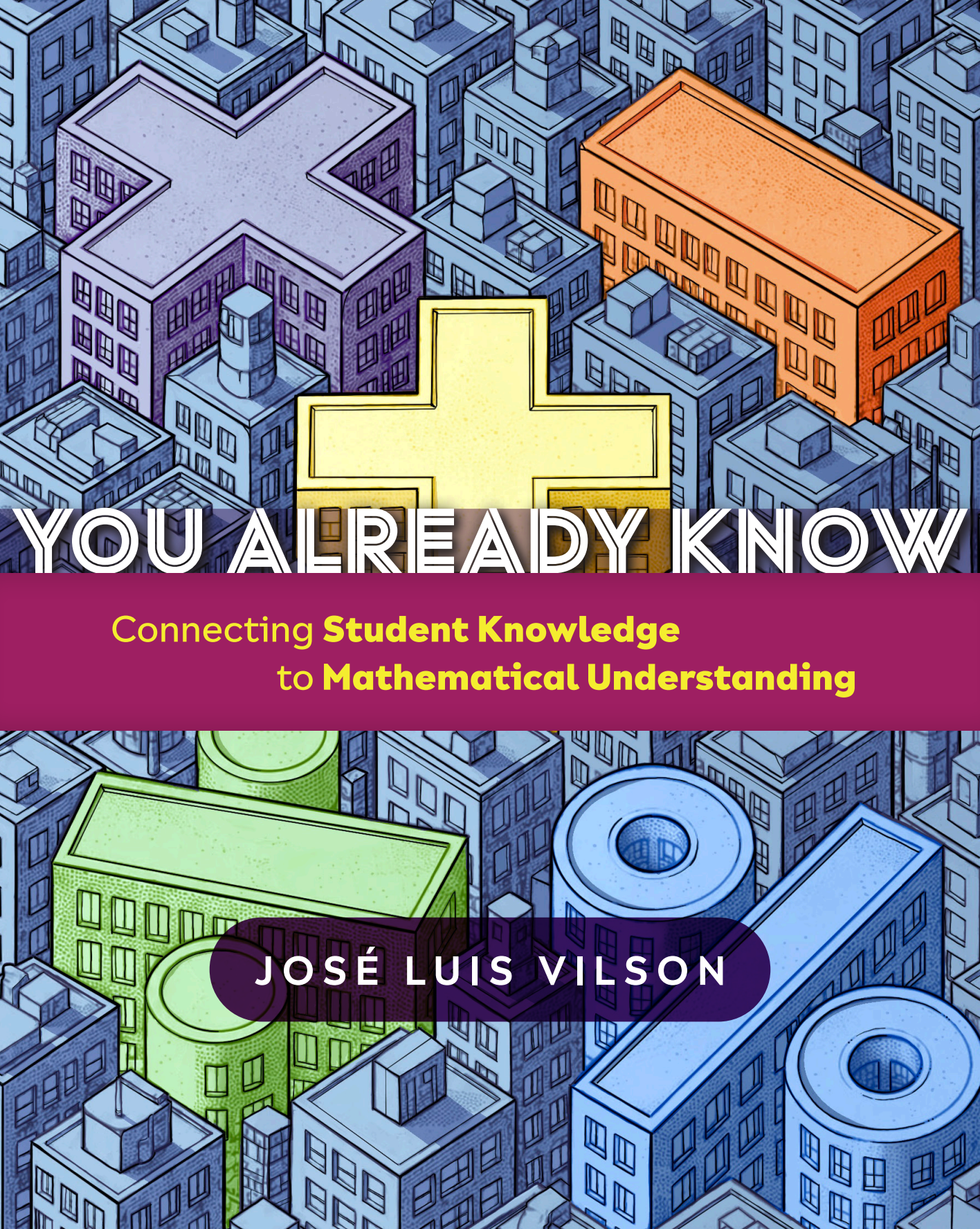


YOU ALREADY KNOW

Connecting **Student Knowledge** to **Mathematical Understanding**

JOSÉ LUIS VILSON

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Acknowledgments:
Author photo courtesy of Ernesto Guadalupe.
"The Five-Step Curricular Process," Robert P. Moses, The Algebra Project, Inc.

Library of Congress Control Number: 2026945530

Printed in the United States on acid-free paper

ISBN-13: 978-0-358-97306-5

r8.26

PO #4500938999

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To those who've moved us along our coordinate planes in an asymptotic relationship to freedom

To all the teachers and other educators who told me I'd amount to something

To all the students I taught a few blocks away from the 200th St. stop

And to the two who make my three a magic number . . .

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ACKNOWLEDGMENTS

This time, I'd like to thank my family first. LuzMaria, I couldn't do what I get to do without you, not just as my wife, but as a brilliant mind and heart in your own right. Alejandro, my son and favorite student, thank you for your love and righteousness. I couldn't be the man I am without you making me more courageous at every step. Saludos a Porfiria. Gracias por ayudarnos a criar a nuestro hijo, pero también por su apoyo a Luz y a mí. Special shout-out to all my family and friends, particularly my nieces and nephews official or unofficial: Renzo and Rumi, Brandon, Josiah, Gianna, Jaiden, Miles, Liam, and Maya, and all the children in our extended family trees. And love to my mother, Milagros Berroa, brother, Ralf Balbi Jr., all my cousins, in-laws, and all the parents and guardians of the children in our care.

I also want to thank the wonderful team at Heinemann. Louisa Irele, thank you for the patience, care, and attention to my wild ideas in my work. I knew you were the right editor to bring this wonderful work to. We withstood a whole pandemic and other calamities and still pulled it all together. You, too, are a math person! Caritza Berlioz, thank you for your time, knowledge, and the tres leches cake. Your attention to nudging me toward completing was a godsend. Thanks to the rest of the Heinemann team: Catrina Swasey, Sarah Fournier, Roderick Spelman, Ellery Harvey,

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David Stockdale, Sonja Chapman, Elizabeth Silvis, Amanda Nieman, Tara Baliat, Lesley Sullivan, Straive, and Lana Barnes.

Throughout my years of service, I scribbled a few ideas down about what this imaginary math book could be. Yet, after a few false starts, I was drawn to the idea that I had something to say as scripted curriculum and narrower forms of math education became popular again. Luckily, then NCTM president Robert Berry invited me, a practicing teacher, to deliver the Iris M. Carl Equity Address for the NCTM Annual Meeting in San Diego, California. His belief that a current (at the time) classroom teacher could provide some insight that the field was missing felt transformative. His work, across multiple spaces, to get it right for our kids most in need of this type of math needs more attention. Thank you, Dr. Berry.

Thank you, Deborah Ball, for pulling me into your public works, particularly the AERA Annual Meeting where I met Bob Moses for the first and only time. Your scholarship and conversations have pushed me forward. Similarly, thank you to the many scholars of various sorts who, through your scholarship and/or conversations with me, carried this book home: Imani Goffney, Pedro Noguera, Linda Darling-Hammond, Prudence Carter, Lisa Delpit, Rochelle Gutierrez, Cathery Yeh, and so many others. I also want to thank people whose conversations, pedagogy, and pieces throughout the years have pushed my thinking about math and/or pedagogy: Annie Fetter, Sara Rezvi, Wendy Menard (RIP), Sendy Keenan, Shaniece Mosley, Lynn Shon, Kristopher Childs, Juliana Urtubey, Akil Bello, Christopher Danielson, Jonathan Halabi, Tina Cardone, Xinyu Pan, Porscha Burke, Marian Dingle, hema khodai, Betina Hsieh, Tyrone Martinez-Black, Guy Brandenburg, Danny Rice and so many others along the way!

I'd like to thank the thousand-plus students (and their families) who I taught at Intermediate School 52, now known as the Harold O. Levy School in Washington Heights, New York City. (Note: I spent fifteen years in the school, but with all the different changes over the years, it felt like nine different schools.) It's cliché to say that I learned more from you than you did from me, but I really couldn't have been the person I am now without having had each of you in my classroom. I did what I could, however

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imperfectly. I also want to thank my peers and administrators, particularly those who welcomed me into their classrooms just to observe their brilliance, even when we didn't share the same content area. I also thank the many families throughout the years that gave me the opportunity to tutor or speak to their children about math in any capacity.

At least twice a week, I see students on the crosstown bus doing their math homework, either digitally or on paper. Typically, the young people can hear babies crying, strangers arguing, horns blaring, passengers rushing in and out, and the cacophony of New York City during early rush hour. Nevertheless, the students lose themselves in their mini problem sets, everything from a worksheet of multiplication fill-in-the-blanks to derivatives and all points in between. I watch with curiosity that one might disrupt their alone time to tend to this labor, but it reminded me of two dynamics. One, it might offer a student the opportunity to focus intentionally. Secondly, these problems have *solutions* of some nature and, given their earnest attempts to solve the problems, they feel a responsibility to do just that. I wrote this book out of a deep sense of responsibility to the profession, to the work, and most importantly, to the hundreds of children who I had the honor of teaching. And even the ones I didn't, particularly those on public transit with me. Thank you.

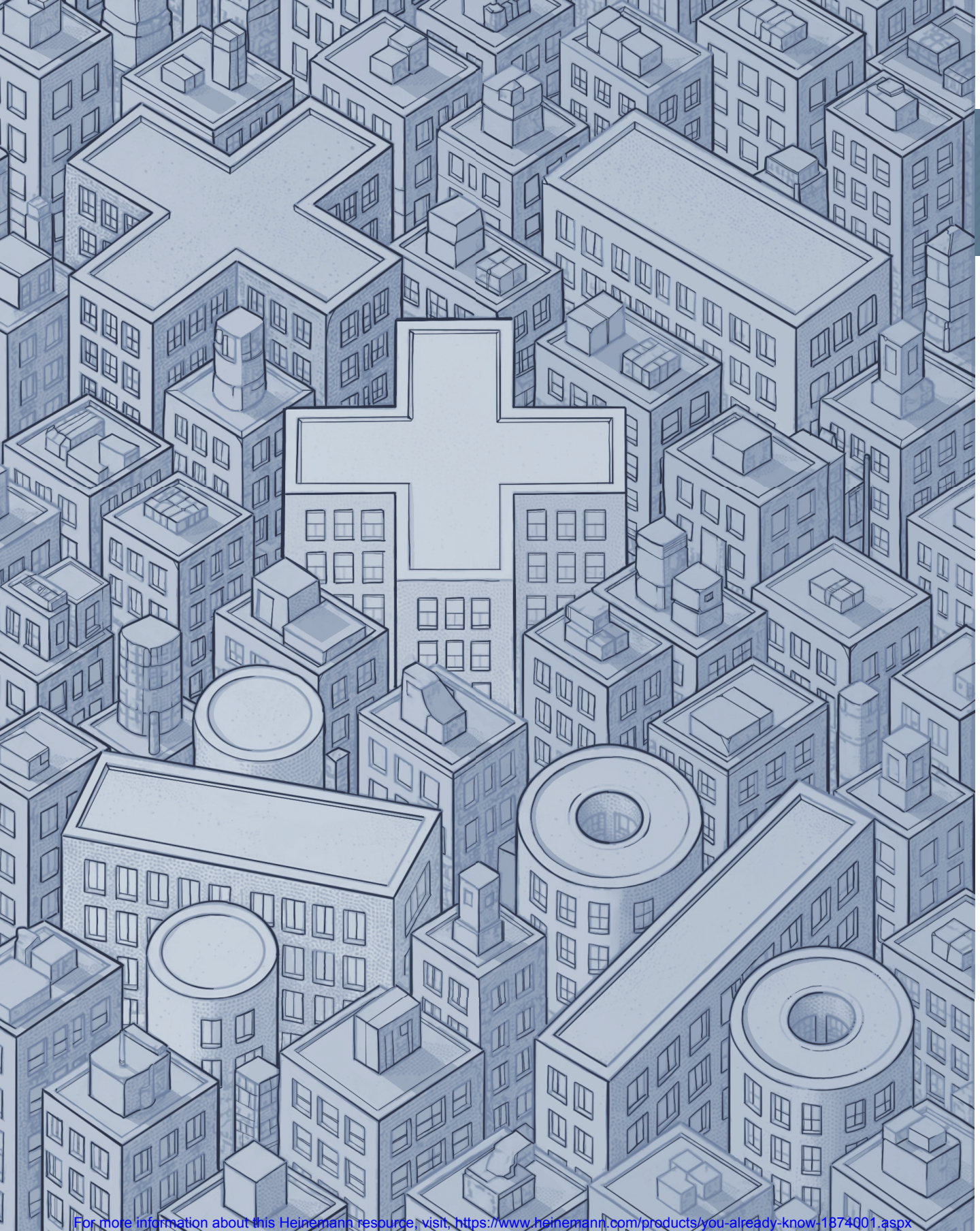
Special shouts to my EduColor collective with special love for the working group: Yamil Baez, Estefania Hereira, Trisha Wright, and the board of directors: Christopher Thinnes, Theresa Collins, Chris Lehmann, James E. Ford, Zakiya Jackson, Crystal Cubbage, Alex Rias, and Caroline Blackwell. Thank you to my dissertation committee (yes, I wrote a whole dissertation at around the same time as this book): Drs. Amy Stuart Wells, Ansley Erickson, Yolanda Sealey-Ruiz, Marcelle Mentor, and Terrenda Corisa White. Thanks to Na Lor for your guidance in my process as well. To Tori Marra, Sandy Ion, Tara Berthier, and the good folks at Outspoken Agency, thank you for representing me and putting me in the best position possible to do my work. Special shout-outs to people and collectives that have built community with me toward greater goals over the last few years: Math for America, the New Yorkers for Racially Just Public Schools

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coalition, the HEAL Collaborative team, the generative (see what I did there?) New America team, the National Education Association, National Board for Professional Teaching Standards, Edutopia, PowerMyLearning, Just Equations, Creators for NYC, and the Mayor's Office of New York City (yes, including Mayor Zohran Kwame Mamdani himself). Also, thank you to the 2026 New York Knicks for winning a championship as I'm writing this. It wasn't just the ways you all won, but also how you've healed the inner teen frustrated with decades of watching the team we love fail time and again. Thanks to the New York Liberty for the Big W in 2024, too. Seafoam looks good on us! If I forgot anyone that was influential in helping me write this book, blame my mind, not my heart.

As I scratched my mind for words and concepts, I'd take a step back and close my eyes to tap into ancestral and contemporary sources to converse with, that this volume might help leave some people's worlds better for having read it. This book is a conversation with them and you. While I wrote this book, I met the specter of existential crises with a flourishing hope, a sentiment I carry with me. I haven't left the classroom; the world is the classroom now.

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INTRODUCTION

Are We Really All Math People?

As I write this, millions marvel at the launch of Artemis II, NASA's latest mission that sent a team of astronauts to circle the moon and return home in a span of ten days. The intense preparation, technical acumen, and physical ardor of becoming an astronaut often gets lost in the awesome spectacle of space travel. In the *New York Times* article "They're Going to the Moon and They Know Not Everyone Is with Them," Timothy Bella (2026) laments the rancor amid America and the world. As physical, technical, and ideological conflicts wither societal trust domestically and internationally, the writer wonders whether American society can collectively appreciate the achievement of Artemis II or whether our society is too fractured to do so.

To that end, I thought about the moments in my classroom where my school, at times, didn't feel possible, much less space travel beyond us. Since the start of my teaching career, the prospect of school closure was openly discussed. When I began teaching, I quickly saw the value in observing my veteran colleagues, which, along with my graduate courses at City College of New York, helped me build a tapestry of best practices in my own lessons. I also realized that there are many schools of thought about how teachers

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should teach. It's difficult to agree on the best techniques and practices without first coming to some consensus about our intentions in American classrooms. Schools and districts display their mission and vision statements, but people struggle to describe those values in practice. With so many disparate approaches to teaching, it's important to recognize that teachers are products of the teaching they've witnessed. We not only pick up habits from when we were students ourselves but also adjust our pedagogies to align with teachers in our immediate surroundings. Our practice is constantly changing in response to the students and other educators we encounter. However, I believe that a lesson plan always boils down to three basic parts: 1) the content, 2) how you introduce it, and 3) how you plan to share that knowledge. It took me some time as a math teacher to come to this realization, but when I did, it changed much of how I developed lessons and how I talked to each and every learner. At the same time, whether I could name it or not, one thing has stayed with me for all my years of teaching: my deep belief that *all* students need people (along with supportive systems) to believe they have gifts to unearth.

In 2004, I felt a deep sense of gratitude for earning a degree in computer science from a rigorous program at Syracuse University. But I was also frustrated with how ill-prepared I had been for my studies. Few of my peers who started out with me made it to graduation. It became even more apparent as much of the news highlighted high school dropout rates and how they were disproportionately affecting Black and Latino students. Thus, this lack of preparation felt less like individual choices and more like structural obstacles to overcome.

My interest was in becoming part of the solution. Math was often the subject that made students feel like they had no business in high school, much less going on to college. As I pivoted to K–12 teaching, I realized that to make an impact on high school dropout rates, I'd have to catch students in middle school. My teacher prep program, NYC Teaching Fellows, offered me a few choices, but once they recognized that I spoke Spanish fluently (don't ask my mother or mother-in-law, though, thanks), I was steered toward a middle school in Washington Heights, New York City. I had eight weeks of teacher training: a two-week seventh-to-twelfth-grade intensive math class

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to prepare incoming teachers for a math content specialist exam and another six weeks of teaching summer school paired with college courses.

None of that prepared me for being alone with three classes of thirty students with various academic and socioemotional needs, who, at any moment, given the slightest whiff of insecurity, could stage a rebellion. Or, at least that's what I would tell myself at night as I completed my lesson plans. The tough and solemn exterior I presented to my students belied the deeply conflicted ways I thought about teaching them.

Despite a decent first few months of teaching, I always felt like I was one lesson away from failing at the whole enterprise. Around November, I started teaching a unit on ratios. I remember looking at the standards and thinking, *Yeah, my students should pick this up quickly because they've already been exposed to ratios over the last few years.* I know what you're thinking, but I had even more naiveté to come. For that first lesson, I offered a refresher on ratios and let students get to work. The class had a strong rapport, so they generally worked together to problem solve.

However, one of the students, Sofia, came up to me and asked me to explain ratios. The exchange went something like this:

> Sofia: Mr. Vilson, I don't get this.
>
> Me: What do you mean by that?
>
> Sofia: I don't get it.

Here, I leaned into what I and so many other teachers in my school felt appropriate. If you haven't heard the phrase "three before me," it's the idea that teachers delegate expertise to students so students can resolve problems among themselves.

> Me: Did you try asking other students?
>
> Sofia: No.
>
> Me: Why not?
>
> Sofia: Because.
>
> Me: Yes?
>
> Sofia: I don't need them to know that I don't know.

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Even when we think we've done a good job with our lessons and that we've checked for understanding, it still might not give students the confidence they need to work things out with their peers. Even if they've spent multiple grades with each other, I understood viscerally that many of our classrooms aren't set up for students to be academically vulnerable with each other. But as a newbie teacher, I could only muster:

ME: But did you try? So why are you here? Fine, how can I help you?

SOFIA: OK, so I don't get it.

ME: What about it?

SOFIA: I don't know?

ME: Do you know what a ratio is?

SOFIA: I guess, but no.

ME: What is it, then?

SOFIA: A relationship between two numbers, like a fraction or something, I don't know.

In that moment, I saw she knew more than she initially let on. The "I don't know" was less about her conceptual knowledge and more about the certainty she had of her ideas. But this was an opportunity to build on what she already knew. I pressed on:

ME: OK, let's try a comparison.

SOFIA: OK.

ME: So if you have two apples and five oranges, what's the ratio of apples to oranges?

SOFIA: I don't know.

ME: OK, I'm going to draw it for you and you tell me if it works.

I quickly drew two weird-looking apples and five weird-looking oranges (see Figure I.1).

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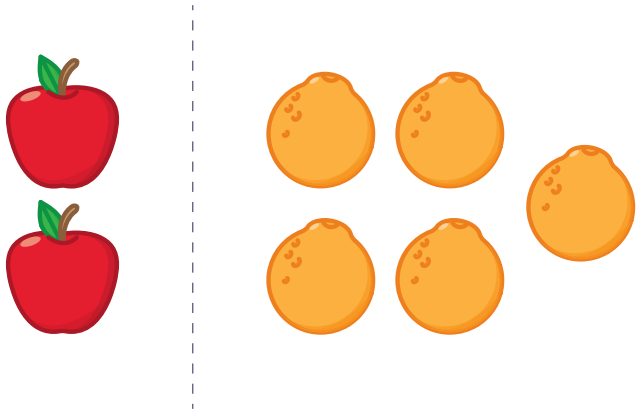


Figure I.1 Apples to Oranges Ratio Drawing

Sofia: Nope.

Me: OK, imagine you have two apples in one hand and five oranges in another, does that work?

Sofia: Nope.

This went on for another three minutes, at which point other students got frustrated that I was only attending to Sofia. Suddenly, I asked, "Is the confusion about fruit?"

Sofia: Maybe?

Me: So here's the thing, Sofia. We're not adding fruit together because we know apples and oranges are fruit. We're asking you to compare one type of fruit that you have to another type of fruit, right?

Sofia: Right.

Me: So for now, if I wanted to represent the apples and the oranges in a way that makes that clear, how would I do that?

Sofia: (looking at my drawing again and looking back at me with an eye roll) Mr. Vilson, why didn't you just say that?

She runs off and starts to piece together her work. The misconception was less about the drawing or the fruits, but the level of representation.

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I probably rolled my eyes back a bit and laughed, but it taught me two things. First, I'm glad to have had the deep content knowledge to pull out six different ways of explaining the same concept to middle schoolers. Second, foundational to the conversations about pedagogy, curriculum, and assessment is a set of virtues and ways of being that guide our behaviors and interactions with students in how they do their work. It would have been easy to give up after Sofia's rapid-fire I-don't-knows, which I could have interpreted as a tool she used to escape from an undesired academic challenge. But the reward for my patience was that Sofia, from that moment on, could answer a wide set of problems involving ratios, including fractions and other division-based representations, in multiple ways. As I reflect on my practice now, the delayed gratification—a feeling all too familiar for educators—held true for so many other students like Sofia.

I now know that a crucial part of my work as a math teacher isn't teaching the math itself. It's helping students *believe* that they can do math. There was a time when I was focused on sorting out the tension between different pedagogical stances. Was direct teaching or exploratory teaching the most effective approach? Which of these methods proved more effective at achievement on standardized tests? Is the standardized test the most important measure of attending to student learning? Do students aspire to become better-informed citizens or better test takers? Every few months, a new article came out urging teachers to change their practice with unbridled confidence but without a clear rationale other than "because someone more powerful than you told you so." In some ways, administrators, education professors, and other professional development providers only left me with more questions about my internal arguments on best math teaching practices.

We were also in an era of market-based solutions that paved the way for schools, particularly those with more students of color in impoverished conditions, to go from a pedagogical trial-and-error approach to a trial-and-failure approach. If students failed, a storm of accountability would rain on teachers individually and collectively. Education policy based on fear of reprisal undermined the ostensibly altruistic motives of a slogan like "No Child Left Behind." It eroded the reasons that led someone like me to choose to go to a school that was under resourced, over scrutinized,

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and consistently threatened with closure. The country wasn't ready for a nuanced conversation about this drive toward punitive accountability, either. In actuality, it was possible to critique teachers who needed the support and motivation to improve without belittling the entire teaching profession for doing much more with increasingly *less*.

If that's the message teachers across the nation felt, the students in schools like the ones where I taught felt this tension exponentially. My students often came to my classroom with a depressed level of confidence in their mathematical abilities. They'd initially scoff at the idea that they were good at math. The compliant students would plod through the sets of problems while the more vocal students would cry, "Why are we doing this anyway?" In my first years, I came up with some rationales, ranging from wholesome to ridiculous:

Because math is great.

This is going to help you later in life!

It's so much fun when you try it!

You wanna get robbed?

Admittedly, I had some reflecting to do. Of course, I would never advocate on behalf of larcenists—our school systems did enough of this, particularly stealing students' academic identity formation. Even the honors students often received messages from multiple sides about the limits to their intellectual capabilities. They had teachers and principals who believed they "wouldn't amount to nothin'" when they were just trying to work toward making some money in the future to feed their families. Their families—often victims of inadequate schooling themselves—would excuse their children's lack of mathematical aptitude with "Well, I wasn't a math person either, so that's fine." Society sent the same messages about math, too. My students didn't see themselves in the stereotypical nerdy math students shown in mass media. But I also argue that white and Asian students who saw these stereotypes got hurt by them as well. These academic myths kept many people from understanding why math mattered until much later in life.

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A Special Shout-Out to Bob Moses

What does it mean to move math from a framework of access to a framework of self-actualization? Of the many books I studied for my own professional development, I found Bob Moses and Charles E. Cobb's *Radical Equations: Civil Rights from Mississippi to the Algebra Project* (2002) most remarkable. The book details Moses' life from New York City teacher to Mississippi Freedom Rider to his well-regarded Algebra Project. A giant in two fields (civil rights and mathematics education), he doesn't get enough credit for the ways he saw the present and future for students who often didn't see themselves in the math that was already theirs. A New Yorker who started his career teaching math in the Bronx, Moses was inspired by civil rights activist Ella Baker to organize for voting rights as part of the Student Nonviolent Coordinating Committee (SNCC) in Mississippi. In one of his more famous quotes, he asserts that "Leadership is there in the people. You don't have to worry about where your leaders are, how are we going to get some leaders . . . If you go out and work with your people, then the leadership will emerge" (SNCC Legacy Project and Duke University n.d.). He went on to form the Algebra Project, an organization that uses a combination of community organizing and student agency to bolster mathematics education for students, particularly in schools with a lack of access to higher-order math.

In *Radical Equations*, he discusses how he and others applied the lessons he garnered in organizing for Black voters to teaching children math through a community-based approach. Among the many gems in this phenomenal work, the book posits:

> So algebra, once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, now is the gatekeeper for citizenship; and people who don't have it are like the people who couldn't read and write in the industrial age. But because of how access to—the learning of—algebra was organized in the industrial era, its place in

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society under the old jurisdiction, it has become not a barrier to college entrance, but a barrier to citizenship. (Moses and Cobb 2002, 14)

While this book was published in 2002, the message feels more relevant than ever. With the technological advances we're seeing now and the ways math gets used to manipulate everything from economics to public opinion, it matters that our students learn math well. Like Moses, I don't believe that teaching math justly and teaching math well are mutually exclusive. If anything, we, as teachers, know the stakes of treating these endeavors as mutually inclusive projects. All of our students deserve the breadth of knowledge we have to offer, and artificial intelligence can't offer that promise.

Moses' organization, the Algebra Project, still stands and provides a framework (see Figure I.2) for students to integrate their experiences into

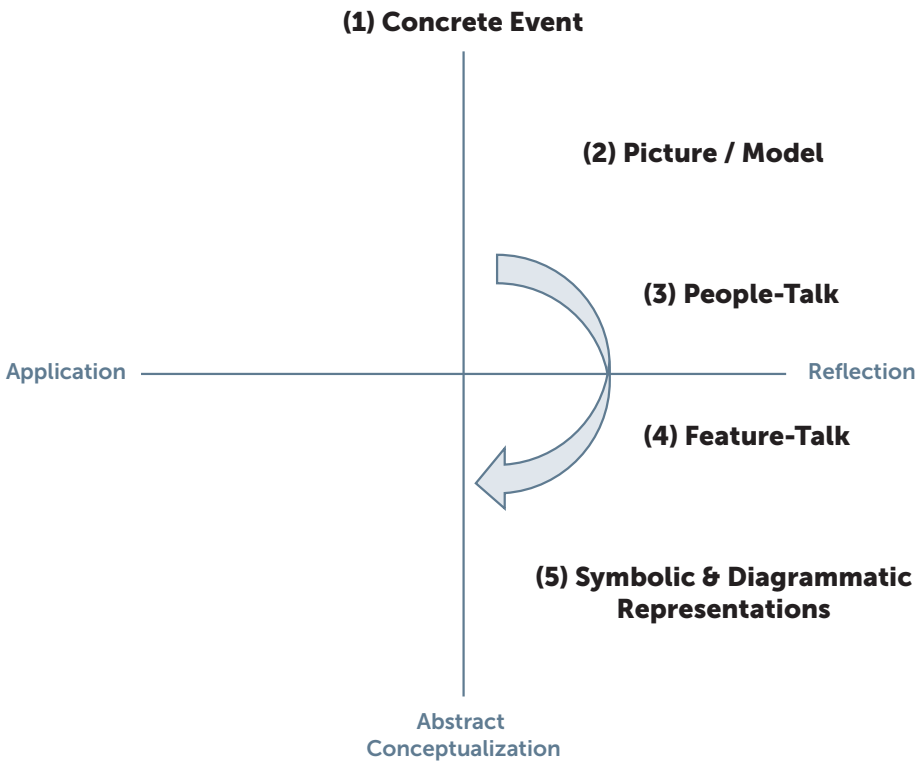


Figure I.2 The Five-Step Curricular Process

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the math they must learn. The framework starts with a concrete and shared experience (their most popular example is a ride on a bus or train), followed by modeling, partner and class dialogue, and then symbolic/algebraic representation. Even though I only had the opportunity to meet Moses in person once and never attended a professional development from the Algebra Project, my work stands on his and so many others' shoulders.

In 2021 Bob Moses and I were supposed to be speaking at the same virtual event for the National Council of Teachers of Mathematics. I opened the event and paid homage to his work, noting that in one of his best teachings, he said that the leadership we seek for any community is already in the community. We just need to listen. The closing panel was to feature Moses and his colleagues in the work, including actor Danny Glover. Unfortunately, we found out that he was not feeling well, and, a few months later, passed away.

With advancements in artificial intelligence and shifts in labor, the easy argument to make is that we may not really need students to attain fluency in trigonometry, calculus, and other math. Yet, what Moses and so many interested citizens have pointed to is, regardless of students' career-based aspirations, math attainment shouldn't be limited to post-secondary financial prospects. To this I would add that the presence and relevance of mathematics in people's lives remains foundational to the rights and liberties of humanity more generally.

Skills including the ability to interpret patterns, verify figures, and recognize shapes at higher levels ought to raise our social consciousness about the relevance and urgency of mathematical literacy. At the same time, what counts as *math* has been under intense debate since at least 1900. We've seen various labels for the types of math we teach students: New Math, Common Core Math, Basic Math. Many publishers have also

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sought to brand their type of curriculum as a special math that attends to student achievement and learning. As new frameworks for how we discuss the content emerge and change, we've had few opportunities to discuss the values that undergird the content and processes that compose what we call mathematical learning.

Teaching Math from the Knowledge Within

The myriad ways that we discuss math doesn't account for how humans already have math within them. Taking a more culturally responsive approach, history tells us that several civilizations have had sophisticated concepts for enumeration, measurement, and structure since time immemorial. Dissidents who decry how advocates have "inserted culture" into math teaching negate how culture has already informed the types of math we learn, how we learn math, and who gets to learn it. The narrowing of math to a specialty as opposed to a broader body of knowledge has given many people the idea that the math isn't already theirs.

Thus, I argue for tapping into the math that students already know. In this book, I will outline ways for teachers and parents to rethink how they teach math using some time-tested processes from my classroom. Years ago, Annie Fetter (2011) of the Math Forum developed a protocol named "What Do You Notice? What Do You Wonder?" As an opening activity, teachers would present an item, whether it's a picture of a sky or a problem the class hasn't solved yet, and have students share their observations. The "notice" is a naming of the present: to see characteristics that they may immediately recognize. The "wonder" is a spark for future considerations: to develop questions that may have more possibilities, intended or not.

I would also add, "What do you already know?" This is a call to the past: to tap into what students may already have an awareness of even if they can't outright name it. Thus, the five principles I will outline in this book are shown in Figure I.3.

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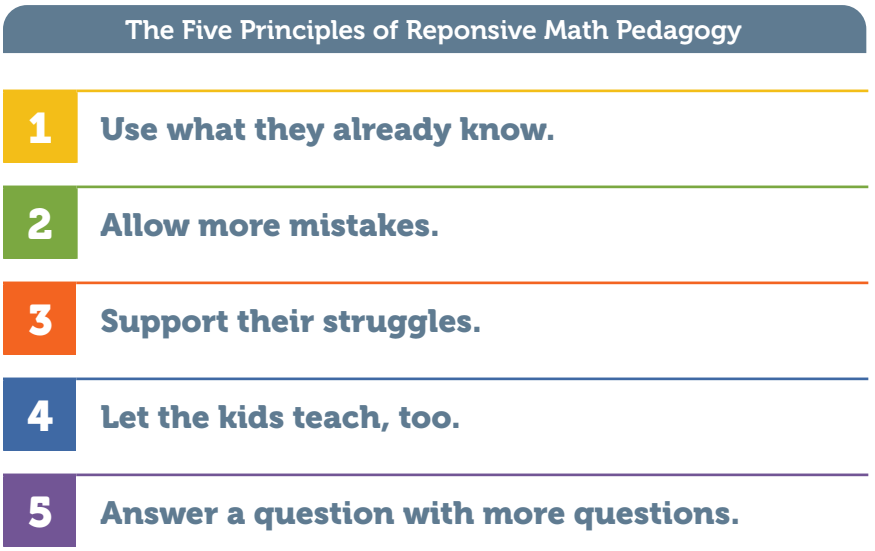


Figure I.3 The Five Principles of Reponsive Math Pedagogy

While there is research to support the application of these principles in other content areas, few books have actually shown what that looks like in a math classroom. Through stories and solid research, this book will solidify a transformative way of looking at math pedagogy in the twenty-first century. Although these principles overlap in their application and relevance, each of the five chapters will bring a fresh voice to the five principles and demonstrate how they look in the classroom. In Chapter 1, I frame the importance of using what students already know. In this chapter, I discuss percents and slopes. In Chapter 2, I model the transformative power of allowing more mistakes. In this chapter, I elevate exponents, scientific notation, and formulas more generally. In Chapter 3, I demonstrate how to support students' struggles. Here, I bring together a more robust thought process about operations with fractions as well as functions. In Chapter 4, I provide a model for letting students teach, too. The content here includes equations, angles, and making sense of bivariate data. In Chapter 5, I make a case for answering students' questions with more questions. I build bridges to algebra including operations with polynomials. Throughout, I'll share

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activities and sample dialogue from various classrooms that highlight good teaching practices. While this book pulls in Common Core content and practice standards, as well as culturally responsive math teaching, it also seeks to position math teachers as expert practitioners rather than bystanders in developing content knowledge.

The Influence of US Policies and Culture on Math Education

"Formal education in colonial America was primarily limited to teaching literacy and training the elite college-bound in the classics." (Furr 1996). If anything, schools saw higher-order math as the province of the college-bound elite, whereas everyone else would receive simple skills to enhance the labor market at best. For example, in the nineteenth century, mathematics teaching for girls and women was focused mainly on arithmetic (d'Enfert 2025). While people rightly point to the Reagan-era *A Nation at Risk* report (Gardner 1983) for pivoting our country to standards-based reforms, the National Defense Education Act (NDEA) of 1958 provided funding for math, science, and foreign language for our K–12 schools. This bill came after the launching of *Sputnik 1* by the Soviet Union in 1957 (Clowse 1977) and came with a series of hearings and debates about America's progress in the Cold War.

But, as we've seen time and again, the expansion of math learning came with ideas of standardized testing as a neutral testing instrument. Intense debates over the role of testing have been around for decades. In Jim Wynter Porter's (2018) historical analysis and reflections on the impact of the NDEA, he notes how Cold War fears ran concurrently with an emphasis on the individualization of intelligence and race-based testing. Where the transformative *Brown v. Board of Education* decision of 1954 broke ground on the federal role in enforcing equity in education and other social spheres, Porter argued that the NDEA proliferated notions of *who*

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could be educated and to what degree with an intense focus on "gifted" children. In other words, the expansion of mathematics education wasn't just a "national security" issue, but more broadly a cultural issue related to citizenship and national aspirations.

The fact that this history dovetails well with the history of the civil rights movement, an era of groundbreaking educational opportunity, tells us how much our work as math educators matters. While the language of NDEA and subsequent policies like it is race-neutral, the effects left large communities of children who ought to have been the beneficiaries of equity reforms in the dust. As we know from Berry, Ellis, and Hughes' (2014) work, the waves of education reforms since the 1940s and '50s to expand mathematics education did not reach historically marginalized children, particularly Black children. According to Berry, Ellis, and Hughes, Black children are more likely to experience:

A. reduced access to advanced mathematics courses that prepare them for higher education and improved career options;

B. routine exposure to activities that focus primarily on rote, decontextualized learning through drills and practice with little to no engagement in activities that promote reasoning and position mathematics as a tool to analyze social and economic issues, critique power dynamics, and build advocacy; and

C. less access to qualified teachers of mathematics who both understand mathematics deeply and understand their students' cultural and community context deeply in order to give learners access to mathematical knowledge.

As much of the country continues to take a deficit framework about historically marginalized children, especially in the age of standardized testing, it's important to take a step back and recognize the ways that culture necessarily informs school policies and what we abide by. By the 1980s, *A Nation at Risk*, one of the most important reports on the state of education

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in the past fifty years, paved the way for a bipartisan effort to further raise standards, but did so under the assertion that American schools were failing. By 2000, the urgency embedded in that message allowed for the No Child Left Behind Act. This act pushed policies that concretized the idea of testing as *the* mechanism for raising educational expectations. Where the triumvirate of high-quality teaching centers curriculum, pedagogy, and assessment, high-stakes standardized testing took an outsized role in the teaching and learning process. Schools focused much of their energy on narrow and, in many cases, scripted renditions of math and English/language arts with some attention to science and little attention to social studies.

Mathematics education, whether holistic or teacher-directed, places great emphasis on assuring that students see math as a one-way transfer of information instead of a series of understandings developed through true inquiry and mistakes along the way. In the twenty first century, we have to reform our pedagogy to fit the needs of our student population, already connected to a deluge of digital tools that give them a direct answer without the "middleman." Emphasizing the ways and means by which students learn math may help students implicitly understand how math applies to their everyday lives and give them the resources necessary to interpret and analyze implicitly mathematical situations.

Every teacher, new or veteran, wants to find ways to make seemingly abstract and intangible concepts concrete. This book speaks to developing a culture for math classrooms that provides for the five principles, with a specific emphasis on confidence and investigation. Teachers who find ways to pull away from taking over the class develop pathways for student empowerment and ownership. We, as teachers, create intrinsically motivated, self-monitoring learners who can do so in community with others.

This book also prompts administrators of all levels to have a clear vision for this newer process of learning. In an environment driven by high-stakes assessments, educators, including principals, classroom pedagogues, and specialists, and students have an expectation that getting things wrong does not bode well for assessment purposes. We can both attend to notions of

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achievement while creating a values-based system of inclusive, generative learning that is accessible. The temptation is to hold steadfast to ways of thinking about schooling that is irresponsive to the moment we're in now. I propose the use of fresh ideas that build bridges from centuries past to a new world. This book gives clear examples for administrators to demonstrate to teachers what accountable talk or formative assessment looks like.

In addition to some of the lessons I've learned from my classroom and through research I'll be highlighting some of the best practices from my colleagues in my school as part of the Common Core Learning Lab here in New York City. As part of the pilot, we've received training from the Institute for Learning, Marilyn Burns, and others.

Although this book contains lessons for students who may already feel confident in their math learning, I gear much of this work to students who need an approach that may unlock this knowledge without losing the connections and depth of the content itself. When we say "high expectations" about students' academic capabilities, we should recognize that none of our students come as blank slates, a basis for the past few decades of education reform. Rather, they come bearing gifts and we have several opportunities to surface them.

I've used these ideas in parallel with multiple curricula. As any veteran teacher knows, administrators and policymakers have an unhelpful habit of changing whole curricula without giving everyone time or space to acclimate to the demands. All the while, they expect immediate and positive results. The values and practices in this book are ones I've used in my fifteen years of teaching math and shared in presentations, workshops, and conversations with other math teachers across the country.

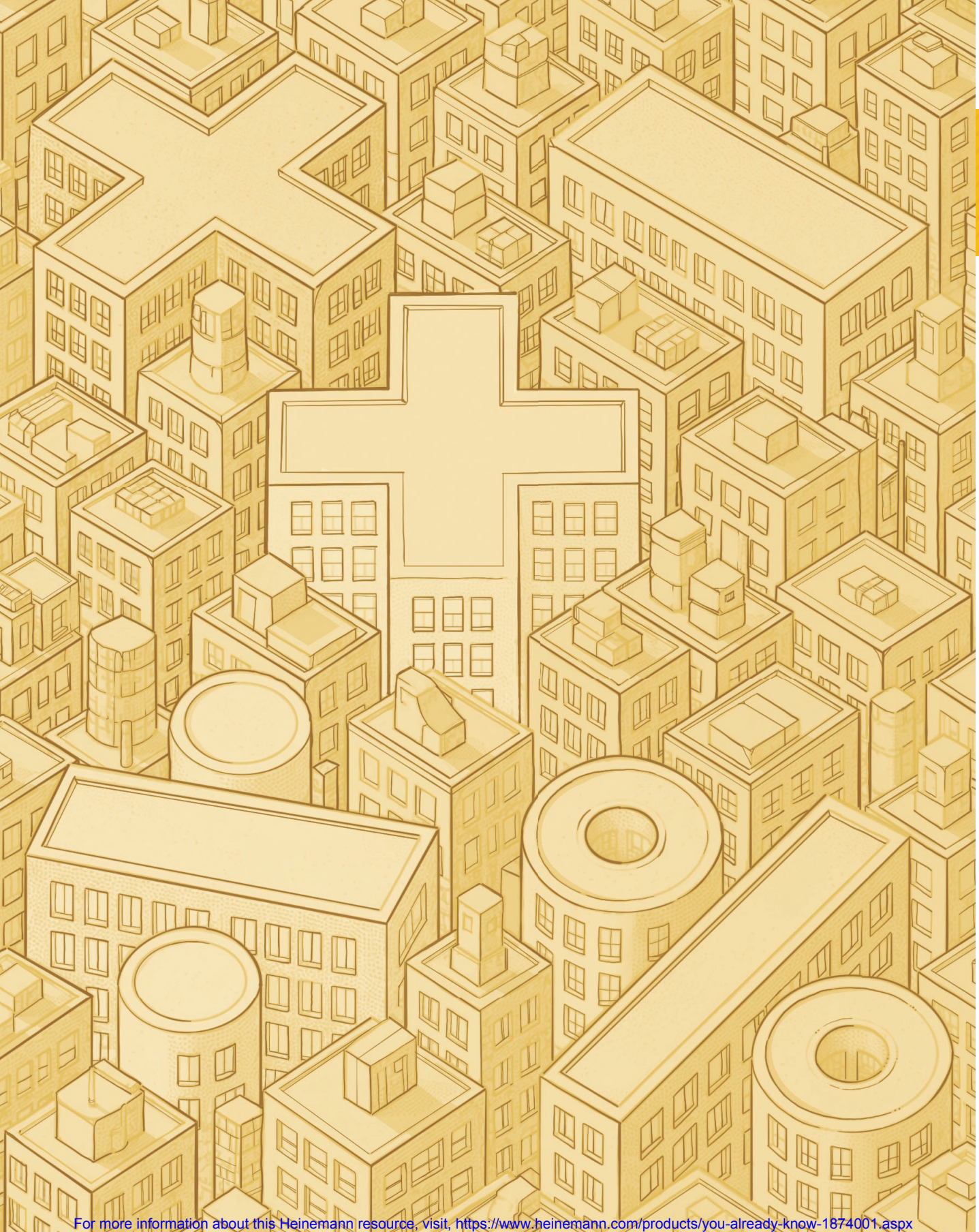
Over time I've tried this with more than a thousand students giving me examples to elevate where I could have done better. So this is also an invitation to rethink math education more generally. Contrary to the ethos we've constructed around math, math comes alive best when done in community and conversation. From my classrooms and those I've visited to

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the collaborative spaces such as Math for America, Twitter Math Camp, and others, math done socially makes for better engagement. In fact, I've often seen adults vociferously argue with each other over some of the simplest math concepts. Why do we not afford similar opportunities for us to learn with students and each other?

The probability that *all* of our students will become astronauts is slim. But if they can't become physicists, mathematicians, and engineers in their own right, they can help engineer a society in which all of our visions for living can be prosperous, joyous, and more compassionate through math. We can start now. Join us.

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CHAPTER 1

Use What Students Already Know

Whenever I do workshops or talks related to math and culture, I usually get a comment along the lines of "That's not part of my work." Despite all evidence to the contrary, educators in STEM-related fields dispute the notion that culture has anything to do with learning math. For most folks, math arose from Greco-Roman geniuses who, in their egalitarian stances, created objective axioms and frameworks for the rest of the world. Everyone else simply saw their wisdom and agreed over the course of the last two millennia that whole societies would incorporate their math in all schools.

On occasion, someone might bring up al-Khwarizmi, the Persian mathematician whose namesake forms the bridge to higher-order math. Even then, it's a cursory mention and otherwise, math is (maths? are?) a content area with traditions we shouldn't interrupt because, among other things, it helps us determine which students have the intellect and wherewithal to go on to higher levels of education and become the idealized leaders of the country and potentially the world. Even if teachers don't believe in everything I've mentioned here, the implicit belief of math as a

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gatekeeper to intellectual spaces persists in our schools, including higher education institutions.

Unfortunately, the stories we tell ourselves about math education limit the expansiveness of the math we've seen since the dawn of human civilization. Societies across time and space have found innumerate ways to count, make shapes, and model abstractly (see Boyer and Merzbach 1991, Stedall and Robson 2011, Struik 1987). Historians have covered at length how multiple civilizations have had concepts of zero, formed different types of calendars based on the rotations of the sun or moon, and used more efficient measurement systems than the United States' customary system of measurement. Other civilizations had already conceptualized what is commonly known as the Pythagorean theorem before Pythagoras (or whoever wrote among Pythagoras' disciples). Evidence of the relationship between the sides of a right triangle and its hypotenuse have been known in Babylon, Egypt, China, and India (Delbert 2021, Rajendran 2015). Many of the ancient artifacts the world reveres came well before the times of Greco-Roman math that Euro-American culture exalts. Understanding that we came to our current notions of math not just through some benevolent intelligentsia, but through social, physical, and economic conflict and conquest helps us make meaning of the role of culture in the way we do math.

Through a brief examination of the history of mathematics education and culture, I know that no history can be fully inclusive. Yet, it can tap into important themes for us to recognize, including:

- None of our math is neutral. Everything from the content we teach to how our students learn math came through conflict and conquest as much as it came from culture and agreement.
- Our experiences with math matter as much as the difficulty and comprehensiveness of the content itself.
- The triangle of teaching (content, pedagogy, and assessment) should happen evenly, none outweighing the other.
- Being culturally responsive/relevant/sustaining doesn't mean we completely disregard everything we've taught.

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- Representation matters, but only if it's done with connection and responsiveness in mind.

At this point, I'm not asking us to discard our current math as there is no such thing as an apolitical math (though I won't squabble if we disband customary units of measurement). I'm asking us to use the math in front of us to pull as many humans into the conversation as possible. I believe our work as mathematicians can point us to a shared humanity, and we as teachers can model that, too.

Representation and Cultural Responsiveness in STEM

A plethora of folks have explored the intersection of math and culture. In the last forty or so years, we've seen important contributions from scholars such as Gloria Ladson-Billings, Geneva Gay, and H. Samy Alim and Django Paris. Their contributions in culturally relevant pedagogy, culturally responsive teaching, and culturally sustaining pedagogy, respectively, have brought potent and useful language to the ways that culturally invested teachers have sought to reflect a more inclusive world in their classrooms, even for teachers who may not have heard of these frameworks. State governments have made efforts over the last couple of decades to empower educators to engage in this work and hold space for educators to learn more about authentic inclusivity across the content areas.

However, despite the best intentions, some calls for cultural responsiveness have given way to shallow and ephemeral notions of diversity. We've seen them all. Textbooks replete with human characters of different shades and phenotypes flooded the market to the delight of people who needed to address and redress the diversity conversation. Even more recent efforts to ban books and censor anti-racist talk fail to differentiate between diversifying the characters in books and ensuring that our ideas of pedagogy, curriculum, and assessment are as inclusive and expansive as possible for our students.

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Ultimately, the deeper work of culturally responsive teaching doesn't set aside students' individual and collective identities. It gets to the heart of why students want to get an education in the first place. One of the reasons I became a teacher was to open more doors for students with similar racial, cultural, and economic backgrounds as me. They deserved to major in STEM fields if they chose to go that route, and, if not, at least have the confidence to use their innate math abilities to move about the world. On the surface, it should be enough that I connected my identities to at least one of theirs. Identity includes class, (dis)ability, gender, sexual orientation, religion, and all the ways our students show up for and with us. However, the relationships, attention to connectivity and relevance, and my belief in their own capacity to learn mattered more than me simply walking into the classroom as their teacher.

Percents

There's a joke out there about how only 25% of people understand percents while the other 80% are just guessing. I never understood it, but I could see how many of my students did.

Prior to arriving in my seventh-grade classroom, the students had several examples of adults using percents without concretizing why that number mattered. When a meteorologist says, "There's a 70% chance of rain," does it mean that there's a good likelihood that it will rain throughout the day, that it will rain for 70% of the day, or that there's a 70% chance of precipitation occurring at any one spot in the area covered by the forecast? Turns out it's the latter of the three, yet the umbrellas and raincoats still spring up on said occasion.

When the principal says a class had 70% attendance, does it mean that 70% of the students showed up 100% of the time, that 100% of the students showed up 70% of the time, or that the number of days present divided by the number of total days in that span and converted to a percentage was equivalent to 70%? Was it accurate or an estimate? If it was a decimal, how does one split a child? (Please don't!) If the threshold for good attendance is

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90%, how does a class meaningfully interpret the goal if they're not given the collective means to do so nor the context for why the attendance is at 70%?

Or when the dean hands a first-year teacher a roster with fifteen kids, and the first-year teacher asks, "Hey, is this the whole roster?" To which the dean says, "Yes, consider yourself lucky." Of course, that number was bound to expand once the first weeks' rosters shook out. In this massive middle school in Washington Heights, a class of fifteen seventh graders turned into thirty-one when two classes collapsed into one. This is not a hypothetical. *I* was that first-year teacher who was told they would have a manageable number of students with various needs and languages including English, Spanish, and Hindi. Suddenly, I had more than double the original class size. While I never got a direct answer about why the classes were being combined, my best guess was that from an administration standpoint, combining the two smaller groups may have made sense. However, my students now had the same needs but also the bitter taste of knowing they were now all crowded in one classroom. As you can imagine, day one's 100% attendance felt a bit different from day three's 100%—by 200%.

Supporting Multiple Learning Approaches

The vast majority of the student body in my school identified as Latinx, specifically Dominican (from the Dominican Republic). While my Spanish was better than conversational, I didn't have the mastery to teach the lesson on percents in Spanish, much less a whole year's worth of lessons in both English and Spanish with fluency. However, what I lacked in linguistic adaptability, I made up for in inquisitiveness and hard work. Thus, I took time to visit as many classrooms as I could, picking up (yes, stealing) teaching techniques I liked and discarding those that didn't align with who I was.

In particular, I appreciated how the teachers who taught in Spanish and English used literacy as an entryway into math. They'd highlight a word problem on their boards using different colors to show parts where the students understood and parts where the students got stuck. Then they'd use think-aloud techniques to show students not *what* to think, but *how* to think through it with the language they already had within them. Instead of simply

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putting an addition problem on the board and directly showing students the procedure to solve it, the teacher put the numbers on the board and asked themselves questions, modeling aloud for the students, and thinking through the problem the way their students might. The students could then help guide the teacher on how to complete the problem. This set an example where the whole class would collectively arrive at an answer. Some of the teachers would have vocabulary words in both English and Spanish, color coded according to the language.

I didn't embrace the value of various learning styles that were commonly touted at the time because in my experience the majority of us are visual learners. (The work of cognitive researchers such as Dan Willingham ultimately affirmed my beliefs.) I did believe that the delivery of content needed to attend to multiple approaches. *That's* a learning style I can get behind.

So, back to my class, where most students couldn't describe what a percent was with clarity and understanding. Meeting them where they were was a powerful disposition to adapt. Whether I was conflicted about teaching math in isolation was immaterial to what I needed to do for my students when it came to percents. The night before I started my unit, I created a basic lesson plan: standards, Do Now (opening activity), a definition, some examples, and then some work for them to try. (I was terrible at closings.) I thought I had my students in mind as I started the lesson that morning. To their credit, the students were doing their part in the schooling process: taking notes, trying to work through the numbers, and generally piecing together what I was trying to convey. But I don't think they were actually learning, and I had strong evidence for that. When I asked my students to tell me what a percent was in their own words, I got a few random answers—many of them involving the number 100. But when I pressed them to tell me what that meant (and, at least implicitly, why we should care), I got no responses. Suddenly, it hit me: my students knew how to do school, but they didn't necessarily know what it meant to fully learn content in the way I would have liked them to learn it. I then stopped my lesson and borrowed a bit from the literacy teachers down the hall.

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Me: OK, so look at the word *percent*. Does anyone see a word they might recognize?

(*A few students scratched their heads, but a few waved their hands enthusiastically.*)

Juan: Cent! I see cent!

Me: OK, tell me more.

Juan: I don't know what you mean, Mr. Vilson.

Me: Well, there's the cent. What does it mean to you?

Juan: *Cent* is *centavo* in Spanish.

(*Some students nodded in agreement. I underlined the* cent *part of the word.*)

Me: So what's a *centavo*?

Kelly: Oh, it means a penny.

Me: OK, what do we do with pennies?

(A few students shared their ideas, just grazing the surface of understanding.)

Me: OK, fine. What can I compare a penny to that'll give me more value?

Kelly: Well, if we collect 100 of these, I can get a dollar.

Me: Oh, so this cent, this penny, if I get one out of a hundred, it's how I know this is a cent, right?

(*The class let out a collective "OHH!"*)

I'm not always a fan of using finances, but I needed a hook at that moment, and I found it. Ultimately, it might have been one of the better moves in this particular situation. The root *cent* is useful not only in math but also in many other contexts. People believe a centipede to have 100 legs (though usually untrue). A centimeter is one hundredth of a meter (always true). A century has 100 years (also always true). A centennial is a 100th

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anniversary. Thus, I'm addressing all the content areas at once, or at least giving students a tool to navigate a common Latin root in all their other classes.

What's more, it led me down a path that ensured I would be more critical and thus more engaged with how the textbooks guide us through the math we had to teach. At the time, scripted lessons were at the precipice of becoming commonplace in the No Child Left Behind era, much to the chagrin of the veteran educators in my math department. While it's important to embrace change, the experienced teachers had a point: scripted lessons don't offer opportunities like the aforementioned dialogue to respond with intuition, to make cultural and personal adjustments, or to disband notions of timing in favor of letting the learning marinate.

The reasons I, not the standards, want students to understand percents comes down to comparison. Generally, if we can find a common point of comparison between two different quantities, we can better analyze the relationship between the two. One hundred is a nice number because it's a multiple of 10, and it's not as small as 10 nor as large as 1,000. We can divide it by 2, 4, 5, 10, 20, 25, and 50, and get some nice estimates when we divide it by 3, 6, and 8. One hundred is also a number that's useful for comparing larger numbers. If 156,382 out of 1,293,201 people in a given city like the color green, that might not mean much, but if I tell someone that about 12 out of 100 people in that same city like the color green, there's a firmer grasp on that statistic. I might also naturally expect that person to reply, "Oh, that's about 12%." Because the percent concept is so prevalent in our society, we need some concept and understanding of it to get by in multiple social contexts.

Percents as a basis for culturally responsive math works given its several entry points, questions to ask, and applications from fourth grade onward. *Percent* has a word akin to it in multiple languages because base 10 is the most common base used in the world, though multiple bases are also worth exploring. (Yes, there are good lessons to write about this. We will explore it later in this book.)

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Using Questions for Informal Assessments

As the lesson proceeded, I pulled in some visual representations, asked students to use manipulatives, and had them draw a square to lay the foundation for multiple representations within my lesson. I then instructed students to work in small groups on a set of problems. Jasmine, finding herself stumped by a problem, exclaimed, "I don't get it." While I appreciated her courage and candor, I also recognized it as an opportunity to help her problem solve. When I hear "I don't get it," I ask three to five questions about the problem that are formulaic enough for the student to see where my thinking is going. The problem for an introductory lesson would usually go like this:

> Joaquin ate 3 of the 4 pieces of candy in his pocket. If he continues that pattern, how many will he have eaten if he has 100 pieces of candy?

This might be hyperbolic if I wasn't also the kid eating too much candy over the course of a year. The table shown in Figure 1.1 provides guidance on formulating questions for your informal assessment. Most of the questions can be asked in any order depending on the needs of your students. What's critical is that, when we ask what the student doesn't get, they don't just get to reply "Just everything!" while waving their hands frantically. While that's a response born of frustration, it doesn't give us entry into authentic learning. The questions here are content-specific, but we can and should have questions like these ready in the back of our minds for on-the-spot informal assessments.

The questions aren't as much for teachers as they're for the students. Most of our assessment methods should give students the ability to ask themselves and each other the same questions. In our everyday lives, we question things all the time; for example, when we're trying to find missing things or when we're trying to build routines. When we model this type of thinking for students—aloud and with mistakes along the way—we break free from notions of education as an exercise in compliance and present it more as a nonlinear marination of processes inside the mind. In other words, the routines can free students and teachers up to do more imaginative thinking and problem-solving.

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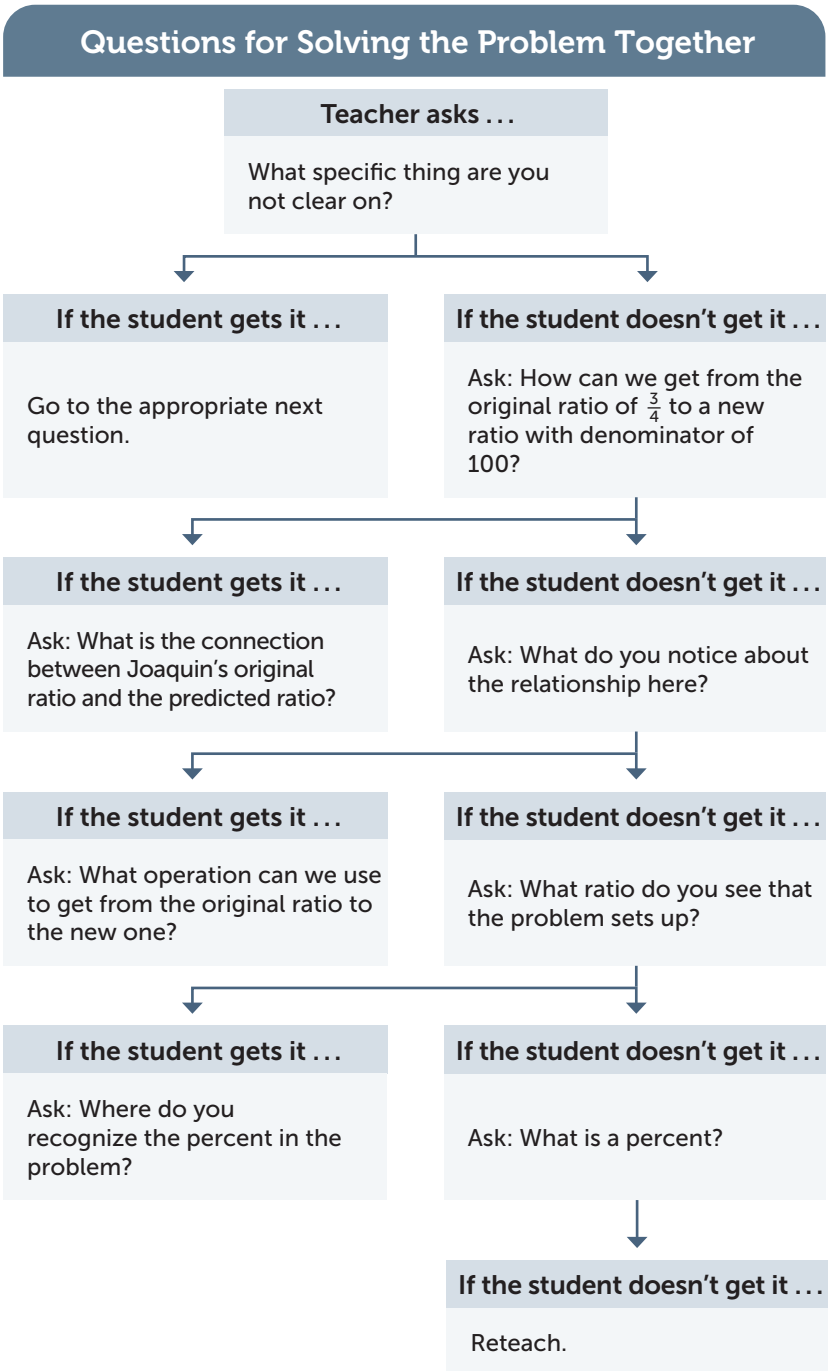


Figure 1.1

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The first time I taught Joaquin's candy problem, a lightning bolt must have struck me mid-lesson. After misplacing my lesson plan, I found myself working with the students on their understandings of the term *percent*. They had been taught that *percent* meant "out of one hundred," but perhaps hadn't yet fully integrated the meaning into their ownership of the concept. In the midst of my excitement, I lost track of time and the bell rang before I did a closing. The next day, I made sure to put the word *percent* on the word wall with *cent* written in a different-color marker before I started the lesson.

An Aside: About Taxes, Tips, and Financial Math

A common refrain on social media is that schools should teach less algebra and more about "real math" like taxes, mortgages, and interest rates. While I would never oppose classes dedicated to financial literacy, it's important for people to understand a few things. First, K–12 schools already have too much content to teach and not enough time. Secondly, taxes don't get taught in schools due to our intentionally complex and ever-changing tax system. If taxes were simple, whole industries would be wiped out. Lastly, the larger concept of taxes lies in our understanding of percents, and every set of math standards attempts to address taxes and interest directly.

Yet and still, when I talk to tax prep agencies, many of them advise LLCs and small businesses to save about 33 percent of their annual income in case they don't get taxes collected up front. This type of math doesn't come from a precise calculation or a mandate from the IRS. It comes from experience and noticing trends. We rely on their accounting expertise to simplify an otherwise complicated set of calculations. Having a good understanding of percentages allows for a level of transparency between members of a society and tax professionals.

To calculate tip amounts, we also use generalizable conventions to help us attend to accuracy. When I go to restaurants with math-inclined people, they look at the receipt and usually throw down a few bucks in a matter of seconds. When I ask them how they know that they've given the right amount of money, they'll respond with:

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"I just look at the sum, move the decimal once, then multiple by two." or

"I check the tax, then double that and add a little more depending on the service." or

"I use a calculator, but I look at the waiter's face to see if that made sense."

The third response is more of a joke, but the first two responses show how estimation is the "real math" that is often the subject of whataboutism. Nowadays, restaurants may either include the tip in the total or show a list of tip calculations on the receipt. Even so, it behooves consumers to check those calculations on their own or at least protect themselves with strong estimations.

About Percentages

Having a strong sense of what percents do helps students solve problems involving percentages. Most textbooks I've seen do a good job explaining why we need to understand percents. Not enough explain the difference between a percent and a percentage. A percent refers strictly to a specific number (12%, 300%, and so on). A percentage is about finding a relationship between two numbers so that you can map that ratio to 100. This nuance requires some number sense, but also some conceptual understanding from the beginning. Consider the following question:

What is 40% of 60?

Rather than focus on the procedural parts of solving a problem like this, I might start off with an abstraction to get the conversation going. Given that this is a sixth-grade math standard, I might start with a discussion-based question:

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You have a full water bottle in your bag. If someone asks you for 25% of your water, are they asking you for:

A. Some of it
B. Half of it
C. All of it

How do you know?

A question like this prompts students to think about the idea of "25%" so they get a smoother transition into percentages. From there, I would build a lesson around the question: What is 40% of 60? The typical approach to this would ask students to put it into a structured equation like:

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

It's tempting because the formula allows students to substitute ("plug in") the parts and voilà! A result. Unfortunately, it also allows students to skip over the core of the math, which doesn't serve them well with understanding the larger concept. Instead, the approach I would take is a series of think-aloud questions:

If I had 60 seats in a show and I wanted 40% of them, is that some of them, half of them, most of them, or all of them?

Hopefully, students get that it's less than half of the seats. I then might ask them to guess a number that makes sense.

Do you think 30 seats would be 40% of 60? Do you think 50 seats makes sense? How about 20?

Of course, some of the students who pick up on what I'm doing may find the exact answer, which is to the class's benefit. But, for those who don't, this type of thinking allows them to check if their answers make sense when we get to the *actual* answer. Guesses and precise answers don't have to contradict each other. When done correctly, they can work hand in hand.

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I also should point out what this means as we attend to language. This attention to relationships and problem structure matters more than the operation initially because word orders may shift in translation, but the "answer" doesn't. Giving students access to the problem means that, regardless of the language, we can still show multiple ways of representing the same question.



If the circle represents 60, does this look 40% filled? Why or why not? Would I need to fill in more? Less?

Figure 1.2 Does This Pie Chart Represent 40% of 60?

This dynamic sets up a way for students to solve these types of problems because they can see the relationship in their minds. In fact, a closing question for this class session would bring us to the prompt: 24 is what percent of 60?

We can ask whether 24 is some, half, most, or all of 60. Given that, at this point, students already solved for the missing number here, we can ask, "Why does 40% make sense here?" Answers like "Because it's some, but not all of the whole," "Because it's not the whole thing," or "Because 40 is less than half of 100 like 24 is less than half of 60" spark conversations. We build the bridge by starting with concepts.

These types of questions build out to when the numbers get more complicated in their explanation. Asking "What is 27% of 45?" is fine numerically, but weird conceptually. The answer is 12.15. If we ask this as

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a water bottle problem, then the answer is more fluid. (Yes, pun intended.) If we ask this as a seat problem, then the answer makes less sense. Context, again, matters. But launching from there:

> If we have 45 seats in a plane, and 27% of them are filled, how many seats are filled?

Middle schoolers will come up with the wildest explanations: "The .15 is a bag/dog/thicker thigh." Whatever their explanation, this type of discussion prompts the difference between the abstract (the math expression) and the concrete (the real-life example). Good mathematicians do both with fluidity.

If fractions are the most discussed topic in mathematics, percentages seem like a close second. People use percents in so many different cases: polls, interest rates, discounts, and the list goes on. Yet, without a good understanding of how percents work, people continually use percents to manipulate or deceive the reader. For instance, if a poll shows that 45% of people disagree with a specific policy, is this 45% of everyone, 45% of people who are likely affected by this policy, or 45% of the people who've been polled? Understanding the 45% and its relationship to the rest of the context matters. Further, it allows for people to make sense of the inverse. Does this mean that 55% of people agree with the policy? This is a good activity for any classroom. Science and social studies also use applied mathematics in that sense. Percentages provide that lens. But percents are far from the only topic that leverage what students already know. In fact, all the "real-world math" people keep asking us to do lives and breathes around us if we just take some inspiration from our environments. An example in my case is slopes, which include the places where I walk and take the train.

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