What to Look For 2

Multiplicative Reasoning Curriculum Correlation with Ontario Grades 3–4

Making a match between the lessons, which reflect a research-based developmental trajectory, with a ministry list of grade-level curricular expectations is a challenge. The ministry has the difficult task of mapping out, from K to 12, all the mathematics students should know and when they should know it. We, on the other hand, are looking much more narrowly, and therefore granularly, at the development of multiplicative reasoning over time in Grades 3 and 4. Most of the expectations are not dealt with in one or two lessons but gradually over many lessons. For example, the expectation B1.5 *to use place value to describe multi-digit numbers,* starts in Lesson 4 with skip counting by 10 up over 100 and builds on this in different ways over the course of many lessons (Lessons 5, 9, 11, 14, 15, 17, 18, and 20) without being explicitly mentioned until Lesson 21. I have therefore placed the match with that lesson but know that students who have not built up their understanding in earlier lessons of multiplication by 10, or the distributive property, or flexible modelling on the array are less likely to be ready for this lesson. Additionally, most of the lessons after 21 (Lessons 22–26) also address place value in different situations and activities but are not listed for this expectation in this correlation. Note that Lessons 21 to 26 appear in the Additional Resources portion of the website rather than in the print resource.

The second thing you will notice is that parts of some expectations are greyed out. We do not address this aspect of the expectation, sometimes because we think it does not represent the most effective teaching practice (for example, the use of base ten materials with place value) or we address this aspect of the expectation later in the lessons, or not at all because we have not yet done that research (for example, fractions and decimal fractions).

Section 1: The Beginning of the Multiplication and Division Continuum			
***WTLF2* Lesson**	**Lesson Summary**	**ON Grade 3 Expectation**	**ON Grade 4 Expectation**
1 Counting Collections	To introduce intensive quantities, pairs count large collections of items by organizing the items into containers with the same number of items in each container.	**B1.4** Count to 1000, including by 50s, 100s, and 200s, using a variety of tools and strategies. [Because Lesson 1 is an inquiry, the final counts and count-by numbers vary. It is foundational to larger counting that can be done as skip counting practice, which begins in Lesson 4.]	
2 Working with the Intensive Quantity	Students build on their learning from Lesson 1: • through quick image sequences • by counting a new collection of items • by playing Multiplication Match Bingo		
3 Writing the Math Sentence (Equation)	• Students continue to practise skip counting or repeatedly adding the intensive quantity rather than counting by ones. • They also work on recording their mathematical thinking.		**B2.8** Show simple multiplicative relationships involving whole-number rates, using various tools and drawings.

Section 2: Building Fluency with 10× and 2× Facts			
***WTLF2* Lesson**	**Lesson Summary**	**ON Grade 3 Expectation**	**ON Grade 4 Expectation**
4 Using Strings to Develop Multiplication of 10	• The lesson begins with practice skip counting by 10. • Students compare two of the word problems they completed in Lesson 3 in a class discussion. • Two different strings are used to develop multiplication by 10. • Students consolidate their learning by playing Connect 4 Products.	**B2.3** Use mental math strategies to multiply whole numbers by 10, 100, and 1000, divide whole numbers by 10, and add and subtract decimal tenths, and explain the strategies used.	
5 Discovering the Commutative Property of Multiplication	• Students extend the Timbits quick images from Lesson 4 and play a new version of Connect 4 Products. • A new Timbits string is used to help students begin to develop an understanding of the commutative property. • Students consolidate their learning by completing word problems.		
6 Two Ways to Think About 2× Facts	• The lesson begins with a review of two of the word problems students completed in Lesson 5. • To get students thinking about commutativity, they write equations describing two images from the Double Fun Bingo game that involve the same factors. • Students then play the game for additional practice writing equations and gain further exposure to the commutative property.		

Section 3: Automatizing 2× Facts, Solving Word Problems, and Introducing Division			
***WTLF2* Lesson**	• **Lesson Summary**	**ON Grade 3 Expectation**	**ON Grade 4 Expectation**
7 Automatizing 2× Facts	• The lesson begins with the class playing a game of Double Fun Bingo to review 2× facts (including commuted facts). • Each student creates a set of thinking cards they can use to practise 2× facts and sorts the cards into facts they know automatically, and those they need to work on.	**B2.2** Recall and demonstrate multiplication facts of 2, 5, and 10, and related division facts. [Automatization begins with this lesson and continues in subsequent lessons until students master them.]	
8 Using 2× Facts	• Students complete two word problems that are more challenging than those they have encountered in earlier lessons. One is a two-part problem designed to work on the commutative property and the other involves two operations. • Students continue to practise with their 2× thinking cards. • Students play a version of Connect 4 Products that focuses on 2× facts. • The lesson ends with a debrief of one of the word problems.		**B2.1** Use the properties of operations, and the relationships between addition, subtraction, multiplication, and division, to solve problems involving whole numbers, including those requiring more than one operation, and check calculations.
9 Introducing Division Represented as a Missing Factor	• Quotative division is introduced by writing it as a missing factor solution. Students work on a sample problem independently and discuss solution strategies as a class. • The class plays Make It True Bingo to practise 1×, 2×, 5×, and 10× facts with missing factors.	**B2.1** Use the properties of operations, and the relationships between multiplication and division, to solve problems and check calculations. **B2.7** Represent and solve problems involving multiplication and division,	

	• Students then complete two additional word problems with a partner.	including problems that involve groups of one half, one fourth, and one third, using tools and drawings.	
10 Continuing to Work with Division Represented as a Missing Factor	• Students play another round of Make it True Bingo. • The class debriefs the two quotative division problems from the previous lesson. • Students practise their 2× thinking cards and those who are ready take an assessment. • Students work independently on two new division problems.		

Section 4: Building and Working with the Array			
***WTLF2* Lesson**	**Lesson Summary**	**ON Grade 3 Expectation**	**ON Grade 4 Expectation**
11 Introducing the Array	• Students learn about arrays by observing and then performing a card trick. • They solve a division problem about an array of windows with 5 windows per column. • They use an array model to solve a two-part problem involving multiplication by 10 and by 5.	**B2.6** Represent multiplication of numbers up to 10 × 10 and division up to 100 ÷ 10, using a variety of tools and drawings, including arrays.	
12 Problem Solving on the Array and 5× Facts	• The lesson begins with a review of the first word problem solved in Lesson 11. • Students then develop strategies for solving a "numberless" word problem that involves an array. • Students explore patterns in multiples of 5. • They use these patterns to help create a set of thinking cards to automatize 5× facts.		
13 Shifting to the Graph-Paper Closed Array	• To begin, students determine a 10× and a 5× product shown on quick image arrays of dots. • Students then use closed arrays drawn on grids to determine products involving multiplication by 10 and 11. • Students consolidate their learning by playing a game called Target 10× or 5×.		

14 Using and Modelling the Distributive Property on the Array	• The lesson begins with a guided discussion during which students solve a two-part word problem. • At their desks, students complete a worksheet that focuses on building on an existing array to determine a new product. • Finally, students continue to work on automatizing their 2× and 5× facts, taking assessments when ready.		
14B Supplementary Practice: 10×, 0×, 1×, 2×, and 5× Anchor Facts	• Suggestions are provided for activities students can complete to help them consolidate their understanding of the commutative property and automatize their 10×, 0×, 1×, 2×, and 5× number facts before moving on to Lesson 15.		
15 Using the Distributive Property on the Array to Model ×9 Facts	• To begin, students use a playing card array to determine a ×9 product by starting with the related ×10 product. • Students continue this strategy as they determine a string of products by drawing and modifying closed array models. • At their desks, students consolidate their learning by completing a worksheet. • The lesson ends with an investigation of patterns in products of 9.		

Section 5: Introducing Partitive Division and Building on Anchor Facts			
***WTLF2* Lesson**	**Lesson Summary**	**ON Grade 3 Expectation**	**ON Grade 4 Expectation**
16 Practicing 9× Facts	• Students discuss patterns in products of 9 with a particular focus on using a related 10× anchor fact to determine a 9× fact. • They create a set of 9× thinking cards and use them with a partner. • The lesson ends with a new game that involves 9× facts.		**B2.2** Recall and demonstrate multiplication facts for 1 × 1 to 10 × 10, and related division facts. [Automatization of 9x begins here followed by other numbers throughout the lessons.]
17 Introducing Partitive Division Problems	• Students work together to solve a quotative division problem involving an array. • They then solve their first partitive division problem and share solution strategies. • Pairs work on automatizing 9× facts using thinking cards and a game, taking an assessment when they feel ready. • Finally, each student independently solves a partitive division problem.		**B2.6** Represent and solve problems involving the division of two- or three-digit whole numbers by one-digit whole numbers, expressing any remainder as a fraction when appropriate, using appropriate tools, including arrays
18 More Partitive Division Problems	• Students work at their desks to solve a partitive division problem involving a two-digit number, then share their solution strategies. • They work independently to solve another partitive problem and complete an array worksheet. • Students continue to work on automatizing 9× facts using thinking cards and a game.		

18B Supplementary Learning: Building 4× and 3× Facts from the 2× Anchor	• Activities are provided to help students automatize 3× and 4× facts. For each set of facts, student begin by investigating patterns in products. • They also complete a worksheet in which they build on 2× arrays to create 3× and 4× arrays. • Students create sets of thinking cards and use them to practise the facts. They also practise by playing new versions of games and take assessments when they feel ready.		**B2.2** Recall and demonstrate multiplication facts for 1 × 1 to 10 × 10, and related division facts.
19: *Six Dinner Sid* and More Partitive Division Problems	• The lesson begins with a reading from the children's book *Six Dinner Sid*. Part way through the book, a rate problem based on the story is introduced. Students solve the problem and share their strategies, then the rest of the book is read aloud. • Students work at their desks to solve a new partitive division problem with an intensive quantity greater than 10. Solutions are shared in a class discussion, and a new problem is introduced for students to solve independently.		
19B Supplementary Learning: 6× and 7× Facts	• Activities are provided to help students automatize 6× and 7× facts. For each set of facts, student begin by investigating patterns in products. They complete a worksheet in which they build on 5× arrays to create 6× and 7× arrays.		

	• Students create sets of thinking cards and use them to practise the facts. They also practise by playing new versions of games and take assessments when they feel ready.		
20 Building Multiplication by 10 and 100 and a Shift to the Open Array	• The class completes an investigation into multiplication by 10 and by 100 where you model and students work on their own grid arrays. There is an option to extend the activity to multiplying by 1000. • Students use arrays to determine products involving 101 and 99 and independently complete an open array worksheet. • Students play a new game to practise their mental math skills multiplying by 9, 10, 11, 99, 100, and 101.	**B1.5** Use place value when describing and representing multi- digit numbers in a variety of ways, including with base ten materials.	**B2.3** Use mental math strategies to multiply whole numbers by 10, 100, and 1000, divide whole numbers by 10, and add and subtract decimal tenths, and explain the strategies used.

Section 6: Introducing Multi-Digit Multiplication and Division with Remainders			
***WTLF2* Lesson**	**Lesson Summary**	**ON Grade 3 Expectation**	**ON Grade 4 Expectation**
21 Mental Multiplication of 1-Digit by 2-Digit Numbers Modelled as an Open Array	• Students model 4 x 10 and then 4 x 100 as open arrays. • The class completes two multiplication strings designed to have them use a known fact and build on it using the associative property and then the distributive property to calculate another product more easily. They model the steps using open arrays. ▪ Students independently complete a work sheet about modelling products as open arrays. ▪ Pairs of students play a new game called Hack It, Stack It! to consolidate their mental math 1-digit by 2-digit multiplication making.	**B1.1** Read, represent, compose, and decompose whole numbers up to and including 1000, using a variety of tools and strategies, and describe various ways they are used in everyday life.	**B1.1** Read, represent, compose, and decompose whole numbers up to and including 10 000, using a variety of tools and strategies, and describe various ways they are used in everyday life.
22 Mental Multiplication of 1-Digit by 3-Digit Numbers Modelled as an Open Array	• After working through the first example together, students independently complete an open array worksheet. • Students complete two word problems that involve multiplication of 1-digit by 2- or 3-digit numbers. The class identifies the numbers, unknown, and referents for the initial part of the first problem but students do this independently for the next part and the second problem.		

	• Students who finish the word problems play Hack It, Stack It! with a partner. • The class debriefs the first word problem.		
23 Multiplication of 1-Digit by 3-Digit Numbers: Shifting to the Accessible Algorithm	• The class debriefs the second word problem from Lesson 22, which involves 1-digit by 3-digit multiplication. • During the debrief, the partial products open array is used to introduce an accessible vertical algorithm for multi-digit multiplication. • Students complete a worksheet that involves modelling 1-digit by 3-digit multiplication as an array and recording the calculation using an accessible algorithm. • Pairs of students practise 1-digit by 2-digit multiplication by playing new versions of Two Moves Connect 4.	**B1.5** Use place value when describing and representing multi- digit numbers in a variety of ways, including with base ten materials.	**B2.5** Represent and solve problems involving the multiplication of two- or three-digit whole numbers by one-digit whole numbers and by 10, 100, and 1000, using appropriate tools, including arrays.
24 Extending Multi-Digit Multiplication to Products Greater than 1000	• The class completes a string to extend their knowledge of 1-digit by 3-digit multiplication to products greater than 1000. • Students complete another 1-digit by 3-digit calculation independently. • Students complete a worksheet that involves modelling 1-digit by 3-digit multiplication with products greater than 1000 as an array and recording the calculations using an accessible algorithm.	**B1.1** Read, represent, compose, and decompose whole numbers up to and including 1000, using a variety of tools and strategies, and describe various ways they are used in everyday life.	**B1.1** Read, represent, compose, and decompose whole numbers up to and including 1000, using a variety of tools and strategies, and describe various ways they are used in everyday life.

	• Students practise any number facts they are still working on and take assessments when they feel ready. • Students who have completed all their assessments can play the new versions of Two Moves Connect 4 that were introduced in Lesson 23.		
25 Extending to Modelling 2-Digit by 2-Digit Multiplication as an Open Array	*Day 1* • The lesson begins with a brief time for thinking-card practise. • The class works together to interpret and solve a word problem by creating a closed array with different coloured sections that represent the partial products in a 2-digit by 2-digit multiplication. • Students work independently to record the solution as an open array and as an equation, then debrief the solution as a class. *Day 2* • Students solve another word problem involving 2-digit by 2-digit multiplication. This can be debriefed before students begin the worksheet if you feel it would be helpful. • Students complete an open array worksheet about 2-digit by 2-digit multiplication. The worksheet has fewer helper parts than previous worksheets so the first example can be completed as a class.		

26 Dealing with Remainders	*Day 1* • Students independently solve two quotative division word problems that use the same numbers but involve remainders that will be treated differently. • The class debriefs the word problems, comparing the different treatment of the remainders based on the problem situations. • If time permits, students play Hack It, Stack It!, which was introduced in Lesson 21. *Day 2* • Students solve a third related word problem that involves the same numbers but is a partitive division problem. In this problem, the remainder can be shared. • The class debriefs the word problem. • Students work independently on four new word problems that involve remainders.		**B2.6** Represent and solve problems involving the division of two- or three-digit whole numbers by one-digit whole numbers, expressing any remainder as a fraction when appropriate, using appropriate tools, including arrays.
Multiplicative Comparison Problems	This material appears in the Additional Resources portion of the website rather than in the print resource. It can be accessed by clicking on the *Multiplicative Comparison Problem Family* card.	**B2.9** Use the ratios of 1 to 2, 1 to 5, and 1 to 10 to scale up numbers and to solve problems.	