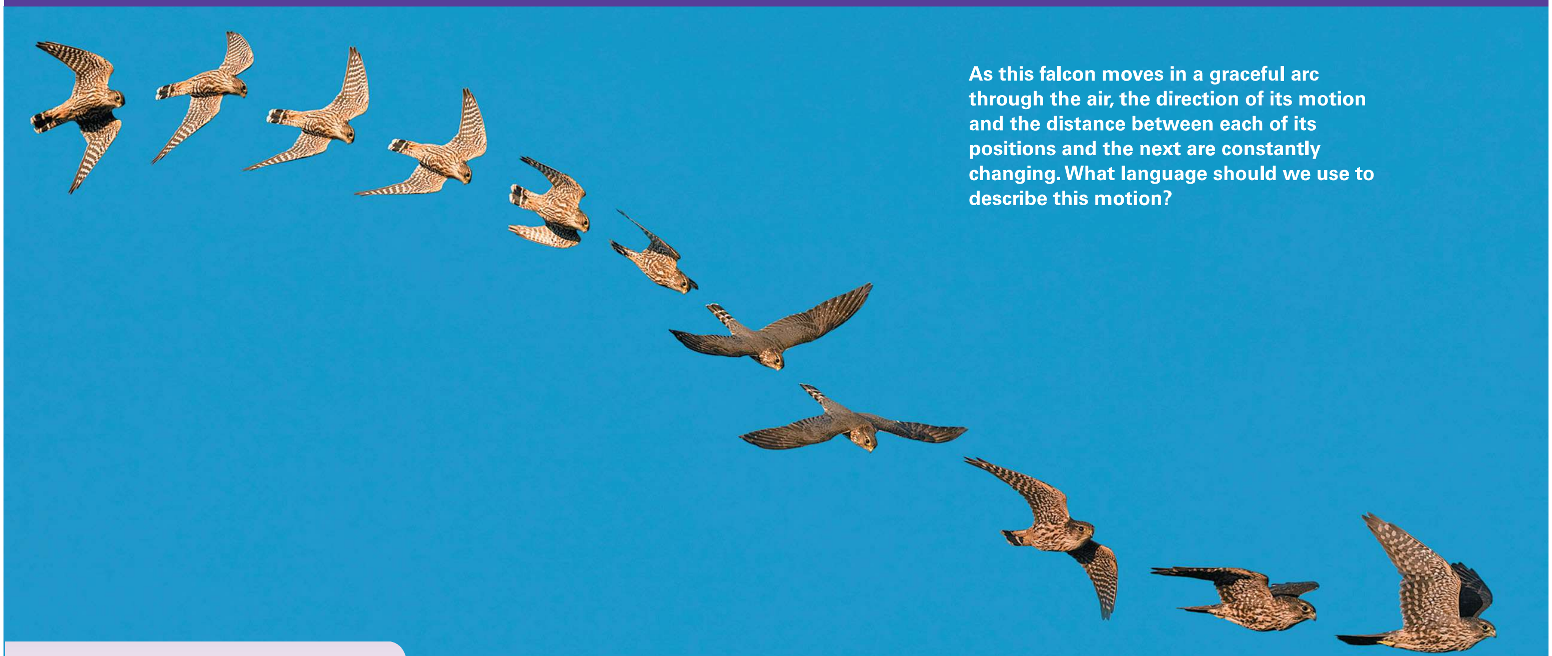
1 Representing Motion

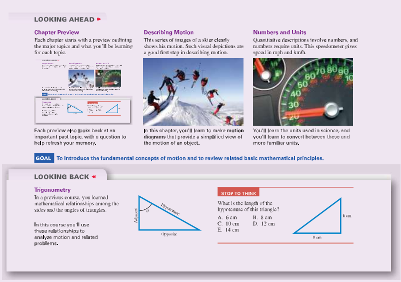


As this falcon moves in a graceful arc through the air, the direction of its motion and the distance between each of its positions and the next are constantly changing. What language should we use to describe this motion?

LOOKING AHEAD

Chapter Preview

Each chapter starts with a preview outlining the major topics and what you'll be learning for each topic.



Each preview also looks back at an important past topic, with a question to help refresh your memory.

Describing Motion

This series of images of a skier clearly shows his motion. Such visual depictions are a good first step in describing motion.

In this chapter, you'll learn to make **motion diagrams** that provide a simplified view of the motion of an object.

Numbers and Units

Quantitative descriptions involve numbers, and numbers require units. This speedometer gives speed in mph and km/h.

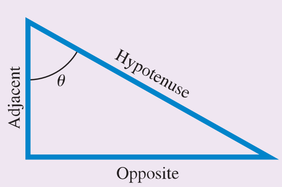
You'll learn the units used in science, and you'll learn to convert between these and more familiar units.

GOAL To introduce the fundamental concepts of motion and to review related basic mathematical principles.

LOOKING BACK

Trigonometry

In a previous course, you learned mathematical relationships among the sides and the angles of triangles.

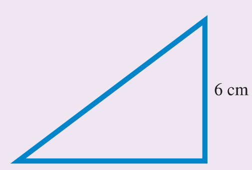


In this course you'll use these relationships to analyze motion and related problems.

STOP TO THINK

What is the length of the hypotenuse of this triangle?

A. 6 cm B. 8 cm
C. 10 cm D. 12 cm
E. 14 cm



1.1 Motion: A First Look

Motion is a theme that will appear in one form or another throughout this entire text. You have a well-developed intuition about motion based on your experiences, but we'll find that some of the most important aspects of motion can be rather subtle. We need to develop some tools to help us explain and understand motion, so rather than jumping immediately into a lot of mathematics and calculations, this first chapter focuses on *visualizing* motion and becoming familiar with the *concepts* needed to describe a moving object.

One key difference between physics and other sciences is how we set up and solve problems. We'll often use a two-step process to solve motion problems. The first step is to develop a simplified *representation* of the motion so that key elements stand out. For example, the photo of the falcon at the start of the chapter allows us to observe its position at many successive times. We will begin our study of motion by considering this sort of picture. The second step is to analyze the motion with the language of mathematics. The process of putting numbers on nature is often the most challenging aspect of the problems you will solve. In this chapter, we will explore the steps in this process as we introduce the basic concepts of motion.

Types of Motion

As a starting point, let's define **motion** as the change of an object's position or orientation with time. Examples of motion are easy to list. Bicycles, baseballs, cars, airplanes, and rockets are all objects that move. The path along which an object moves, which might be a straight line or might be curved, is called the object's **trajectory.**

FIGURE 1.1 shows four basic types of motion that we will study in this text. In this chapter, we will focus on the first type of motion in the figure, motion along a straight line, or *straight-line motion*. In later chapters, we will learn about *circular motion*, which is the motion of an object along a circular path; *projectile motion*, the motion of an object through the air; and *rotational motion*, the spinning of an object about an axis.

FIGURE 1.1 Four basic types of motion.

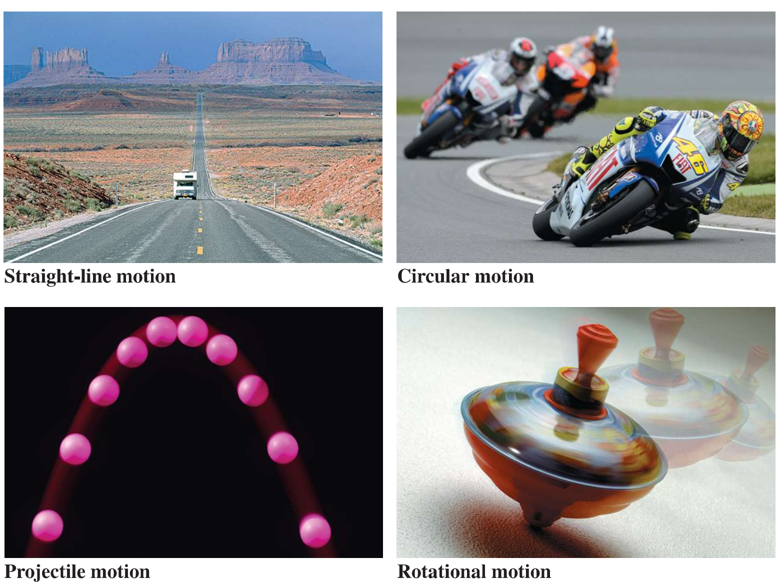
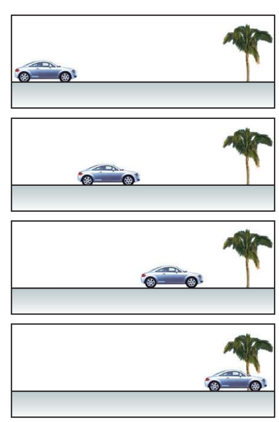


FIGURE 1.2 Several frames from the video of a car.



Making a Motion Diagram

An easy way to study motion is to record a video of a moving object with a stationary camera. A video camera takes images at a fixed rate, typically 30 images every second. Each separate image is called a *frame*. As an example, FIGURE 1.2 shows several frames from a video of a car going past, with the camera in a fixed position. Not surprisingly, the car is in a different position in each frame.

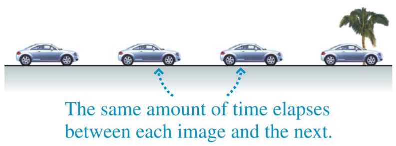
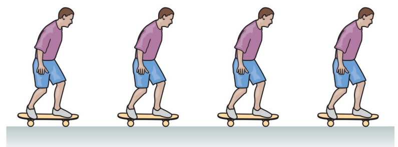


FIGURE 1.3 A motion diagram of the car shows all the frames simultaneously.

Suppose we now edit the video by layering the frames on top of each other. We end up with the picture in **FIGURE 1.3.** This composite image, showing an object's positions at several *equally spaced instants of time,* is called a **motion diagram.** As simple as motion diagrams seem, they will turn out to be powerful tools for analyzing motion.

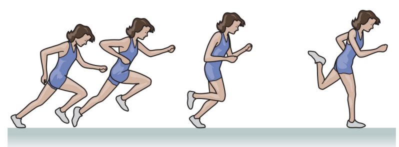
Now let's take our camera out into the world and make some motion diagrams. The following table illustrates how a motion diagram shows important features of different kinds of motion.

Examples of motion diagrams



Images that are *equally spaced* indicate an object moving with *constant speed*.

A skateboarder rolling down the sidewalk.



An *increasing distance* between the images shows that the object is *speeding up*.

A sprinter starting the 100 meter dash.

A *decreasing distance* between the images shows that the object is *slowing down*.

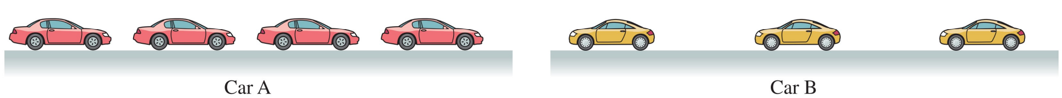
A car stopping for a red light.

A more complex motion diagram shows changes in speed and direction.

A basketball free throw.

We have defined several concepts (constant speed, speeding up, and slowing down) in terms of how the moving object appears in a motion diagram. These are called **operational definitions,** meaning that the concepts are defined in terms of a particular procedure or operation. For example, we could answer the question Is the airplane speeding up? by checking whether the images in the plane's motion diagram are getting farther apart. Many of the concepts in physics will be introduced as operational definitions. This reminds us that physics is an experimental science.

STOP TO THINK 1.1 Which car is going faster, A or B? Assume there are equal intervals of time between the frames of both videos.



NOTE ▶ Each chapter in this text has several *Stop to Think* questions. These questions are designed to see if you've understood the basic ideas that have just been presented. The answers are given at the end of the chapter, but you should make a serious effort to think about these questions before turning to the answers. ◀

1.2 Models and Modeling

The real world is messy and complicated. Our goal in studying physics is to brush aside many of the real-world details in order to discern patterns that occur over and over. For example, a swinging pendulum, a vibrating guitar string, a sound wave, and jiggling atoms in a crystal are all very different—yet they share a common core characteristic: Each is an example of an *oscillating system*, something that moves back and forth around an equilibrium position. If we focus on understanding a very simple oscillating system, such as a block (generically, a "mass") attached to a spring, we'll automatically understand quite a bit about the many real-world examples of oscillations.

Stripping away the details to focus on essential features is a process called *modeling*. A **model** is a highly simplified picture of reality, but one that still captures the essence of what we want to study. Thus a mass attached to a spring is a simple but realistic model of many oscillating systems.

Models allow us to make sense of complex situations by providing a framework for thinking about them. One could go so far as to say that developing and testing models is at the heart of the scientific process. Albert Einstein once said, "Physics should be as simple as possible—but not simpler." We want to find the simplest model that allows us to understand the phenomenon we're studying, but we can't make the model so simple that key aspects of the phenomenon get lost.

We'll develop and use many models throughout this text; they'll be one of our most important thinking tools. These models will be of two types:

- *Descriptive models:* What are the essential characteristics and properties of a phenomenon? How do we describe it in the simplest possible terms? For example, the mass-on-a-spring model of an oscillating system is a descriptive model.
- *Explanatory models:* Why do things happen as they do? Explanatory models, based on the laws of physics, have predictive power. They allow us to test—against experimental data—whether a model provides an adequate explanation of our observations. For example, the *charge model* that we will introduce in Chapter 20 helps us explain and predict a wide range of experimental outcomes related to electric forces.

When we solve physics problems, one of the most important steps is choosing an appropriate model for the system we are studying. In the worked examples in this text, in the first "Strategize" step, we'll point out the model being used, when appropriate.

The Particle Model

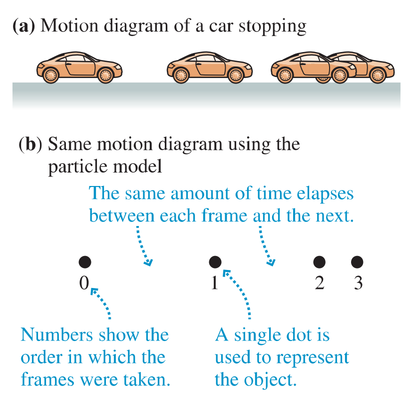
For many objects, the motion of the object *as a whole* is not influenced by the details of the object's size and shape. To describe the object's motion, all we really need to keep track of is the motion of a single point: You could imagine looking at the motion of a dot painted on the side of the object.

In fact, for the purposes of analyzing the motion, we can often consider the object *as if* it were just a single point. We can also treat the object *as if* all of its mass were concentrated into this single point. An object that can be represented as a mass at a single point in space is called a **particle.**

If we treat an object as a particle, we can represent the object in each frame of a motion diagram as a simple dot. **FIGURE 1.4** shows how much simpler motion diagrams appear when the object is represented as a particle. Note that the dots have been numbered 0, 1, 2, . . . to tell the sequence in which the frames were exposed. These diagrams still convey a complete understanding of the object's motion.

In representing the car in Figure 1.4 as a particle, we have discarded many of the details of the car, such as the shape of its body and the motion of its wheels, which are unimportant in understanding its overall motion. In other words, we have developed a model for moving objects, the **particle model,** that allows us to see

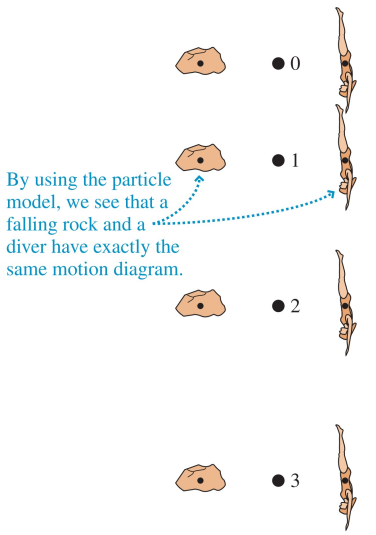
FIGURE 1.4 Simplifying a motion diagram using the particle model.



A video to support a section's topic is embedded in the eText.

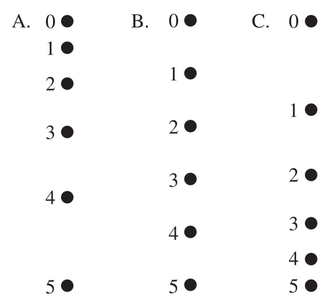
Video Figure 1.4

FIGURE 1.5 The particle model for two falling objects.



connections that are very important but that are obscured or lost by examining all the parts of an extended, real object. Consider the motion of the rock and the diver shown in **FIGURE 1.5.** These two very different objects have exactly the same motion diagram. As we will see, all objects falling under the influence of gravity move in exactly the same manner if no other forces act. The simplification of the particle model has revealed something about the physics that underlies both of these situations.

STOP TO THINK 1.2 Three motion diagrams are shown. Which is a dust particle settling to the floor at constant speed, which is a ball dropped from the roof of a building, and which is a descending rocket slowing to make a soft landing on Mars?

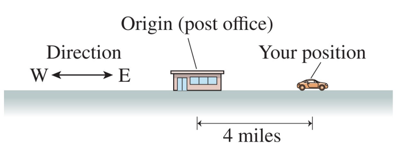


1.3 Position and Time: Putting Numbers on Nature

To develop our understanding of motion further, we need to be able to make quantitative measurements: We need to use numbers. As we analyze a motion diagram, it is useful to know where the object is (its *position*) and when the object was at that position (the *time*). We'll start by considering the motion of an object that can move only along a straight line. Examples of this **one-dimensional** or "1-D" motion are a car moving along a long, straight road; an airplane taxiing down a runway; and an elevator moving up and down a shaft.

Position and Coordinate Systems

FIGURE 1.6 Describing your position.



Suppose you are driving along a long, straight country road, as in **FIGURE 1.6,** and your friend calls and asks where you are. You might reply that you are 4 miles east of the post office, and your friend would then know just where you were. Your location at a particular instant in time (when your friend phoned) is called your **position.** Notice that to know your position along the road, your friend needed three pieces of information. First, you had to give her a reference point (the post office) from which all distances are to be measured. We call this fixed reference point the **origin.** Second, she needed to know how far you were from that reference point or origin—in this case, 4 miles. Finally, she needed to know which side of the origin you were on: You could be 4 miles to the west of it or 4 miles to the east.

We will need these same three pieces of information in order to specify any object's position along a line. We first choose our origin, from which we measure the distance to the object. The position of the origin is arbitrary, and we are free to place it where we like. Usually, however, there are certain points (such as the well-known post office) that are more convenient choices than others.

This gauge's vertical scale measures the depth of snow when it falls. It has a natural origin at the level of the road.

In order to specify how far our object is from the origin, we lay down an imaginary axis along the line of the object's motion. Like a ruler, this axis is marked off in equally spaced divisions of distance, perhaps in inches, meters, or miles, depending on the problem at hand. We place the zero mark of this ruler at the origin, allowing us to locate the position of our object by reading the ruler mark where the object is.

Finally, we need to be able to specify which side of the origin our object is on. To do this, we imagine the axis extending from one side of the origin with increasing positive markings; on the other side, the axis is marked with increasing *negative* numbers. By reporting the position as either a positive or a negative number, we know on what side of the origin the object is.

These elements—an origin and an axis marked in both the positive and negative directions—can be used to unambiguously locate the position of an object. We call this a **coordinate system.** We will use coordinate systems throughout this text, and we will soon develop coordinate systems that can be used to describe the positions of objects moving in more complex ways than just along a line. **FIGURE 1.7** shows a coordinate system that we can use to locate various objects along the country road discussed earlier.

Although our coordinate system works well for describing the positions of objects located along the axis, our notation is somewhat cumbersome because we keep needing to say things like "the car is at position +4 miles." A better notation, and one that will become particularly important when we study motion in two dimensions, is to use a symbol such as x or y to represent the position along the axis. Then we can say "the cow is at $x = -5$ miles." The symbol that represents a position along an axis is called a **coordinate.** The introduction of symbols to represent positions (and, later, velocities and accelerations) also allows us to work with these quantities mathematically.

FIGURE 1.8 shows how we would set up a coordinate system for a sprinter running a 50 meter race (we use the standard abbreviation "m" for meters). For horizontal motion like this we usually use the coordinate x to represent the position.

FIGURE 1.7 The coordinate system used to describe objects along a country road.

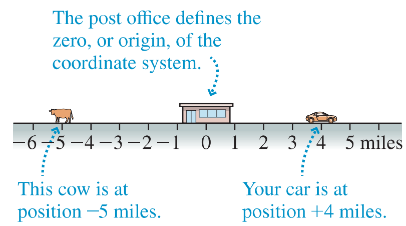
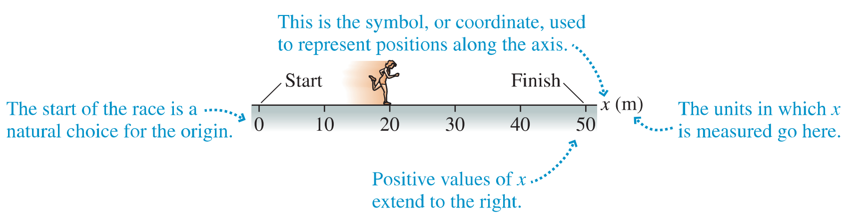
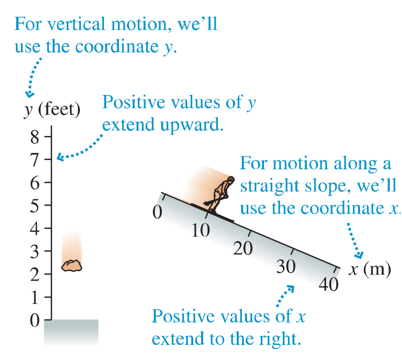


FIGURE 1.8 A coordinate system for a 50 meter race.



Motion along a straight line need not be horizontal. As shown in **FIGURE 1.9,** a rock falling vertically downward and a skier skiing down a straight slope are also examples of straight-line or one-dimensional motion.

FIGURE 1.9 Examples of one-dimensional motion.



Time

The pictures in Figure 1.9 show the position of an object at just one instant of time. But a full motion diagram represents how an object moves as time progresses. So far, we have labeled the dots in a motion diagram by the numbers 0, 1, 2, . . . to indicate the order in which the frames were taken. But to fully describe the motion, we need to indicate the *time,* as read off a clock or a stopwatch, at which each frame of a video was made. This is important, as we can see from the motion diagram of a stopping car in **FIGURE 1.10.** If the frames were taken 1 second apart, this motion diagram shows a leisurely stop; if 1/10 of a second apart, it represents a screeching halt.

For a complete motion diagram, we thus need to label each frame with its corresponding time (symbol t) as read off a clock. But when should we start the clock? Which frame should be labeled $t = 0$? This choice is much like choosing the origin $x = 0$ of a coordinate system: You can pick any arbitrary point in the motion and label it "$t = 0$ seconds." This is simply the instant you decide to start your clock or stopwatch, so it is the origin of your time coordinate. A video frame labeled "$t = 4$ seconds" means it was taken 4 seconds after you started your clock. We typically choose $t = 0$ to represent the "beginning" of a problem, but the object may have been moving before then.

FIGURE 1.10 Is this a leisurely stop or a screeching halt?

FIGURE 1.11 The motion diagram of a car that travels at constant speed and then brakes to a halt.

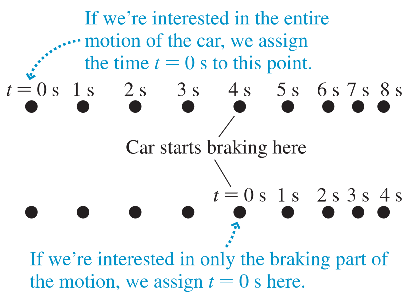


FIGURE 1.12 Sam undergoes a displacement Δx from position x_i to position x_f.

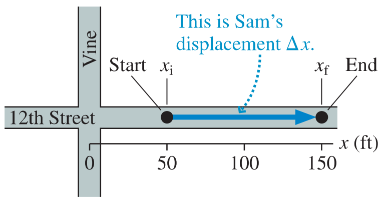


FIGURE 1.13 A displacement is a signed quantity. Here Δx is a negative number.

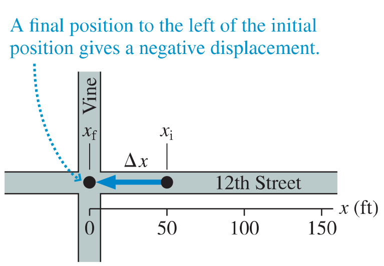
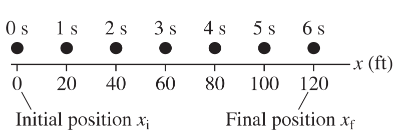


FIGURE 1.14 The motion diagram of a bicycle moving to the right at a constant speed.



To illustrate, **FIGURE 1.11** shows the motion diagram for a car moving at a constant speed and then braking to a halt. Two possible choices for the frame labeled $t = 0$ seconds are shown; our choice depends on what part of the motion we're interested in. Each successive position of the car is then labeled with the clock reading in seconds (abbreviated by "s").

Changes in Position: Displacement

Now that we've seen how to measure position and time, let's return to the problem of motion. To describe motion we'll need to measure the *changes* in position that occur with time. Consider the following:

> Sam is standing 50 feet (ft) east of the corner of 12th Street and Vine. He then walks to a second point 150 ft east of Vine. What is Sam's change of position?

FIGURE 1.12 shows Sam's motion on a map. We've placed a coordinate system on the map, using the coordinate x. We are free to place the origin of our coordinate system wherever we wish, so we have placed it at the intersection. Sam's initial position is then at $x_i = 50$ ft. The positive value for x_i tells us that Sam is east of the origin.

NOTE ▶ We will label special values of x or y with subscripts. The value at the start of a problem is usually labeled with a subscript "i," for *initial,* and the value at the end is labeled with a subscript "f," for *final.* For cases having several special values, we will usually use subscripts "1," "2," and so on. ◀

Sam's final position is $x_f = 150$ ft, indicating that he is 150 ft east of the origin. You can see that Sam has changed position, and a *change* of position is called a **displacement.** His displacement is the distance labeled Δx in Figure 1.12. The Greek letter delta (Δ) is used in math and science to indicate the *change* in a quantity. Thus Δx indicates a change in the position x.

NOTE ▶ Δx is a *single* symbol. You cannot cancel out or remove the Δ in algebraic operations. ◀

To get from the 50 ft mark to the 150 ft mark, Sam clearly had to walk 100 ft, so the change in his position—his displacement—is 100 ft. We can think about displacement in a more general way, however. Displacement is the *difference* between a final position x_f and an initial position x_i. Thus we can write

$$\Delta x = x_f - x_i = 150 \text{ ft} - 50 \text{ ft} = 100 \text{ ft}$$

NOTE ▶A general principle, used throughout this text, is that the change in any quantity is the final value of the quantity minus its initial value. ◀

Displacement is a *signed quantity;* that is, it can be either positive or negative. If, as shown in **FIGURE 1.13,** Sam's final position x_f had been at the origin instead of the 150 ft mark, his displacement would have been

$$\Delta x = x_f - x_i = 0 \text{ ft} - 50 \text{ ft} = -50 \text{ ft}$$

The negative sign tells us that he moved to the *left* along the x-axis, or 50 ft *west.*

Changes in Time

A displacement is a change in position. In order to quantify motion, we'll need to also consider changes in *time,* which we call **time intervals.** We've seen how we can label each frame of a motion diagram with a specific time, as determined by our stopwatch. **FIGURE 1.14** shows the motion diagram of a bicycle moving at a constant speed, with the times of the measured points indicated.

The displacement between the initial position x_i and the final position x_f is

$$\Delta x = x_f - x_i = 120 \text{ ft} - 0 \text{ ft} = 120 \text{ ft}$$

Similarly, we define the time interval between these two points to be

$$\Delta t = t_f - t_i = 6\text{ s} - 0\text{ s} = 6\text{ s}$$

A time interval Δt measures the elapsed time as an object moves from an initial position x_i at time t_i to a final position x_f at time t_f. Note that, unlike Δx, Δt is always positive because t_f is always greater than t_i.

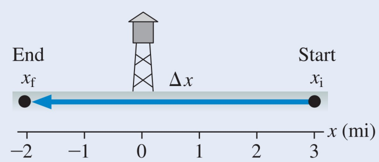
EXAMPLE 1.1 **How long a ride?**

Emily is enjoying a bicycle ride on a country road that runs east-west past a water tower. At noon, Emily is 3 miles (mi) east of the water tower. A half-hour later, she is 2 mi west of the water tower. What is her displacement during that half-hour?

STRATEGIZE We will use the particle model to represent Emily. She is riding along a straight east-west road, so we will make use of a one-dimensional coordinate system to describe her motion.

PREPARE Although it may seem like overkill for such a simple problem, you should start by making a drawing, like the one in FIGURE 1.15, with the x-axis along the road. We choose our coordinate system so that increasing x means moving to the east. Distances are measured with respect to the water tower, so it is a natural origin for the coordinate system. Once the coordinate system is established, we can show Emily's initial and final positions and her displacement between the two.

FIGURE 1.15 A drawing of Emily's motion.



SOLVE We've specified values for Emily's initial and final positions in our drawing. We can thus compute her displacement:

$$\Delta x = x_f - x_i = (-2\text{ mi}) - (3\text{ mi}) = -5\text{ mi}$$

ASSESS Once we've completed the solution to the problem, we need to go back to see if it makes sense. Emily is moving to the west, so we expect her displacement to be negative—and it is. We can see from our drawing in Figure 1.15 that she has moved 5 miles from her starting position, so our answer seems reasonable. As part of the Assess step, we also check our answers to see if they make physical sense. Emily travels 5 miles in a half-hour, quite a reasonable pace for a cyclist.

NOTE ▸ All of the numerical examples in the text are worked out with the same four-step process: Strategize, Prepare, Solve, Assess. It's tempting to cut corners, especially for the simple problems in these early chapters, but you should take the time to do all of these steps now, to practice your problem-solving technique. We'll have more to say about our general problem-solving approach in Chapter 2. ◂

STOP TO THINK 1.3 Sarah starts at a positive position along the x-axis. She then undergoes a negative displacement. Her final position

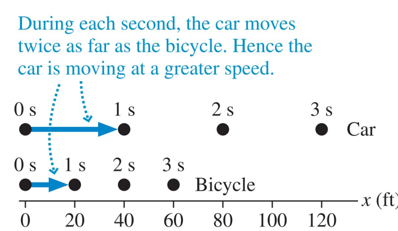
A. Is positive. B. Is negative. C. Could be either positive or negative.

1.4 Velocity

We all have an intuitive sense of whether something is moving very fast or just cruising slowly along. To make this intuitive idea more precise, let's start by examining the motion diagrams of some objects moving along a straight line at a *constant* speed, objects that are neither speeding up nor slowing down. This motion at a constant speed is called **uniform motion.** As we saw for the skateboarder in Section 1.1, for an object in uniform motion, successive frames of the motion diagram are *equally spaced*, so the object's displacement Δx is the *same* between successive frames.

To see how an object's displacement between successive frames is related to its speed, consider the motion diagrams of a bicycle and a car, traveling along the same street as shown in FIGURE 1.16. Clearly the car is moving faster than the bicycle: In any 1 second time interval, the car undergoes a displacement $\Delta x = 40$ ft, while the bicycle's displacement is only 20 ft.

FIGURE 1.16 Motion diagrams for a car and a bicycle.



The greater the distance traveled by an object in a given time interval, the greater its speed. This idea leads us to define the speed of an object as

$$\text{speed} = \frac{\text{distance traveled in a given time interval}}{\text{time interval}} \tag{1.1}$$

Speed of an object in uniform motion

For the bicycle, this equation gives

$$\text{speed} = \frac{20\ \text{ft}}{1\ \text{s}} = 20\ \frac{\text{ft}}{\text{s}}$$

while for the car we have

$$\text{speed} = \frac{40\ \text{ft}}{1\ \text{s}} = 40\ \frac{\text{ft}}{\text{s}}$$

The speed of the car is twice that of the bicycle, which seems reasonable.

NOTE ▶ The division gives units that are a fraction: ft/s. This is read as "feet per second," just like the more familiar "miles per hour." ◀

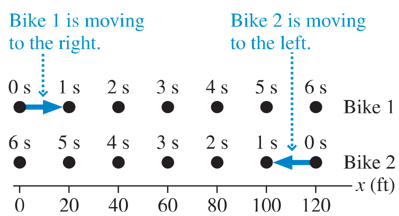


FIGURE 1.17 Two bicycles traveling at the same speed, but with different velocities.

To fully characterize the motion of an object, we must specify not only the object's speed but also the *direction* in which it is moving. **FIGURE 1.17** shows the motion diagrams of two bicycles traveling at 20 ft/s. The two bicycles have the same speed, but something about their motion is different—the *direction* of their motion.

The "distance traveled" in Equation 1.1 doesn't capture any information about the direction of travel. But we've seen that the *displacement* of an object does contain this information. We can introduce a new quantity, the **velocity,** as

$$\text{velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{\Delta x}{\Delta t} \tag{1.2}$$

Velocity of a moving object

The velocity of bicycle 1 in Figure 1.17, computed using the 1 second time interval between the $t = 0$ s and $t = 1$ s positions, is

$$v = \frac{\Delta x}{\Delta t} = \frac{x_1 - x_0}{1\ \text{s} - 0\ \text{s}} = \frac{20\ \text{ft} - 0\ \text{ft}}{1\ \text{s}} = +20\ \frac{\text{ft}}{\text{s}}$$

while the velocity of bicycle 2, during the same time interval, is

$$v = \frac{\Delta x}{\Delta t} = \frac{x_1 - x_0}{1\ \text{s} - 0\ \text{s}} = \frac{100\ \text{ft} - 120\ \text{ft}}{1\ \text{s}} = -20\ \frac{\text{ft}}{\text{s}}$$

NOTE ▶ We have used x_0 for the position at time $t = 0$ seconds and x_1 for the position at time $t = 1$ second. The subscripts serve the same role as before—identifying particular positions—but in this case the positions are identified by the time at which each position is reached. ◀

The two velocities have opposite signs because the bicycles are traveling in opposite directions. **Speed measures only how fast an object moves, but velocity tells us both an object's speed *and its direction*.** In this text, we'll use a positive velocity to indicate motion to the right or, for vertical motion, upward. We'll use a negative velocity for an object moving to the left, or downward.

NOTE ▶ Learning to distinguish between speed, which is always a positive number, and velocity, which can be either positive or negative, is one of the most important tasks in the analysis of motion. ◀

The velocity as defined by Equation 1.2 is actually what is called the *average* velocity. On average, over each 1 s interval bicycle 1 moves 20 ft, but we don't know if it was moving at exactly the same speed at every moment during this time interval. In Chapter 2, we'll develop the idea of *instantaneous* velocity, the velocity of an object at a particular instant in time. Since our goal in this chapter is to *visualize* motion with motion diagrams, we'll somewhat blur the distinction between average and instantaneous quantities, refining these definitions in Chapter 2.

The "Per" in Miles Per Hour

The units for speed and velocity are a unit of distance, such as feet, meters, or miles, divided by a unit of time, such as seconds or hours. Thus we could measure velocity in units of mi/h (or mph) or m/s, pronounced "miles *per* hour" or "meters *per* second." The word "per" will often arise in physics when we consider the ratio of two quantities. What do we mean, exactly, by "per"?

If a car moves with a speed of 23 m/s, we mean that it travels 23 meters *for each* second of elapsed time. The word "per" thus associates the number of units in the numerator (23 m) with *one* unit of the denominator (1 s). We'll see many other examples of this idea as the text progresses. You may already know a bit about *density;* you can look up the density of gold and you'll find that it is 19.3 g/cm^3 ("grams *per* cubic centimeter"). This means that there are 19.3 grams of gold *for each* cubic centimeter of the metal.

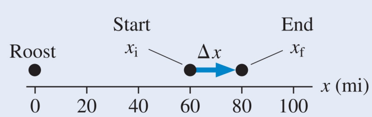
EXAMPLE 1.2 Finding the speed of a seabird BIO

Albatrosses are seabirds that spend most of their lives flying over the ocean looking for food. With a stiff tailwind, an albatross can fly at high speeds. Satellite data on one particularly speedy albatross showed it 60 miles east of its roost at 3:00 PM and then, at 3:15 PM, 80 miles east of its roost. What was its velocity?

STRATEGIZE We will assume that the albatross is flying at a constant speed; that is, it is in uniform motion. Using the particle model, we can represent its motion on a coordinate system.

PREPARE The statement of the problem provides us with a natural coordinate system: We can measure distances with respect to the roost, with distances to the east as positive. With this coordinate system, the motion of the albatross appears as in FIGURE 1.18. The motion takes place between 3:00 and 3:15, a time interval of 15 minutes. If we want our final velocity to be in the familiar units of miles per hour, or mph, we need this time interval in hours (abbreviated "h"). So we make the conversion 15 min = 0.25 h.

FIGURE 1.18 The motion of an albatross at sea.



SOLVE We know the initial and final positions, and we know the time interval, so we can calculate the velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{0.25\ \text{h}} = \frac{20\ \text{mi}}{0.25\ \text{h}} = 80\ \text{mph}$$

ASSESS The velocity is positive, which makes sense because Figure 1.18 shows that the motion is to the right. A speed of 80 mph is certainly fast, but the problem said it was a "particularly speedy" albatross, so our answer seems reasonable. (Indeed, albatrosses have been observed to fly at such speeds in the very fast winds of the Southern Ocean. This problem is based on real observations, as will be our general practice in this text.)

STOP TO THINK 1.4 Jane starts from her house to take a stroll in her neighborhood. After walking for 2 hours at a steady pace, she has walked 4 miles and is 2 miles from home. For this time interval, what was her speed?

A. 4 mph B. 3 mph C. 2 mph D. 1 mph

1.5 A Sense of Scale: Significant Figures, Scientific Notation, and Units

Physics attempts to explain the natural world, from the very small to the exceedingly large. And in order to understand our world, we need to be able to *measure* quantities both minuscule and enormous. A properly reported measurement has three elements. First, we can measure our quantity with only a certain precision. To make this precision clear, we need to make sure that we report our measurement with the correct number of *significant figures.*

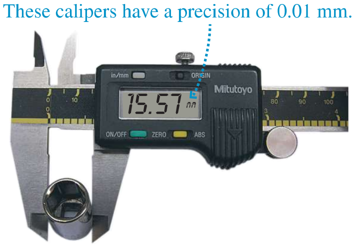
Second, writing down the really big and small numbers that often come up in physics can be awkward. To avoid writing all those zeros, scientists use *scientific notation* to express numbers both big and small.

Finally, we need to choose an agreed-upon set of *units* for the quantity. For speed, common units include meters per second and miles per hour. For mass, the kilogram is the most commonly used unit. Every physical quantity that we can measure has an associated set of units.

Measurements and Significant Figures

When we measure any quantity, such as the length of a bone or the weight of a specimen, we can do so with only a certain *precision.* If you make a measurement with the ruler shown in **FIGURE 1.19,** you probably can't be more accurate than about ± 1 mm, so the ruler has a precision of 1 mm. The digital calipers shown can make a measurement to within ± 0.01 mm, so it has a precision of 0.01 mm. The precision of a measurement can also be affected by the skill or judgment of the person performing the measurement. A stopwatch might have a precision of 0.001 s, but, due to your reaction time, your measurement of the time of a sprinter would be much less precise.

FIGURE 1.19 The precision of a measurement depends on the instrument used to make it.



Walter Davis's best long jump on this day was reported as 8.24 m. This implies that the actual length of the jump was between 8.235 m and 8.245 m, a spread of only 0.01 m, which is 1 cm. Does this claimed accuracy seem reasonable?

It is important that your measurement be reported in a way that reflects its actual precision. Suppose you use a ruler to measure the length of a particular frog. You judge that you can make this measurement with a precision of about 1 mm, or 0.1 cm. In this case, the frog's length should be reported as, say, 6.2 cm. We interpret this to mean that the actual value falls between 6.15 cm and 6.25 cm and thus rounds to 6.2 cm. If you reported the frog's length as simply 6 cm, you would be saying less than you know; you would be withholding information. If you reported the number as 6.213 cm, however, anyone reviewing your work would interpret this to mean that the actual length falls between 6.2125 cm and 6.2135 cm, a precision of 0.001 cm. In this case, you would be claiming to have more information than you really possessed.

The way to state your knowledge precisely is through the proper use of **significant figures.** You can think of a significant figure as a digit that is reliably known. A measurement such as 6.2 cm has *two* significant figures, the 6 and the 2. The next decimal place—the hundredths—is not reliably known and is thus not a significant figure. Similarly, a time measurement of 34.62 s has four significant figures, implying that the 2 in the hundredths place is reliably known.

When we perform a calculation such as adding or multiplying two or more measured numbers, we can't claim more accuracy for the result than was present in the initial measurements. Determining the proper number of significant figures is straightforward, but there are a few definite rules to follow. We will often spell out such technical details in what we call a "Tactics Box." A Tactics Box is designed to teach you particular skills and techniques. Each Tactics Box will include the icon to designate exercises in the *Student Workbook* that you can use to practice these skills.

TACTICS BOX 1.1 Using significant figures

1. When you multiply or divide several numbers, or when you take roots, the number of significant figures in the answer should match the number of significant figures of the *least* precisely known number used in the calculation:

Three significant figures

$$3.73 \times 5.7 = 21$$

Two significant figures

Answer should have the *lower* of the two, or two significant figures.

Continued

❷ When you add or subtract several numbers, the number of decimal places in the answer should match the *smallest* number of decimal places of any number used in the calculation:

$$\begin{array}{r} 18.54 \\ +106.6 \\ \hline 125.1 \end{array}$$

18.54 — Two decimal places
+106.6 — One decimal place
125.1 — Answer should have the *lower* of the two, or one decimal place.

❸ **Exact numbers** have no uncertainty and, when used in calculations, do not change the number of significant figures of measured numbers. Examples of exact numbers are π and the number 2 in the relation $d = 2r$ between a circle's diameter and radius.

There is one notable exception to these rules:

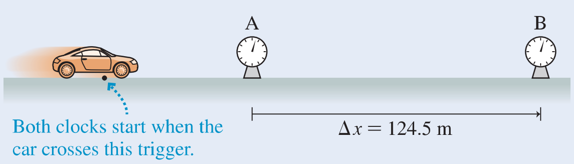
- It is acceptable to keep one or two extra digits during *intermediate* steps of a calculation to minimize round-off errors in the calculation. But the *final* answer must be reported with the proper number of significant figures.

Exercise 15

EXAMPLE 1.3 **Using significant figures when measuring the velocity of a car**

To measure the velocity of a car, clocks A and B are set up at two points along the road, as shown in **FIGURE 1.20**. Clock A is precise to 0.01 s, while clock B is precise to only 0.1 s. The distance between these two clocks is carefully measured to be 124.5 m. The two clocks are automatically started when the car passes a trigger in the road; each clock stops automatically when the car passes that clock. After the car has passed both clocks, clock A is found to read $t_A = 1.22$ s, and clock B to read $t_B = 4.5$ s. The time from the less-precise clock B is correctly reported with fewer significant figures than that from A. What is the velocity of the car, and how should it be reported with the correct number of significant figures?

FIGURE 1.20 Measuring the velocity of a car.



STRATEGIZE To find the car's velocity with the correct precision, we will need to take into account the significant figures in the measured quantities and then apply the rules of Tactics Box 1.1.

PREPARE To calculate the velocity, we need the displacement Δx and the time interval Δt as the car moves between the two clocks. The displacement is given as $\Delta x = 124.5$ m; we can calculate the time interval as the difference between the two measured times.

SOLVE The time interval is

$$\Delta t = t_B - t_A = (4.5\ \text{s}) - (1.22\ \text{s}) = 3.3\ \text{s}$$

This number has one decimal place. This number has two decimal places. By rule 2 of Tactics Box 1.1, the result should have *one* decimal place.

We can now calculate the velocity with the displacement and the time interval:

$$v = \frac{\Delta x}{\Delta t} = \frac{124.5\ \text{m}}{3.3\ \text{s}} = 38\ \text{m/s}$$

The displacement has four significant figures. The time interval has two significant figures. By rule 1 of Tactics Box 1.1, the result should have *two* significant figures.

ASSESS Our final value has two significant figures. Suppose you had been hired to measure the speed of a car this way, and you reported 37.72 m/s. It would be reasonable for someone looking at your result to assume that the measurements you used to arrive at this value were correct to four significant figures and thus that you had measured time to the nearest 0.001 second. Our correct result of 38 m/s has all of the accuracy that you can claim, but no more!

Scientific Notation

It's easy to write down measurements of ordinary-sized objects: Your height might be 1.72 meters, the weight of an apple 0.34 pound. But the radius of a hydrogen atom is 0.000 000 000 053 m, and the distance to the moon is 384,000,000 m. Keeping track of all those zeros is quite cumbersome.

Beyond requiring you to deal with all the zeros, writing quantities this way makes it unclear how many significant figures are involved. For the distance to the moon, how many of those digits are significant? Three? Four? All nine?

Writing numbers using **scientific notation** avoids both these problems. A value in scientific notation is a number with one digit to the left of the decimal point and zero or more to the right of it, multiplied by a power of ten. This solves the problem of writing so many zeros and makes the number of significant figures immediately apparent. In scientific notation, writing the distance to the moon as 3.84×10^8 m indicates that three digits are significant; writing it as 3.8×10^8 m indicates that only two digits are.

Even for smaller values, scientific notation can clarify the number of significant figures. Suppose a distance is reported as 1200 m. How many significant figures does this measurement have? It's ambiguous, but using scientific notation can remove any ambiguity. If this distance is known to within 1 m, we can write it as 1.200×10^3 m, showing that all four digits are significant; if it is accurate to only 100 m or so, we can report it as 1.2×10^3 m, indicating two significant figures.

TACTICS BOX 1.2 **Using scientific notation**

To convert a number into scientific notation:

❶ For a number greater than 10, move the decimal point to the left until only one digit remains to the left of the decimal point. The remaining number is then multiplied by 10 to a power; this power is given by the number of spaces the decimal point was moved. Here we convert the radius of the earth to scientific notation:

We move the decimal point until there is only one digit to its left, counting the number of steps.

Since we moved the decimal point 6 steps, the power of ten is 6.

$$6\,370\,000 \text{ m} = 6.37 \times 10^6 \text{ m}$$

The number of digits here equals the number of significant figures.

❷ For a number less than 1, move the decimal point to the right until it passes the first digit that isn't a zero. The remaining number is then multiplied by 10 to a negative power; the power is given by the number of spaces the decimal point was moved. For the diameter of a red blood cell we have:

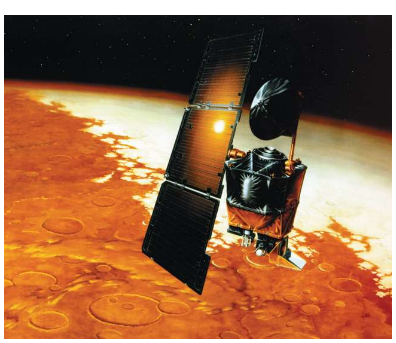
We move the decimal point until it passes the first digit that is not a zero, counting the number of steps.

Since we moved the decimal point 6 steps, the power of ten is −6.

$$0.000\,007\,5 \text{ m} = 7.5 \times 10^{-6} \text{ m}$$

The number of digits here equals the number of significant figures.

Exercise 16



◂ **The importance of units** In 1999, the $125 million Mars Climate Orbiter burned up in the Martian atmosphere instead of entering a safe orbit from which it could perform observations. The problem was faulty units! An engineering team had provided critical data on spacecraft performance in English units, but the navigation team assumed these data were in metric units. As a consequence, the navigation team had the spacecraft fly too close to the planet, and it burned up in the atmosphere.

Units

As we have seen, in order to measure a quantity we need to give it a numerical value. But a measurement is more than just a number—it requires a *unit* to be given. You can't go to the deli and ask for "three quarters of cheese." You need to use a unit—here, one of weight, such as pounds—in addition to the number.

In your daily life, you probably use the English system of units, in which distances are measured in inches, feet, and miles. These units are well adapted for daily life, but they are rarely used in scientific work. Given that science is an international discipline, it is also important to have a system of units that is recognized around the world. For these reasons, scientists use a system of units called *le Système Internationale d'Unités,* commonly referred to as **SI units.** We often refer to these as *metric units* because the meter is the basic standard of length.

The three basic SI quantities, shown in TABLE 1.1, are time, length (or distance), and mass. The SI units for these quantities are meters, seconds, and kilograms, respectively. Other quantities needed to understand motion can be expressed as combinations of these basic units. For example, speed and velocity are expressed in meters per second or m/s. This combination is a ratio of the length unit (the meter) to the time unit (the second).

TABLE 1.1 Common SI units

Quantity	Unit	Abbreviation
Time	second	s
Length	meter	m
Mass	kilogram	kg

Using Prefixes

We will have many occasions to use lengths, times, and masses that are either much less or much greater than the standards of 1 meter, 1 second, and 1 kilogram. We will do so by using *prefixes* to denote various powers of ten. For instance, the prefix "kilo" (abbreviation k) denotes 10^3, or a factor of 1000. Thus 1 km equals 1000 m, 1 MW (megawatt) equals 10^6 watts, and 1 μV (microvolt) equals 10^{-6} volts. TABLE 1.2 lists the common prefixes that will be used frequently throughout this text. A more extensive list of prefixes is shown at the front of the text.

Although prefixes make it easier to talk about quantities, those given with prefixed units are usually converted to base SI units of seconds and meters before any calculations are done. Thus 23.0 cm should be converted to 0.230 m before starting calculations. (The exception is the kilogram, which is already the base SI unit.)

TABLE 1.2 Common prefixes for powers of 10

Prefix	Abbreviation	Power of 10
mega-	M	10^6
kilo-	k	10^3
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}

Unit Conversions Between Measurement Systems

Although SI units are our standard, we cannot entirely forget that the United States still uses English units. Even after repeated exposure to metric units in classes, most of us "think" in English units. Thus it remains important to be able to convert back and forth between SI units and English units. TABLE 1.3 shows some frequently used conversions that will come in handy.

One effective method of performing unit conversions begins by noticing that since, for example, 1 mi = 1.609 km, the ratio of these two distances—*including their units*—is equal to 1, so that

$$\frac{1\ \text{mi}}{1.609\ \text{km}} = \frac{1.609\ \text{km}}{1\ \text{mi}} = 1$$

A ratio of values equal to 1 is called a **conversion factor.** The following Tactics Box shows how to make a unit conversion.

TABLE 1.3 Useful unit conversions

1 inch (in) = 2.54 cm
1 foot (ft) = 0.305 m
1 mile (mi) = 1.609 km
1 mile per hour (mph) = 0.447 m/s
1 m = 39.37 in
1 km = 0.621 mi
1 m/s = 2.24 mph

TACTICS BOX 1.3 **Making a unit conversion**

❶ Start with the quantity you wish to convert.

❷ Multiply by the appropriate conversion factor. Because this conversion factor is equal to 1, multiplying by it does not change the value of the quantity—only its units.

❺ Remember to convert your final answer to the correct number of significant figures!

$$60\ \text{mi} = 60\ \cancel{\text{mi}} \times \frac{1.609\ \text{km}}{1\ \cancel{\text{mi}}} = 96.54\ \text{km} = 97\ \text{km}$$

❸ You can cancel the original unit (here, miles) because it appears in both the numerator and the denominator.

❹ Calculate the answer; it is in the desired units. Remember, 60 mi and 96.54 km are the same distance; they are simply in different units.

Exercise 17

More complicated conversions can be done with several successive multiplications of conversion factors, as we see in the example on the next page.

EXAMPLE 1.4 **Can a bicycle go that fast?**

In Section 1.4, we calculated the speed of a bicycle to be 20 ft/s. Is this a reasonable speed for a bicycle?

STRATEGIZE In order to determine whether or not this speed is reasonable, we will convert it to more familiar units. For speed, the unit you are most familiar with is likely miles per hour.

PREPARE We need the following unit conversions:

$$1 \text{ mi} = 5280 \text{ ft} \qquad 1 \text{ hour (1 h)} = 60 \text{ min} \qquad 1 \text{ min} = 60 \text{ s}$$

SOLVE We then multiply our original value by successive factors of 1 in order to convert the units:

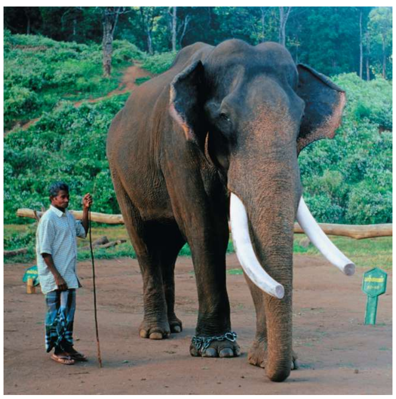
We want to cancel feet here in the numerator . . .

. . . so we multiply by $1 = \frac{1 \text{ mi}}{5280 \text{ ft}}$ to get the feet in the denominator.

$$20\,\frac{\text{ft}}{\text{s}} = 20\,\frac{\text{ft}}{\text{s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 14\,\frac{\text{mi}}{\text{h}} = 14 \text{ mph}$$

The unwanted units cancel in pairs, as indicated by the colors.

ASSESS Our final result of 14 miles per hour (14 mph) is a very reasonable speed for a bicycle, which gives us confidence in our answer. If we had calculated a speed of 140 miles per hour, we would have suspected that we had made an error because this is quite a bit faster than the average bicyclist can travel!



◀ The man has a mass of 70 kg. What is the mass of the elephant standing next to him? By thinking about the relative dimensions of the two, you can make a reasonable one-significant-figure *estimate*.

Estimation

Precise calculations are appropriate when we have precise data, but there are many times when just a rough estimate is sufficient. Suppose you saw a rock fall off a cliff and wanted to know how fast it was going when it hit the ground. By doing a mental comparison with the speeds of familiar objects, such as cars and bicycles, you might judge that the rock was traveling at about 20 mph.

This is a one-significant-figure estimate: You can probably distinguish 20 mph from either 10 mph or 30 mph, but you certainly cannot distinguish 20 mph from 21 mph just from an observation. A one-significant-figure estimate or calculation, such as this estimate of speed, is called an **order-of-magnitude estimate.** An order-of-magnitude estimate is indicated by the symbol $\sim$, which indicates even less precision than the "approximately equal" symbol $\approx$. You would report your estimate of the speed of the falling rock as $v \sim 20$ mph.

It's a useful skill to make reliable order-of-magnitude estimates on the basis of known information (or information found on the Internet), simple reasoning, and common sense. It may help to convert from SI units to more familiar units to make such estimates. You can also do this to assess problem solutions given in SI units. TABLE 1.4 lists some approximate conversion factors to apply in such cases.

TABLE 1.4 Some approximate conversion factors

Quantity	SI unit	Approximate conversion
Mass	kg	1 kg ≈ 2 lb
Length	m	1 m ≈ 3 ft
	cm	3 cm ≈ 1 in
	km	5 km ≈ 3 mi
Speed	m/s	1 m/s ≈ 2 mph
	km/h	10 km/h ≈ 6 mph

EXAMPLE 1.5 **How fast do you walk?**

Estimate how fast you walk, in meters per second.

STRATEGIZE In this example we're asked for an *estimate* of your walking speed, so we'll need to use only rough values obtained from our everyday experience of walking.

PREPARE In order to compute speed, we need a distance and a time. If you walked a mile to campus, how long would this take? You'd probably say 30 minutes or so—half an hour. Let's use this rough number in our estimate.

SOLVE Given this estimate, we compute your speed as

$$\text{speed} = \frac{\text{distance}}{\text{time}} \sim \frac{1 \text{ mile}}{1/2 \text{ hour}} = 2\,\frac{\text{mi}}{\text{h}}$$

But we want the speed in meters per second. Since our calculation is only an estimate, we use an approximate conversion factor from Table 1.4:

$$1\,\frac{\text{mi}}{\text{h}} \sim 0.5\,\frac{\text{m}}{\text{s}}$$

This gives an approximate walking speed of 1 m/s.

ASSESS Is this a reasonable value? Let's do another estimate. Your stride is probably about 1 yard long—about 1 meter. And you take about one step per second; next time you are walking, you can count and see. So a walking speed of 1 meter per second sounds pretty reasonable.

STOP TO THINK 1.5 Rank in order, from the most to the fewest, the number of significant figures in the following numbers. For example, if B has more than C, C has the same number as A, and A has more than D, give your answer as B > C = A > D.

A. 0.43   B. 0.0052   C. 0.430   D. 4.321×10^{-10}

1.6 Vectors and Motion: A First Look

Many physical quantities, such as time, temperature, and mass, can be described completely by a number with a unit. For example, the mass of an object might be 6 kg and its temperature 30°C. When a physical quantity is described by a single number (with a unit), we call it a **scalar quantity.** A scalar can be positive, negative, or zero.

Many other quantities, however, have a directional quality and cannot be described by a single number. To describe the motion of a car, for example, you must specify not only how fast it is moving, but also the *direction* in which it is moving. A **vector quantity** is a quantity that has both a *size* (How far? or How fast?) and a *direction* (Which way?). The size or length of a vector is called its **magnitude.** The magnitude of a vector can be positive or zero, but it cannot be negative.

We graphically represent a vector as an *arrow*, as illustrated for the velocity and force vectors. The arrow is drawn to point in the direction of the vector quantity, and the *length* of the arrow is proportional to the magnitude of the vector quantity.

When we want to represent a vector quantity with a *symbol,* we need somehow to indicate that the symbol is for a vector rather than for a scalar. We do this by drawing an arrow over the letter that represents the quantity. Thus $\vec{r}$ and $\vec{A}$ are symbols for vectors, whereas r and A, without the arrows, are symbols for scalars. In handwritten work you *must* draw arrows over all symbols that represent vectors. This may seem strange until you get used to it, but it is very important because we will often use both r and $\vec{r}$ or both A and $\vec{A}$ in the same problem, and they mean different things!

NOTE ▸ The arrow over the symbol always points to the right, regardless of which direction the actual vector points. Thus we write $\vec{r}$ or $\vec{A}$, never $\overleftarrow{r}$ or $\overleftarrow{A}$. ◂

Scalars and vectors

Scalars

Time, temperature, and mass are all *scalar* quantities. To specify the current time, the temperature outside, or your mass, we need only a single number.

Vectors

The velocity of the race car is a *vector*. To fully specify a velocity, we need to give its magnitude (e.g., 120 mph) *and* its direction (e.g., west).

The force with which the boy pushes on his friend is another example of a vector. To completely specify this force, we must know not only how hard he pushes (the magnitude) but also in which direction.

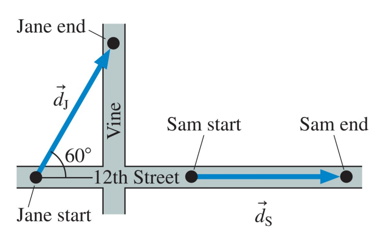
Displacement Vectors

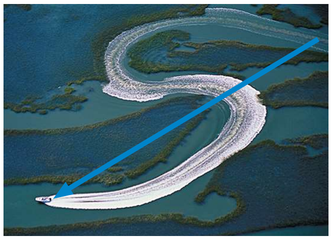
For motion along a line, we found in Section 1.3 that the displacement is a quantity that specifies not only how *far* an object moves but also the *direction*—to the left or to the right—that the object moves. Since displacement is a quantity that has both a magnitude (How far?) and a direction, it can be represented by a vector, the **displacement vector.** FIGURE 1.21 shows the displacement vector for Sam's trip that we discussed earlier. We've simply drawn an arrow—the vector—from his initial to his final position and assigned it the symbol $\vec{d}_S$. Because $\vec{d}_S$ has both a magnitude and a direction, it is convenient to write Sam's displacement as $\vec{d}_S = (100\text{ ft, east})$. The first value in the parentheses is the magnitude of the vector (i.e., the size of the displacement), and the second value specifies its direction.

Also shown in Figure 1.21 is the displacement vector $\vec{d}_J$ for Jane, who started on 12th Street and ended up on Vine. As with Sam, we draw her displacement vector as an arrow from her initial to her final position. In this case, $\vec{d}_J = (100\text{ ft, } 60° \text{ north of east})$.

Jane's trip illustrates an important point about displacement vectors. Jane started her trip on 12th Street and ended up on Vine, leading to the displacement vector

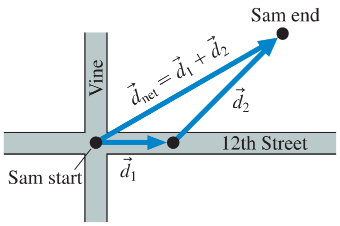
FIGURE 1.21 Two displacement vectors.





The boat's displacement is the straight-line connection from its initial to its final position.

FIGURE 1.22 Sam undergoes two displacements.



shown. But to get from her initial to her final position, she didn't have to walk along the straight-line path denoted by $\vec{d}_J$. If she walked east along 12th Street to the intersection and then headed north on Vine, her displacement would still be the vector shown. **An object's displacement vector is drawn from the object's initial position to its final position, regardless of the actual path followed between these two points.**

Vector Addition

Let's consider one more trip for the peripatetic Sam. In **FIGURE 1.22**, he starts at the intersection and walks east 50 ft; then he walks 100 ft to the northeast through a vacant lot. His displacement vectors for the two legs of his trip are labeled $\vec{d}_1$ and $\vec{d}_2$ in the figure.

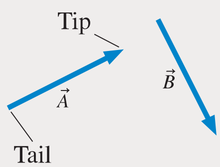
Sam's trip consists of two legs that can be represented by the two vectors $\vec{d}_1$ and $\vec{d}_2$, but we can represent his trip as a whole, from his initial starting position to his overall final position, with the *net* displacement vector labeled $\vec{d}_{net}$. Sam's net displacement is in a sense the *sum* of the two displacements that made it up, so we can write

$$\vec{d}_{net} = \vec{d}_1 + \vec{d}_2$$

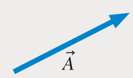
Sam's net displacement thus requires the *addition* of two vectors, but vector addition obeys different rules from the addition of two scalar quantities. The directions of the two vectors, as well as their magnitudes, must be taken into account. Sam's trip suggests that we can add vectors together by putting the "tail" of one vector at the tip of the other. This idea, which is reasonable for displacement vectors, in fact is how *any* two vectors are added. Tactics Box 1.4 shows how to add two vectors $\vec{A}$ and $\vec{B}$ to get their **vector sum** $\vec{A} + \vec{B}$.

TACTICS BOX 1.4 **Adding vectors**

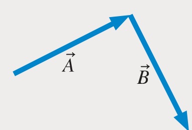
To add $\vec{B}$ to $\vec{A}$:



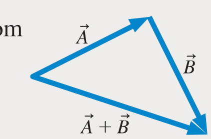
1. Draw $\vec{A}$.



2. Place the tail of $\vec{B}$ at the tip of $\vec{A}$.



3. Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A} + \vec{B}$.



Exercise 21

Vectors and Trigonometry

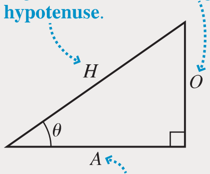
When we need to add displacements or other vectors in more than one dimension, we'll end up computing lengths and angles of triangles. This is the job of trigonometry. **FIGURE 1.23** reviews the basic ideas of trigonometry.

KEY CONCEPT **FIGURE 1.23** Relating sides and angles of right triangles using trigonometry.

We specify the sides of a right triangle in relation to one of the angles.

The longest side, opposite to the right angle, is the **hypotenuse**.

This is the side **opposite** to angle θ.



This is the side **adjacent** to angle θ.

The three sides are related by the *Pythagorean theorem*:

$$H = \sqrt{A^2 + O^2}$$

The sine, cosine, and tangent of angle θ are defined as ratios of the side lengths.

$$\sin\theta = \frac{O}{H}$$

$$\cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A}$$

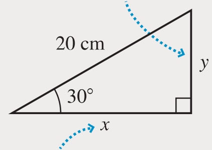
We can rearrange these equations in useful ways:

$$O = H\sin\theta$$

$$A = H\cos\theta$$

Given the length of the hypotenuse and one angle, we can find the side lengths.

y is opposite to the angle; use the sine formula.



x is adjacent to the angle; use the cosine formula.

$$x = (20\text{ cm})\cos(30°) = 17\text{ cm}$$

$$y = (20\text{ cm})\sin(30°) = 10\text{ cm}$$

Inverse trig functions let us find angles given lengths.

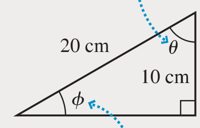
$$\theta = \sin^{-1}\left(\frac{O}{H}\right)$$

$$\theta = \cos^{-1}\left(\frac{A}{H}\right)$$

$$\theta = \tan^{-1}\left(\frac{O}{A}\right)$$

If we are given the lengths of the triangle's sides, we can find angles.

θ is adjacent to the 10 cm side; use the $\cos^{-1}$ formula.



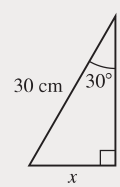
ϕ is opposite to the 10 cm side; use the $\sin^{-1}$ formula.

$$\theta = \cos^{-1}\left(\frac{10\text{ cm}}{20\text{ cm}}\right) = 60°$$

$$\phi = \sin^{-1}\left(\frac{10\text{ cm}}{20\text{ cm}}\right) = 30°$$

STOP TO THINK 1.6 Using the information in Figure 1.23, what is the distance x, to the nearest cm, in the triangle at the right?

A. 26 cm B. 20 cm C. 17 cm D. 15 cm



EXAMPLE 1.6 **How far north and east?**

Suppose Alex is navigating using a compass. She starts walking at an angle 60° north of east and walks a total of 100 m. How far north is she from her starting point? How far east?

STRATEGIZE We'll need to use trigonometry to solve this problem. To do so, we'll need to sketch the situation so that we can identify a right triangle along with its hypotenuse and adjacent and opposite sides.

PREPARE A sketch of Alex's motion is shown in **FIGURE 1.24a**. We've shown north and east as they are on a map, and we've noted Alex's displacement as a vector, giving its magnitude and direction. **FIGURE 1.24b** shows a triangle with this displacement as the hypotenuse. Alex's distance north of her starting point is this triangle's opposite side, and her distance east of her starting point is its adjacent side.

SOLVE Because we want to find O and A for a triangle with $\theta = 60°$ and $H = 100$ m, we use the equations $O = H\sin\theta$ and $A = H\cos\theta$, giving

$$\text{distance north of start} = (100\text{ m})\sin(60°) = 87\text{ m}$$

$$\text{distance east of start} = (100\text{ m})\cos(60°) = 50\text{ m}$$

ASSESS Both of the distances we calculated are less than 100 m, as they must be, and the distance east is less than the distance north, as our diagram in Figure 1.24b shows it should be. Our answers seem reasonable. In finding the solution to this problem, we "broke down" the displacement into two different distances, one north and one east. This hints at the idea of the *components* of a vector, something we'll explore in the next chapter.

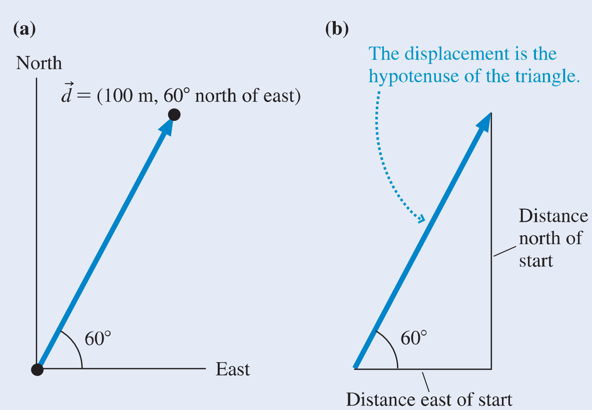


FIGURE 1.24 An analysis of Alex's motion.

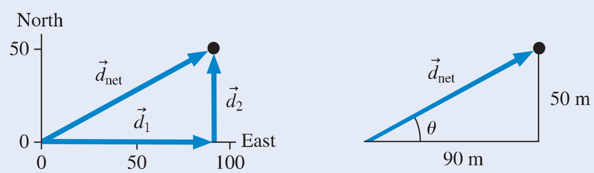
EXAMPLE 1.7 **How far away is Anna?**

Anna walks 90 m due east and then 50 m due north. What is her displacement from her starting point?

STRATEGIZE Again, we will need to use trigonometry to solve this problem, so we'll draw a right triangle and identify its sides.

PREPARE Let's start with the sketch in FIGURE 1.25a. We set up a coordinate system with Anna's original position as the origin, and then we drew her two subsequent motions as the two displacement vectors $\vec{d}_1$ and $\vec{d}_2$.

FIGURE 1.25 Analyzing Anna's motion.



SOLVE We drew the two vector displacements with the tail of one vector starting at the tip of the previous one—exactly what is needed to form a vector sum. The vector $\vec{d}_{net}$ in Figure 1.25a is the vector sum of the successive displacements and thus represents Anna's net displacement from the origin.

Anna's distance from the origin is the length of this vector $\vec{d}_{net}$. FIGURE 1.25b shows that this vector is the hypotenuse of a right triangle with sides 50 m (because Anna walked 50 m north) and 90 m (because she walked 90 m east). We can compute the magnitude of this vector, her net displacement, using the Pythagorean theorem:

$$d_{net} = \sqrt{(50\text{ m})^2 + (90\text{ m})^2} = 103\text{ m} \approx 100\text{ m}$$

We have rounded off to the appropriate number of significant figures, giving us 100 m for the magnitude of the displacement vector. How about the direction? Figure 1.25b identifies the angle that gives the angle north of east of Anna's displacement. In the right triangle, 50 m is the opposite side and 90 m is the adjacent side, so the angle is

$$\theta = \tan^{-1}\left(\frac{50\text{ m}}{90\text{ m}}\right) = \tan^{-1}\left(\frac{5}{9}\right) = 29°$$

Putting it all together, we get a net displacement of

$$\vec{d}_{net} = (100\text{ m}, 29°\text{ north of east})$$

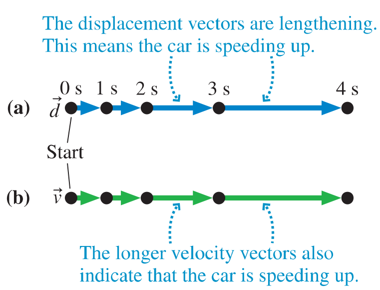
ASSESS We can use our drawing to assess our result. If the two sides of the triangle are 50 m and 90 m, a length of 100 m for the hypotenuse seems about right. The angle is certainly smaller than 45°, but not too much smaller, so 29° seems reasonable.

Velocity Vectors

We've seen that a basic quantity describing the motion of an object is its velocity. Velocity is a vector quantity because its specification involves how fast an object is moving (its speed) and also the direction in which the object is moving. We thus represent the velocity of an object by a **velocity vector** $\vec{v}$ that points in the direction of the object's motion, and whose magnitude is the object's speed.

FIGURE 1.26a shows the motion diagram of a car accelerating from rest. We've drawn vectors showing the car's displacement between successive positions in the motion diagram. To draw the velocity vectors, first note that the direction of the displacement vector is the direction of motion between successive points in the motion diagram. The velocity of an object also points in the direction of motion, so the velocity vector points in the same direction as its displacement vector. Next, note that the magnitude of the velocity vector—How fast?—is the object's speed. Higher speeds imply greater displacements, so the length of the velocity vector should be proportional to the length of the displacement vector between successive points on a motion diagram. All this means that the vectors connecting each dot of a motion diagram to the next dot, which we have labeled as displacement vectors, could equally well be identified as velocity vectors, as shown in FIGURE 1.26b. **From now on, we'll show and label velocity vectors on motion diagrams rather than displacement vectors.**

FIGURE 1.26 The motion diagram for a car starting from rest.



NOTE ► The velocity vectors shown in Figure 1.26b are actually *average* velocity vectors. Because the velocity is steadily increasing, it's a bit less than this average at the start of each time interval, and a bit greater at the end. In Chapter 2 we'll refine these ideas as we develop the idea of instantaneous velocity. ◄

EXAMPLE 1.8 **Drawing a ball's motion diagram**

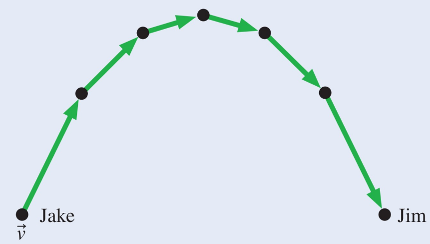
Jake hits a ball at a 60° angle from the horizontal. It is caught by Jim. Draw a motion diagram of the ball that shows velocity vectors rather than displacement vectors.

STRATEGIZE This example is typical of how many problems in physics are worded. The problem does not give a clear statement of where the motion begins or ends. Are we interested in the motion of the ball only during the time it is in the air between Jake and Jim? What about the motion *as* Jake hits it (ball rapidly speeding up) or *as* Jim catches it (ball rapidly slowing down)? Should we include Jim dropping the ball after he catches it? The point is that *you* will often be called on to make a *reasonable interpretation* of a problem statement. In this problem, the details of hitting and catching the ball are complex. The motion of the ball through the air is easier to describe, and it's a motion you might expect to learn about in a physics class. So our *interpretation* is that the motion diagram should start as the ball leaves Jake's bat (ball already moving) and should end the instant it touches Jim's hand (ball still moving).

PREPARE We model the ball as a particle, and sketch a motion diagram that represents the motion of a thrown ball along an arc.

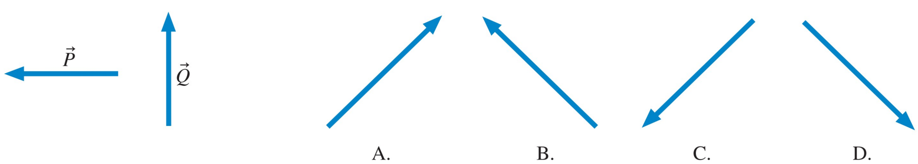
SOLVE **FIGURE 1.27** shows the motion diagram of the ball. Notice how, in contrast to the car of Figure 1.26, the ball is already moving as the motion diagram begins. As before, the velocity vectors are shown by connecting the dots with arrows. You can see that the velocity vectors get shorter (ball slowing down), get longer (ball speeding up), and change direction. Each $\vec{v}$ is different, so this is *not* constant-velocity motion.

FIGURE 1.27 The motion diagram of a ball traveling from Jake to Jim.



ASSESS We haven't learned enough to make a detailed analysis of the motion of the ball, but it's still worthwhile to do a quick assessment. Does our diagram make sense? Think about the velocity of the ball—we show it moving upward at the start and downward at the end. This does match what happens when you toss a ball back and forth, so our answer seems reasonable.

STOP TO THINK 1.7 $\vec{P}$ and $\vec{Q}$ are two vectors of equal length but different direction. Which vector shows the sum $\vec{P} + \vec{Q}$?

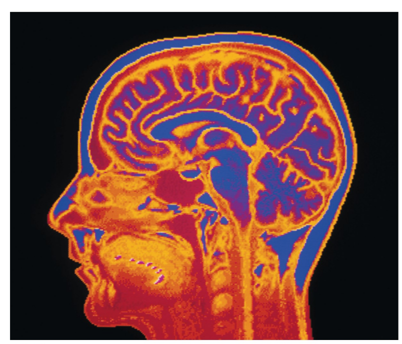


1.7 Where Do We Go from Here?

This first chapter has been an introduction to some of the fundamental ideas about motion and some of the basic techniques that you will use in the rest of the course. You have seen some examples of how to make *models* of a physical situation, thereby focusing on the essential elements of the situation. You have learned some practical ideas, such as how to convert quantities from one set of units to another. The rest of this text—and the rest of your course—will extend these themes.

In each chapter of this text, you'll learn both new principles and more tools and techniques. As you proceed, you'll find that each new chapter depends on those that preceded it. The principles and the problem-solving strategies you learned in this chapter will still be needed in Chapter 30.

We'll give you some assistance integrating new ideas with the material of previous chapters. When you start a chapter, the **chapter preview** will let you know which topics are especially important to review. And the last element in each chapter will be an **integrated example** that brings together the principles and techniques you have just learned with those you learned previously. The integrated nature of



BIO Chapter 28 ends with an integrated example that explores the basic physics of magnetic resonance imaging (MRI), explaining how the interaction of magnetic fields with the nuclei of atoms in the body can be used to create an image of the body's interior.

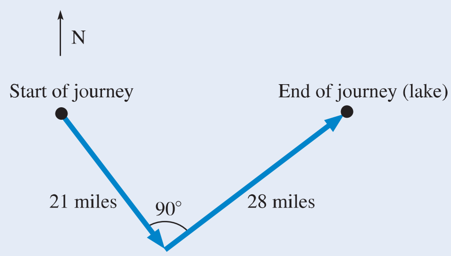
these examples will also be a helpful reminder that the problems of the real world are similarly complex, and solving such problems requires you to do just this kind of integration.

Our first integrated example is reasonably straightforward because there's not much to integrate yet. The examples in future chapters will be much richer.

INTEGRATED EXAMPLE 1.9 A goose gets its bearings BIO

Migrating geese determine direction using many different tools: by noting local landmarks, by following rivers and roads, and by using the position of the sun in the sky. When the weather is overcast so that they can't use the sun's position to get their bearings, geese may start their day's flight in the wrong direction. **FIGURE 1.28** shows the path of a Canada goose that flew in a straight line for some time before making a corrective right-angle turn. One hour after beginning, the goose made a rest stop on a lake due east of its original position.

FIGURE 1.28 Trajectory of a misdirected goose.

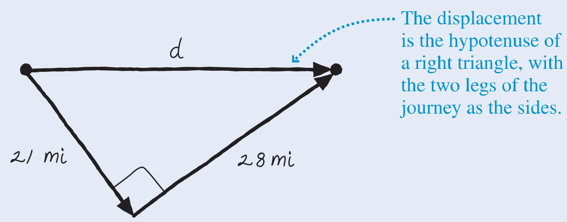


a. How much extra distance did the goose travel due to its initial error in flight direction? That is, how much farther did it fly than if it had simply flown directly to its final position on the lake?
b. What was the flight speed of the goose?
c. A typical flight speed for a migrating goose is 80 km/h. Given this, does your result seem reasonable?

STRATEGIZE In this integrated example, we'll need to pull together all our knowledge about right triangles, speed, and significant figures.

PREPARE Figure 1.28 shows the trajectory of the goose, but it's worthwhile to redraw Figure 1.28 and note the displacement from the start to the end of the journey, the shortest distance the goose could have flown. (The examples in the chapter to this point have used professionally rendered drawings, but these are much more careful and detailed than you are likely to make. **FIGURE 1.29** shows a drawing that is more typical of what you might actually do when working problems yourself.) Drawing and labeling the displacement between the starting and ending points in Figure 1.29 show that it is the hypotenuse of a right triangle, so we can use our rules for triangles as we look for a solution.

FIGURE 1.29 A typical student sketch shows the motion and the displacement of the goose.



SOLVE

a. The minimum distance the goose *could* have flown, if it flew straight to the lake, is the hypotenuse of a triangle with sides 21 mi and 28 mi. This straight-line distance is

$$d = \sqrt{(21 \text{ mi})^2 + (28 \text{ mi})^2} = 35 \text{ mi}$$

The actual distance the goose flew is the sum of the distances traveled for the two legs of the journey:

$$\text{distance traveled} = 21 \text{ mi} + 28 \text{ mi} = 49 \text{ mi}$$

The extra distance flown is the difference between the actual distance flown and the straight-line distance—namely, 14 miles.

b. To compute the flight speed, we need to consider the distance that the bird actually flew. The flight speed is the total distance flown divided by the total time of the flight:

$$v = \frac{49 \text{ mi}}{1.0 \text{ h}} = 49 \text{ mi/h}$$

c. To compare our calculated speed with a typical flight speed, we must convert our solution to km/h, rounding off to the correct number of significant figures:

$$49 \frac{\text{mi}}{\text{h}} \times \frac{1.61 \text{ km}}{1.00 \text{ mi}} = 79 \frac{\text{km}}{\text{h}}$$

A calculator will return many more digits, but the original data had only two significant figures, so we report the final result to this accuracy.

ASSESS In this case, an assessment was built into the solution of the problem. The calculated flight speed matches the expected value for a goose, which gives us confidence that our answer is correct. As a further check, our calculated net displacement of 35 mi seems about right for the hypotenuse of the triangle in Figure 1.29.

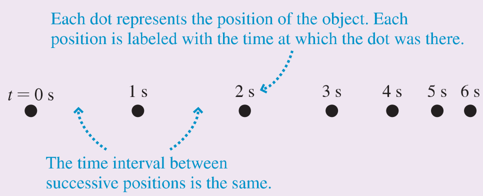
SUMMARY

GOAL **To introduce the fundamental concepts of motion and to review related basic mathematical principles.**

IMPORTANT CONCEPTS

Motion Diagrams

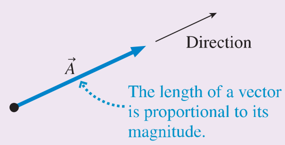
The **particle model** represents a moving object as if all its mass were concentrated at a single point. Using this model, we can represent motion with a **motion diagram,** where dots indicate the object's positions at successive times. In a motion diagram, the time interval between successive dots is always the same.



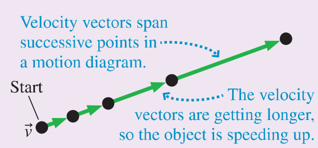
Scalars and Vectors

Scalar quantities have only a magnitude and can be represented by a single number. Temperature, time, and mass are scalars.

A **vector** is a quantity described by both a magnitude and a direction. Velocity and displacement are vectors.

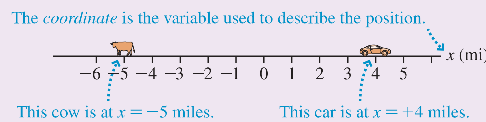


Velocity vectors can be drawn on a motion diagram by connecting successive points with a vector.



Describing Motion

Position locates an object with respect to a chosen coordinate system. It is described by a **coordinate**.



A change in position is called a **displacement**. For motion along a line, a displacement is a signed quantity. The displacement from x_i to x_f is $\Delta x = x_f - x_i$.

Time is measured from a particular instant to which we assign $t = 0$. A **time interval** is the elapsed time between two specific instants t_i and t_f. It is given by $\Delta t = t_f - t_i$.

Velocity is the ratio of the displacement of an object to the time interval during which this displacement occurs:

$$v = \frac{\Delta x}{\Delta t}$$

Units

Every measurement of a quantity must include a **unit.**

The standard system of units used in science is the **SI system.** Common SI units include:

- Length: meters (m)
- Time: seconds (s)
- Mass: kilograms (kg)

APPLICATIONS

Working with Numbers

In **scientific notation,** a number is expressed as a decimal number between 1 and 10 multiplied by a power of ten. In scientific notation, the diameter of the earth is 1.27×10^7 m.

A **prefix** can be used before a unit to indicate a multiple of 10 or 1/10. Thus we can write the diameter of the earth as 12,700 km, where the k in km denotes 1000.

We can perform a **unit conversion** to convert the diameter of the earth to a different unit, such as miles. We do so by multiplying by a conversion factor equal to 1, such as 1 = 1 mi/1.61 km.

Significant figures are reliably known digits. The number of significant figures for:

- **Multiplication, division, and powers** is set by the value with the fewest significant figures.
- **Addition and subtraction** is set by the value with the smallest number of decimal places.

An **order-of-magnitude estimate** is an estimate that has an accuracy of about one significant figure. Such estimates are usually made using rough numbers from everyday experience.

Learning Objectives After studying this chapter, you should be able to:

- Draw and interpret motion diagrams to represent motion. *Conceptual Questions 1.2, 1.13; Problems 1.1, 1.2, 1.3*
- Describe motion in terms of position, velocity, and time. *Conceptual Question 1.8; Problems 1.5, 1.6, 1.7, 1.8, 1.9*
- Calculate the speed and velocity of an object. *Conceptual Question 1.5; Problems 1.11, 1.12, 1.13, 1.15*
- Use scientific notation. *Problems 1.23, 1.24, 1.25*
- Express quantities with the appropriate units and the proper number of significant figures. *Problems 1.19, 1.20, 1.21, 1.22*
- Perform unit conversions. *Conceptual Question 1.17; Problems 1.16, 1.17, 1.18*
- Describe motion using vectors and trigonometry. *Conceptual Question 1.16; Problems 1.27, 1.28, 1.32, 1.35, 1.38*

STOP TO THINK ANSWERS

Chapter Preview Stop to Think: C. The sides of a right triangle are related by the Pythagorean theorem. The length of the hypotenuse is thus $\sqrt{(6\text{ cm})^2 + (8\text{ cm})^2} = 10$ cm. Note that this triangle is a version of a 3-4-5 right triangle; the lengths of the sides are in this ratio.

Stop to Think 1.1: B. The images of B are farther apart, so B travels a greater distance than does A during the same intervals of time.

Stop to Think 1.2: A. Dropped ball. B. Dust particle. C. Descending rocket.

Stop to Think 1.3: C. Depending on her initial positive position and how far she moves in the negative direction, she could end up on either side of the origin.

Stop to Think 1.4: C. Her speed is given by Equation 1.1. Her speed is the distance traveled (4 miles) divided by the time interval (2 hours), or 2 mph.

Stop to Think 1.5: D $>$ C $>$ B $=$ A.

Stop to Think 1.6: D. x is the length of the side opposite the 30° angle, so $x = (30\text{ cm})\sin 30° = 15$ cm.

Stop to Think 1.7: B. The vector sum is found by placing the tail of one vector at the tip of the other vector.

Video Tutor Solution Chapter 1

QUESTIONS

Conceptual Questions

1. A softball player slides into second base. Use the particle model to draw a motion diagram of the player from the time he begins to slide until he reaches the base. Number the dots in order, starting with zero.
2. A car travels to the left at a steady speed for a few seconds, then brakes for a stop sign. Use the particle model to draw a motion diagram of the car for the entire motion described here. Number the dots in order, starting with zero.
3. BIO The bush baby, a small African mammal, is a remarkable jumper. Although only about 8 inches long, it can jump, from a standing start, straight up to a height of over 7 feet! Use the particle model to draw a motion diagram for a bush baby's jump, from its start until it reaches its highest point.
4. A ball is dropped from the roof of a tall building and students in a physics class are asked to sketch a motion diagram for this situation. A student submits the diagram shown in Figure Q1.4. Is the diagram correct? Explain.

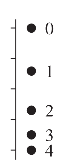


FIGURE Q1.4

5. Mark and Sofia walk together down a long, straight road. They walk without stopping for 4 miles. At this point Sofia says their displacement during the trip must have been 4 miles; Mark says their current position must be 4 miles. Who, if either, is correct? Explain.
6. Give an example of a trip you might take in your car for which the distance traveled as measured on your car's odometer is not equal to the displacement between your initial and final positions.
7. Write a sentence or two describing the difference between speed and velocity. Give one example of each.
8. The motion of a skateboard along a horizontal axis is observed for 5 s. The initial position of the skateboard is negative with respect to a chosen origin, and its velocity throughout the 5 s is also negative. At the end of the observation time, is the skateboard closer to or farther from the origin than initially? Explain.
9. You are standing on a straight stretch of road and watching the motion of a bicycle; you choose your position as the origin. At one instant, the position of the bicycle is negative and its velocity is positive. Is the bicycle getting closer to you or farther away? Explain.
10. Two friends watch a jogger complete a 400 m lap around the track in 100 s. One of the friends states, "The jogger's velocity was 4 m/s during this lap." The second friend objects, saying, "No, the jogger's speed was 4 m/s." Who is correct? Justify your answer.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging). INT Problems labeled integrate significant material from earlier chapters; BIO are of biological or medical interest.

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11. A softball player hits the ball and starts running toward first base. Draw a motion diagram, using the particle model, showing her velocity vectors during the first few seconds of her run.
12. A child is sledding on a smooth, level patch of snow. She encounters a rocky patch and slows to a stop. Draw a motion diagram, using the particle model, showing her velocity vectors.
13. A skydiver jumps out of an airplane. Her speed steadily increases until she deploys her parachute, at which point her speed quickly decreases. She subsequently falls to earth at a constant rate, stopping when she lands on the ground. Draw a motion diagram, using the particle model, that shows her position at successive times and includes velocity vectors.
14. Your roommate drops a tennis ball from a third-story balcony. It hits the sidewalk and bounces as high as the second story. Draw a motion diagram, using the particle model, showing the ball's velocity vectors from the time it is released until it reaches the maximum height on its bounce.
15. A car is driving north at a steady speed. It makes a gradual 90° left turn without losing speed, then continues driving to the west. Draw a motion diagram, using the particle model, showing the car's velocity vectors as seen from a helicopter hovering over the highway.
16. Three displacement vectors have lengths 1 m, 2 m, and 4 m. Could they possibly add together to get a vector of length zero?
17. Your friend Travis claims to have set the new world speed record for riding a unicycle. His top speed, he says, was 55 m/s. Do you believe him? Explain.

Multiple-Choice Questions

18. | A student walks 1.0 mi west and then 1.0 mi north. Afterward, how far is she from her starting point?
A. 1.0 mi B. 1.4 mi C. 1.6 mi D. 2.0 mi
19. | You throw a rock upward. The rock is moving upward, but it is slowing down. If we define the ground as the origin, the position of the rock is ______ and the velocity of the rock is ______.
A. positive, positive B. positive, negative
C. negative, positive D. negative, negative
20. | Which of the following motions could be described by the motion diagram of Figure Q1.20?
A. A hockey puck sliding across smooth ice.
B. A cyclist braking to a stop.
C. A sprinter starting a race.
D. A ball bouncing off a wall.

FIGURE Q1.20

21. | Which of the following motions is described by the motion diagram of Figure Q1.21?
A. An ice skater gliding across the ice.
B. An airplane braking to a stop after landing.
C. A car pulling away from a stop sign.
D. A pool ball bouncing off a cushion and reversing direction.

FIGURE Q1.21

22. | A bird flies 3.0 km due west and then 2.0 km due north. What is the magnitude of the bird's displacement?
A. 2.0 km B. 3.0 km C. 3.6 km D. 5.0 km
23. || Weddell seals make holes in sea ice so that they can swim down to forage on the ocean floor below. Measurements for one seal showed that it dived straight down from such an opening, reaching a depth of 0.30 km in a time of 5.0 min. What was the speed of the diving seal?
BIO
A. 0.60 m/s B. 1.0 m/s C. 1.6 m/s D. 6.0 m/s
E. 10 m/s
24. || A bird flies 3.0 km due west and then 2.0 km due north. Another bird flies 2.0 km due west and 3.0 km due north. What is the angle between the net displacement vectors for the two birds?
A. 23° B. 34° C. 56° D. 90°
25. || Hicham El Guerrouj of Morocco holds the world record in the 1500 m running race. He ran the final 400 m in a time of 51.9 s. What was his average speed in mph over the last 400 m?
A. 14.2 mph B. 15.5 mph
C. 17.2 mph D. 23.9 mph
26. | Compute 3.24 m + 0.532 m to the correct number of significant figures.
A. 3.7 m B. 3.77 m
C. 3.772 m D. 3.7720 m
27. || An American football field is 109.7 m long and 48.8 m wide. To the correct number of significant figures, what is its area?
A. 5351 m^2 B. 5.35×10^3 m^2
C. 5351.17 m^2 D. 5400 m^2
28. | The earth formed 4.57×10^9 years ago. What is this time in seconds?
A. 1.67×10^{12} s B. 4.01×10^{13} s
C. 2.40×10^{15} s D. 1.44×10^{17} s
29. ||| An object's average density ρ is defined as the ratio of its mass to its volume: $\rho = M/V$. The earth's mass is 5.94×10^{24} kg, and its volume is 1.08×10^{12} km^3. What is the earth's average density?
A. 5.50×10^3 kg/m^3 B. 5.50×10^6 kg/m^3
C. 5.50×10^9 kg/m^3 D. 5.50×10^{12} kg/m^3

PROBLEMS

Section 1.1 Motion: A First Look

1. | A car skids to a halt to avoid hitting an object in the road. Draw a motion diagram of the car from the time the skid begins until the instant the car stops.
2. | A man rides a bike along a straight road for 5 min, then has a flat tire. He stops for 5 min to repair the flat, but can't fix it. He walks the rest of the way, which takes him another 10 min. Use the particle model to draw a motion diagram of the man for the entire motion described here. Number the dots in order, starting with zero.
3. | Amanda has just entered an elevator. The elevator rises and stops at the third floor. Use the particle model to draw a motion diagram of Amanda during her entire ride on the elevator. Number the dots in order, starting from zero. (Be sure to consider how the elevator speeds up and slows down.)

Watch Video Solution Problem 1.25

Section 1.2 Models and Modeling

Section 1.3 Position and Time: Putting Numbers on Nature

4. | Figure P1.4 shows Sue along the straight-line path between her home and the cinema. What is Sue's position x if
   a. Her home is the origin?
   b. The cinema is the origin?

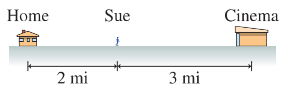


FIGURE P1.4

5. | Figure P1.4 shows Sue along the straight-line path between her home and the cinema. Now Sue walks home. What is Sue's displacement if
   a. Her home is the origin?
   b. The cinema is the origin?
6. | Logan observes a paramecium under a microscope. The eye-
BIO piece of the microscope has a horizontal scale marked in mm. The paramecium starts at the 65 mm mark and ends up at the 42 mm mark. What is the paramecium's displacement?
7. | Keira starts at position $x = 23$ m along a coordinate axis. She then undergoes a displacement of -45 m. What is her final position?
8. | A car travels along a straight east-west road. A coordinate system is established on the road, with x increasing to the east. The car ends up 14 mi west of the origin, which is defined as the intersection with Mulberry Road. If the car's displacement was -23 mi, what side of Mulberry Road did the car start on? How far from the intersection was the car at the start?
9. | Foraging bees often move in straight lines away from and
BIO toward their hives. Suppose a bee starts at its hive and flies 500 m due east, then flies 400 m west, then 700 m east. How far is the bee from the hive?

Section 1.4 Velocity

10. | A security guard walks at a steady pace, traveling 110 m in one trip around the perimeter of a building. It takes him 240 s to make this trip. What is his speed?
11. || List the following items in order of decreasing speed, from greatest to least: (i) A wind-up toy car that moves 0.15 m in 2.5 s. (ii) A soccer ball that rolls 2.3 m in 0.55 s. (iii) A bicycle that travels 0.60 m in 0.075 s. (iv) A cat that runs 8.0 m in 2.0 s.
12. || Figure P1.12 shows the motion diagram for a horse galloping in one direction along a straight path. Not every dot is labeled, but the dots are at equally spaced instants of time. What is the horse's velocity
    a. During the first 10 seconds of its gallop?
    b. During the interval from 30 s to 40 s?
    c. During the interval from 50 s to 70 s?

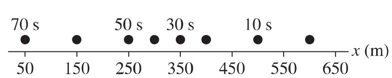


FIGURE P1.12

13. || It takes Harry 35 s to walk from $x = -12$ m to $x = -47$ m. What is his velocity?
14. | A dog trots from $x = -12$ m to $x = 3$ m in 10 s. What is its velocity?
15. || In Michael Johnson's world-record 400 m sprint, he ran the
BIO first 100 m in 11.20 s; then he reached the 200 m mark after a total time of 21.32 s had elapsed, reached the 300 m mark after 31.76 s, and finished in 43.18 s.
    a. During what 100 m segment was his speed the highest?
    b. During this segment, what was his speed in m/s?

Section 1.5 A Sense of Scale: Significant Figures, Scientific Notation, and Units

16. || Convert the following to SI base units:
    a. 9.12 μs b. 3.42 km
    c. 44 cm/ms d. 80 km/h
17. | Convert the following to SI units:
    a. 8.0 in b. 66 ft/s c. 60 mph
18. | Convert the following to SI units:
    a. 1.0 hour b. 1.0 day c. 1.0 year
19. || How many significant figures does each of the following numbers have?
    a. 6.21 b. 62.1
    c. 0.620 d. 0.062
20. | How many significant figures does each of the following numbers have?
    a. 0.621 b. 0.006200
    c. 1.0621 d. 6.21×10^3
21. | Compute the following numbers to three significant figures.
    a. 33.3×25.4 b. $33.3 - 25.4$
    c. $\sqrt{33.3}$ d. $333.3 \div 25.4$
22. || If you make multiple mea-
BIO surements of your height, you are likely to find that the results vary by nearly half an inch in either direction due to measurement error and actual variations in height. You are slightly shorter in the evening, after gravity has compressed and reshaped your spine over the course of a day. One measurement of a man's height is 6 feet and 1 inch. Express his height in meters, using the appropriate number of significant figures.

23. | Mount Everest has a height of 29,029 ft above sea level. Express this height in meters, giving your result in scientific notation with the correct number of significant figures.
24. ||| Blades of grass grow from the bottom, so, as growth occurs,
BIO the top of the blade moves upward. During the summer, when your lawn is growing quickly, estimate this speed, in m/s. Make this estimate from your experience noting, for instance, how often you mow the lawn and what length you trim. Express your result in scientific notation.
25. || Estimate the average speed, in m/s, with which the hair on
BIO your head grows. Make this estimate from your own experience noting, for instance, how often you cut your hair and how much you trim. Express your result in scientific notation.

Section 1.6 Vectors and Motion: A First Look

26. | Loveland, Colorado, is 18 km due south of Fort Collins and 31 km due west of Greeley. What is the distance between Fort Collins and Greeley?

27. || A city has streets laid out in a square grid, with each block 135 m long. If you drive north for three blocks, then west for two blocks, how far are you from your starting point?

28. | Joe and Max shake hands and say goodbye. Joe walks east 0.55 km to a coffee shop, and Max flags a cab and rides north 3.25 km to a bookstore. How far apart are their destinations?

29. || In downtown Chicago, the east-west blocks are 400 ft long while the north-south blocks are 280 ft long. Because of the many one-way streets, it can be challenging to get around. Veronica starts at the corner of Dearborn and Ohio Streets. She drives four blocks north to Superior, two blocks east to Wabash, then a block south to get to her destination at Wabash and Huron. What is the straight-line distance from her starting point?

30. || A butterfly flies from the top of a tree in the center of a garden to rest on top of a red flower at the garden's edge. The tree is 8.0 m taller than the flower, and the garden is 12 m wide. Determine the magnitude of the butterfly's displacement.

31. ||| A garden has a circular path of radius 50 m. John starts at the easternmost point on this path, then walks counterclockwise around the path until he is at its southernmost point. What is John's displacement? Use the (magnitude, direction) notation for your answer.

32. ||| Luis is visiting a public garden that has a large, circular path. When he has walked one-quarter of the distance around the path, the magnitude of his displacement is 180 m. What is the diameter of the path?

33. | Migrating geese tend to travel at approximately constant
BIO speed, flying in segments that are straight lines. A goose flies 32 km south, then turns to fly 20 km west. Afterward, how far is the goose from its original position?

34. || A circular test track for cars in England has a circumference of 3.2 km. A car travels around the track from the southernmost point to the northernmost point.
   a. What distance does the car travel?
   b. What is the car's displacement from its original position?

35. || Black vultures excel at gliding flight; they can move long
BIO distances through the air without flapping their wings while undergoing only a modest drop in height. A vulture in a typical glide in still air moves along a path tipped 3.5° below the horizontal. If the vulture moves a horizontal distance of 100 m, how much height does it lose?

36. ||| Figure P1.36 shows a map of Olivia's trip to a coffee shop. She gets on her bike at Loomis and then rides south 0.8 mi to Broadway. She turns east onto Broadway, rides 0.8 mi to where Broadway turns, and then continues another 1.2 mi to the shop. What is the total displacement of her trip, in (magnitude, direction) form?

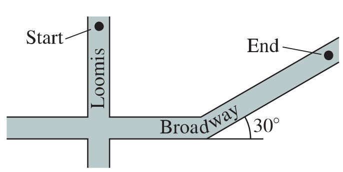


FIGURE P1.36

37. || The Great Pyramid of Giza is 139 m tall, with a slope of 51.8°. If you were to climb the pyramid from base to top (which is forbidden!), what distance along the face of the pyramid would you travel?

38. | A hiker is climbing a steep 10° slope. Her pedometer shows that she has walked 1500 m along the slope. How much elevation has she gained?

39. ||||| A ball on a porch rolls 60 cm to the porch's edge, drops 40 cm, continues rolling on the grass, and eventually stops 80 cm from the porch's edge. What is the magnitude of the ball's net displacement, in centimeters?

40. || A kicker punts a football from the very center of the field to the sideline 43 yards downfield. What is the net displacement of the ball? (A football field is 53 yards wide.)

Problems 41 and 42 relate to the gliding flight of flying squirrels. These squirrels glide from tree to tree at a constant speed, moving in a straight line tipped below the vertical and steadily losing altitude as they move forward. Short and long glides have different profiles.

41. || A squirrel completing a short glide travels in a straight line
BIO tipped 40° below the horizontal. The squirrel starts 9.0 m above the ground on one tree and glides to a second tree that is a horizontal distance of 3.5 m away.
   a. What is the length of the squirrel's glide path?
   b. What is the squirrel's height above the ground when it lands?

42. || A squirrel in a typical long glide covers a horizontal distance
BIO of 16 m while losing 8.0 m of elevation. During this glide,
   a. What is the angle of the squirrel's path below the horizontal?
   b. What is the total distance covered by the squirrel?

General Problems

Problems 43 through 49 are motion problems similar to those you will learn to solve in Chapter 2. For now, simply *interpret* the problem by drawing a motion diagram showing the object's position and its velocity vectors. **Do *not* solve these problems** or do any mathematics.

43. || In a typical greyhound race, a dog accelerates to a speed of
BIO 20 m/s over a distance of 30 m. It then maintains this speed. What would be a greyhound's time in the 100 m dash?

44. || Billy drops a watermelon from the top of a three-story building, 10 m above the sidewalk. How fast is the watermelon going when it hits?

45. || A skateboarder starts from rest at the top of a ramp. He rolls down the ramp and then continues rolling on the smooth, horizontal floor.

46. || A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?

47. || The giant eland, an African antelope, is an exceptional
BIO jumper, able to leap 1.5 m off the ground. To jump this high, with what speed must the eland leave the ground?

48. || A ball rolls along a smooth horizontal floor at 10 m/s, then starts up a 20° ramp. How high does it go before rolling back down?

49. || A motorist is traveling at 20 m/s. He is 60 m from a stop light when he sees it turn yellow. His reaction time, before stepping on the brake, is 0.50 s. What steady deceleration while braking will bring him to a stop right at the light?

Watch Video Solution Problem 1.61

Problems 50 through 54 show a motion diagram. For each of these problems, write a one or two sentence "story" about a *real object* that has this motion diagram. Your stories should talk about people or objects by name and say what they are doing. Problems 43 through 49 are examples of motion short stories.

50. I

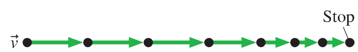


FIGURE P1.50

51. I

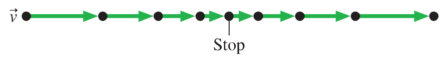


FIGURE P1.51

52. I

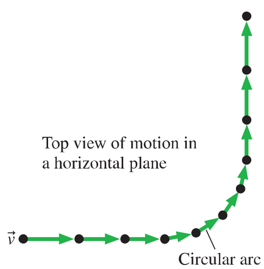


FIGURE P1.52

53. I

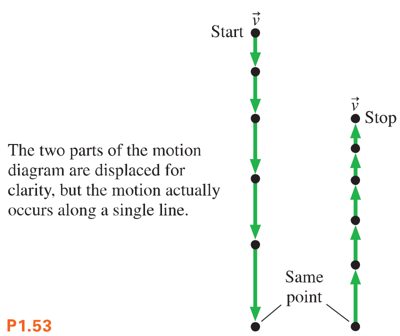


FIGURE P1.53

54. I

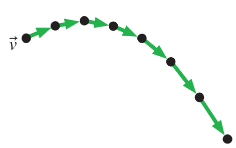
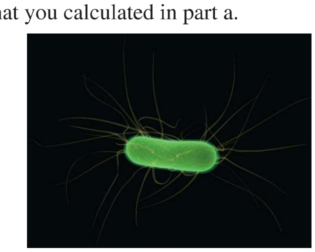


FIGURE P1.54

55. II Estimate the length of a human lifetime, in seconds.
56. III On a highway trip, Joseph drives the first 25 miles at 55 mph, and the next 15 miles at 70 mph. What is his average speed for this trip?
57. III Evan is just leaving his house to visit his grandmother. Normally, the trip takes him 25 minutes on the freeway, going 55 mph. But tonight he's running 5 minutes late. How fast will he need to drive on the freeway to make up the 5 minutes?
58. III Gretchen runs the first 4.0 km of a race at 5.0 m/s. Then a stiff wind comes up, so she runs the last 1.0 km at only 4.0 m/s. If she later ran the same course again, what constant speed would let her finish in the same time as in the first race?
59. IIII If you swim with the current in a river, your speed is increased by the speed of the water; if you swim against the current, your speed is decreased by the water's speed. The current in a river flows at 0.52 m/s. In still water you can swim at 1.78 m/s. If you swim downstream a certain distance, then back again upstream, how much longer, in percent, does it take compared to the same trip in still water?
60. II The end of Hubbard Glacier in Alaska advances by an average of 105 feet per year. What is the speed of advance of the glacier in m/s?
61. I The earth completes a circular orbit around the sun in one year. The orbit has a radius of 93,000,000 miles. What is the speed of the earth around the sun in m/s? Report your result using scientific notation.
62. II BIO The Greenland shark is thought to be the longest-living vertebrate on earth. Early estimates of its maximum age were based on the fact that such sharks grow in length only about a centimeter per year, and yet an adult shark can reach a length of 15 feet. Estimate how long a 15 foot shark might have lived. (A newborn shark is about 1 foot long.)
63. III The winner of the 2016 Keystone (Colorado) Uphill/Downhill mountain bike race finished in a total time of 47 minutes and 25 seconds. The uphill leg was 4.6 miles long, and on this leg his average speed was 8.75 mph. The downhill leg was 6.9 miles. What was his average speed on this leg?
64. III Shannon decides to check the accuracy of her speedometer. She adjusts her speed to read exactly 70 mph on her speedometer and holds this steady, measuring the time between successive mile markers separated by exactly 1.00 mile. If she measures a time of 54 s, is her speedometer accurate? If not, is the speed it shows too high or too low?
65. II The Nardo ring is a circular test track for cars. It has a circumference of 12.5 km. Cars travel around the track at a constant speed of 100 km/h. A car starts at the easternmost point of the ring and drives for 15 minutes at this speed.
    a. What distance, in km, does the car travel?
    b. What is the magnitude of the car's displacement, in km, from its initial position?
    c. What is the speed of the car in m/s?
66. II BIO Motor neurons in mammals transmit signals from the brain to skeletal muscles at approximately 25 m/s. Estimate how much time in ms (10^{-3} s) it will take for a signal to get from your brain to your hand.
67. III BIO Satellite data taken several times per hour on a particular albatross showed travel of 1200 km over a time of 1.4 days.
    a. Given these data, what was the bird's average speed in mph?
    b. Data on the bird's position were recorded only intermittently. Explain how this means that the bird's actual average speed was higher than what you calculated in part a.
68. II BIO The bacterium *Escherichia coli* (or *E. coli*) is a single-celled organism that lives in the gut of healthy humans and animals. Its body shape can be modeled as a 2-μm-long cylinder with a 1 μm diameter, and it has a mass of 1×10^{-12} g. Its chromosome consists of a single double-stranded chain of DNA 700 times longer than its body length. The bacterium moves at a constant speed of 20 μm/s, though not always in the same direction. Answer the following questions about *E. coli* using SI base units (unless specifically requested otherwise) and correct significant figures.



    a. What is its length?
    b. Diameter?
    c. Mass?
    d. What is the length of its DNA, in millimeters?
    e. If the organism were to move along a straight path, how many meters would it travel in one day?

69. || The bacterium *Escherichia coli* (or *E. coli*) is a single-celled organism that lives in the gut of healthy humans and animals. When grown in a uniform medium rich in salts and amino acids, it swims along zig-zag paths at a constant speed changing direction at varying time intervals. Figure P1.69 shows the positions of an *E. coli* as it moves from point A to point J. Each segment of the motion can be identified by two letters, such as segment BC. During which segments, if any, does the bacterium have the same
BIO
    a. Displacement? b. Speed? c. Velocity?

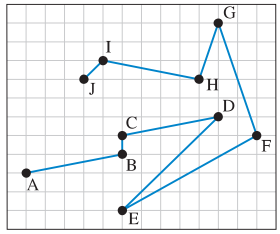


FIGURE P1.69

70. ||| The sun is 30° above the horizon. It makes a 52-m-long shadow of a tall tree. How high is the tree?

71. ||| Weddell seals foraging in open water dive toward the ocean bottom by swimming forward in a straight-line path tipped below the horizontal. The tracking data for one seal showed it taking 4.0 min to descend 360 m below the surface while moving 920 m horizontally.
BIO
    a. What was the angle of the seal's path below the horizontal?
    b. What distance did the seal cover in making this dive?
    c. What was the seal's speed, in m/s?

72. ||| Erica is participating in a road race. The first part of the race is on a 5.2-mile-long straight road oriented at an angle of 25° north of east. The road then turns due north for another 4.0 mi to the finish line. In miles, what is the straight-line distance from the starting point to the end of the race?

73. ||| Whale sharks swim forward while ascending or descending. They swim along a straight-line path at a shallow angle as they move from the surface to deep water or from the depths to the surface. In one recorded dive, a shark started 50 m below the surface and swam at 0.85 m/s along a path tipped at a 13° angle above the horizontal until reaching the surface.
BIO
    a. What was the horizontal distance between the shark's starting and ending positions?
    b. What was the total distance that the shark swam?
    c. How much time did this motion take?

74. ||| Starting from its nest, an eagle flies at constant speed for 3.0 min due east, then 4.0 min due north. From there the eagle flies directly to its nest at the same speed. How long is the eagle in the air?

75. ||| John walks 1.00 km north, then turns right and walks 1.00 km east. His speed is 1.50 m/s during the entire stroll.
    a. What is the magnitude of his displacement, from beginning to end?
    b. If Jane starts at the same time and place as John, but walks in a straight line to the endpoint of John's stroll, at what speed should she walk to arrive at the endpoint just when John does?

MCAT-Style Passage Problems

Growth Speed

The images of trees in Figure P1.76 come from a catalog advertising fast-growing trees. If we mark the position of the top of the tree in the successive years, as shown in the graph in the figure, we obtain a motion diagram much like ones we have seen for other kinds of motion. The motion isn't steady, of course. In some months the tree grows rapidly; in other months, quite slowly. We can see, though, that the average speed of growth is fairly constant for the first few years.

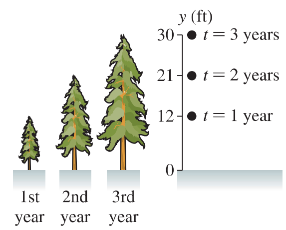


FIGURE P1.76

76. | What is the tree's speed of growth, in feet per year, from $t = 1$ yr to $t = 3$ yr?
    A. 12 ft/yr B. 9 ft/yr
    C. 6 ft/yr D. 3 ft/yr

77. | What is this speed in m/s?
    A. 9×10^{-8} m/s B. 3×10^{-9} m/s
    C. 5×10^{-6} m/s D. 2×10^{-6} m/s

78. | At the end of year 3, a rope is tied to the very top of the tree to steady it. This rope is staked into the ground 15 feet away from the tree. What angle does the rope make with the ground?
    A. 63° B. 60°
    C. 30° D. 27°