

Exploring Volume

About the Math

5.MD.3.A; 5.MD.3.B; 5.MD.4; 5.MD.5.A; 5.MD.5.B; 5.MD.5.C

Although students measured liquid volume, or capacity, in third and fourth grades, the study of volume as an attribute of solid figures begins in fifth grade. At this level, students develop an understanding of volume, as well as explore ways to measure it.

The key ideas focused on in this module include:

- understanding the concept of volume, and measuring volume by counting the number of cubes it takes to fill a figure
- understanding that volume is measured in cubic units
- exploring the volume of rectangular prisms and making connections between volume and area
- discovering the formula for determining the volume of a rectangular prism
- solving problems about volume
- recognizing volume as additive and finding the volumes of complex figures.

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Volume is a measure of the space inside a figure, typically measured in cubic units. At this grade level, volume is measured in whole number units. Initial explorations have students filling shapes with cubes and counting the number of cubes to find the volume. As they explore the concept further, they select appropriate units (e.g., cubic inches or cubic centimeters) and decide on appropriate strategies and tools (e.g., counting cubes or using addition or multiplication methods for finding the total volume). They use their experiences to estimate volume and solve problems related to volume.



Figure 13.1 Students pack boxes with cubes and then count cubes to find the volume.

Although students can count to find the total number of cubic units, they discover that there are other ways to find the volume without counting each cube. Rather than telling students the formula for finding the volume of a right rectangular prism, providing opportunities for students to gather data, observe those data, and gain insights of their own allows them to make sense of the formulas they discover (multiplying $\text{length} \times \text{width} \times \text{height}$ or multiplying $\text{base} \times \text{height}$).

As students explore volume, they revisit other math concepts. Students recognize that when calculating the volume, the order of the three measurements (length, width, height) is irrelevant because of their understanding of the associative property. Whether they calculate $\text{length} \times \text{width} \times \text{height}$ or $\text{height} \times \text{width} \times \text{length}$, the

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volume will be the same. Students see connections with the formula for calculating area as they figure out why, for a right rectangular prism, multiplying the area of the base by the height would be the same as multiplying the length by the width by the height.

Decomposition is a common theme in mathematics. Students decompose numbers by place value in expanded form. They decompose monetary amounts into various coin denominations. And they decompose three-dimensional figures into other figures in order to determine the volume. Through this work, students recognize that volume is additive. They discover that they can split some shapes into separate rectangular prisms, find the volume of each one, and put those volumes together to find the volume of the original figure.



As important as understanding *how* to calculate volume is the understanding of *when* and *why* we calculate volume. Posing problems related to volume and having students explore, discuss, and solve the problems deepens their understanding of volume in real-world situations. These problems include situations where the volume is unknown, as well as situations in which one of the dimensions (length, width, or height) might be unknown.

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Learning Goals

I can explain what it means to find the volume of a solid figure and how volume is measured.

I can find the volume of a rectangular prism by using a formula.

I can find the volume of complex figures.

I can solve math problems using my understanding of volume.

Lessons in This Module

Exploring the Concept of Volume

Introduction to Volume:
Counting Cubes

Build It!

Base $\times$ Height

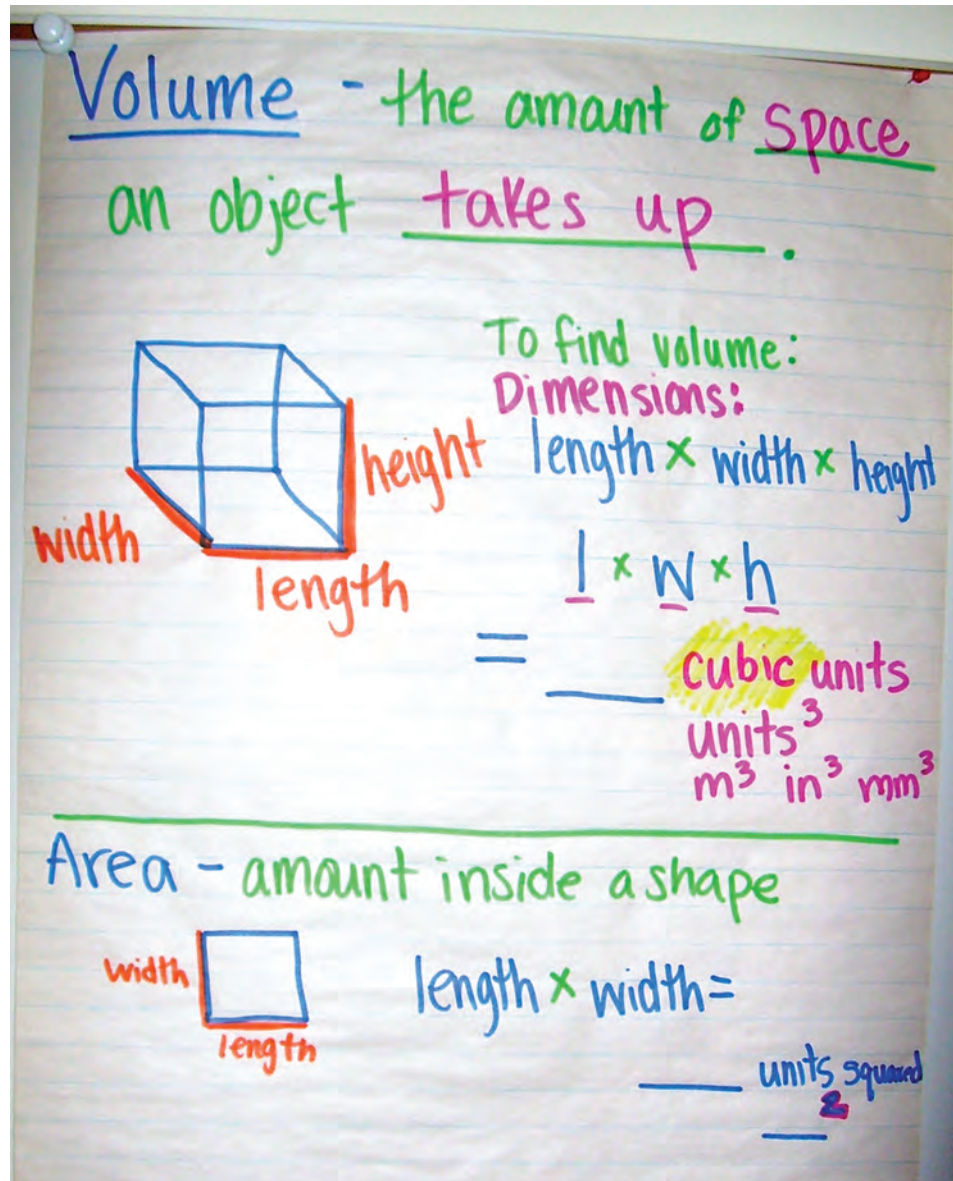
What's the Rule?

Problem Solving

Understanding Volume as Additive

Volume Is Additive

The Birthday Cake



This anchor chart shows the labeling of area in square units for two-dimensional shapes and the labeling of volume in cubic units for three-dimensional shapes.

Exploring the Progression

PREVIOUS

Grade 4

Exploring area; calculating area of rectangles



NOW

Grade 5

Exploring volume; calculating volume of rectangular prisms



NEXT

Grade 6

Calculating volume of rectangular prisms with fractional measurements; calculating surface area

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Ideas for Instruction and Assessment

EXPLORING THE CONCEPT OF VOLUME

Students explore the concept of volume by first filling solid containers with cubes and then observing and discovering the mathematical formula.

Thinking Through a Lesson

SMP4, SMP5, SMP6

Introduction to Volume: Counting Cubes

Students fill boxes with cubes and count them to explore volume.

Ask students about situations when it would be important to know how much something could hold, or how much it would take to fill an object.

Can you think of times when you might need to know how much could fit in a container?

When might you need to know how much it would take to fill something?

Have partners brainstorm ideas of real-world situations related to volume.

Make a list on a chart.

Have students work in groups of four.

It is a good practice to have students work in groups so they can share their thinking and strategies.

Common Error or Misconception

Students sometimes confuse the concepts of area and volume and the units used to measure them (square or cubic units respectively). Introductory investigations with materials (squares or cubes) help them develop deeper understandings of the differences between the concepts.

Connecting to the real world makes the concept more meaningful for students. Real-world situations may include amount of cereal in a box, sand in a sandbox, or potting soil in a planter.

Prepare centimeter grids copied on card stock paper for each student.

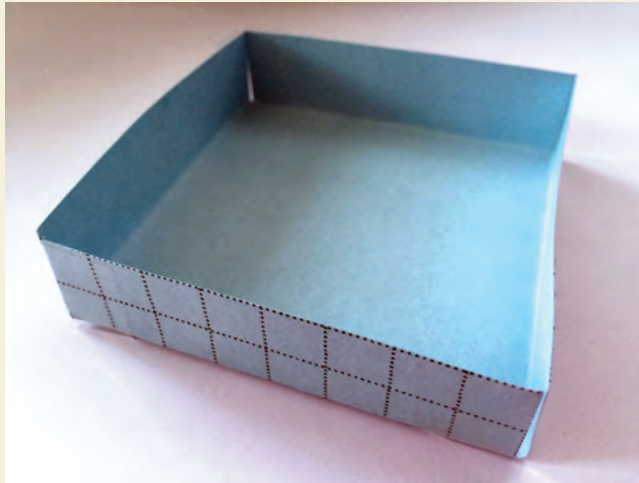
Have each student cut out a 12×12 square.

Have each of the four students cut out a different square section from each corner of his square (one cuts out a 2×2 section of each corner, another cuts a 3×3 , another cuts a 4×4 , and another cuts a 5×5).

Then students fold and tape the sides (grid side facing out) to make open boxes of various sizes.

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This student makes a box, making sure the grid lines are facing out.

Compare your boxes with the other boxes in your group.

Which box is biggest?

Talk with your team members and be ready to share and justify your answer.

Have students share their ideas. Some may talk about the tallest or widest.

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Is the tallest box the biggest? Why or why not? (It's the tallest, but it is not very wide.)

Is the widest box the biggest, why or why not? (It's the widest, but it's not very tall.)

Is there another way we might measure the boxes? (We could see how much they hold inside.)

Provide each team with a set of centimeter cubes.

Have students fill each box with cubes and count the number of cubes that fill the box.

How many cubes filled each box?

Which box has the most cubes?

Which box has the fewest cubes?

Explain that they just measured the volume of these boxes.

Hold up a flat rectangle.

What did we call the amount of space that covered the rectangle? (The area.)

How did we label the units for area? (Units squared, square units.) Why? (It was like counting squares.)

How might we tell someone the volume of our box?

What unit are we using to measure? (Cubes.)

Turn and talk to your partner about how we might label volume. (We might use cubic units.) Why? (We are using cubes. Or, It is a three-dimensional figure. Or, The cube is 1 unit on each side.)

So the tall box has a volume of 20 cubic units since 20 cubes filled it.

Add the phrase *cubic units* to the word wall or Math Talk chart.

Can we be more precise with the cubic unit? Turn and talk to your team.

Did anyone come up with a more precise label? (Yes. We said cubic centimeters since we used centimeter cubes.)

Turn and talk to your team about the volume of each box.

Write the word *volume* on the word wall or Math Talk chart.

Turn to your partner. How would you describe volume? What is it? (It is the amount something can hold. Or, It's the amount of space something takes up. Or, It's how many cubes fit in the box.)

How is it like area? How is it different? (It measures how much space something takes; area measures how much flat space something covers, while volume measures how much three-dimensional space something takes up.)

Write the word volume in your journal.

In your own words, tell what volume is.

Tell how you measure volume.

Ask students for their ideas before sharing the units used. They are more likely to remember it if they make sense of it.

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Build It!

Students compare volume by using cubes to build rectangular prisms.

Give each team of four students a set of multilink cubes.

What is a rectangular prism?

Use your cubes to make rectangular prisms. Each team member should make one that is a different size.

Allow time for students to build their rectangular prisms.

Talk to your team and predict which one will have the greatest volume.

Students may predict the tallest or widest, or one with the most cubes.

Predict which one will have the least volume.

Students may predict the shortest, or one with the fewest cubes.

Once students have made their predictions, have them find the volume of each one by counting the cubes. They may have to disassemble parts of the prism to do this, but can reassemble after they count the cubes.

Write the volume of each rectangular prism on a sticky note and place it by the prism.

How should you label your volume measurement?

Were your predictions correct?

Which has the greatest volume?

Which has the least volume?

Have students do a walk around the room to see the rectangular prisms made by each team.

As you look at each team's rectangular prisms, think about the ones with the greatest and least volumes.

What do you notice about them? Are they always the tallest? Are they always the widest?

Ask students to share their ideas about volume. (They might notice that the tallest doesn't always mean the volume is greater. Or that some of the shorter ones had a large volume because they were wide.)

Order the class' rectangular prisms from least to greatest volume, using the measurements on the sticky notes.

To end the lesson, have students discuss the following:

Do any of our rectangular prisms have the same volume but look different? Is that possible? Why or why not?

Differentiation

Challenge a group of students to work together to make prisms that look as different as possible but have the same volume.

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Students use cubes to build rectangular prisms of various sizes.

Base $\times$ Height

Students discover that the volume of a right rectangular prism can be calculated by multiplying the area of its base by its height.

Have the students work in groups of four.

Provide each member of the group with centimeter grid paper on cardstock and centimeter cubes.

Have each student cut a 10×10 square out of the grid paper.

Then have them cut a 4×4 square out of each corner.

Have them fold and tape the sides (grid side facing out) to make an open box.

Have each student cover the bottom of their box with one layer of cubes.

How many cubes do you have?

Student 1 will keep his box as it is.

Student 2 will add 1 more layer.

Student 3 will add 2 more layers.

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Student 4 will add 3 more layers.

Find the volume of the part of your box that is filled with cubes.

Record the data on the board.

Student 1: 4 cubic units

Student 2: 8 cubic units

Student 3: 12 cubic units

Student 4: 16 cubic units

What do you notice?

Does that make sense?

Have students flip over the box.

What would I be measuring if I measured just the flat bottom, the base, of the box?

How could you find the area of the base?

What is the area of the base? Think about the units you use for this measurement.

Turn and talk: How does the area relate to your volume measurements?

Have students repeat the layering activity with a box that is made from a 12×12 square with 4×4 cutouts in each corner.

Then, have them share their data and record it on the board on a chart like the following:

Area of Base	Number of Layers Tall	Volume
16 square units	1	16 cubic units
16 square units	2	32 cubic units
16 square units	3	48 cubic units
16 square units	4	64 cubic units

What do you notice about the area and the volume?

Does this make sense? Explain.

Have students make one more box with a 14×14 square with 4×4 cutouts in each corner.

Have students fill in the chart.

Turn and talk: Is there a way you could find the volume of all 4 layers of the box without filling it with cubes and counting them? Explain.

Have partners share their ideas.



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What's the Rule?

Students discover that the volume of a right rectangular prism can be found by multiplying its length, width, and height.

Give each pair of students some multilink cubes. Pose the question:

How many different rectangular prisms might you build with a volume of 18 cubic units?

Build as many as you can and be ready to tell me the volume, the length of the base, the width of the base, and the height.

Make a class chart and have partners share the dimensions of their prisms.

Record the dimensions of each one.

Length of Base	Width of Base	Height	Volume
			18 units ³
			18 units ³
			18 units ³

What do you notice?

Are there any connections between the numbers in each row on our chart?

Is there a way to find the volume without counting every cube?

Students may suggest that multiplying the three measurements gives the same result as counting all of the cubes.

Do you think that will always work?

Build as many different rectangular prisms as you can with a volume of 24 units³.

Record the dimensions for their rectangular prisms to test the conjecture.

What do you notice?

Does multiplying the dimensions give us the volume?

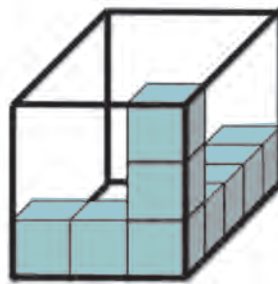
Have students practice finding volume by making a few more rectangular prisms of any size, recording the dimensions, finding the volume by multiplication, and then counting the cubes to check the volume.

FORMATIVE ASSESSMENT

Pose the following:

This prism is partly filled with cubes. What is the volume of the whole prism?

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Look for students who understand that the dimensions are $3 \times 3 \times 5 = 45$ cubic units. If students are having difficulty, give them cubes to build the figure and count the cubes.

Problem Solving

Students solve problems using volume concepts.

Pose this problem for students:

Mr. Sines wants to take 3 boxes of materials home from school. He needs to know if they will all fit in his truck, or if he needs to make two trips to get all the boxes home. Here is some information you will need:

- 2 of the boxes are the same size. (2 ft. long, 3 ft. wide, and 2 ft. high)
- 1 box is larger than the others. (3 ft. long, 3 ft. wide, and 3 ft. high)
- Mr. Sines' truck has 60 cubic feet of space.

Can Mr. Sines fit all 3 boxes in his truck?

Have students work in pairs to solve.

Have them present their findings to another partner pair.

Does the other pair agree or disagree? Why?

Pose another problem and have students work with a new partner or group to solve and present their findings.

Mrs. Petti is planning to ship a package. She needs a box that will hold at least 30 cubic feet.

Which box will be large enough? Prepare an argument to convince the class which box will work.



Box A		Box B	
Length	7'	Length	6'
Width	2'	Width	2'
Height	2'	Height	3'

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Additional Ideas for Support and Practice

The following ideas extend students' understanding of volume and provide meaningful practice.

CHECK THE FORMULA

Students fill open boxes (made with centimeter grid paper) with centimeter cubes.

They use the grids on the outside of the box to determine the length, width, and height.

Students then find volume by multiplying the dimensions and by counting cubes.

They check to see if the two volumes are the same.

They record the actual data for $V = l \times w \times h$ in their journals.

PASS-A-PROBLEM

Tape one problem on the front of a manila envelope.

Prepare enough envelopes so that each group of four students will have an envelope with a different problem.

Students will solve a problem, place their solutions and work in the envelope, and pass it to the next group.

After the last rotation of problems, students will take out all of the solutions for the problem they are holding.

Each group will present that problem, solution, and strategies to the class.

Problems may include:

Tom built a rectangular prism using 40 centimeter cubes. The length was 4 cm, the height was 5 cm, and the width was 2 cm. He took it apart and used the same 40 cubes to build another rectangular prism with different dimensions. What could the dimensions be?

Heidi's new shoes came in a box that has a volume of 240 cubic inches. The length is 12 inches. What could the height and width be?

A cereal box has a volume of 128 cubic centimeters. What could the dimensions be?

For additional problems, see the online resources.

TALK ABOUT IT/WRITE ABOUT IT 🗨️📝

You have been asked to design a toy box for a child's bedroom. The toy box needs to hold 30 cubic feet. What could the dimensions be? Justify your answer.

How is volume different from area? Explain.

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What is the difference between cm^2 and cm^3 ? Explain.

How can you find the volume of a rectangular prism without filling it with cubes and counting them?

A rectangular prism has a volume of 48 cubic units and one of its dimensions is 4. What might the other dimensions be? Justify your answer.

THE SAME, BUT DIFFERENT

Have students work with a partner to build two rectangular prisms with the same volume, but different dimensions.

Have them record the volume and the different dimensions in their journals.

Students may choose the number of cubes to use. Do they recognize that they cannot always build more than one rectangular prism with the same volume? This is a good connection to finding the factors of a number.

PACK IT UP

Students construct a box to hold a small item.

Place small items such as a glue stick, stapler, small stuffed animal, or small toy in a basket on the table.

Provide grid paper, ruler, and scissors.

Have students build an open-top box to hold the item.

Have them record the dimensions in their journals and calculate the volume.

Have them explain how they determined what dimensions to make the box and if the box was too small, too large, or the perfect size.

ROLL A RECTANGULAR PRISM

Students play with a partner to calculate the volume of rectangular prisms.

Each player rolls a 10-sided die three times to find the dimensions of each side of a rectangular prism.

Each student calculates the volume of the rectangular prism.

The student with the greater volume gets one point.

Repeat ten times. The student with the most points is the winner.

Provide calculators for students who need them, to keep the focus on the measurement concepts rather than on the calculations.



UNDERSTANDING VOLUME AS ADDITIVE

Students recognize that volume is additive. They split a shape into two or more rectangular prisms, find the volume of each one, and put those volumes together to find the volume of the original figure.

Volume Is Additive

Students find the volume of a complex shape by decomposing it into two rectangular prisms.

Have students work in pairs and each make a rectangular prism out of connecting cubes.

What is the volume of your prism?

Put your prisms together to make a new figure.

What do you think the volume of the new figure is? Why?

How would we find the volume of the figure?

Have students take their original figure back.

Combine your figure with a different partner's figure.

What is the volume of the new figure?

Have partners swap their composite figures with another team.

Find the volume of the figure you just received.

What did you have to do to find the volume of the complex figure made up of rectangular prisms?

Turn and talk to your partner about how you would find the volume of a complex figure.

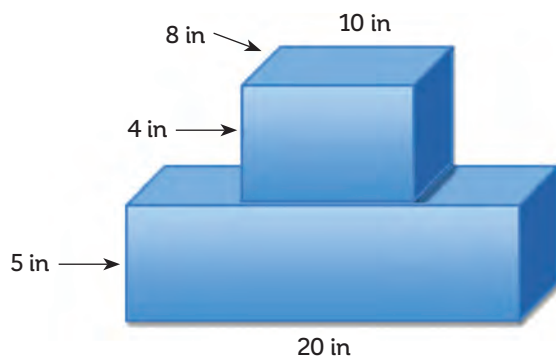
Review the definition of a composite or complex figure. It is a figure made from two or more geometric figures.

The Birthday Cake

Students solve problems about complex figures.

Post this visual and problem on the board:

Daniele is baking a cake for Grandpa Bill's ninetieth birthday party. She needs the cake to have a volume of 1,000 cubic inches so that there will be enough for all the guests. Here is an image of the cake. Will there be enough cake?



Work with a partner to solve the problem.

Observe as students solve the problem.

What strategies did you use to solve the problem?

What was the top layer's volume?

What was the bottom layer's volume?

Common Error or Misconception

Some students may multiply all the numbers together ($5 \times 4 \times 10 \times 8 \times 20$) instead of finding the volume of each rectangular prism. Ask students to explain why they made the calculations they did.

How did you find the volume of the whole cake?

Did Daniele bake a large enough cake?

Turn and talk to your partner about how you find the volume of a complex shape made up of rectangular prisms.

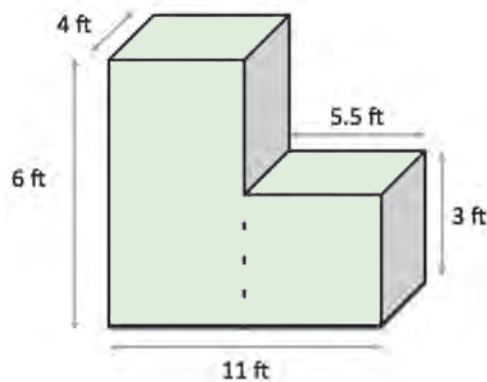
FORMATIVE ASSESSMENT

Pose:

Find the volume of this figure. Explain how you figured it out.

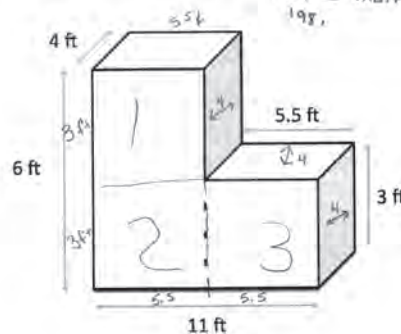
Differentiation

For a more challenging problem, use a figure involving decimals.



The volume is $\rightarrow 198 \text{ ft}^3$

I got 198 ft^3 by splitting the shape into 3 cubes. One cube was 66 so then I multiplied $66 \times 3 = 198$.

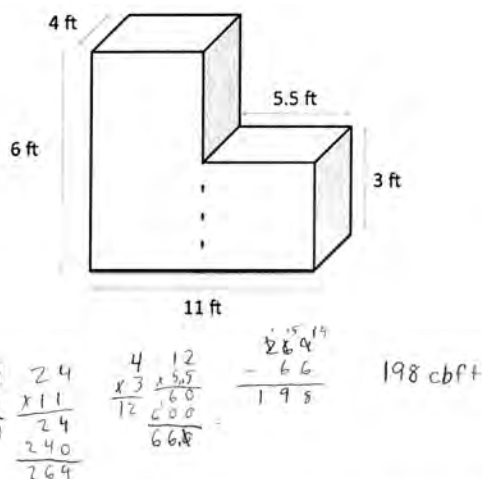


$$W \times L \times H \\ 4 \times 5.5 \times 3 = 66 \times 3 = 198 \\ \text{answer}$$

$$\begin{array}{r} 22 \\ \times 3 \\ \hline 66 \\ \times 3 \\ \hline 198 \end{array}$$

This student splits the figure into 3 equivalent figures. The student calculates the volume of one of the rectangular prisms and then multiplies it by 3 but does not explain her work or label her answer.

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I got my answer by know if I did $4 \times 6 \times 11$ with equals the whole square including the empty space then subtract $4 \times 3 \times 5.5$ by 264 you get the answer 198.

$$(4 \times 6 \times 11) - (4 \times 3 \times 5.5) = 198 \text{ cubic feet}$$

This student finds the volume of the entire figure, including the empty space. The student then finds the volume of the empty space and subtracts it from the total.

For additional formative assessment techniques, see Math in Practice: A Guide for Teachers, Chapter 5.

Additional Ideas for Support and Practice

The following ideas extend students' understanding of volume of complex figures and provide meaningful practice.

TALK ABOUT IT/WRITE ABOUT IT 🗨️📝

Does it matter which way you break apart solid figures to determine volume? Explain.

Identify some real-world figures that can be decomposed into rectangular prisms to calculate volume.

BUILDING SKYSCRAPERS 🖐️

Students find the volume of complex figures, using premade paper grid boxes to build a skyscraper.

As a class, have each student make one box from grid paper to place in the center.

Students may also build prisms from connecting cubes and use them to build the skyscrapers.

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Vocabulary

base	height
complex figure	length
cube	unit
cubic units ³	volume
	width

Remind students that using a numbered list to explain each step might help them organize their ideas.

ONLINE RESOURCES

General Resources

Additional Problems
Grid Paper

Resources for Specific Activities

Base $\times$ Height Chart

These resources are available at [REDACTED]

Have each student cut a square out of the grid paper. Make sure sizes vary (7×7 , 8×8 , $9 \times 9 \dots$)

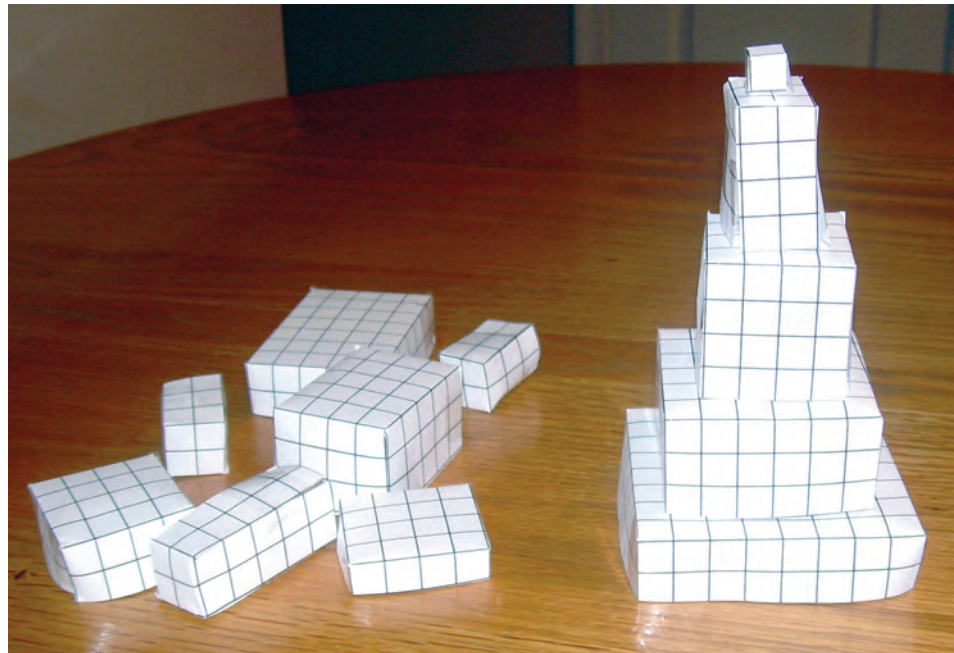
Then have them cut out the same size square from each corner on their paper. Students may choose the size (2×2 , 3×3 , 4×4).

They fold and tape the sides (grid side facing out) to make an open box.

Students work with partners to build a skyscraper with three or more boxes.

Partners figure out the volume of each level and then find the volume of their skyscraper.

Partners then write how they found the volume.



This is a sample of a figure that a student built using a variety of paper boxes.

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