume that the student moved *continuously* through all intervening points of space, so we can represent her motion as a continuous curve that passes through the measured points, as shown in FIGURE 2.4. Such a continuous curve that shows an object's position as a function of time is called a **position-versus-time graph** or, sometimes, just a *position graph.*

FIGURE 2.4 Extending the graph of Figure 2.3 to a position-versus-time graph.

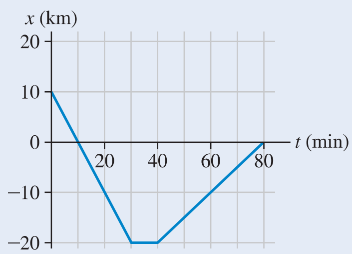


NOTE ▶ A graph is *not* a "picture" of the motion. The student is walking along a straight line, but the graph itself is not a straight line. Further, we've graphed her position on the vertical axis even though her motion is horizontal. A graph is an *abstract representation* of motion. ◀

CONCEPTUAL EXAMPLE 2.1 **Interpreting a car's position-versus-time graph**

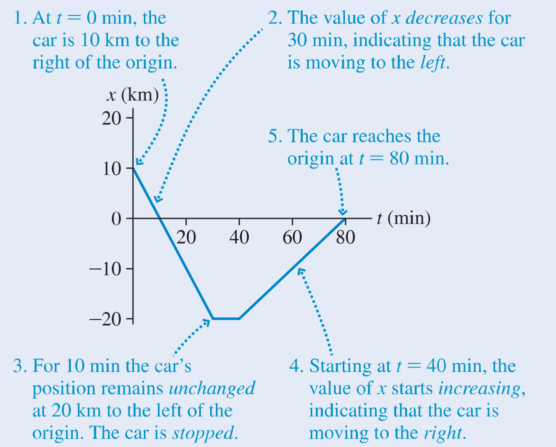
The graph in FIGURE 2.5 represents the motion of a car along a straight road. Describe (in words) the motion of the car.

FIGURE 2.5 Position-versus-time graph for the car.



REASON The vertical axis in Figure 2.5 is labeled "x (km)," so the car's position is measured in kilometers. Our convention for motion along the x-axis given in Figure 2.1 tells us that x increases as the car moves to the right and x decreases as the car moves to the left. As FIGURE 2.6 explains in detail, the graph thus shows that the car travels to the left for 30 minutes, stops for 10 minutes, then travels to the right for 40 minutes. It ends up 10 km to the left of where it began.

FIGURE 2.6 Looking at the position-versus-time graph in detail.



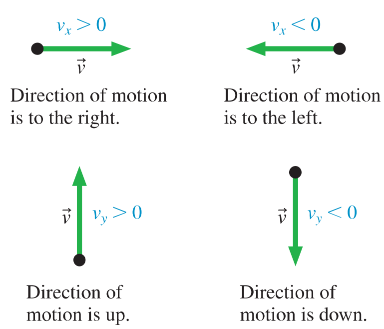
ASSESS The car travels to the left for 30 minutes and to the right for 40 minutes. Nonetheless, it ends up to the left of where it started. This means that the car was moving faster when it was moving to the left than when it was moving to the right. We can deduce this fact from the graph as well, as we will see in the next section.

Representing Velocity

Velocity is a vector, having both a magnitude and a direction. When we draw a general velocity vector on a diagram, we use an arrow labeled with the symbol $\vec{v}$.

For motion in one dimension, however, velocity vectors are restricted to point only forward or backward for horizontal motion, or up or down for vertical motion. This restriction lets us simplify our notation for velocity vectors in one dimension. When we solve problems for motion along an x-axis, we represent the velocity with the simple variable v_x. As FIGURE 2.7 shows, we adopt the convention that for an object moving to the right, v_x is positive, whereas for motion to the left, v_x is negative.

FIGURE 2.7 Sign conventions for velocity.



For vertical motion along the y-axis, we use the symbol v_y to represent the velocity. The sign conventions for this vertical motion are also shown in Figure 2.7.

We use the symbol v, with no subscript, to represent the *speed* of an object. **Speed is the *magnitude* of the velocity vector** and is thus always positive.

In Chapter 1 we defined an object's velocity as $\Delta x/\Delta t$, where $\Delta x = x_f - x_i$ is the *displacement*, or change in position, as the object moves from an initial position x_i to a final position x_f, and Δt is the interval of time during which the motion occurs. For motion along a horizontal line, we can write

$$v_x = \frac{\Delta x}{\Delta t} \tag{2.1}$$

This agrees with the sign conventions in Figure 2.7. If Δx is positive, x is increasing, the object is moving to the right, and Equation 2.1 gives a positive value for velocity. If Δx is negative, x is decreasing, the object is moving to the left, and Equation 2.1 gives a negative value for velocity.

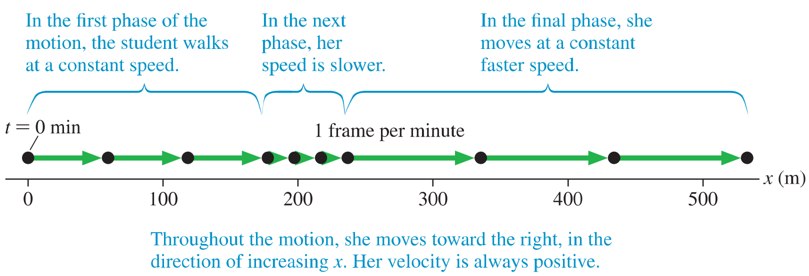
Equation 2.1 is the first of many *kinematic equations* we'll see in this chapter. We'll often specify equations in terms of the coordinate x, but if the motion is vertical, in which case we use the coordinate y, the equations can be easily adapted. For example, Equation 2.1 for motion along a vertical axis becomes

$$v_y = \frac{\Delta y}{\Delta t} \tag{2.2}$$

From Position to Velocity

How is an object's velocity related to its position-versus-time graph? To find out, let's take another look at the motion diagram of the student walking to school. As we see in FIGURE 2.8, where we have repeated the motion diagram of Figure 2.2, her motion has three clearly defined phases. In each phase her speed is constant (because the velocity vectors have the same length) but the speed varies from phase to phase.

FIGURE 2.8 Revisiting the motion diagram of the student walking to school.



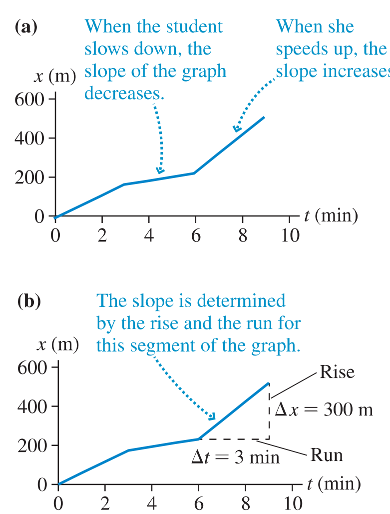
Just as her motion has three different phases, her position-versus-time graph (redrawn in FIGURE 2.9a) has three clearly defined segments with three different *slopes*. We can see that there's a relationship between her speed and the slope of the graph: **A faster speed corresponds to a steeper slope.**

The correspondence is actually deeper than this. Let's look at the slope of the third segment of the position-versus-time graph, as shown in FIGURE 2.9b. The slope of a graph is defined as the ratio of the "rise," the vertical change, to the "run," the horizontal change. For the segment of the graph shown, the slope is

$$\text{slope of graph} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

This ratio has a physical meaning—it's the velocity, exactly as we defined it in Equation 2.1. We've shown this correspondence for one particular graph, but it is a general principle: **The slope of an object's position-versus-time graph is the object's velocity at that point in the motion.** This principle also holds for negative slopes, which correspond to negative velocities. We can associate the slope of a position-versus-time graph, a *geometrical* quantity, with velocity, a *physical* quantity.

FIGURE 2.9 Interpreting the slope of the position graph for the student walking to school.



TACTICS BOX 2.1 Interpreting position-versus-time graphs

Information about motion can be obtained from position-versus-time graphs as follows:

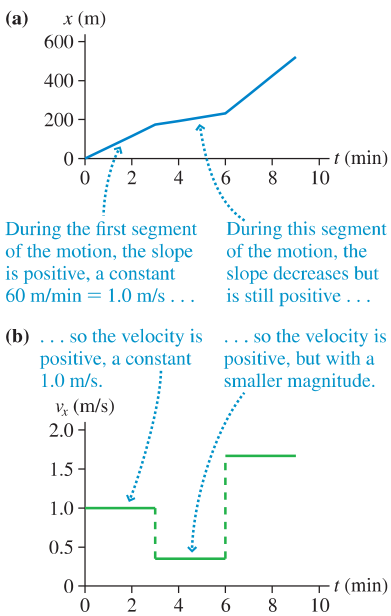
1. Determine an object's *position* at time t by reading the graph at that instant of time.
2. Determine the object's *velocity* at time t by finding the slope of the position graph at that point. Steeper slopes correspond to faster speeds.
3. Determine the *direction of motion* by noting the sign of the slope. Positive slopes correspond to positive velocities and, hence, to motion to the right (or up). Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).

Exercises 3,4

NOTE ▶ The slope is a ratio of intervals, $\Delta x/\Delta t$, not a ratio of coordinates; that is, the slope is *not* simply x/t. ◀

NOTE ▶ We are distinguishing between the actual slope—the slope on the graph—and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope we are referring to when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope. ◀

FIGURE 2.10 Deducing the velocity-versus-time graph from the position-versus-time graph.



We can now use the approach of Tactics Box 2.1 to analyze the student's position-versus-time graph, redrawn in FIGURE 2.10a. We can determine her velocity during the first phase of her motion by measuring the slope of the line:

$$v_x = \text{slope} = \frac{\Delta x}{\Delta t} = \frac{180\text{ m}}{3.0\text{ min}} = 60\,\frac{\text{m}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = 1.0\text{ m/s}$$

In completing this calculation, we've converted to more usual units for speed, m/s. During this phase of the motion, her velocity is constant, so a graph of velocity versus time appears as a horizontal line at 1.0 m/s, as shown in FIGURE 2.10b. We can do similar calculations to show that her velocity during the second phase of her motion is +0.33 m/s, and then increases to +1.7 m/s during the final phase. We combine this information to create the **velocity-versus-time graph** shown in Figure 2.10b.

An inspection of the velocity-versus-time graph shows that it matches our understanding of the student's motion: There are three phases of the motion, each with constant speed. In each phase, the velocity is positive because she is always moving to the right. The second phase is slow (low velocity) and the third phase is fast (high velocity). All of this can be clearly seen on the velocity-versus-time graph, which is yet another way to represent her motion.

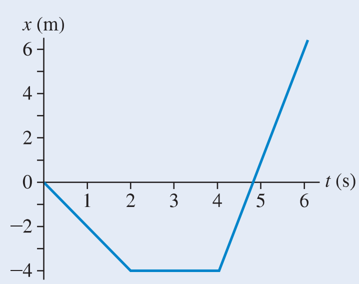
NOTE ▶ The velocity-versus-time graph in Figure 2.10b includes vertical segments in which the velocity changes instantaneously. Such rapid changes are an idealization; it actually takes at least a small amount of time to change velocity. ◀

EXAMPLE 2.2 Finding a car's velocity graph from its position graph

FIGURE 2.11 gives the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.
b. Describe the car's motion in words.

FIGURE 2.11 The position-versus-time graph of a car.



STRATEGIZE We will use the steps from Tactics Box 2.1 to understand the car's motion and to draw its velocity-versus-time graph based on its position graph.

PREPARE Figure 2.11 is a graphical representation of the motion. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

SOLVE

a. From $t = 0$ s to $t = 2$ s $(\Delta t = 2\text{ s})$ the car's displacement is $\Delta x = -4\text{ m} - 0\text{ m} = -4\text{ m}$. The velocity during this interval is

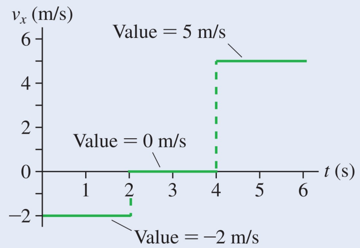
$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4\text{ m}}{2\text{ s}} = -2\text{ m/s}$$

The car's position does not change from $t = 2$ s to $t = 4$ s $(\Delta x = 0\text{ m})$, so $v_x = 0$ m/s. Finally, the displacement between $t = 4$ s and $t = 6$ s $(\Delta t = 2\text{ s})$ is $\Delta x = 10$ m. Thus the velocity during this interval is

$$v_x = \frac{10\text{ m}}{2\text{ s}} = 5\text{ m/s}$$

These velocities are represented graphically in **FIGURE 2.12.**

FIGURE 2.12 The velocity-versus-time graph for the car.



b. The velocity-versus-time graph of Figure 2.12 shows the motion in a way that we can describe in a straightforward manner: The car backs up for 2 s at 2 m/s, sits at rest for 2 s, then drives forward at 5 m/s for 2 s.

ASSESS Notice that the velocity graph and the position graph look completely different. They should! The value of the velocity graph at any instant of time equals the *slope* of the position graph. Since the position graph is made up of segments of constant slope, the velocity graph should be made up of segments of constant *value*, as it is. This gives us confidence that the graph we have drawn is correct.

From Velocity to Position

We've now seen how to move between different representations of uniform motion. There's one last issue to address: If you have a graph of velocity versus time, how can you determine the position graph?

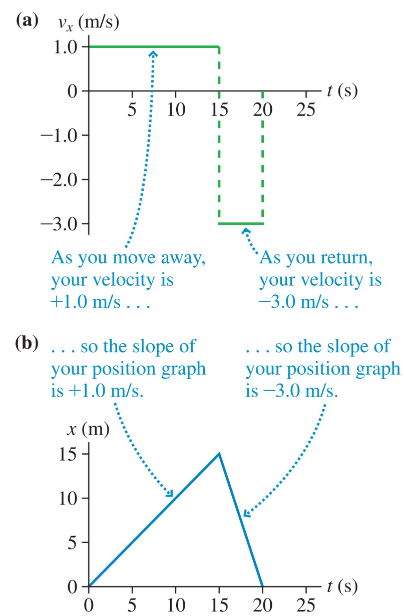
Suppose you leave a lecture hall and begin walking toward your next class, which is down the hall to the east. You then realize that you left your textbook at your seat. You turn around and run back to the lecture hall to retrieve it. A velocity-versus-time graph for this motion appears in **FIGURE 2.13a.** There are two clear phases to the motion: walking away from the lecture hall (velocity +1.0 m/s) and running back (velocity −3.0 m/s). How can we deduce your position-versus-time graph?

As before, we can analyze the graph segment by segment, as shown in Figure 2.13. For the first segment, the velocity graph in Figure 2.13a indicates motion with a constant velocity of +1.0 m/s. This tells us that the corresponding position graph must be a straight line with a positive slope of +1.0 m/s, as shown in the position graph of **FIGURE 2.13b.** For the second segment, where the velocity is −3.0 m/s, as in Figure 2.13a, the position graph must be a line with a negative slope of −3.0 m/s, also shown in the position graph of Figure 2.13b.

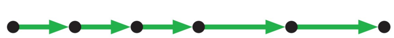
The position graph makes sense: It shows 15 seconds of slowly increasing position (walking away from the lecture hall) and then 5 seconds of rapidly decreasing position (running back). And you end up back where you started.

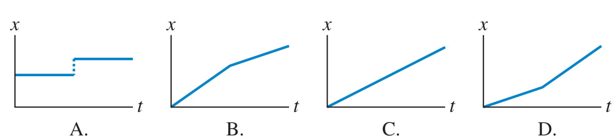
There's one important detail that we didn't talk about in the preceding paragraph: How did we know that the position graph started at $x = 0$ m? The velocity graph tells us the *slope* of the position graph, but it doesn't tell us where the position graph should start. Although you're free to select any point you choose as the origin of the coordinate system, here it seems reasonable to set $x = 0$ m at your starting point in the lecture hall; as you walk away, your position increases.

FIGURE 2.13 Deducing a position graph from a velocity-versus-time graph.



STOP TO THINK 2.1 Which position-versus-time graph best describes the motion diagram at left?





2.2 Uniform Motion

If you drive your car on a straight road at a perfectly steady 60 miles per hour (mph), you will cover 60 mi during the first hour, another 60 mi during the second hour, yet another 60 mi during the third hour, and so on. This is an example of what we call *uniform motion.* **Straight-line motion in which equal displacements occur during any successive equal-time intervals is called uniform motion or constant-velocity motion.**

NOTE ▶ The qualifier "any" is important. If for each successive hour of a trip you drive 120 mph for 30 min and stop for 30 min, you will cover 60 mi during each successive 1 hour interval. But you will *not* have equal displacements during successive 30 min intervals, so this motion is not uniform. ◀

FIGURE 2.14 Motion diagram and position-versus-time graph for uniform motion.

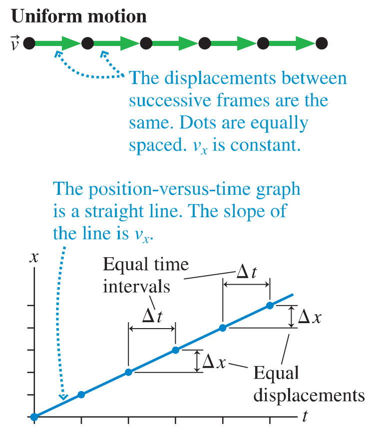


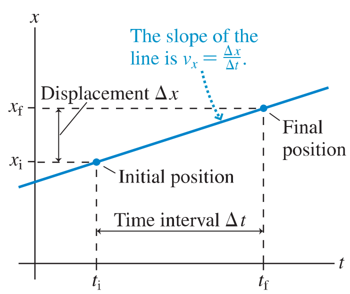
FIGURE 2.14 shows a motion diagram and a position-versus-time graph for an object in uniform motion. Notice that the position-versus-time graph for uniform motion is a straight line. This follows from the requirement that all values of the displacement Δx corresponding to the same time interval Δt be equal. In fact, an alternative definition of uniform motion is: **An object's motion is uniform if and only if its position-versus-time graph is a straight line.**

Equations of Uniform Motion

We've just learned that an object in uniform motion along the x-axis will have a linear (straight-line) position-versus-time graph like the one shown in FIGURE 2.15. Recall from Chapter 1 that we denote the object's initial position as x_i at time t_i. The term "initial" refers to the starting point of our analysis or the starting point in a problem. The object may or may not have been in motion prior to t_i. We use the term "final" for the ending point of our analysis or the ending point of a problem, and denote the object's final position x_f at the time t_f. As we've seen, the object's velocity v_x along the x-axis can be determined by finding the slope of the graph:

$$v_x = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \tag{2.3}$$

FIGURE 2.15 Position-versus-time graph for an object in uniform motion.



Here, the displacement $\Delta x = x_f - x_i$ is the change in position that occurs during the time interval $\Delta t = t_f - t_i$. Equation 2.3 can be rearranged to give

$$x_f = x_i + v_x \, \Delta t \tag{2.4}$$

Position equation for an object in uniform motion (v_x is constant)

where Δt is the interval of time in which the object moves from position x_i to position x_f. Equation 2.4 applies to any time interval Δt during which the velocity is constant. We can also write this in terms of the object's displacement Δx:

$$\Delta x = v_x \, \Delta t \tag{2.5}$$

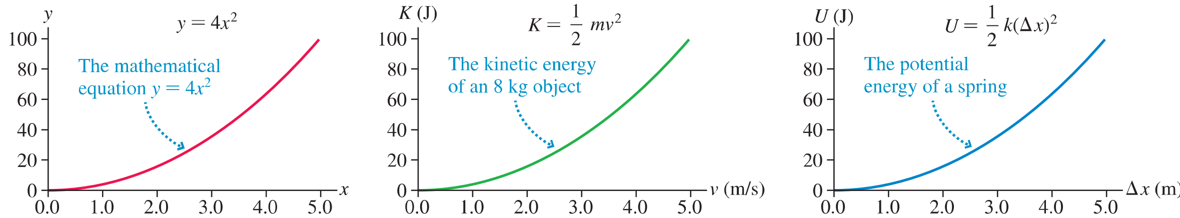
The velocity of an object in uniform motion tells us the amount by which its position changes during each second. An object with a velocity of 20 m/s *changes* its position by 20 m during every second of motion: by 20 m during the first second of its motion, by another 20 m during the next second, and so on. We say that position is changing at the *rate* of 20 m/s. If the object starts at $x_i = 10$ m, it will be at $x = 30$ m after 1 s of motion and at $x = 50$ m after 2 s of motion. Thinking of velocity like this will help you develop an intuitive understanding of the connection between velocity and position.

Mathematical Relationships

Physics may seem densely populated with equations, but most equations follow a few basic forms. FIGURE 2.16 shows three graphs: a mathematical equation, the kinetic energy of a moving object versus its speed, and the potential energy of a spring versus how far the spring is compressed.

All of these graphs have the same overall appearance. The three expressions differ in their variables, but all three equations have the same **mathematical**

FIGURE 2.16 Three graphs with the same mathematical relationship.



relationship. We'll use only a handful of different mathematical relationships in this text. As we meet each relationship for the first time, we will give an overview of its most important properties. When you see the relationship again in a new equation, we'll insert an icon, such as , that refers back to the overview so that you can remind yourself of the key details.

For instance, the mathematical form of Equation 2.5 is a type that we will see often: The displacement Δx is *proportional* to the time interval Δt. The following proportional relationships overview gives the details.

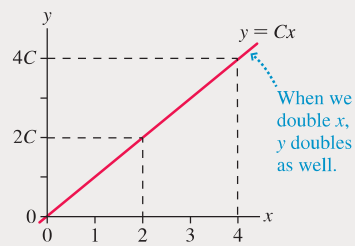
Proportional relationships

We say that y is **proportional** to x if they are related by an equation of the form

$$y = Cx$$

y is proportional to x

We call C the **proportionality constant.** A graph of y versus x is a straight line that passes through the origin.



SCALING If x has the initial value x_1, then y has the initial value $y_1 = Cx_1$. Changing x from x_1 to x_2 changes y from y_1 to y_2. The ratio of y_2 to y_1 is

$$\frac{y_2}{y_1} = \frac{Cx_2}{Cx_1} = \frac{x_2}{x_1}$$

The ratio of y_2 to y_1 is exactly the same as the ratio of x_2 to x_1. If y is proportional to x, which is often written $y \propto x$, then x and y change by the same factor:

- If you double x, you double y.
- If you decrease x by a factor of 3, you decrease y by a factor of 3.

If two variables have a proportional relationship, we can draw important conclusions from ratios without knowing the value of the proportionality constant C. We can often solve problems in a very straightforward manner by looking at such ratios. This is an important skill called *ratio reasoning*.

Exercise 11

EXAMPLE 2.3 **If a train leaves Cleveland at 2:00 ...**

A train is moving due west at a constant speed. A passenger notes that it takes 10 minutes to travel 12 km. How long will it take the train to travel 60 km?

STRATEGIZE For an object in uniform motion, Equation 2.5 shows that the distance traveled Δx is proportional to the time interval Δt, so this is a good problem to solve using ratio reasoning.

PREPARE We are comparing two cases: the time $\Delta t_1 = 10$ min it takes to travel the distance $\Delta x_1 = 12$ km, and the (unknown) time Δt_2 it will take to travel $\Delta x_2 = 60$ km. Ratio reasoning tells us that $\Delta x_2/\Delta x_1 = \Delta t_2/\Delta t_1$.

Continued

SOLVE The ratio of the distances is

$$\frac{\Delta x_2}{\Delta x_1} = \frac{60\text{ km}}{12\text{ km}} = 5$$

This is equal to the ratio of the times:

$$\frac{\Delta t_2}{\Delta t_1} = \frac{\Delta t_2}{10\text{ min}} = \frac{\Delta x_2}{\Delta x_1} = 5$$

$$\Delta t_2 = 5 \times (10\text{ min}) = 50\text{ min}$$

It takes 10 minutes to travel 12 km, so it will take 50 minutes—5 times as long—to travel 60 km.

ASSESS For an object in steady motion, it makes sense that 5 times the distance requires 5 times the time. We can see that using ratio reasoning is a straightforward way to solve this problem. We don't need to know the proportionality constant (in this case, the velocity); we just used ratios of distances and times.

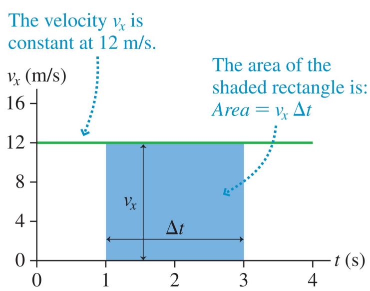
A Second Way to Find Position from Velocity

Earlier, we saw that we could deduce an object's position graph from its velocity graph by drawing a position graph in which the slopes everywhere matched the velocity graph. But there's another way to understand the relationship between velocity and position graphs—by looking at what we call the *area under the graph*. Let's look at an example.

Suppose a car is in uniform motion at 12 m/s. How far does it travel—that is, what is its displacement—during the time interval between $t = 1.0$ s and $t = 3.0$ s?

Equation 2.5, $\Delta x = v_x \Delta t$, describes the displacement mathematically; for a graphical interpretation, consider the graph of velocity versus time in FIGURE 2.17. In the figure, we've shaded a rectangle whose height is the velocity v_x and whose base is the time interval Δt. The area of this rectangle is $v_x \Delta t$. Looking at Equation 2.5, we see that the quantity is also equal to the displacement of the car. The area of this rectangle is the area between the axis and the line representing the velocity; we call it the "area under the graph." We see that **the displacement Δx is equal to the area under the velocity graph during interval Δt.**

FIGURE 2.17 Displacement is the area under a velocity-versus-time graph.



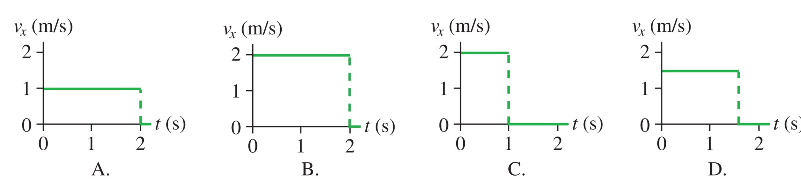
Whether we use Equation 2.5 or the area under the graph to compute the displacement, we get the same result:

$$\Delta x = v_x \Delta t = (12\text{ m/s})(2.0\text{ s}) = 24\text{ m}$$

Although we've shown that the displacement is the area under the graph only for uniform motion, where the velocity is constant, we'll soon see that this result applies to any one-dimensional motion.

NOTE ▶ Wait a minute! The displacement $\Delta x = x_f - x_i$ is a length. How can a length equal an area? Recall that earlier, when we found that the velocity is the slope of the position graph, we made a distinction between the *actual* slope and the *physically meaningful* slope? The same distinction applies here. The velocity graph does indeed bound a certain area on the page. That is the actual area—say, in square inches of paper—but it is *not* the area to which we are referring. Once again, we need to measure the quantities we are using, v_x and Δt, by referring to the scales on the axes. Δt is some number of seconds, while v_x is some number of meters per second. When these are multiplied, the *physically meaningful* area has units of meters, appropriate for a displacement. ◀

STOP TO THINK 2.2 Four objects move with the velocity-versus-time graphs shown. Which object has the largest displacement between $t = 0$ s and $t = 2$ s?



2.3 Instantaneous Velocity

The objects we've studied so far have moved with a constant, unchanging velocity or, like the car in Example 2.1, have a velocity that changes abruptly from one constant value to another. This is not very realistic. Real moving objects speed up and slow down, with their velocity changing smoothly. Suppose you're sitting at a red light at the start of a freeway ramp. When the light turns green, you increase your speed steadily from 0 mph to 60 mph until you merge onto the freeway.

A car's speed increases smoothly as it enters the freeway.

Perhaps as you speed down the ramp you glance at your speedometer and notice that, at that specific instant, it reads 40 mph. The speedometer indicates how fast you're moving *at a particular instant* in time. An object's velocity—its speed and direction—at a specific instant of time t is called the object's **instantaneous velocity.**

But what does it mean to have a velocity "at an instant"? An instantaneous velocity with a speed of 40 mph means that the rate at which your car's position is changing—at that exact instant—is such that it would travel a distance of 40 miles in 1 hour if it continued at that rate without change. If you speed up to pass a car that is traveling at a steady 40 mph, then at the very moment that your instantaneous velocity is 40 mph, your speed will match that of the other car—but an instant later, you'll be moving faster than 40 mph.

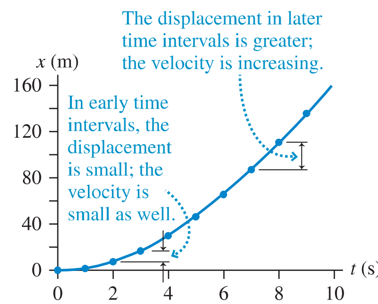
The velocity we introduced in Section 1.4 is really the *average velocity*; it is the velocity *averaged* over a *finite* time interval, such as 1 s or 1 min. **From now on, though, the word "velocity" will always mean instantaneous velocity**—the velocity at a single instant of time.

For uniform motion, an object's position-versus-time graph is a straight line and the object's velocity is the slope of that line. In contrast, FIGURE 2.18 shows that the position-versus-time graph for a car entering a freeway is a *curved* line. The displacement Δx during equal intervals of time gets greater as the car speeds up. Even so, we can use the slope of the position graph to measure the car's velocity. We can say that

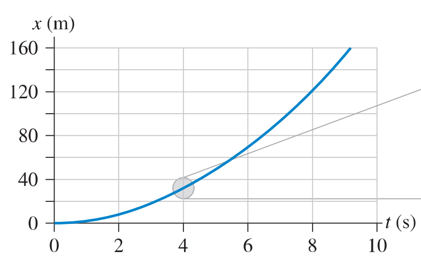
$$\text{instantaneous velocity } v_x \text{ at time } t = \text{slope of position graph at time } t \qquad (2.6)$$

But how do we determine the slope of a curved line at a particular point? The following table shows how.

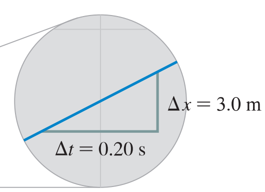
FIGURE 2.18 Position-versus-time graph for a car entering a freeway.



Finding the instantaneous velocity



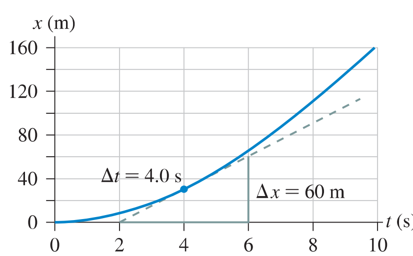
If the velocity changes, the position graph is a curved line. But we can compute a slope at a point by considering a small segment of the graph. Let's look at the motion in a very small time interval right around $t = 4.0$ s. This is highlighted with a circle, and we show a closeup in the next graph at the right.



In this magnified segment of the position graph, the curve isn't apparent. It appears to be a line segment. We can find the slope by calculating the rise over the run, just as before:

$$v_x = (3.0\text{ m})/(0.20\text{ s}) = 15\text{ m/s}$$

This is the slope at $t = 4.0$ s and thus the velocity at this instant of time.



Graphically, the slope of the curve at a point is the same as the slope of a straight line drawn *tangent* to the curve at that point. Calculating rise over run for the tangent line, we get

$$v_x = (60\text{ m})/(4.0\text{ s}) = 15\text{ m/s}$$

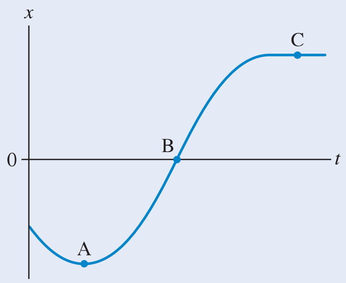
This is the same value we obtained from the close-up view. **The slope of the tangent line is the instantaneous velocity at that instant of time.**

CONCEPTUAL EXAMPLE 2.4 Analyzing a hockey player's position graph

A hockey player moves in a straight line along the length of the ice in a game. We measure position from the center of the rink. FIGURE 2.19 shows a position-versus-time graph for his motion.

a. Sketch an approximate velocity-versus-time graph.
b. At which point or points is the player moving the fastest?
c. Is the player ever at rest? If so, at which point or points?

FIGURE 2.19 The position-versus-time graph for a hockey player.



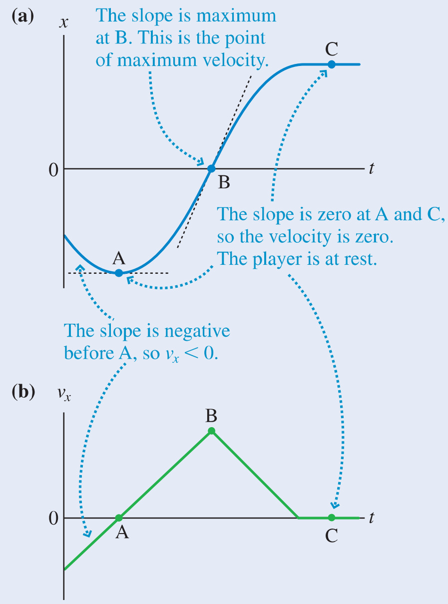
REASON a. The velocity at a particular instant of time is the slope of the tangent line to the position-versus-time graph at that time. We can move point-by-point along the position-versus-time graph, noting the slope of the tangent at each point to find the velocity at that point.

Initially, to the left of point A, the slope is negative and thus the velocity is negative (i.e., the player is moving to the left). But the slope decreases as the curve flattens out, and by the time the graph gets to point A, the slope is zero. The slope then increases to a maximum value at point B, decreases back to zero a little before point C, and remains at zero thereafter. This reasoning process is outlined in FIGURE 2.20a, and FIGURE 2.20b shows the approximate velocity-versus-time graph that results.

The other questions were answered during the construction of the graph:

b. The player moves the fastest at point B where the slope of the position graph is the steepest.
c. If the player is at rest, $v_x = 0$. Graphically, this occurs at points where the line tangent to the position-versus-time graph is horizontal and thus has zero slope. Figure 2.20 shows that the slope is zero at point A and for a small range of times near point C. At point A, the velocity is only instantaneously zero—the player is reversing direction, changing from moving to the left to moving to the right. Near point C, he has stopped moving and stays at rest.

FIGURE 2.20 Finding a velocity graph from a position graph.



ASSESS The best way to check our work is to look at different segments of the motion and see if the velocity and position graphs match. Until point A, x is decreasing. The player is moving to the left, so the velocity should be negative, which our graph shows. Between points A and C, x is increasing, so the velocity should be positive, which is also a feature of our graph. The steepest slope is at point B, so this should be the high point of our velocity graph, as it is.

FIGURE 2.21 Velocity-versus-time graph for a lion pursuing prey.

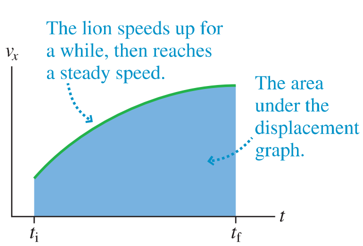


FIGURE 2.21 shows a velocity-versus-time graph for a lion speeding up to pursue prey. Even though the speed varies, we can still use the graph to determine how far the lion moves during the time interval t_i to t_f. For uniform motion we showed that the displacement Δx is the area under the velocity-versus-time graph during the time interval. But there was nothing special about the type of motion: We can generalize this idea to the case of an object whose velocity varies. If we draw a velocity graph for the motion, the object's displacement is given by

$$x_f - x_i = \text{area under the velocity graph between } t_i \text{ and } t_f \tag{2.7}$$

The area under the graph in Figure 2.21 tells us how far the lion ran during this segment of the chase.

In many cases, as in the next example, the area under the graph is a simple shape whose area we can easily compute. If the shape is complex, however, we can approximate the area using a number of simpler shapes that closely match it.

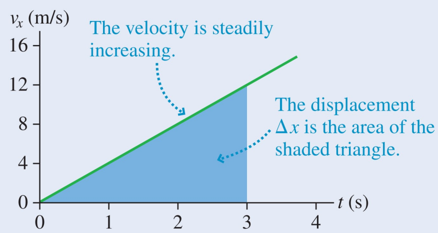
EXAMPLE 2.5 Calculating the displacement of a car during a rapid start

FIGURE 2.22 shows the velocity-versus-time graph of a car pulling away from a stop. How far does the car move during the first 3.0 s?

STRATEGIZE The question How far? indicates that we need to find a displacement Δx rather than a position x. Graphically, the displacement is given by the area under the velocity-versus-time graph.

PREPARE In Figure 2.22 we have shaded the area we need to find. It is the area between the straight line of the velocity graph and the t-axis, between $t_i = 0$ s and $t_f = 3.0$ s.

FIGURE 2.22 Velocity-versus-time graph for the car of Example 2.5.



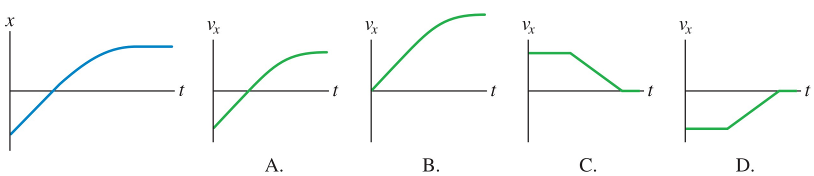
SOLVE The graph in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3.0 \text{ s}$$
$$= \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times 3.0 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The car moves 18 m during the first 3 seconds as its velocity changes from 0 to 12 m/s.

ASSESS The physically meaningful area is a product of a time in s and a velocity in m/s, so Δx has the proper units of m. Let's check the numbers to see if they make physical sense. The final velocity, 12 m/s, is about 25 mph. Pulling away from a stop, you might expect to reach this speed in about 3 s—at least if you have a reasonably sporty vehicle! Another check is to realize that if the car had moved at a constant 12 m/s (the final velocity) during these 3 s, the distance would be 36 m. The actual distance traveled during the 3 s is 18 m—half of 36 m. This makes sense, as the velocity was 0 m/s at the start of the problem and increased steadily to 12 m/s.

STOP TO THINK 2.3 Which velocity-versus-time graph goes with the position-versus-time graph on the left?



2.4 Acceleration

The goal of this chapter is to describe motion. We've seen that velocity describes the rate at which an object changes position. We need one more motion concept to complete the description, one that will describe an object whose velocity is changing.

As an example, let's look at a frequently quoted measurement of car performance, the time it takes the car to go from 0 to 60 mph. TABLE 2.2 shows this time for two different cars, a sporty Corvette and a compact Sonic with a much more modest engine.

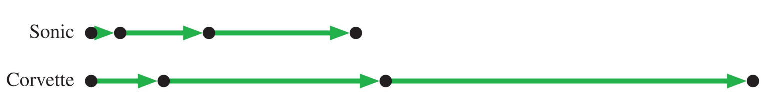
TABLE 2.2 Performance data for vehicles

Vehicle	Time to go from 0 to 60 mph
2016 Chevy Corvette	3.6 s
2016 Chevy Sonic	9.0 s

Let's look at motion diagrams for the Corvette and the Sonic in FIGURE 2.23. We can see two important facts about the motion. First, the lengths of the velocity vectors are increasing, showing that the speeds are increasing. Second, the velocity vectors for the Corvette are increasing in length more rapidly than those of the Sonic. The quantity we seek is one that measures how rapidly an object's velocity vectors change in length.

When we wanted to measure changes in position, the ratio $\Delta x/\Delta t$ was useful. This ratio, which we defined as the velocity, is the *rate of change of position*. Similarly, we

FIGURE 2.23 Motion diagrams for the Corvette and Sonic.



can measure how rapidly an object's velocity changes with the ratio $\Delta v_x/\Delta t$. Given our experience with velocity, we can say a couple of things about this new ratio:

- The ratio $\Delta v_x/\Delta t$ is the *rate of change of velocity.*
- The ratio $\Delta v_x/\Delta t$ is the *slope of a velocity-versus-time graph.*

We will define this ratio as the **acceleration,** for which we use the symbol a_x:

$$a_x = \frac{\Delta v_x}{\Delta t} \tag{2.8}$$

Definition of acceleration as the rate of change of velocity

Cushion kinematics When a car hits an obstacle head-on, the damage to the car and its occupants can be reduced by making the acceleration as small as possible. As we can see from Equation 2.8, acceleration can be reduced by making the *time* for a change in velocity as long as possible. This is the purpose of the yellow crash cushion barrels you may have seen in work zones on highways—to lengthen the time of a collision with a barrier.

Similarly, $a_y = \Delta v_y/\Delta t$ for vertical motion.

As an example, let's calculate the acceleration for the Corvette and the Sonic. For both, the initial velocity $(v_x)_i$ is zero and the final velocity $(v_x)_f$ is 60 mph. Thus the *change* in velocity is $\Delta v_x = 60$ mph. In m/s, our SI unit of velocity, $\Delta v_x = 27$ m/s.

Now we can use Equation 2.8 to compute the acceleration. Let's start with the Corvette, which speeds up to 27 m/s in $\Delta t = 3.6$ s:

$$a_{\text{Corvette } x} = \frac{\Delta v_x}{\Delta t} = \frac{27 \text{ m/s}}{3.6 \text{ s}} = 7.5 \, \frac{\text{m/s}}{\text{s}}$$

Here's the meaning of this final figure: Every second, the Corvette's velocity changes by 7.5 m/s. In the first second of motion, the Corvette's velocity increases by 7.5 m/s; in the next second, it increases by another 7.5 m/s, and so on. Thus after 1 second, the velocity is 7.5 m/s; after 2 seconds, it is 15 m/s. We thus interpret the units as 7.5 meters per second, per second—7.5 (m/s)/s.

The Sonic's acceleration is

$$a_{\text{Sonic } x} = \frac{\Delta v_x}{\Delta t} = \frac{27 \text{ m/s}}{9.0 \text{ s}} = 3.0 \, \frac{\text{m/s}}{\text{s}}$$

In each second, the Sonic changes its speed by 3.0 m/s. This is only 2/5 the acceleration of the Corvette! The reason the Corvette is capable of greater acceleration has to do with what causes the motion. We will explore the reasons for acceleration in Chapter 4. For now, we will simply note that the Corvette is capable of much greater acceleration, something you would have suspected.

NOTE ▸ It is customary to abbreviate the acceleration units (m/s)/s as m/s^2, which we say as "meters per second squared." For example, the Sonic has an acceleration of 3.0 m/s^2. When you use this notation, keep in mind its *meaning* as "(meters per second) per second." ◂

EXAMPLE 2.6 Animal acceleration BIO

Lions, like most predators, are capable of very rapid starts. From rest, a lion can sustain an acceleration of 9.5 m/s^2 for up to one second. How much time does it take a lion to go from rest to a typical recreational runner's top speed of 10 mph?

STRATEGIZE The lion's speed increases by 9.5 m/s^2 each second. Once we know the runner's speed in m/s, we will calculate the time it would take for the lion to reach that speed.

PREPARE We start by converting to SI units. The speed the lion must reach is

$$v_f = 10 \text{ mph} \times \frac{0.45 \text{ m/s}}{1.0 \text{ mph}} = 4.5 \text{ m/s}$$

The lion can accelerate at 9.5 m/s^2, changing its speed by 9.5 m/s per second, for only 1.0 s—long enough to reach 9.5 m/s. It will take the lion less than 1.0 s to reach 4.5 m/s, so we can use $a_x = 9.5 \text{ m/s}^2$ in our solution.

SOLVE We know the acceleration and the desired change in velocity, so we can rearrange Equation 2.8 to find the time:

$$\Delta t = \frac{\Delta v_x}{a_x} = \frac{4.5 \text{ m/s}}{9.5 \text{ m/s}^2} = 0.47 \text{ s}$$

ASSESS The lion changes its speed by 9.5 meters per second in one second. So it's reasonable (if a bit intimidating) that it will reach 4.5 m/s in just under half a second.

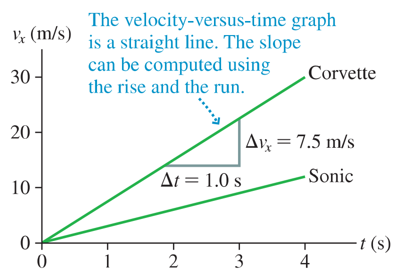
From Velocity to Acceleration

Let's use the values we have computed for acceleration to make a table of velocities for the Corvette and the Sonic we considered earlier. **TABLE 2.3** uses the idea that the Sonic's velocity increases by 3.0 m/s every second while the Corvette's velocity increases by 7.5 m/s every second. The data in Table 2.3 are the basis for the velocity-versus-time graphs in **FIGURE 2.24**. As you can see, **an object undergoing constant acceleration has a straight-line velocity graph.**

TABLE 2.3 Velocity data for the Sonic and the Corvette

Time (s)	Velocity of Sonic (m/s)	Velocity of Corvette (m/s)
0	0	0
1	3.0	7.5
2	6.0	15.0
3	9.0	22.5
4	12.0	30.0

FIGURE 2.24 Velocity-versus-time graphs for the two cars.



The slope of either of these lines—the rise over the run—is $\Delta v_x / \Delta t$. Comparing this with Equation 2.8, we see that the equation for the slope is the same as that for the acceleration. That is, **an object's acceleration is the slope of its velocity-versus-time graph:**

$$\text{acceleration } a_x \text{ at time } t = \text{slope of velocity graph at time } t \qquad (2.9)$$

The Sonic has a smaller acceleration, so its velocity graph has a smaller slope.

CONCEPTUAL EXAMPLE 2.7 Analyzing a car's velocity-versus-time graph

FIGURE 2.25a is a graph of velocity versus time for a car. Sketch a graph of the car's acceleration versus time.

REASON The graph can be divided into three sections:

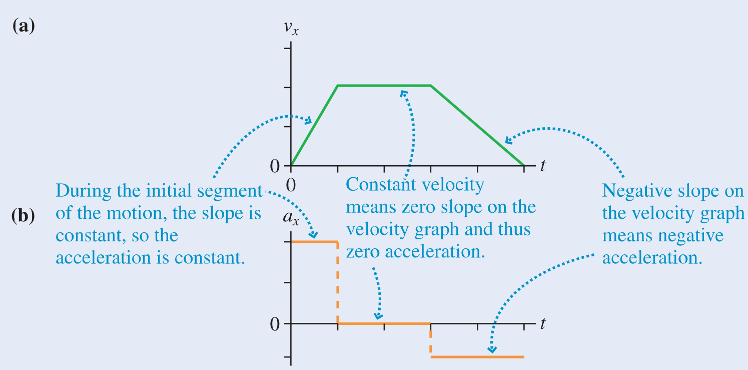
- An initial segment, in which the velocity increases at a steady rate
- A middle segment, in which the velocity is constant
- A final segment, in which the velocity decreases at a steady rate

In each section, the acceleration is the slope of the velocity-versus-time graph. Thus the initial segment has constant, positive acceleration, the middle segment has zero acceleration, and the final segment has constant, *negative* acceleration. The acceleration graph appears in **FIGURE 2.25b.**

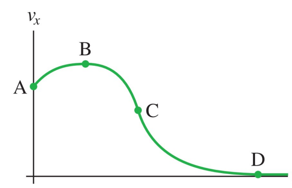
ASSESS This process is analogous to finding a velocity graph from the slope of a position graph. It is important to understand that the zero acceleration in the middle segment does *not* mean that the velocity there is zero. In this segment the velocity is *constant*, which means that it is *not changing* and thus the car is not accelerating.

In the first and last segments, the velocity is changing, and so the car does have a nonzero acceleration. In the first segment, the acceleration is positive; in the last segment, it is negative. What does the *sign* of the acceleration tell us? We will address this issue in the next section.

FIGURE 2.25 Finding an acceleration graph from a velocity graph.



STOP TO THINK 2.4 A particle moves with the velocity-versus-time graph shown here. At which labeled point is the magnitude of the acceleration the greatest?



The Sign of the Acceleration

A video to support a section's topic is embedded in the eText.

Video Motion Along a Straight Line

It's a natural tendency to think that a positive value of a_x or a_y describes an object that is speeding up while a negative value describes an object that is slowing down. Unfortunately, this simple interpretation *is not correct.*

Acceleration, like velocity, is a vector. Specifically, **the acceleration vector points in the *same* direction as the velocity vector for an object that is speeding up, and *opposite* to the velocity vector for an object that is slowing down.** Regardless of which way an object moves, an acceleration vector that points in the same direction as the velocity "pulls" the velocity vectors to make them longer and longer—speeding up—while an acceleration vector that points opposite to the velocity "pushes against" the velocity vectors to make them shorter and shorter—slowing down. You can see this in the four situations shown in FIGURE 2.26.

Just as we did with velocity, we can simplify our analysis of one-dimensional motion by using an ordinary variable a_x, which can be positive or negative, to represent the one-dimensional acceleration. The sign convention for a_x (and a_y) is exactly

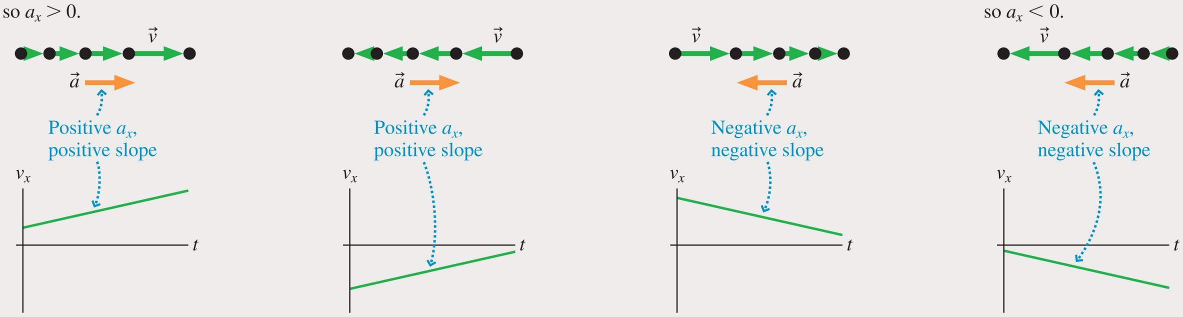
KEY CONCEPT FIGURE 2.26 Determining the sign of the acceleration.

The object is moving to the right, so $v_x > 0$. Because it is speeding up, its acceleration vector points in the same direction as its velocity (i.e., to the right), so $a_x > 0$.

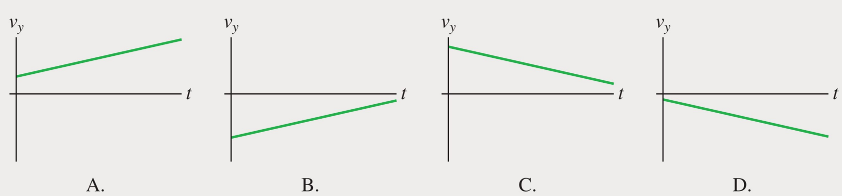
The object is moving to the left, so $v_x < 0$. Because it is slowing down, its acceleration vector points opposite to its velocity (i.e., to the right), so $a_x > 0$.

The object is moving to the right, so $v_x > 0$. Because it is slowing down, its acceleration vector points opposite to its velocity (i.e., to the left), so $a_x < 0$.

The object is moving to the left, so $v_x < 0$. Because it is speeding up, its acceleration vector points in the same direction as its velocity (i.e., to the left), so $a_x < 0$.



STOP TO THINK 2.5 An elevator is moving downward. It is slowing down as it approaches the ground floor. Adapt the information in Figure 2.26 to determine which of the following velocity graphs best represents the motion of the elevator.



the same as the sign convention for velocity that was shown in Figure 2.7: a_x (or a_y) is positive when the acceleration vector points to the right (or up), negative when the acceleration vector points to the left (or down).

Notice that the first two situations in Figure 2.26, where the acceleration vectors point to the right, both have positive values of the acceleration a_x even though one shows an object speeding up and the other an object slowing down. **The sign of a_x is based on the direction the acceleration vector points,** not on whether the object is speeding up or slowing down.

Figure 2.26 illustrates two more ideas: First, our convention for the sign of the acceleration is consistent with what you just saw about an object's acceleration being the slope of its velocity graph. Second, **an object is speeding up if v_x and a_x have the same sign, and slowing down if they have opposite signs.**

2.5 Motion with Constant Acceleration

For uniform motion—motion with constant velocity—we found in Equation 2.3 a simple relationship between position and time. It's no surprise that there are also simple relationships that connect the various kinematic variables in constant-acceleration motion. We will start with a concrete example, the launch of a Saturn V rocket like the one that carried the Apollo astronauts to the moon in the 1960s and 1970s. FIGURE 2.27 shows one frame from a video of a rocket lifting off the launch pad. The red dots show the positions of the top of the rocket at equally spaced intervals of time in earlier frames of the video. This is a motion diagram for the rocket, and we can see that the velocity is increasing. The graph of velocity versus time in FIGURE 2.28 shows that the velocity is increasing at a fairly constant rate. We can approximate the rocket's motion as having constant acceleration.

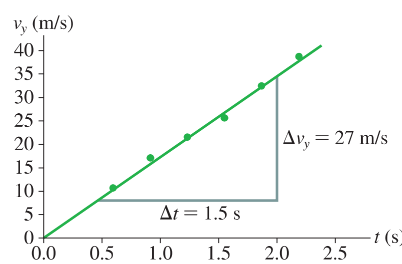
We can use the slope of the graph in Figure 2.28 to determine the acceleration of the rocket:

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{27 \text{ m/s}}{1.5 \text{ s}} = 18 \text{ m/s}^2$$

This acceleration is more than double the acceleration of the Corvette we discussed earlier, and it goes on for a long time—the first phase of the launch lasts over 2 minutes! How fast is the rocket moving at the end of this acceleration, and how far has it traveled? To answer questions like these, we first need to work out some basic kinematic equations for motion with constant acceleration.

FIGURE 2.27 The red dots show the positions of the top of the Saturn V rocket at equally spaced intervals of time during liftoff.

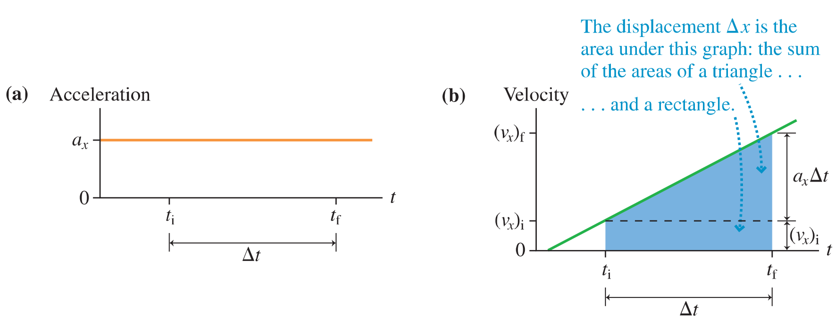
FIGURE 2.28 A graph of the rocket's velocity versus time.



Constant-Acceleration Equations

Consider an object whose acceleration a_x remains constant during the time interval $\Delta t = t_f - t_i$. At the beginning of this interval, the object has initial velocity $(v_x)_i$ and initial position x_i. Note that t_i is often zero, but it need not be. FIGURE 2.29a shows the acceleration-versus-time graph. It is a horizontal line between t_i and t_f, indicating a *constant* acceleration.

FIGURE 2.29 Acceleration and velocity graphs for motion with constant acceleration.



The object's velocity is changing because the object is accelerating. We can use the acceleration to find $(v_x)_f$ at a later time t_f. We defined acceleration as

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t} \tag{2.10}$$

which is rearranged to give

$$(v_x)_f = (v_x)_i + a_x \, \Delta t \tag{2.11}$$

Velocity equation for an object with constant acceleration

NOTE ► We have expressed this equation for motion along the x-axis, but it is a general result that will apply to any axis. ◄

The velocity-versus-time graph for this constant-acceleration motion, shown in FIGURE 2.29b, is a straight line with value $(v_x)_i$ at time t_i and with slope a_x.

We would also like to know the object's position x_f at time t_f. As you learned earlier, the displacement Δx during a time interval Δt is the area under the velocity-versus-time graph. This area is shown shaded in Figure 2.29b. The shaded area can be divided into a rectangle of area $(v_x)_i \, \Delta t$ and a triangle of area $\frac{1}{2}(a_x \, \Delta t)(\Delta t) = \frac{1}{2} a_x (\Delta t)^2$. Adding these gives

$$x_f = x_i + (v_x)_i \, \Delta t + \frac{1}{2} a_x (\Delta t)^2 \tag{2.12}$$

Position equation for an object with constant acceleration

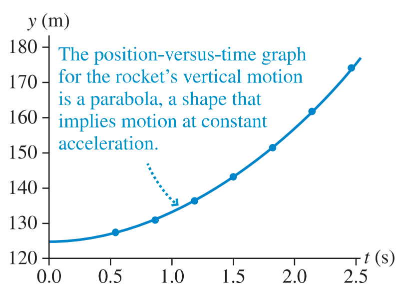


FIGURE 2.30 Position-versus-time graph for the Saturn V rocket launch.

where $\Delta t = t_f - t_i$ is the elapsed time. The fact that the time interval Δt appears in the equation as $(\Delta t)^2$ causes the position-versus-time graph for constant-acceleration motion to have a parabolic shape. For the rocket launch of Figure 2.27, a graph of the position of the top of the rocket versus time appears as in FIGURE 2.30.

Equations 2.11 and 2.12 are two of the basic kinematic equations for motion with constant acceleration. They allow us to predict an object's position and velocity at a future instant of time. We need one more equation to complete our set, a direct relationship between displacement and velocity. To derive this relationship, we first use Equation 2.11 to write $\Delta t = ((v_x)_f - (v_x)_i)/a_x$. We can substitute this into Equation 2.12 to obtain

$$(v_x)_f^{\,2} = (v_x)_i^{\,2} + 2a_x \, \Delta x \tag{2.13}$$

Relating velocity and displacement for constant-acceleration motion

In Equation 2.13, $\Delta x = x_f - x_i$ is the *displacement* (not the distance!), so it can be positive or negative. Notice that Equation 2.13 does not require that we know the time interval Δt. This is an important equation in problems where we're not given information about times.

At this point, it's worthwhile to summarize the relationships among kinematic variables that we've seen. This will help you solve problems by gathering together the most important information that you'll use in your solutions. But, more important, gathering this information together allows you to compare graphs, equations, and details from different parts of the chapter in one place. This will help you make important connections. The emphasis is on *synthesis*—hence the title of this box. You'll find other such synthesis boxes in most chapters.

SYNTHESIS 2.1 Describing motion in one dimension

We describe motion in terms of position, velocity, and acceleration.

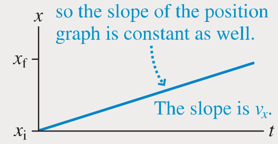
For **all motion:**

Velocity is the rate of change of position, in m/s. $v_x = \dfrac{\Delta x}{\Delta t}$

Acceleration is the rate of change of velocity, in m/s^2. $a_x = \dfrac{\Delta v_x}{\Delta t}$

For **uniform motion:**

- acceleration is zero
- velocity is constant
- position changes steadily



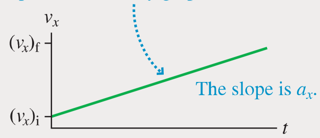
Final and initial position (m)

$$x_f = x_i + v_x \Delta t \qquad (2.4)$$

Velocity (m/s) Time interval (s)

For **motion with constant acceleration:**

- acceleration is steady; it does not change



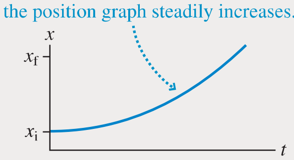
- velocity changes steadily

Final and initial velocity (m/s)

$$(v_x)_f = (v_x)_i + a_x \Delta t \qquad (2.11)$$

Acceleration (m/s^2) Time interval (s)

- the position changes as the square of the time interval



Final and initial position (m) Time interval (s)

$$x_f = x_i + (v_x)_i \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \qquad (2.12)$$

Initial velocity (m/s) Acceleration (m/s^2)

- we can also express the change in velocity in terms of **displacement, not time**

This gives us a third equation, which is useful for many kinematics problems.

Final and initial velocity (m/s)

$$(v_x)_f^2 = (v_x)_i^2 + 2 a_x \Delta x \qquad (2.13)$$

Acceleration (m/s^2) Change in position (m)

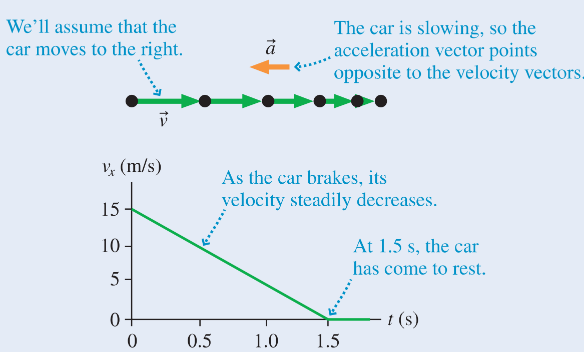
EXAMPLE 2.8 Coming to a stop in a car

As you drive in your car at 15 m/s (just a bit under 35 mph), you see a child's ball roll into the street ahead of you. You hit the brakes and stop as quickly as you can. In this case, you come to rest in 1.5 s. How far does your car travel as you brake to a stop?

STRATEGIZE The problem states that your car begins to slow down when you hit the brakes; we'll model this as constant-acceleration motion. We know the initial and final speeds, and we want to find the distance traveled. These observations suggest that we use Equation 2.12 of Synthesis 2.1.

PREPARE The problem gives us a description of motion in words. To help us visualize the situation, FIGURE 2.31 illustrates the key features of the motion with a motion diagram and a velocity graph. The graph is based on the car slowing from 15 m/s to 0 m/s in 1.5 s.

FIGURE 2.31 Motion diagram and velocity graph for a car coming to a stop.



SOLVE We've assumed that your car is moving to the right, so its initial velocity is $(v_x)_i = +15$ m/s. After you come to rest, your final velocity is $(v_x)_f = 0$ m/s. We use the definition of acceleration from Synthesis 2.1:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t} = \frac{0\text{ m/s} - 15\text{ m/s}}{1.5\text{ s}} = -10\text{ m/s}^2$$

Now that we know the acceleration, we can compute the distance that the car moves as it comes to rest using Equation 2.12:

$$x_f - x_i = (v_x)_i \Delta t + \tfrac{1}{2} a_x (\Delta t)^2$$
$$= (15\text{ m/s})(1.5\text{ s}) + \tfrac{1}{2}(-10\text{ m/s}^2)(1.5\text{ s})^2 = 11\text{ m}$$

ASSESS 11 m is a little over 35 feet. That's a reasonable distance for a quick stop while traveling at about 35 mph.

We found that the acceleration a_x is negative. This makes sense from two perspectives. First, as we learned in Figure 2.26, an object moving to the right and slowing down has a negative acceleration. Second, the slope of the velocity graph in Figure 2.31 is negative, again indicating a negative acceleration.

Getting up to speed BIO A bird must have a minimum speed to fly. Generally, the larger the bird, the faster the takeoff speed. Small birds can get moving fast enough to fly with a vigorous jump, but larger birds may need a running start. This swan must accelerate for a long distance in order to achieve the high speed it needs to fly, so it makes a frenzied dash across the frozen surface of a pond. Swans require a long, clear stretch of water or land to become airborne.

As we've noted, for motion at constant acceleration, the position changes as the square of the time interval. If the initial velocity $(v_x)_i$ is zero, Equation 2.12 from Synthesis 2.1 can be written as

$$x_f - x_i = \Delta x = \frac{1}{2}a_x(\Delta t)^2$$

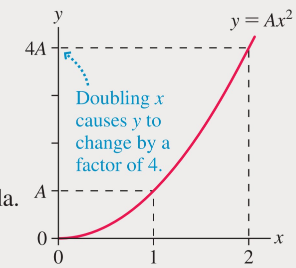
This is a new mathematical relationship—a *quadratic relationship*—that we will see again and one that we can use as the basis of reasoning to solve problems.

Quadratic relationships

Two quantities are said to have a **quadratic relationship** if y is proportional to the square of x. We write the mathematical relationship as

$$y = Ax^2$$

y is proportional to x^2



The graph of a quadratic relationship is a parabola.

SCALING If x has the initial value x_1, then y has the initial value $y_1 = A(x_1)^2$. Changing x from x_1 to x_2 changes y from y_1 to y_2. The ratio of y_2 to y_1 is

$$\frac{y_2}{y_1} = \frac{A(x_2)^2}{A(x_1)^2} = \left(\frac{x_2}{x_1}\right)^2$$

The ratio of y_2 to y_1 is the square of the ratio of x_2 to x_1. If y is a quadratic function of x, a change in x by some factor changes y by the square of that factor:

- If you increase x by a factor of 2, you increase y by a factor of $2^2 = 4$.
- If you decrease x by a factor of 3, you decrease y by a factor of $3^2 = 9$.

Generally, we can say that:

Changing x by a factor of c changes y by a factor of c^2.

Exercise 19

EXAMPLE 2.9 **Finding the displacement of a drag racer**

A drag racer, starting from rest, travels 6.0 m in 1.0 s. Suppose the car continues this acceleration for an additional 4.0 s. How far from the starting line will the car be?

STRATEGIZE We assume that the acceleration is constant. Because the initial position and velocity are zero, the displacement will then scale as the square of the time; we can then use ratio reasoning to solve the problem.

PREPARE After 1.0 s, the car has traveled 6.0 m; after another 4.0 s, a total of 5.0 s will have elapsed.

SOLVE The initial elapsed time was 1.0 s, so the elapsed time increases by a factor of 5. The displacement thus increases by a factor of 5^2, or 25. The total displacement is

$$\Delta x = 25(6.0 \text{ m}) = 150 \text{ m}$$

ASSESS This is a big distance in a short time, but drag racing is a fast sport, so our answer makes sense.

STOP TO THINK 2.6 A cyclist is at rest at a traffic light. When the light turns green, he begins accelerating at 1.2 m/s^2. How many seconds after the light turns green does he reach his cruising speed of 6.0 m/s?

A. 1.0 s B. 2.0 s C. 3.0 s D. 4.0 s E. 5.0 s

2.6 Solving One-Dimensional Motion Problems

The big challenge when solving a physics problem is to translate the words into symbols that can be manipulated, calculated, and graphed. This translation from words to symbols is the heart of problem solving in physics. Ambiguous words and phrases must be clarified, the imprecise must be made precise, and you must arrive at an understanding of exactly what the question is asking.

PROBLEM-SOLVING APPROACH

The first step in solving a seemingly complicated problem is to break it down into a series of smaller steps. In worked examples in the text, we use a problem-solving approach that consists of four steps: *strategize, prepare, solve,* and *assess*. Each of these steps has important elements that you should follow when you solve problems on your own.

STRATEGIZE The Strategize step of the solution is where you address the *big-picture* questions about the problem. Here, you take a step back from the details of the problem to ask:

- **What kind of problem is this?** From reading the problem statement, try to categorize the problem in terms of what you've learned in the chapter. If, for instance, the problem refers to a bicyclist riding at a constant 7.0 m/s, this suggests the problem is about uniform motion.
- **What's the correct general approach?** What principles, strategies, and tactics that you've learned are relevant in solving this problem? For example, if you're given a position-versus-time graph and are asked to find the velocity, the principle that the velocity is related to the slope of the position graph is likely to be important.
- **What should the answer look like?** Is a numerical answer asked for? Do you need a graph or a sketch?

PREPARE The Prepare step of a solution is where you identify important elements of the problem and collect information. It's tempting to jump right to the Solve step, but a skilled problem solver spends the most time on preparation, which includes:

- **Drawing a picture.** This is often the most important part of a problem. The picture lets you model the problem and identify the important elements. As you add information to your picture, the outline of the solution will take shape. For the problems in this chapter, a picture could be a motion diagram or a graph—or perhaps both.
- **Collecting necessary information.** The problem's statement may give you some values of variables. Other information may be implied, or looked up in a table, or estimated or measured.
- **Doing preliminary calculations.** Some calculations, such as unit conversions, are best done in advance.

SOLVE The Solve step of a solution is where you actually do the mathematics or reasoning necessary to arrive at the answer. This is the part of the problem-solving approach that you likely think of as "solving problems." The Strategize and Prepare steps help you be certain you understand the problem before you start putting numbers in equations.

ASSESS The Assess step of your solution is very important. Once you have an answer, you should check to see whether it makes sense. Ask yourself:

- **Does my solution answer the question that was asked?** Make sure you have addressed all parts of the question and clearly written down your solutions.
- **Does my answer have the correct units and number of significant figures?**
- **Does the value I computed make physical sense?** In this book all calculations use physically reasonable numbers. If your answer seems unreasonable, go back and check your work.
- **Can I estimate what the answer should be to check my solution?**
- **Does my final solution make sense in the context of the material I am learning?**

The Pictorial Representation

Many physics problems, including one-dimensional motion problems, have several variables and other pieces of information to keep track of. The best way to tackle such problems is to draw a picture, as we noted when we introduced a general problem-solving approach. But what kind of picture should you draw?

In this section, we will begin to draw **pictorial representations** as an aid to solving problems. A pictorial representation shows all of the important details that we need to keep track of and will be very important in solving motion problems.

Dinner at a distance BIO A chameleon's tongue is a powerful tool for catching prey. Certain species can extend the tongue to a distance of over 1 ft in less than 0.1 s! A study of the kinematics of the motion of the chameleon tongue reveals that the tongue has a period of rapid acceleration followed by a period of constant velocity. This knowledge is a very valuable clue in the analysis of the evolutionary relationships between chameleons and other animals.

TACTICS BOX 2.2 Drawing a pictorial representation

❶ **Sketch the situation.** Not just any sketch: Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Very simple drawings are adequate.

❷ **Establish a coordinate system.** Select your axes and origin to match the motion.

❸ **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch.

We will generally combine the pictorial representation with a **list of values.** In this list, you should:

❹ *List the known information.* Make a table of the quantities whose values you can determine from the problem statement or that you can find quickly with simple geometry or unit conversions.

❺ *Identify the desired unknowns.* What quantity or quantities will allow you to answer the question?

Exercise 21

EXAMPLE 2.10 Drawing a pictorial representation

Complete a pictorial representation and a list of values for the following problem: A rocket sled accelerates at 50 m/s^2 for 5 s. What are the total distance traveled and the final velocity?

STRATEGIZE We'll prepare the pictorial representation and list of values according to the steps of Tactics Box 2.2.

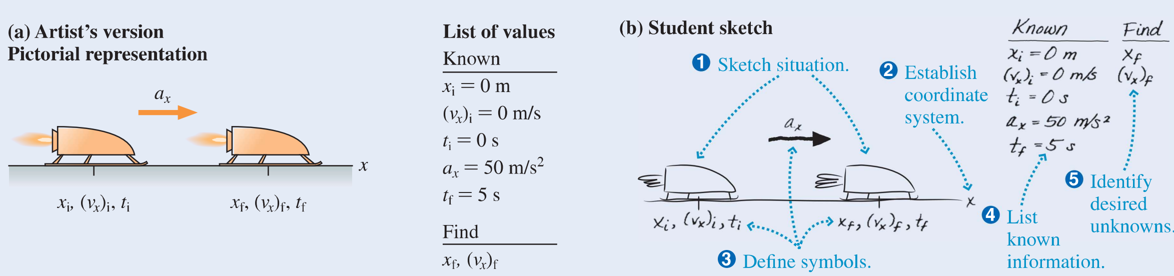
PREPARE FIGURE 2.32a shows a pictorial representation as drawn by an artist in the style of the figures in this book. This is certainly neater and more artistic than the sketches you will make when solving problems yourself! FIGURE 2.32b shows a sketch like one you might actually draw. It's less formal, but it contains all of the important information you need to solve the problem. The circled numbers in the sketch correspond to the steps in Tactics Box 2.2.

NOTE ▶ Throughout this book we will illustrate select examples with actual hand-drawn figures so that you have them to refer to as you work on your own pictures for homework and practice. ◀

Let's look at how these pictures were constructed. The motion has a clear beginning and end; these are the points in the motion that we've sketched. A coordinate system has been chosen with the origin at the starting point. The quantities x, v_x, and t are needed at both points, so these have been defined on the sketch and distinguished by subscripts. The acceleration is associated with an interval between these points. Values for two of these quantities are given in the problem statement. Others, such as $x_i = 0$ m and $t_i = 0$ s, are inferred from our choice of coordinate system. The value $(v_x)_i = 0$ m/s is part of our *interpretation* of the problem. Finally, we identify x_f and $(v_x)_f$ as the quantities that will answer the question. We now understand quite a bit about the problem and would be ready to start a quantitative analysis.

ASSESS We didn't *solve* the problem; that was not our purpose. Constructing a pictorial representation and a list of values is part of a systematic approach to interpreting a problem and getting ready for a mathematical solution.

FIGURE 2.32 Constructing a pictorial representation and a list of values.



The Visual Overview

Video Motion with Constant Acceleration

The pictorial representation and the list of values are a very good complement to the motion diagram and other ways of looking at a problem that we have seen. As we translate a problem into a form we can solve, we will combine these elements into what we will term a **visual overview.** The visual overview will consist of some or all of the following elements:

- A *motion diagram.* A good approach for solving a motion problem is to start by drawing a motion diagram.
- A *pictorial representation,* as defined in Tactics Box 2.2.
- A *list of values,* also described in Tactics Box 2.2. This list should sum up all of the important values in the problem.
- A *graphical representation.* For motion problems, it is often quite useful to include a graph of position and/or velocity.

Future chapters will add other elements to this visual overview of the physics.

EXAMPLE 2.11 **Kinematics of a rocket launch**

A Saturn V rocket is launched straight up with a constant acceleration of 18 m/s^2. After 150 s, how fast is the rocket moving and how far has it traveled?

STRATEGIZE We are given the acceleration and the time interval, suggesting that this is a constant-acceleration problem. We will find the velocity from Equation 2.11 and the position from Equation 2.12.

PREPARE **FIGURE 2.33** shows a visual overview of the rocket launch that includes a motion diagram, a pictorial representation, and a list of values. The visual overview shows the whole problem in a nutshell. The motion diagram illustrates the motion of the rocket. The pictorial representation (produced according to Tactics Box 2.2) shows the axis, identifies the important points of the motion, and defines the variables. Finally, we include a list of values that gives the known and unknown quantities (again according to Tactics Box 2.2). In the visual overview we have taken the statement of the problem in words and made it much more precise. The overview contains everything we need to know about the problem.

SOLVE Our first task is to find the final velocity. Our list of values includes the initial velocity, the acceleration, and the time interval, so we can use Equation 2.11 of Synthesis 2.1 to find the final velocity:

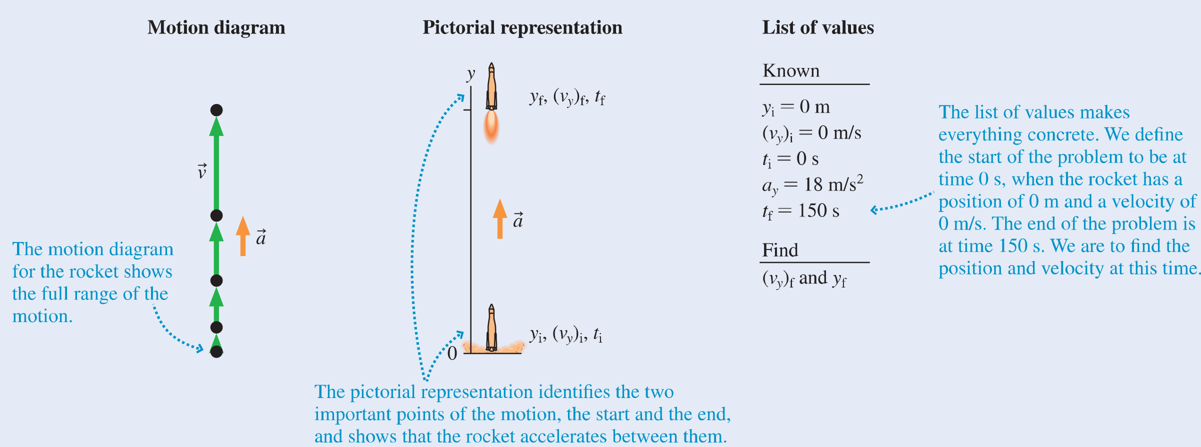
$$(v_y)_f = (v_y)_i + a_y\,\Delta t = 0\text{ m/s} + (18\text{ m/s}^2)(150\text{ s})$$
$$= 2700\text{ m/s}$$

The distance traveled is found using Equation 2.12 of Synthesis 2.1:

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$= 0\text{ m} + (0\text{ m/s})(150\text{ s}) + \tfrac{1}{2}(18\text{ m/s}^2)(150\text{ s})^2$$
$$= 2.0 \times 10^5\text{ m} = 200\text{ km}$$

ASSESS The acceleration is very large, and it goes on for a long time, so the large final velocity and large distance traveled seem reasonable.

FIGURE 2.33 Visual overview of the rocket launch.



Problem-Solving Approach for Motion with Constant Acceleration

Earlier in this section, we introduced a general problem-solving approach. In this and future chapters we will adapt this general approach to specific types of problems.

PROBLEM-SOLVING APPROACH 2.1 Motion with constant acceleration

Problems involving constant acceleration—speeding up, slowing down, vertical motion, horizontal motion—can all be treated with the same problem-solving approach.

STRATEGIZE Identify the problem as one involving constant-acceleration motion: Look for statements that give the acceleration or indicate that the speed or velocity of an object is changing. Free-fall problems, discussed in the next section, are always constant-acceleration problems. Solve constant-acceleration problems using the ideas in Synthesis 2.1.

PREPARE Draw a visual overview of the problem. This should include a motion diagram, a pictorial representation, and a list of values; a graphical representation may be useful for certain problems.

SOLVE The mathematical solution is based on the three constant-acceleration equations in Synthesis 2.1.

- Though the equations are phrased in terms of the variable x, it's customary to use y for motion in the vertical direction.
- Use the equation that best matches what you know and what you need to find. For example, if you know acceleration and time and are looking for a change in velocity, the first equation is the best one to use.
- Uniform motion with constant velocity has $a = 0$.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

Exercise 25

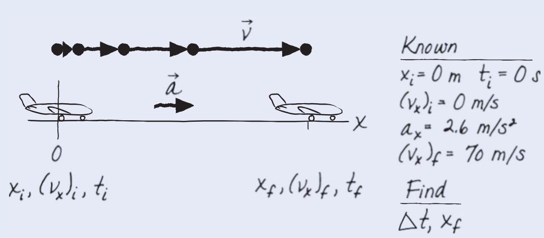
EXAMPLE 2.12 Calculating the minimum length of a runway

A fully loaded Boeing 747 with all engines at full thrust accelerates at 2.6 m/s^2. Its minimum takeoff speed is 70 m/s. How much time will the plane take to reach its takeoff speed? What minimum length of runway does the plane require for takeoff?

STRATEGIZE The acceleration of the plane is given, which directly tells us that this is a constant-acceleration problem. We'll need to use material from Synthesis 2.1.

PREPARE The visual overview of FIGURE 2.34 summarizes the important details of the problem. We set x_i and t_i equal to zero at the starting point of the motion, when the plane is at rest and the acceleration begins. The final point of the motion is when the plane achieves the necessary takeoff speed of 70 m/s. The plane is accelerating to the right, so we will compute the time for the plane to reach a velocity of 70 m/s and the position of the plane at this time, giving us the minimum length of the runway.

FIGURE 2.34 Visual overview for an accelerating plane.



SOLVE First we solve for the time required for the plane to reach takeoff speed. We can use Equation 2.11 from Synthesis 2.1 to compute this time:

$$(v_x)_f = (v_x)_i + a_x\,\Delta t$$

$$70\text{ m/s} = 0\text{ m/s} + (2.6\text{ m/s}^2)\,\Delta t$$

$$\Delta t = \frac{70\text{ m/s}}{2.6\text{ m/s}^2} = 26.9\text{ s}$$

We keep an extra significant figure here because we will use this result in the next step of the calculation.

Given the time that the plane takes to reach takeoff speed, we can compute the position of the plane when it reaches this speed using Equation 2.12 from Synthesis 2.1:

$$\begin{aligned} x_f &= x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \\ &= 0\text{ m} + (0\text{ m/s})(26.9\text{ s}) + \tfrac{1}{2}(2.6\text{ m/s}^2)(26.9\text{ s})^2 \\ &= 940\text{ m} \end{aligned}$$

Our final answers are thus that the plane will take 27 s to reach takeoff speed, with a minimum runway length of 940 m.

ASSESS Think about the last time you flew; 27 s seems like a reasonable time for a plane to accelerate on takeoff. Actual runway lengths at major airports are 3000 m or more, a few times greater than the minimum length, because they have to allow for emergency stops during an aborted takeoff. (If we had calculated a distance far greater than 3000 m, we would know we had done something wrong!)

EXAMPLE 2.13 **Finding the braking distance of a car on the highway**

A car is traveling at a speed of 30 m/s, a typical highway speed, on wet pavement. The driver sees an obstacle ahead and decides to stop. From this instant, it takes him 0.75 s to begin applying the brakes. Once the brakes are applied, the car experiences an acceleration of -6.0 m/s^2. How far does the car travel from the instant the driver notices the obstacle until it stops?

STRATEGIZE The wording of this problem makes it clear that, while it is braking, the car is experiencing constant acceleration. We'll once more use the information from Synthesis 2.1.

PREPARE This problem is more involved than previous problems we have solved, so we will take more care with the visual overview in **FIGURE 2.35.** In addition to a motion diagram and a pictorial representation, we include a graphical representation. Notice that there are two different phases of the motion: a constant-velocity phase before braking begins, and a steady slowing down once the brakes are applied. We will need to do two different calculations, one for each phase. Consequently, we use numerical subscripts rather than the simple i and f.

SOLVE From t_1 to t_2 the velocity stays constant at 30 m/s. This is uniform motion, so we compute the position at time t_2 using Equation 2.4 from Synthesis 2.1:

$$x_2 = x_1 + (v_x)_1(t_2 - t_1) = 0\text{ m} + (30\text{ m/s})(0.75\text{ s})$$
$$= 22.5\text{ m}$$

Starting at t_2, the velocity begins to decrease at a steady -6.0 m/s^2 until the car comes to rest at t_3. We can compute this time interval using Equation 2.11 from Synthesis 2.1, $(v_x)_3 = (v_x)_2 + a_x\,\Delta t$:

$$\Delta t = t_3 - t_2 = \frac{(v_x)_3 - (v_x)_2}{a_x} = \frac{0\text{ m/s} - 30\text{ m/s}}{-6.0\text{ m/s}^2} = 5.0\text{ s}$$

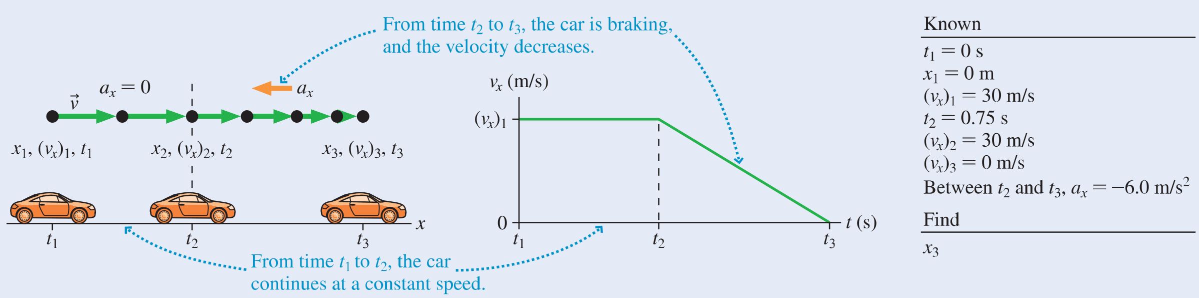
We can compute the position at time t_3 using Equation 2.12 from Synthesis 2.1. We take point 2 as the initial point and point 3 as the final point for this phase of the motion and use $\Delta t = t_3 - t_2$:

$$x_3 = x_2 + (v_x)_2\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
$$= 22.5\text{ m} + (30\text{ m/s})(5.0\text{ s}) + \tfrac{1}{2}(-6.0\text{ m/s}^2)(5.0\text{ s})^2$$
$$= 98\text{ m}$$

x_3 is the position of the car at the end of the problem—and so the car travels 98 m before coming to rest.

ASSESS The numbers for the reaction time and the acceleration on wet pavement are reasonable ones for an alert driver in a car with good tires. The final distance is quite large—more than the length of a football field.

FIGURE 2.35 Visual overview for a car braking to a stop.



2.7 Free Fall

If you drop a hammer and a feather, you know what will happen. The hammer quickly strikes the ground, and the feather drifts slowly downward and lands some time later. But if you do this experiment on the moon, the result is strikingly different: Both the hammer and the feather experience the exact same acceleration, undergo the exact same motion, and strike the ground at the same time.

The moon lacks an atmosphere, and so objects falling to its surface experience no air resistance. They are acted upon by only one force—gravity. If an object moves under the influence of gravity only, and no other forces, we call the resulting motion **free fall.** Many experiments have shown that **all objects in free fall, regardless of their mass, have the same acceleration.** Thus, if you drop two objects and they are both in free fall, they hit the ground at the same time.

On the earth, air resistance is a factor. But when you drop a heavy object like a hammer, air resistance can be ignored, so we make only a slight error in treating the hammer *as if* it were in free fall. Motion with air resistance is a problem we will

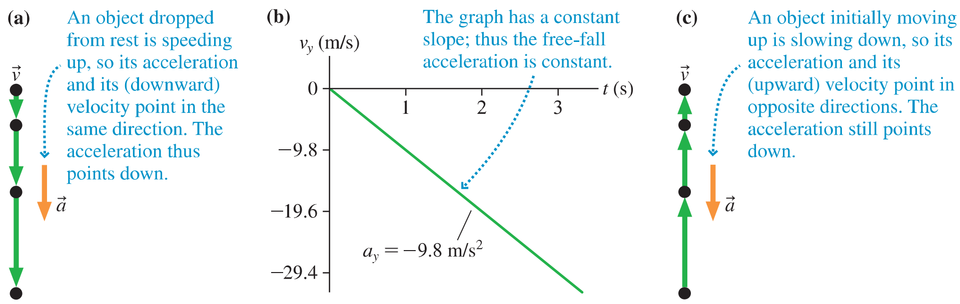
Free-falling feather Apollo 15 lunar astronaut David Scott performed a classic experiment on the moon, simultaneously dropping a hammer and a feather from the same height. Both hit the ground at the exact same time—something that would not happen in the atmosphere of the earth!

study in Chapter 5. Until then, we will restrict our attention to situations in which air resistance can be ignored, and we will assume that falling objects are in free fall.

FIGURE 2.36a shows the motion diagram for an object that was released from rest and falls freely. Since the acceleration is the same for all objects, the diagram and graph would be the same for a falling baseball or a falling boulder! FIGURE 2.36b shows the object's velocity graph. The velocity changes at a steady rate. The slope of the velocity-versus-time graph is the free-fall acceleration $a_{\text{free fall}}$.

Video Figure 2.36

FIGURE 2.36 Motion of an object in free fall.



Instead of dropping the object, suppose we throw it upward. What happens then? You know that the object will move up and that its speed will decrease as it rises. This is illustrated in the motion diagram of FIGURE 2.36c, which shows a surprising result: Even though the object is moving up, its acceleration still points down. In fact, **the free-fall acceleration always points down,** no matter what direction an object is moving.

Video Free Fall

Some of the children on this trampoline are moving up and some are moving down, but all are in free fall—and so are accelerating downward at 9.8 m/s^2.

NOTE ▶ Despite the name, free fall is not restricted to objects that are literally falling. Any object moving under the influence of gravity only, and no other forces, is in free fall. This includes objects falling straight down, objects that have been tossed or shot straight up, objects in projectile motion (such as a basketball free throw), and, as we will see, satellites in orbit. ◀

The value of the free-fall acceleration varies slightly at different places on the earth, but for the calculations in this text we will use the the following average value:

$$\vec{a}_{\text{free fall}} = (9.80 \text{ m/s}^2, \text{ vertically downward}) \tag{2.14}$$

Standard value for the acceleration of an object in free fall

The magnitude of the **free-fall acceleration** has the special symbol g:

$$g = 9.80 \text{ m/s}^2$$

We will generally work with two significant figures and so will use $g = 9.8 \text{ m/s}^2$.

Several points about free fall are worthy of note:

- g, by definition, is *always* positive. **There will never be a problem that uses a negative value for g.**
- The velocity graph in Figure 2.36b has a negative slope. Even though a falling object speeds up, it has *negative* acceleration. Thus g is *not* the object's acceleration, simply the *magnitude* of the acceleration. The one-dimensional acceleration is
$$a_y = -g$$
- Because free fall is motion with constant acceleration, we can use the kinematic equations for constant acceleration with $a_y = -g$.
- Once an object is acted upon by only the force of gravity, it is in free fall with $a_y = -9.8 \text{ m/s}^2$. It doesn't matter how the object entered free fall: Once in the air, a football that was punted straight up has the *same* acceleration of $-g$ as a stone dropped off a bridge.

- g is not called "gravity." Gravity is a force, not an acceleration. g is the *free-fall acceleration.*
- $g = 9.80\ \text{m/s}^2$ only on earth. Other planets have different values of g. You will learn in Chapter 6 how to determine g for other planets.
- We will sometimes compute acceleration of objects not in free fall in units of g. An acceleration of $9.8\ \text{m/s}^2$ is an acceleration of $1g$; an acceleration of $19.6\ \text{m/s}^2$ is $2g$. Generally, we can compute

$$\text{acceleration (in units of } g\text{, or } g\text{'s)} = \frac{\text{acceleration (in units of m/s}^2\text{)}}{9.8\ \text{m/s}^2} \qquad (2.15)$$

This allows us to express accelerations in units that have a definite physical reference.

EXAMPLE 2.14 Analyzing a rock's fall

A heavy rock is dropped from rest at the top of a cliff and falls 100 m before hitting the ground. How long does the rock take to fall to the ground, and what is its velocity when it hits?

STRATEGIZE This is a free-fall problem, so it is a constant-acceleration problem with $a_y = -g$. We will use the constant-acceleration equations from Synthesis 2.1.

PREPARE FIGURE 2.37 shows a visual overview with all necessary data. We have placed the origin at the ground, so that $y_i = 100\ \text{m}$.

SOLVE The first question in the problem statement involves a relationship between time and distance, a relationship expressed by Equation 2.12 in Synthesis 2.1. Using $(v_y)_i = 0\ \text{m/s}$ and $t_i = 0\ \text{s}$, we find

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y\,(\Delta t)^2 = y_i - \tfrac{1}{2}g\,(\Delta t)^2 = y_i - \tfrac{1}{2}g t_f^{\,2}$$

We can now solve for t_f:

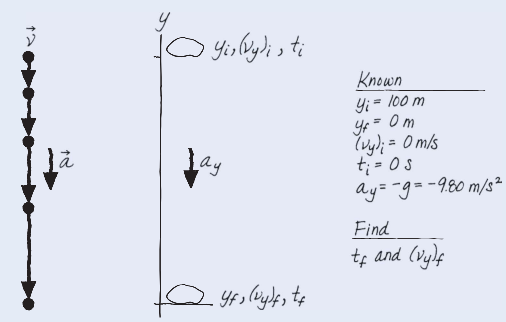
$$t_f = \sqrt{\frac{2(y_i - y_f)}{g}} = \sqrt{\frac{2(100\ \text{m} - 0\ \text{m})}{9.80\ \text{m/s}^2}} = 4.52\ \text{s}$$

Now that we know the fall time, we can use Equation 2.11 to find $(v_y)_f$:

$$(v_y)_f = (v_y)_i - g\,\Delta t = -g t_f = -(9.80\ \text{m/s}^2)(4.52\ \text{s}) = -44.3\ \text{m/s}$$

ASSESS Are the answers reasonable? Well, 100 m is about 300 feet, which is about the height of a 30-floor building. How long does it take an object to fall 30 floors? Four or five seconds seems pretty reasonable. How fast would the object be going at the bottom? Using an approximate version of our conversion factor $1\ \text{m/s} \approx 2\ \text{mph}$, we find that $44.3\ \text{m/s} \approx 90\ \text{mph}$. That also seems like a pretty reasonable speed for something that has fallen 30 floors. Suppose we had made a mistake. If we had misplaced a decimal point, we could have calculated a speed of 443 m/s, or about 900 mph! This is clearly *not* reasonable. If we had misplaced the decimal point in the other direction, we would have calculated a speed of $4.3\ \text{m/s} \approx 9\ \text{mph}$. This is another unreasonable result, because this is slower than a typical bicycling speed.

FIGURE 2.37 Visual overview of a falling rock.



CONCEPTUAL EXAMPLE 2.15 Analyzing the motion of a ball tossed upward

Draw a motion diagram and a velocity-versus-time graph for a ball tossed straight up in the air from the point that it leaves the hand until just before it is caught.

REASON You know what the motion of the ball looks like: The ball goes up, and then it comes back down again. This complicates the drawing of a motion diagram a bit because the ball retraces its route as it falls. A literal motion diagram would show the upward motion and downward motion on top of each other, leading to confusion. We can avoid this difficulty by horizontally separating the upward motion and downward motion diagrams. This will not affect our conclusions because it does not change any of the vectors. The motion diagram and velocity-versus-time graph appear as in FIGURE 2.38.

ASSESS The highest point in the ball's motion, where it reverses direction, is called a *turning point.* What are the velocity and the acceleration at this point? We can see from the motion diagram that the velocity vectors are pointing upward but getting shorter as the ball approaches the top. As it starts to fall, the velocity vectors are pointing downward and getting longer. There must be a moment—just an instant as $\vec{v}$ switches from pointing up to pointing down—when the velocity is zero. Indeed, the ball's velocity *is* zero for an instant at the precise top of the motion! We can also see on the velocity graph

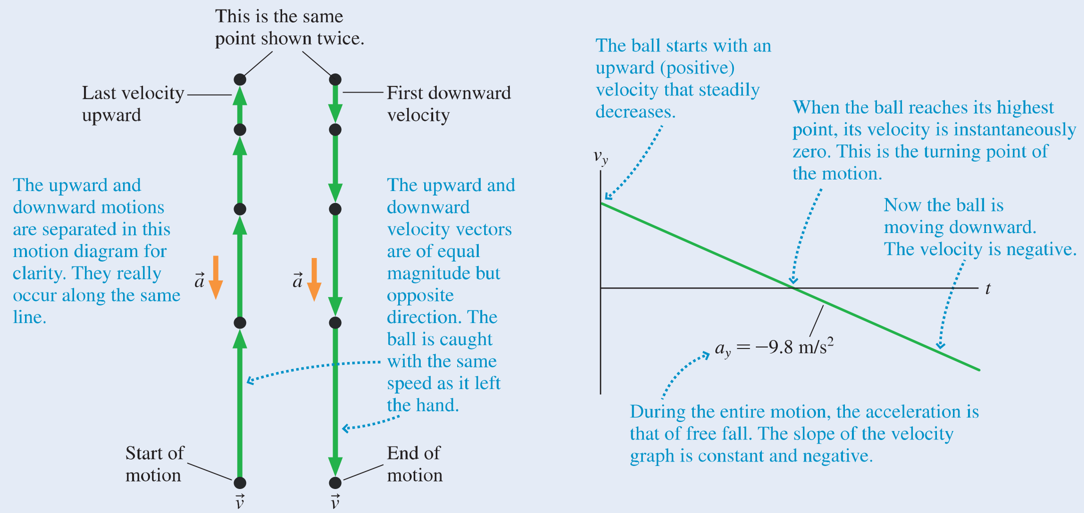
Continued

that there is one instant of time when $v_y = 0$. This is the turning point.

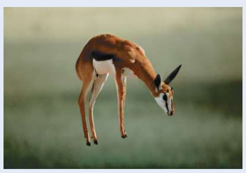
But what about the acceleration at the top? You might expect the acceleration to be zero at the highest point. But recall that the velocity at the top point is changing—from up to down. If the velocity is changing, there *must* be an acceleration. The slope of the velocity graph at the instant when $v_y = 0$—that is, at the highest point—is no different than at any other point in the motion. The ball is still in free fall with acceleration $a_y = -g$!

Another way to think about this is to note that zero acceleration would mean no change of velocity. When the ball reached zero velocity at the top, it would hang there and not fall if the acceleration were also zero!

FIGURE 2.38 Motion diagram and velocity graph of a ball tossed straight up in the air.



EXAMPLE 2.16 Finding the height of a leap BIO



A springbok is an antelope found in southern Africa that gets its name from its remarkable jumping ability. When a springbok is startled, it will leap straight up into the air—a maneuver called a "pronk." A particular springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at $35\ \text{m/s}^2$ for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air.

a. At what speed does the springbok leave the ground?
b. How high does it go?

STRATEGIZE This is a two-part problem. In the first phase of its motion, the springbok accelerates upward, reaching some maximum speed just as it leaves the ground. As soon as it does so, the springbok is subject to only the force of gravity, so it is in free fall. For both phases, we will use the constant-acceleration equations from Synthesis 2.1.

PREPARE We begin with the visual overview shown in **FIGURE 2.39**, where we've identified the two different phases of the motion: the springbok pushing off the ground and the springbok rising into the air. We'll treat these as two separate problems that we solve in turn. We will "re-use" the variables y_i, y_f, $(v_y)_i$, and $(v_y)_f$ for the two phases of the motion.

For the first part of our solution, in Figure 2.39a we choose the origin of the y-axis at the position of the springbok deep in its crouch. The final position is the top extent of the push, at the instant the springbok leaves the ground. We want to find the velocity at this position because that's how fast the springbok is moving as it leaves the ground. Figure 2.39b essentially starts over—we have defined a new vertical axis with its origin at the ground, so the highest point of the springbok's motion is its distance above the ground. The table of values shows the key piece of information for this second part of the problem: The initial velocity for part b is the final velocity from part a.

After the springbok leaves the ground, this is a free-fall problem because the springbok is moving under the influence of gravity only. We want to know the height of the leap, so we are looking for the height at the top point of the motion. This is a turning point of the motion, with the instantaneous velocity equal to zero. Thus y_f, the height of the leap, is the springbok's position at the instant $(v_y)_f = 0$.

SOLVE a. For the first phase, pushing off the ground, we have information about displacement, initial velocity, and acceleration, but we don't know anything about the time interval. Equation 2.13 from Synthesis 2.1 is perfect for this type of situation. We can use it to solve for the velocity with which the springbok lifts off the ground:

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\Delta y$$
$$= (0\ \text{m/s})^2 + 2(35\ \text{m/s}^2)(0.70\ \text{m}) = 49\ \text{m}^2/\text{s}^2$$
$$(v_y)_f = \sqrt{49\ \text{m}^2/\text{s}^2} = 7.0\ \text{m/s}$$

The springbok leaves the ground with a speed of 7.0 m/s.

b. Now we are ready for the second phase of the motion, the vertical motion after leaving the ground. Equation 2.13 is again appropriate because again we don't know the time. Because $y_i = 0$, the springbok's displacement is $\Delta y = y_f - y_i = y_f$, the height of the vertical leap. From part a, the initial velocity is $(v_y)_i = 7.0\ \text{m/s}$, and the final velocity is $(v_y)_f = 0$. This is free-fall motion, with $a_y = -g$; thus

$$(v_y)_f^2 = 0 = (v_y)_i^2 - 2g\Delta y = (v_y)_i^2 - 2gy_f$$

which gives

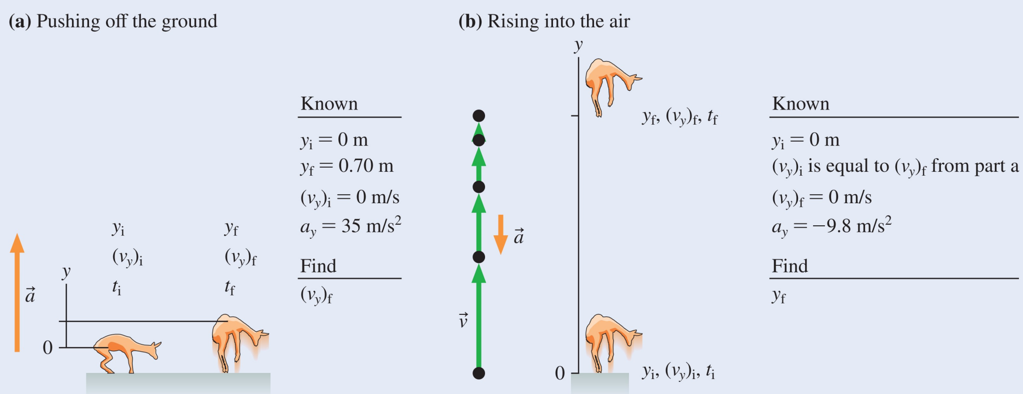
$$(v_y)_i^2 = 2gy_f$$

Solving for y_f, we get a jump height of

$$y_f = \frac{(7.0\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 2.5\ \text{m}$$

ASSESS 2.5 m is a remarkable leap—a bit over 8 ft—but these animals are known for their jumping ability, so this seems reasonable.

FIGURE 2.39 A visual overview of the springbok's leap.

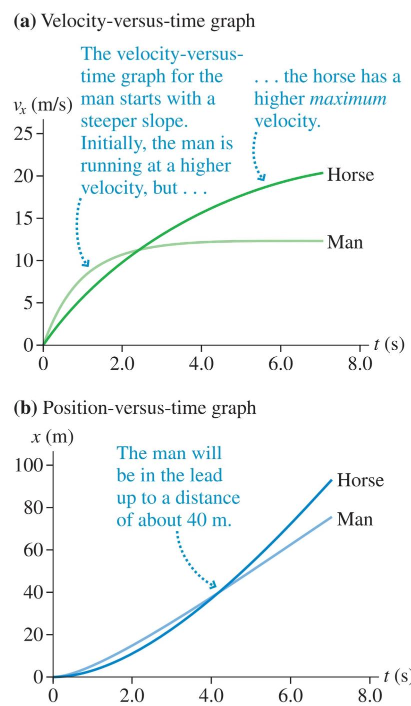


The caption accompanying the photo at the start of the chapter suggested a question about animals and their athletic abilities: Who is the winner in a race between a horse and a man? The surprising answer is "It depends." Specifically, the winner depends on the length of the race.

Some animals are capable of high speed; others are capable of great acceleration. Horses can run much faster than humans, but, when starting from rest, humans are capable of much greater initial acceleration. FIGURE 2.40 shows velocity and position graphs for an elite male sprinter and a thoroughbred racehorse. The horse's maximum velocity is about twice that of the man, but the man's initial acceleration—the slope of the velocity graph at early times—is greater than that of the horse. As the second graph shows, a man could win a short race. For a longer race, the horse's higher maximum velocity will put it in the lead. The men's world-record time for the mile is a bit under 4 min, but a horse can easily run a mile in less than 2 min.

For a race of many miles, another factor comes into play: energy. A very long race is less about velocity and acceleration than about endurance—the ability to continue expending energy for a long time. In such endurance trials, humans often win. We will explore such energy issues in Chapter 11.

FIGURE 2.40 BIO Velocity-versus-time and position-versus-time graphs for a sprint between a man and a horse.



STOP TO THINK 2.7 A volcano ejects a chunk of rock straight up at a velocity of $v_y = 30\ \text{m/s}$. Ignoring air resistance, what will be the velocity v_y of the rock when it falls back into the volcano's crater?

A. > 30 m/s B. 30 m/s C. 0 m/s D. -30 m/s E. < -30 m/s

INTEGRATED EXAMPLE 2.17 Speed versus endurance BIO

Cheetahs have the highest top speed of any land animal, but they usually fail in their attempts to catch their prey because their endurance is limited. They can maintain their maximum speed of 30 m/s for only about 15 s before they need to stop.

Thomson's gazelles, their preferred prey, have a lower top speed than cheetahs, but they can maintain this speed for a few minutes. When a cheetah goes after a gazelle, success or failure is a simple matter of kinematics: Is the cheetah's high speed enough to allow it to reach its prey before the cheetah runs out of steam? The following problem uses realistic data for such a chase.

A cheetah has spotted a gazelle. The cheetah leaps into action, reaching its top speed of 30 m/s in a few seconds. At this instant, the gazelle, 160 m from the running cheetah, notices the danger and heads directly away. The gazelle accelerates at $4.5\ \mathrm{m/s^2}$ for 6.0 s, then continues running at a constant speed. After reaching its maximum speed, the cheetah can continue running for only 15 s. Does the cheetah catch the gazelle, or does the gazelle escape?

STRATEGIZE The example asks, "Does the cheetah catch the gazelle?" Our most challenging task is to translate these words into a mathematical problem that we can solve using the techniques of this chapter. For a problem of this complexity, it will be particularly important to prepare a complete visual overview. An overview lays out all the relevant information in a concise visual form, helping to guide your mathematical solution.

PREPARE This example consists of two related problems, the motion of the cheetah and the motion of the gazelle, for which we'll use the subscripts "C" and "G." Let's take our starting time, $t_1 = 0$ s, as the instant that the gazelle notices the cheetah and begins to run. We'll take the position of the cheetah at this instant as the origin of our coordinate system, so $x_{1C} = 0$ m and $x_{1G} = 160$ m—the gazelle is 160 m away when it notices the cheetah. We've used this information to draw the visual overview in FIGURE 2.41, which includes motion diagrams and velocity graphs for the cheetah and the gazelle. The visual overview sums up everything we know about the problem.

With a clear picture of the situation, we can now rephrase the problem this way: Compute the position of the cheetah and the position of the gazelle at $t_3 = 15$ s, the time when the cheetah needs to break off the chase. If $x_{3G} > x_{3C}$, then the gazelle stays out in front and escapes. If $x_{3G} \le x_{3C}$, the cheetah wins the race—and gets its dinner.

SOLVE The cheetah is in uniform motion for the entire duration of the problem, so we can use Equation 2.4 of Synthesis 2.1 to solve for its position at $t_3 = 15$ s:

$$x_{3C} = x_{1C} + (v_x)_{1C}\Delta t = 0\ \mathrm{m} + (30\ \mathrm{m/s})(15\ \mathrm{s}) = 450\ \mathrm{m}$$

The gazelle's motion has two phases: one of constant acceleration and then one of constant velocity. We can solve for the position and the velocity at t_2, the end of the first phase, using Equations 2.11 and 2.12 of Synthesis 2.1. Let's find the velocity first:

$$(v_x)_{2G} = (v_x)_{1G} + (a_x)_G\Delta t = 0\ \mathrm{m/s} + (4.5\ \mathrm{m/s^2})(6.0\ \mathrm{s}) = 27\ \mathrm{m/s}$$

The gazelle's position at t_2 is

$$x_{2G} = x_{1G} + (v_x)_{1G}\Delta t + \tfrac{1}{2}(a_x)_G(\Delta t)^2$$

$$= 160\ \mathrm{m} + 0 + \tfrac{1}{2}(4.5\ \mathrm{m/s^2})(6.0\ \mathrm{s})^2 = 240\ \mathrm{m}$$

The gazelle has a head start; it begins at $x_{1G} = 160$ m.

Δt is the time for this phase of the motion, $t_2 - t_1 = 6.0$ s.

From t_2 to t_3 the gazelle moves at a constant speed, so we can use the uniform motion equation, Equation 2.4, to find its final position:

The gazelle begins this phase of the motion at $x_{2G} = 240$ m.

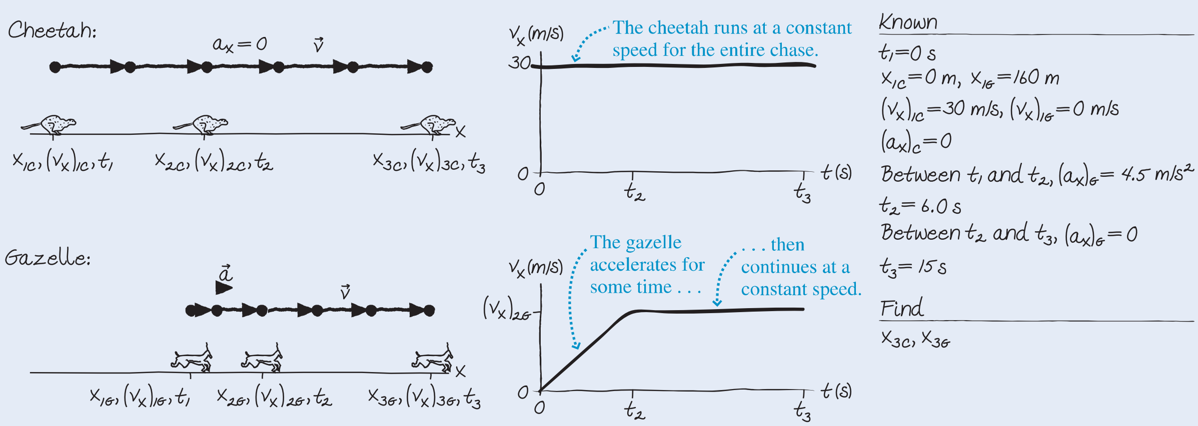
Δt for this phase of the motion is $t_3 - t_2 = 9.0$ s.

$$x_{3G} = x_{2G} + (v_x)_{2G}\ \Delta t = 240\ \mathrm{m} + (27\ \mathrm{m/s})(9.0\ \mathrm{s}) = 480\ \mathrm{m}$$

x_{3C} is 450 m; x_{3G} is 480 m. The gazelle is 30 m ahead of the cheetah when the cheetah has to break off the chase, so the gazelle escapes.

ASSESS Does our solution make sense? Let's look at the final result. The numbers in the problem statement are realistic, so we expect our results to mirror real life. The speed for the gazelle is close to that of the cheetah, which seems reasonable for two animals known for their speed. And the result is the most common occurrence—the chase is close, but the gazelle gets away.

FIGURE 2.41 Visual overview for the cheetah and for the gazelle.



SUMMARY

GOAL To describe and analyze motion along a line.

GENERAL STRATEGIES

Problem-Solving Approach

Our general problem-solving approach has four parts:

STRATEGIZE Think about the big picture: What kind of problem is this? What general approach should be used? What should the answer look like?

PREPARE Set up the problem:

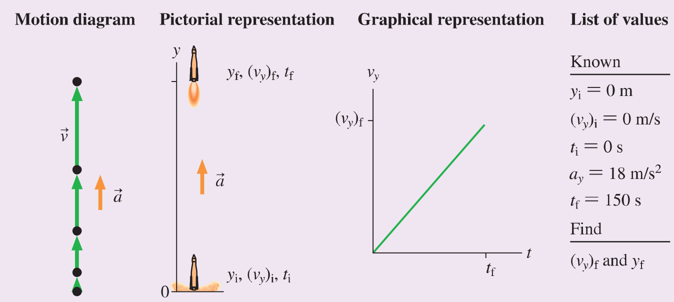
- Draw a picture.
- Collect necessary information.
- Do preliminary calculations.

SOLVE Do the necessary mathematics or reasoning.

ASSESS Check your answer to see if it is complete in all details and makes physical sense.

Visual Overview

A visual overview consists of several parts that completely specify a problem. This may include any or all of the elements below:



IMPORTANT CONCEPTS

Velocity is the rate of change of position:

$$v_x = \frac{\Delta x}{\Delta t}$$

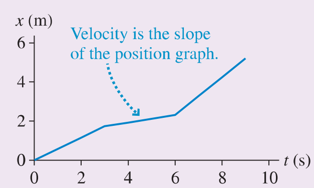
Acceleration is the rate of change of velocity:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

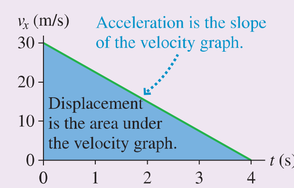
The units of acceleration are m/s^2.

An object is speeding up if v_x and a_x have the same sign, slowing down if they have opposite signs.

A **position-versus-time graph** plots position on the vertical axis against time on the horizontal axis.



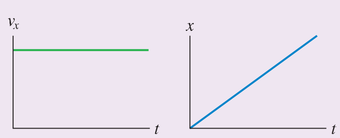
A **velocity-versus-time graph** plots velocity on the vertical axis against time on the horizontal axis.



APPLICATIONS

Uniform Motion

An object in uniform motion has a constant velocity. Its velocity graph is a horizontal line; its position graph is linear.



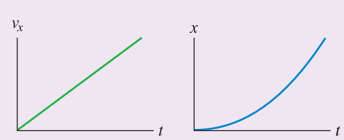
Kinematic equation for uniform motion:

$$x_f = x_i + v_x\,\Delta t$$

Uniform motion is a special case of constant-acceleration motion, with $a_x = 0$.

Motion with Constant Acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.



Kinematic equations for motion with constant acceleration:

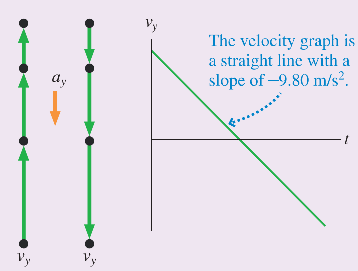
$$(v_x)_f = (v_x)_i + a_x\,\Delta t$$

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$(v_x)_f^{\,2} = (v_x)_i^{\,2} + 2a_x\,\Delta x$$

Free Fall

Free fall is a special case of constant-acceleration motion. The acceleration has magnitude $g = 9.80$ m/s^2 and is always directed vertically downward whether an object is moving up or down.



Learning Objectives After studying this chapter, you should be able to:

- Use motion diagrams to interpret motion. *Conceptual Question 2.3; Problems 2.1, 2.2, 2.59*
- Use and interpret motion graphs. *Conceptual Questions 2.5, 2.13; Problems 2.4, 2.18, 2.19, 2.22, 2.62*
- Calculate the velocity of an object. *Conceptual Question 2.9; Problems 2.8, 2.15, 2.57*
- Solve problems about an object in uniform motion. *Problems 2.9, 2.10, 2.11, 2.13, 2.58*
- Calculate the acceleration of an object. *Problems 2.25, 2.27, 2.32, 2.33, 2.72*
- Determine and interpret the sign of acceleration. *Conceptual Questions 2.2, 2.8; Problem 2.50*
- Use the problem-solving approach to solve problems of motion with constant acceleration and free fall. *Problems 2.36, 2.40, 2.41, 2.47, 2.52, 2.75*

STOP TO THINK ANSWERS

Chapter Preview Stop to Think: B. The bicycle is moving to the left, so the velocity vectors must point to the left. The speed is increasing, so successive velocity vectors must get longer.

Stop to Think 2.1: D. The motion consists of two constant-velocity phases, and the second one has a higher velocity. The correct graph has two straight-line segments, with the second one having a steeper slope.

Stop to Think 2.2: B. The displacement is the area under a velocity-versus-time graph. In all four cases, the graph is a straight line, so the area under the graph is a rectangle. The area is the product of the length and the height, so the largest displacement belongs to the graph with the largest product of the length (the time interval, in s) and the height (the velocity, in m/s).

Stop to Think 2.3: C. Consider the slope of the position-versus-time graph. It starts out positive and constant, then decreases to zero. Thus the velocity graph must start with a constant positive value, then decrease to zero.

Stop to Think 2.4: C. Acceleration is the slope of the velocity-versus-time graph. The largest magnitude of the slope is at point C.

Stop to Think 2.5: B. The elevator is moving down, so $v_y < 0$. It is slowing down, so the magnitude of the velocity is decreasing. As time goes on, the velocity graph should get closer to the origin. This means that the acceleration is positive, and the slope of the graph is positive.

Stop to Think 2.6: E. An acceleration of 1.2 m/s^2 corresponds to an increase of 1.2 m/s every second. At this rate, the cruising speed of 6.0 m/s will be reached after 5.0 s.

Stop to Think 2.7: D. The final velocity will have the same *magnitude* as the initial velocity, but the velocity is negative because the rock will be moving downward.

Video Tutor Solution Chapter 2

QUESTIONS

Conceptual Questions

1. A person gets in an elevator on the ground floor and rides it to the top floor of a building. Sketch a velocity-versus-time graph for this motion.
2. a. Give an example of a vertical motion with a positive velocity and a negative acceleration.
   b. Give an example of a vertical motion with a negative velocity and a negative acceleration.
3. Figure Q2.3 shows growth rings in a tree's trunk. The wide BIO and narrow rings correspond to years of fast and slow growth. Think of the rings as a motion diagram for the tree's growth. If we define an axis as shown, with x measured out from the center of the tree, use the appearance of the rings to sketch a velocity-versus-time graph for the radial growth of the tree.

FIGURE Q2.3

4. Sketch a velocity-versus-time graph for a rock that is thrown straight upward, from the instant it leaves the hand until the instant it hits the ground.
5. You are driving down the road at a constant speed. Another car going a bit faster catches up with you and passes you. Draw a position graph for both vehicles on the same set of axes, and note the point on the graph where the other vehicle passes you.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging). Problems labeled BIO are of biological or medical interest.

The eText icon indicates when there is a video tutor solution available for the chapter or for a specific problem. To launch these videos, log into your eText through Mastering™ Physics or log into the Study Area.

6. Figure Q2.6 shows the velocity-versus-time graphs for two objects A and B. Students Zach and Victoria are asked to tell stories that correspond to the motion of the objects. Zach says, "The graph could represent two cars traveling in opposite directions that pass each other." Victoria says, "No, I think they could be two rocks thrown vertically from a bridge; rock A is thrown upward and rock B is thrown downward." Which student, if either, is correct? Explain.

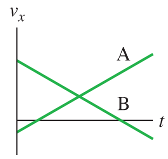


FIGURE Q2.6

7. BIO Certain animals are capable of running at great speeds; other animals are capable of tremendous accelerations. Speculate on which would be more beneficial to a predator—large maximum speed or large acceleration.
8. A ball is thrown straight up into the air. At each of the following instants, is the ball's acceleration a_y equal to g, $-g$, 0, $< g$, or $> g$?
   a. Just after leaving your hand?
   b. At the very top (maximum height)?
   c. Just before hitting the ground?
9. Janelle stands on a balcony, two stories above Michael. She throws one ball straight up and one ball straight down, but both with the same initial speed. Eventually each ball passes Michael. Which ball, if either, is moving faster when it passes Michael? Explain.
10. Figure Q2.10 shows an object's position-versus-time graph. The letters A to E correspond to various segments of the motion in which the graph has constant slope.
    a. Write a realistic motion short story for an object that would have this position graph.
    b. In which segment(s) is the object at rest?
    c. In which segment(s) is the object moving to the right?
    d. Is the speed of the object during segment C greater than, equal to, or less than its speed during segment E? Explain.

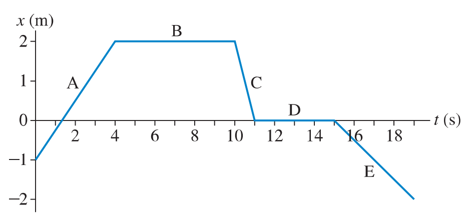


FIGURE Q2.10

11. Figure Q2.11 shows the position graph for an object moving along the horizontal axis.
    a. Write a realistic motion short story for an object that would have this position graph.
    b. Draw the corresponding velocity graph.

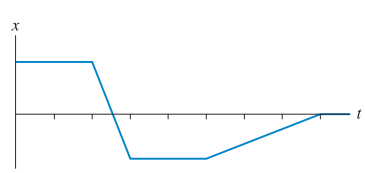


FIGURE Q2.11

12. Figure Q2.12 shows the position-versus-time graphs for two objects, A and B, that are moving along the same axis.
    a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
    b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

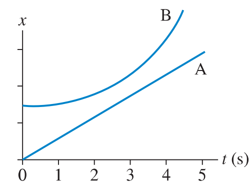


FIGURE Q2.12

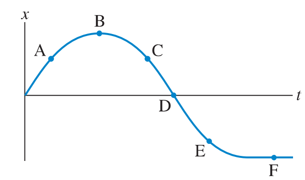


FIGURE Q2.13

13. Figure Q2.13 shows a position-versus-time graph. At which lettered point or points is the object
    a. Moving the fastest?
    b. Moving to the left?
    c. Speeding up?
    d. Slowing down?
    e. Turning around?
14. Figure Q2.14 is the velocity-versus-time graph for an object moving along the x-axis.
    a. During which segment(s) is the velocity constant?
    b. During which segment(s) is the object speeding up?
    c. During which segment(s) is the object slowing down?
    d. During which segment(s) is the object standing still?
    e. During which segment(s) is the object moving to the right?

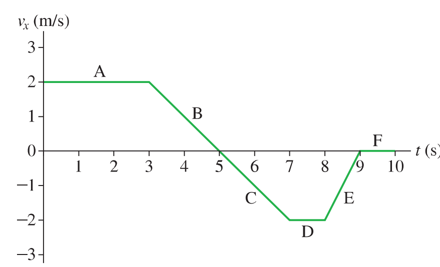


FIGURE Q2.14

Multiple-Choice Questions

15. | Figure Q2.15 shows the position graph of a car traveling on a straight road. At which labeled instant is the speed of the car greatest?

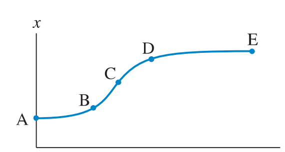


FIGURE Q2.15

16. | Figure Q2.16 shows the position graph of a car traveling on a straight road. The velocity at instant 1 is ________ and the velocity at instant 2 is ________.
    A. positive, negative
    B. positive, positive
    C. negative, negative
    D. negative, zero
    E. positive, zero

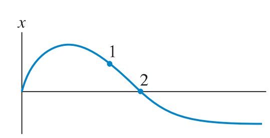


FIGURE Q2.16

17. | Figure Q2.17 shows an object's position-versus-time graph. What is the velocity of the object at $t = 6$ s?
A. 0.67 m/s B. 0.83 m/s
C. 3.3 m/s D. 4.2 m/s
E. 25 m/s

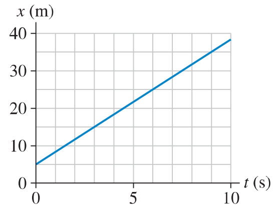


FIGURE Q2.17

18. | The following options describe the motion of four cars A–D. Which car has the largest acceleration?
A. Goes from 0 m/s to 10 m/s in 5.0 s
B. Goes from 0 m/s to 5.0 m/s in 2.0 s
C. Goes from 0 m/s to 20 m/s in 7.0 s
D. Goes from 0 m/s to 3.0 m/s in 1.0 s

19. | A car is traveling at $v_x = 20$ m/s. The driver applies the brakes, and the car slows with $a_x = -4.0$ m/s^2. What is the stopping distance?
A. 5.0 m
B. 25 m
C. 40 m
D. 50 m

20. || Velocity-versus-time graphs for three drag racers are shown in Figure Q2.20. At $t = 5.0$ s, which car has traveled the farthest?
A. Andy
B. Rachel
C. Carl
D. All have traveled the same distance

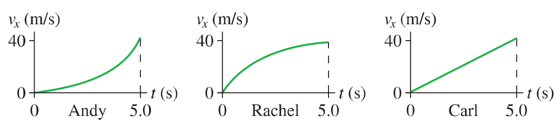


FIGURE Q2.20

21. | Which of the three drag racers in Question 20 had the greatest acceleration at $t = 0$ s?
A. Andy
B. Rachel
C. Carl
D. All had the same acceleration

22. || Chris is holding two softballs while standing on a balcony. She throws ball 1 straight up in the air and, at the same instant, releases her grip on ball 2, letting it drop over the side of the building. Which velocity graph in Figure Q2.22 best represents the motion of the two balls?

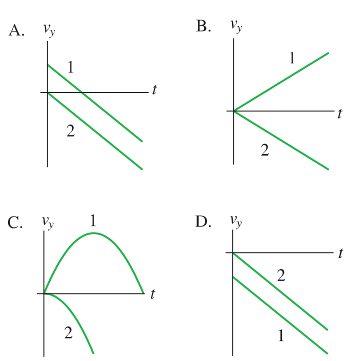


FIGURE Q2.22

23. || Suppose a plane accelerates from rest for 30 s, achieving a takeoff speed of 80 m/s after traveling a distance of 1200 m down the runway. A smaller plane with the same acceleration has a takeoff speed of 40 m/s. Starting from rest, after what distance will this smaller plane reach its takeoff speed?
A. 300 m B. 600 m C. 900 m D. 1200 m

24. || Figure Q2.24 shows a motion diagram with the clock reading (in seconds) shown at each position. From $t = 9$ s to $t = 15$ s the object is at the same position. After that, it returns along the same track. The positions of the dots for $t \geq 16$ s are offset for clarity. Which graph best represents the object's *velocity?*

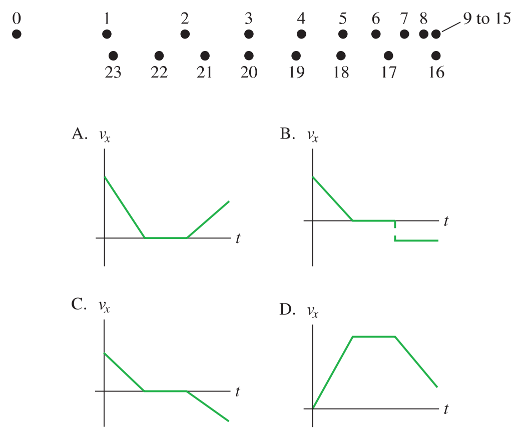


FIGURE Q2.24

25. || Nate throws a ball straight up to Kayla, who is standing on a balcony 3.8 m above Nate. When she catches it, the ball is still moving upward at a speed of 2.8 m/s. With what initial speed did Nate throw the ball?
A. 7.0 m/s B. 12.3 m/s
C. 9.1 m/s D. 10.6 m/s

26. | Starting from rest, a car takes 2.4 s to travel the first 15 m. Assuming a constant acceleration, how long will it take the car to travel the next 15 m?
A. 0.67 s B. 1.0 s C. 1.8 s D. 3.6 s

27. | The velocity-versus-time graph for a car driving down a straight road is shown in Figure Q2.27. What is the acceleration of the car during the period shown?
A. 1.0 m/s^2 B. 2.5 m/s^2
C. 3.8 m/s^2 D. 5.0 m/s^2

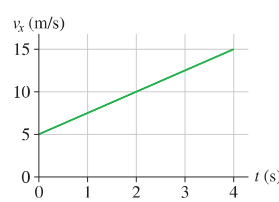


FIGURE Q2.27

28. || The velocity-versus-time graph for a car driving down a straight road is shown in Figure Q2.27. How far does the car travel during the time interval from $t = 0$ s to $t = 4.0$ s?
A. 10 m B. 20 m C. 40 m D. 60 m

PROBLEMS

Section 2.1 Describing Motion

1. ||| Figure P2.1 shows a motion diagram of a car traveling down a street. The camera took one frame every second. A distance scale is provided.
   a. Use the scale to determine the x-value of the car at each dot. Place your data in a table, similar to Table 2.1, showing each position and the instant of time at which it occurred.
   b. Make a graph of x versus t, using the data in your table. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.

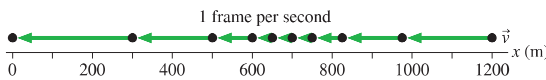


FIGURE P2.1

2. | For each motion diagram in Figure P2.2, determine the sign (positive or negative) of the position and the velocity.

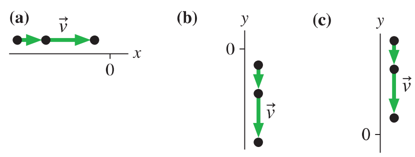


FIGURE P2.2

3. || The position graph of Figure P2.3 shows a dog slowly sneaking up on a squirrel, then putting on a burst of speed.
   a. For how many seconds does the dog move at the slower speed?
   b. Draw the dog's velocity-versus-time graph. Include a numerical scale on both axes.

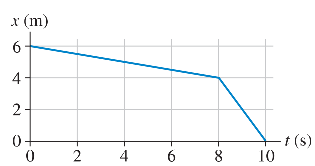


FIGURE P2.3

4. || A rural mail carrier is driving slowly, putting mail in mailboxes near the road. He overshoots one mailbox, stops, shifts into reverse, and then backs up until he is at the right spot. The velocity graph of Figure P2.4 represents his motion.
   a. Draw the mail carrier's position-versus-time graph. Assume that $x = 0$ m at $t = 0$ s.
   b. What is the position of the mailbox?

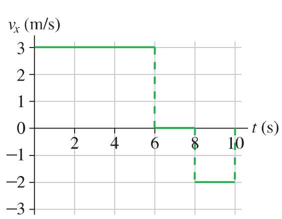


FIGURE P2.4

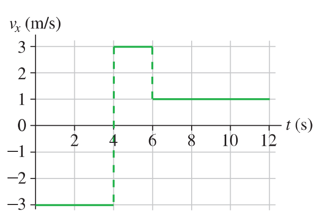


FIGURE P2.5

5. || For the velocity-versus-time graph of Figure P2.5:
   a. Draw the corresponding position-versus-time graph. Assume that $x = 0$ m at $t = 0$ s.
   b. What is the object's position at $t = 12$ s?
   c. Describe a moving object that could have these graphs.
6. || Starting at 48th Street, Dylan rides his bike due east on Meridian Road with the wind at his back. He rides for 20 min at 15 mph. He then stops for 5 min, turns around, and rides back to 48th Street; because of the headwind, his speed is only 10 mph.
   a. How long does his trip take?
   b. Assuming that the origin of his trip is at 48th Street, draw a position-versus-time graph for his trip.
7. ||| An elevator in a high-rise building goes up and down at the same speed. Starting at the ground floor, Rafael and Monica ride up five floors, a vertical rise of 20 m. The elevator stops for 10 s as Monica gets off. Rafael then goes back down two floors. Rafael's entire trip takes 24 s. Taking the origin to be at the ground floor, draw position-versus-time and velocity-versus-time graphs for Rafael's trip.
8. || A bicyclist has the position-versus-time graph shown in Figure P2.8. What is the bicyclist's velocity at $t = 10$ s, at $t = 25$ s, and at $t = 35$ s?

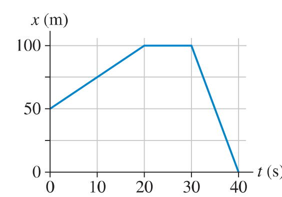


FIGURE P2.8

Section 2.2 Uniform Motion

9. | In major league baseball, the pitcher's mound is 60 feet from the batter. If a pitcher throws a 95 mph fastball, how much time elapses from when the ball leaves the pitcher's hand until the ball reaches the batter?
10. | In college softball, the distance from the pitcher's mound to the batter is 43 feet. If the ball leaves the bat at 100 mph, how much time elapses between the hit and the ball reaching the pitcher?
11. || Alan leaves Los Angeles at 8:00 AM to drive to San Francisco, 400 mi away. He travels at a steady 50 mph. Beth leaves Los Angeles at 9:00 AM and drives a steady 60 mph.
    a. Who gets to San Francisco first?
    b. How long does the first to arrive have to wait for the second?
12. || Richard is driving home to visit his parents. 125 mi of the trip are on the interstate highway where the speed limit is 65 mph. Normally Richard drives at the speed limit, but today he is running late and decides to take his chances by driving at 70 mph. How many minutes does he save?
13. ||| In a 5.00 km race, one runner runs at a steady 12.0 km/h and another runs at 14.5 km/h. How long does the faster runner have to wait at the finish line to see the slower runner cross?
14. |||| In an 8.00 km race, one runner runs at a steady 11.0 km/h and another runs at 14.0 km/h. How far from the finish line is the slower runner when the faster runner finishes the race?

15. || Figure P2.15 shows actual data from Usain Bolt's 2009 word-record run in the 100 m sprint. From this graph, estimate his top speed in m/s and in mph.

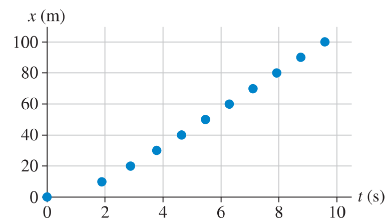


FIGURE P2.15

16. || While running a marathon, a long-distance runner uses a stopwatch to time herself over a distance of 100 m. She finds that she runs this distance in 18 s. Answer the following by considering ratios, without computing her velocity.

a. If she maintains her speed, how much time will it take her to run the next 400 m?

b. How long will it take her to run a mile at this speed?

Section 2.3 Instantaneous Velocity

17. | Figure P2.17 shows the position graph of a particle.

a. Draw the particle's velocity graph for the interval $0\text{ s} \le t \le 4\text{ s}$.

b. Does this particle have a turning point or points? If so, at what time or times?

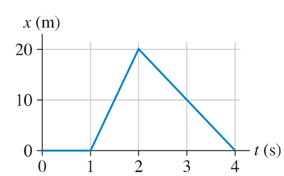


FIGURE P2.17

18. || A somewhat idealized graph of the speed of the blood in
BIO the ascending aorta during one beat of the heart appears as in Figure P2.18.

a. Approximately how far, in cm, does the blood move during one beat?

b. Assume similar data for the motion of the blood in your aorta, and make a rough estimate of the distance from your heart to your brain. Estimate how many beats of the heart it takes for blood to travel from your heart to your brain.

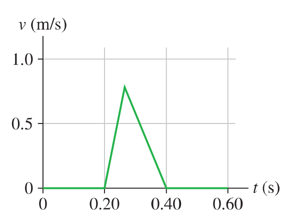


FIGURE P2.18

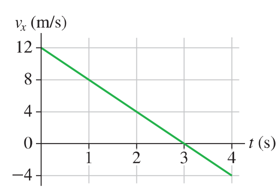


FIGURE P2.19

19. ||| A car starts from $x_i = 10$ m at $t_i = 0$ s and moves with the velocity graph shown in Figure P2.19.

a. What is the car's position at $t = 2$ s, 3 s, and 4 s?

b. Does this car ever change direction? If so, at what time?

20. || Figure P2.20 shows a graph of actual position-versus-time data for a particular type of drag racer known as a "funny car."

a. Estimate the car's velocity at 2.0 s.

b. Estimate the car's velocity at 4.0 s.

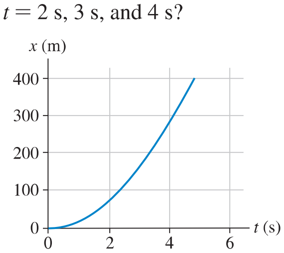


FIGURE P2.20

Section 2.4 Acceleration

21. || Figure P2.21 shows the velocity graph of a bicycle. Draw the bicycle's acceleration graph for the interval $0\text{ s} \le t \le 4\text{ s}$. Give both axes an appropriate numerical scale.

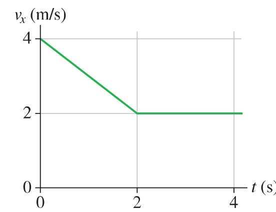


FIGURE P2.21

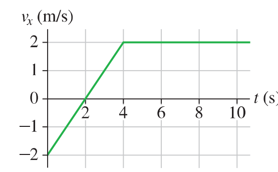


FIGURE P2.22

22. ||| We set the origin of a coordinate system so that the position of a train is $x = 0$ m at $t = 0$ s. Figure P2.22 shows the train's velocity graph.

a. Draw position and acceleration graphs for the train.

b. Find the acceleration of the train at $t = 3.0$ s.

23. ||| An object has the acceleration graph shown in Figure P2.23. Its velocity at $t = 0$ s is $v_x = 2.0$ m/s. Draw the object's velocity graph.

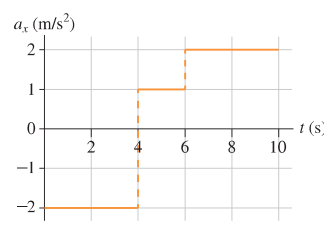


FIGURE P2.23

24. || Figure P2.18 showed data for the speed of blood in the aorta.
BIO Determine the magnitude of the acceleration for both phases, speeding up and slowing down.

25. || Figure P2.25 is a somewhat simplified velocity graph for Olympic sprinter Carl Lewis starting a 100 m dash. Estimate his acceleration during each of the intervals A, B, and C.

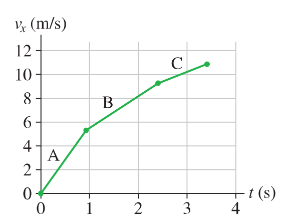


FIGURE P2.25

26. | Small frogs that are good jumpers are capable of remarkable
BIO accelerations. One species reaches a takeoff speed of 3.7 m/s in 60 ms. What is the frog's acceleration during the jump?

27. | A Thomson's gazelle can reach a speed of 13 m/s in 3.0 s. A lion
BIO can reach a speed of 9.5 m/s in 1.0 s. A trout can reach a speed of 2.8 m/s in 0.12 s. Which animal has the largest acceleration?

28. ||| When striking, the pike, a
BIO predatory fish, can accelerate from rest to a speed of 4.0 m/s in 0.11 s.

a. What is the acceleration of the pike during this strike?

b. How far does the pike move during this strike?

Section 2.5 Motion with Constant Acceleration

29. ‖ a. What constant acceleration, in SI units, must a car have to go from zero to 60 mph in 10 s?
b. What fraction of g is this?
c. How far has the car traveled when it reaches 60 mph? Give your answer both in SI units and in feet.

30. ‖ BIO When jumping, a flea rapidly extends its legs, reaching a takeoff speed of 1.0 m/s over a distance of 0.50 mm.
a. What is the flea's acceleration as it extends its legs?
b. How long does it take the flea to leave the ground after it begins pushing off?

31. ‖‖ BIO In a car crash, large accelerations of the head can lead to severe injuries or even death. A driver can probably survive an acceleration of $50g$ that lasts for less than 30 ms, but in a crash with a $50g$ acceleration lasting longer than 30 ms, a driver is unlikely to survive. Imagine a collision in which a driver's head experienced a $50g$ acceleration.
a. What is the highest speed that the car could have had such that the driver survived?
b. What is the shortest survivable distance over which the driver's head could have come to rest?

32. ‖ Light-rail passenger trains that provide transportation within and between cities speed up and slow down with a nearly constant (and quite modest) acceleration. A train travels through a congested part of town at 5.0 m/s. Once free of this area, it speeds up to 12 m/s in 8.0 s. At the edge of town, the driver again accelerates, with the same acceleration, for another 16 s to reach a higher cruising speed. What is the final speed?

33. ‖‖ A cross-country skier is skiing along at a zippy 8.0 m/s. She stops pushing and simply glides along, slowing to a reduced speed of 6.0 m/s after gliding for 5.0 m. What is the magnitude of her acceleration as she slows?

34. ‖ A small propeller airplane can comfortably achieve a high enough speed to take off on a runway that is 1/4 mile long. A large, fully loaded passenger jet has about the same acceleration from rest, but it needs to achieve twice the speed to take off. What is the minimum runway length that will serve? **Hint:** You can solve this problem using ratios without having any additional information.

35. ‖ Formula One racers speed up much more quickly than normal passenger vehicles, and they also can stop in a much shorter distance. A Formula One racer traveling at 90 m/s can stop in a distance of 110 m. What is the magnitude of the car's acceleration as it slows during braking?

36. ‖ Figure P2.36 shows a velocity-versus-time graph for a particle moving along the x-axis. At $t = 0$ s, assume that $x = 0$ m.
a. What are the particle's position, velocity, and acceleration at $t = 1.0$ s?
b. What are the particle's position, velocity, and acceleration at $t = 3.0$ s?

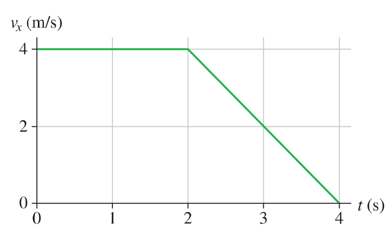


FIGURE P2.36

Section 2.6 Solving One-Dimensional Motion Problems

37. ‖ A driver has a reaction time of 0.50 s, and the maximum deceleration of her car is 6.0 m/s^2. She is driving at 20 m/s when suddenly she sees an obstacle in the road 50 m in front of her. Can she stop the car in time to avoid a collision?

38. ‖ BIO Chameleons catch insects with their tongues, which they can rapidly extend to great lengths. In a typical strike, the chameleon's tongue accelerates at a remarkable 250 m/s^2 for 20 ms, then travels at constant speed for another 30 ms. During this total time of 50 ms, 1/20 of a second, how far does the tongue reach?

39. ‖‖ You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is 10 m/s^2.
a. How much distance is between you and the deer when you come to a stop?
b. What is the maximum speed you could have and still not hit the deer?

40. ‖ BIO Upon impact, bicycle helmets compress, thus lowering the potentially dangerous acceleration experienced by the head. A new kind of helmet uses an airbag that deploys from a pouch worn around the rider's neck. In tests, a headform wearing the inflated airbag is dropped onto a rigid platform; the speed just before impact is 6.0 m/s. Upon impact, the bag compresses its full 12.0 cm thickness, slowing the headform to rest. What is the acceleration, in g's, experienced by the headform? (An acceleration greater than $60g$ is considered especially dangerous.)

41. ‖ A car is traveling at a steady 80 km/h in a 50 km/h zone. A police motorcycle takes off at the instant the car passes it, accelerating at a steady 8.0 m/s^2.
a. How much time elapses before the motorcycle is moving as fast as the car?
b. How far is the motorcycle from the car when it reaches this speed?

42. ‖‖ BIO The velocity-versus-time graph for the vertical jump of a green leafhopper, a small insect, is shown in Figure P2.42. This insect is unusual because it jumps with nearly constant acceleration.
a. Estimate the leafhopper's acceleration.
b. About how far does it move during this phase of its jump?

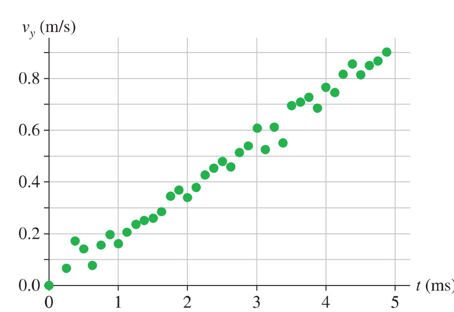


FIGURE P2.42

Watch Video Solution Problems 2.47 and 2.57

43. ||| A simple model for a person running the 100 m dash is to assume the sprinter runs with constant acceleration until reaching top speed, then maintains that speed through the finish line. If a sprinter reaches his top speed of 11.2 m/s in 2.14 s, what will be his total time?

Section 2.7 Free Fall

44. ||| BIO Scientists have investigated how quickly hoverflies start beating their wings when dropped both in complete darkness and in a lighted environment. Starting from rest, the insects were dropped from the top of a 40-cm-tall box. In the light, those flies that began flying 200 ms after being dropped avoided hitting the bottom of the box 80% of the time, while those in the dark avoided hitting only 22% of the time.
   a. How far would a fly have fallen in the 200 ms before it began to beat its wings?
   b. How long would it take for a fly to hit the bottom if it never began to fly?

45. || BIO Here's an interesting challenge you can give to a friend. Hold a \$1 (or larger!) bill by an upper corner. Have a friend prepare to pinch a lower corner, putting her fingers near but not touching the bill. Tell her to try to catch the bill when you drop it by simply closing her fingers. This seems like it should be easy, but it's not. After she sees that you have released the bill, it will take her about 0.25 s to react and close her fingers—which is not fast enough to catch the bill. How much time does it take for the bill to fall beyond her grasp? The length of a bill is 16 cm.

46. ||| BIO In the preceding problem we saw that a person's reaction time is generally not quick enough to allow the person to catch a \$1 bill dropped between the fingers. The 16 cm length of the bill passes through a student's fingers before she can grab it if she has a typical 0.25 s reaction time. How long would a bill need to be for her to have a good chance of catching it?

47. | A gannet is a seabird that fishes by diving from a great height. If a gannet hits the water at 32 m/s (which they do), what height did it dive from? Assume that the gannet was motionless before starting its dive.

48. |||| BIO Steelhead trout migrate upriver to spawn. Occasionally they need to leap up small waterfalls to continue their journey. Fortunately, steelhead are remarkable jumpers, capable of leaving the water at a speed of 8.0 m/s.
   a. What is the maximum height that a steelhead can jump?
   b. Leaving the water vertically at 8.0 m/s, a steelhead lands on the top of a waterfall 1.8 m high. How long is it in the air?

49. ||| In a circus act, an acrobat rebounds upward from the surface of a trampoline at the exact moment that another acrobat, perched 9.0 m above him, releases a ball from rest. While still in flight, the acrobat catches the ball just as it reaches him. If he left the trampoline with a speed of 8.0 m/s, how long is he in the air before he catches the ball?

50. | A student at the top of a building of height h throws ball A straight upward with speed v_0 and throws ball B straight downward with the same initial speed.
   a. Compare the balls' accelerations, both direction and magnitude, immediately after they leave her hand. Is one acceleration larger than the other? Or are the magnitudes equal?
   b. Compare the final speeds of the balls as they reach the ground. Is one larger than the other? Or are they equal?

51. || BIO Excellent human jumpers can leap straight up to a height of 110 cm off the ground. To reach this height, with what speed would a person need to leave the ground?

52. || A football is kicked straight up into the air; it hits the ground 5.2 s later.
   a. What was the greatest height reached by the ball? Assume it is kicked from ground level.
   b. With what speed did it leave the kicker's foot?

53. |||| In an action movie, the villain is rescued from the ocean by grabbing onto the ladder hanging from a helicopter. He is so intent on gripping the ladder that he lets go of his briefcase of counterfeit money when he is 130 m above the water. If the briefcase hits the water 6.0 s later, what was the speed at which the helicopter was ascending?

54. ||| BIO Spud Webb was, at 5 ft 8 in, one of the shortest basketball players to play in the NBA. But he had an amazing vertical leap; he could jump to a height of 1.1 m off the ground, so he could easily dunk a basketball. For such a leap, what was his "hang time"—the time spent in the air after leaving the ground and before touching down again?

55. |||| A rock climber stands on top of a 50-m-high cliff overhanging a pool of water. He throws two stones vertically downward 1.0 s apart and observes that they cause a single splash. The initial speed of the first stone was 2.0 m/s.
   a. How long after the release of the first stone does the second stone hit the water?
   b. What was the initial speed of the second stone?
   c. What is the speed of each stone as it hits the water?

General Problems

56. ||| BIO Actual velocity data for a lion pursuing prey are shown in Figure P2.56. Estimate:
   a. The initial acceleration of the lion.
   b. The acceleration of the lion at 2 s and at 4 s.
   c. The distance traveled by the lion between 0 s and 8 s.

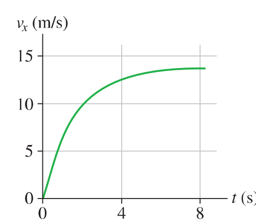


FIGURE P2.56

57. || A truck driver has a shipment of apples to deliver to a destination 440 miles away. The trip usually takes him 8 hours. Today he finds himself daydreaming and realizes 120 miles into his trip that he is running 15 minutes later than his usual pace at this point. At what speed must he drive for the remainder of the trip to complete the trip in the usual amount of time?

58. || Jenny and Alyssa are members of the cross-country team. On a training run, Jenny starts off and runs at a constant 3.8 m/s. Alyssa starts 15 s later and runs at a constant 4.0 m/s. At what time after Jenny's start does Alyssa catch up with Jenny?

59. || Figure P2.59 shows the motion diagram, made at two frames of film per second, of a ball rolling along a track. The track has a 3.0-m-long sticky section.
   a. Use the scale to determine the positions of the center of the ball. Place your data in a table, similar to Table 2.1, showing each position and the instant of time at which it occurred.
   b. Make a graph of x versus t for the ball. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.
   c. What is the *change* in the ball's position from $t = 0$ s to $t = 1.0$ s?
   d. What is the *change* in the ball's position from $t = 2.0$ s to $t = 4.0$ s?
   e. What is the ball's velocity before reaching the sticky section?
   f. What is the ball's velocity after passing the sticky section?
   g. Determine the ball's acceleration on the sticky section of the track.

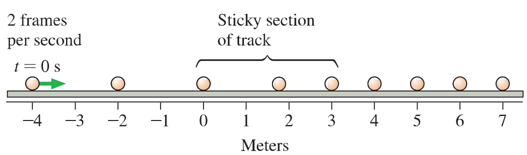


FIGURE P2.59

60. ||| In a 5000 m race, the athletes run $12\frac{1}{2}$ laps; each lap is 400 m. Kara runs the race at a constant pace and finishes in 17.5 min. Hannah runs the race in a blistering 15.3 min, so fast that she actually passes Kara during the race. How many laps has Hannah run when she passes Kara?

61. || The takeoff speed for an Airbus A320 jetliner is 80 m/s. Velocity data measured during takeoff are as shown in the table.

t (s)	v_x (m/s)
0	0
10	23
20	46
30	69

   a. What is the jetliner's acceleration during takeoff, in m/s^2 and in g's?
   b. At what time do the wheels leave the ground?
   c. For safety reasons, in case of an aborted takeoff, the length of the runway must be three times the takeoff distance. What is the minimum length runway this aircraft can use?

62. |||| Does a real automobile have constant acceleration? Measured data for a Porsche 944 Turbo at maximum acceleration are as shown in the table.

t (s)	v_x (mph)
0	0
2	41
4	66
6	83
8	97
10	110

   a. Convert the velocities to m/s, then make a graph of velocity versus time. Based on your graph, is the acceleration constant? Explain.
   b. Estimate how far the car traveled in the first 10 s.
   c. Draw a smooth curve through the points on your graph, then use your graph to *estimate* the car's acceleration at 2.0 s and 8.0 s. Give your answer in SI units. **Hint:** Remember that acceleration is the slope of the velocity graph.

63. || Scientists have studied two species of sand lizards, the Mojave fringe-toed lizard and the western zebra-tailed lizard, to understand the extent to which the different structure of the two species' toes is related to their preferred habitats—fine sand for the Mojave lizard and coarse sand for the zebra-tailed lizard. Figure P2.63 shows a somewhat simplified velocity-versus-time graph for the Mojave fringe-toed lizard.

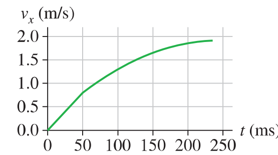


FIGURE P2.63

   a. Estimate the maximum acceleration of the lizard in both m/s^2 and g's.
   b. Estimate its acceleration at $t = 150$ ms.
   c. Estimate how far it travels in the first 50 ms.

64. ||| You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.70 s and that your car brakes with constant acceleration.
   a. How far are you from the intersection when you begin to apply the brakes?
   b. What acceleration will bring you to rest right at the intersection?
   c. How long does it take you to stop?

65. | BIO When you blink your eye, the upper lid goes from rest with your eye open to completely covering your eye in a time of 0.024 s.
   a. Estimate the distance that the top lid of your eye moves during a blink.
   b. What is the acceleration of your eyelid? Assume it to be constant.
   c. What is your upper eyelid's final speed as it hits the bottom eyelid?

66. ||| BIO A bush baby, an African primate, is capable of a remarkable vertical leap. The bush baby goes into a crouch and extends its legs, pushing upward for a distance of 0.16 m. After this upward acceleration, the bush baby leaves the ground and travels upward for 2.3 m. What is the acceleration during the pushing-off phase? Give your answer in m/s^2 and in g's.

67. |||| BIO When jumping, a flea reaches a takeoff speed of 1.0 m/s over a distance of 0.50 mm.
   a. What is the flea's acceleration during the jump phase?
   b. How long does the acceleration phase last?
   c. If the flea jumps straight up, how high will it go? (Ignore air resistance for this problem; in reality, air resistance plays a large role, and the flea will not reach this height.)

68. ||| BIO Certain insects can achieve seemingly impossible accelerations while jumping. The click beetle accelerates at an astonishing $400g$ over a distance of 0.60 cm as it rapidly bends its thorax, making the "click" that gives it its name.

   a. Assuming the beetle jumps straight up, at what speed does it leave the ground?
   b. How much time is required for the beetle to reach this speed?
   c. Ignoring air resistance, how high would it go?

69. |||| A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)

70. |||| A rock is tossed straight up with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.
   a. What is the rock's velocity as it hits the bottom of the hole?
   b. How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?

71. |||| In springboard diving, the diver strides out to the end of the board, takes a jump onto its end, and uses the resultant spring-like nature of the board to help propel him into the air. Assume that the diver's motion is essentially vertical. He leaves the board, which is 3.0 m above the water, with a speed of 6.3 m/s.
   a. How long is the diver in the air, from the moment he leaves the board until he reaches the water?
   b. What is the speed of the diver when he reaches the water?

72. ||| Haley is driving down a straight highway at 75 mph. A construction sign warns that the speed limit will drop to 55 mph in 0.50 mi. What constant acceleration (in m/s) will bring Haley to this lower speed in the distance available?

73. |||| A car starts from rest at a stop sign. It accelerates at 2.0 m/s^2 for 6.0 seconds, coasts for 2.0 s, and then slows down at a rate of 1.5 m/s^2 for the next stop sign. How far apart are the stop signs?

74. ||| Chameleons can rapidly project their very long tongues to
BIO catch nearby insects. The tongue of the tiny Rosette-nosed chameleon has the highest acceleration of a body part of any amniote (reptile, bird, or mammal) ever measured. In a somewhat simplified model of its tongue motion, the tongue, starting from rest, first undergoes a constant-acceleration phase with an astounding magnitude of 2500 m/s^2. This acceleration brings the tongue up to a final speed of 5.0 m/s. It continues at this speed for 22 ms until it hits its target.
   a. How long does the acceleration phase last?
   b. What is the total distance traveled by the chameleon's tongue?

75. ||| Heather and Jerry are standing on a bridge 50 m above a river. Heather throws a rock straight down with a speed of 20 m/s. Jerry, at exactly the same instant of time, throws a rock straight up with the same speed. Ignore air resistance.
   a. How much time elapses between the first splash and the second splash?
   b. Which rock has the faster speed as it hits the water?

76. || A Thomson's gazelle can run at very high speeds, but its
BIO acceleration is relatively modest. A reasonable model for the sprint of a gazelle assumes an acceleration of 4.2 m/s^2 for 6.5 s, after which the gazelle continues at a steady speed.
   a. What is the gazelle's top speed?
   b. A human would win a very short race with a gazelle. The best time for a 30 m sprint for a human runner is 3.6 s. How much time would the gazelle take for a 30 m race?
   c. A gazelle would win a longer race. The best time for a 200 m sprint for a human runner is 19.3 s. How much time would the gazelle take for a 200 m race?

Watch Video Solution Problem 2.75

77. ||| We've seen that a man's higher initial acceleration means
BIO that he can outrun a horse in very short race. A simple—but plausible—model for a sprint by a man and a horse uses these assumptions: The man accelerates at 6.0 m/s^2 for 1.8 s and then runs at a constant speed. A horse accelerates at 5.0 m/s^2 but continues accelerating for 4.8 s and then continues at a constant speed. A man and a horse are competing in a 200 m race. The man is given a 100 m head start, so he begins 100 m from the finish line. How much time does the man take to complete the race? How much time does the horse take? Who wins the race?

78. || A pole-vaulter is nearly motionless as he clears the bar, set 4.2 m above the ground. He then falls onto a thick pad. The top of the pad is 80 cm above the ground, and it compresses by 50 cm as he comes to rest. What is his acceleration as he comes to rest on the pad?

79. || A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of 3.5 m/s^2 is larger than the Honda's 3.0 m/s^2, the Honda gets a 100-m head start—it is only 300 m from the finish line. Assume, somewhat unrealistically, that both cars can maintain these accelerations the entire distance. Who wins, and by how much time?

80. ||||| The minimum stopping distance for a car traveling at a speed of 30 m/s is 60 m, including the distance traveled during the driver's reaction time of 0.50 s.
   a. Draw a position-versus-time graph for the motion of the car. Assume the car is at $x_i = 0$ m when the driver first sees the emergency situation ahead that calls for a rapid halt.
   b. What is the minimum stopping distance for the same car traveling at a speed of 40 m/s?

81. ||||| A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration?

MCAT-Style Passage Problems

Free Fall on Different Worlds

Objects in free fall on the earth have acceleration $a_y = -9.8\ m/s^2$. On the moon, free-fall acceleration is approximately 1/6 of the acceleration on earth. This changes the scale of problems involving free fall. For instance, suppose you jump straight upward, leaving the ground with velocity v_i and then steadily slowing until reaching zero velocity at your highest point. Because your initial velocity is determined mostly by the strength of your leg muscles, we can assume your initial velocity would be the same on the moon. But considering the final equation in Synthesis 2.1 we can see that, with a smaller free-fall acceleration, your maximum height would be greater. The following questions ask you to think about how certain athletic feats might be performed in this reduced-gravity environment.

82. | If an astronaut can jump straight up to a height of 0.50 m on earth, how high could he jump on the moon?
   A. 1.2 m B. 3.0 m C. 3.6 m D. 18 m

83. | On the earth, an astronaut can safely jump to the ground from a height of 1.0 m; her velocity when reaching the ground is slow enough to not cause injury. From what height could the astronaut safely jump to the ground on the moon?
   A. 2.4 m B. 6.0 m C. 7.2 m D. 36 m

84. | On the earth, an astronaut throws a ball straight upward; it stays in the air for a total time of 3.0 s before reaching the ground again. If a ball were to be thrown upward with the same initial speed on the moon, how much time would pass before it hit the ground?
   A. 7.3 s B. 18 s C. 44 s D. 108 s