

Multiplying Fractions by Whole Numbers

About the Math

4.NF.4

In third grade, students began their exploration of multiplication through investigations, models, discussions, and problems focusing on understanding basic multiplication facts. In fourth grade, students extend this understanding to multiply a fraction by a whole number.

The key ideas focused on in this module include:

- understanding nonunit fractions (e.g., $\frac{5}{6}$) as products of unit fractions (e.g., $\frac{5}{6} = 5 \times \frac{1}{6}$)
- finding the product when multiplying a fraction by a whole number
- solving word problems that call for multiplying a fraction by a whole number by using visual models.

Students have experience multiplying whole numbers and are now introduced to multiplying a fraction by a whole number. Beginning with unit fractions (those with a numerator of one, such as $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$) helps to simplify this concept.

Visualizing the multiplication process through manipulatives or diagrams helps our students make connections with what they know about multiplication of whole numbers (see Figures 9.1 and 9.2). Students connect the idea of adding $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ with multiplying $5 \times \frac{1}{4}$, and connect that idea to their growing understanding of how fractions can be composed from unit fractions with the same denominator.

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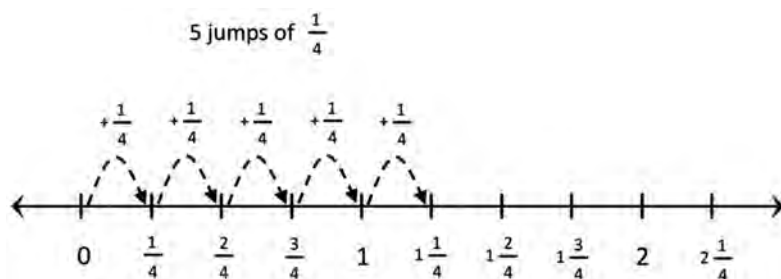


Figure 9.1 Students use a number line to show $5 \times \frac{1}{4}$. The number line shows the multiplication process and clarifies why the product is greater than 1.

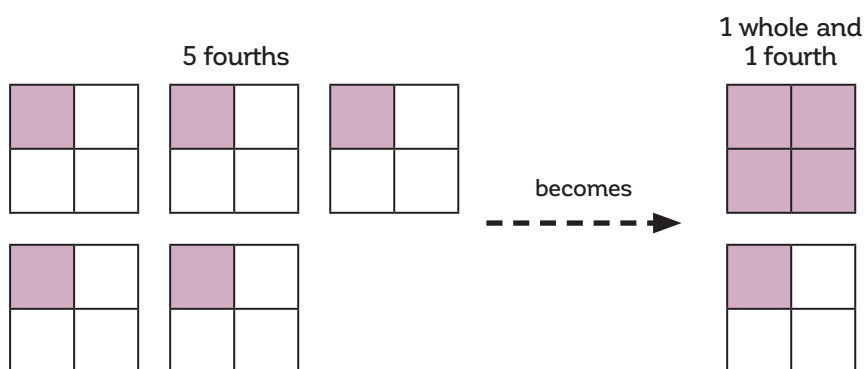
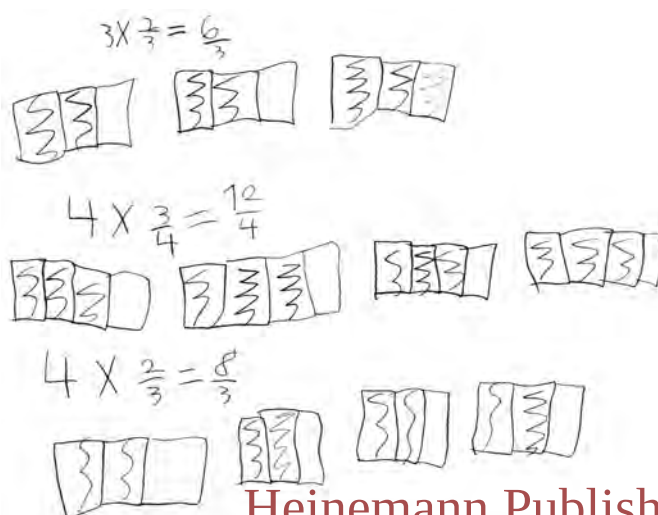


Figure 9.2 Students use an area model to show $5 \times \frac{1}{4}$.

Now that students recognize $6 \times \frac{1}{3}$ as meaning they have 6 one-thirds, they begin to explore multiplication with nonunit fractions. As they model $3 \times \frac{2}{3}$, they realize they have 6 thirds or $\frac{6}{3}$. By creating visual models, they are able to find products of whole numbers and nonunit fractions (see Figure 9.3).

Focusing on fractions with the denominators to 2, 3, 4, 5, 6, 8, 10, 12, and 100 allows fourth-grade students to explore fraction concepts with fractions that are easier to diagram and understand.



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As students are prompted to observe and make sense of the products, they begin to notice interesting patterns (see Figure 9.4).

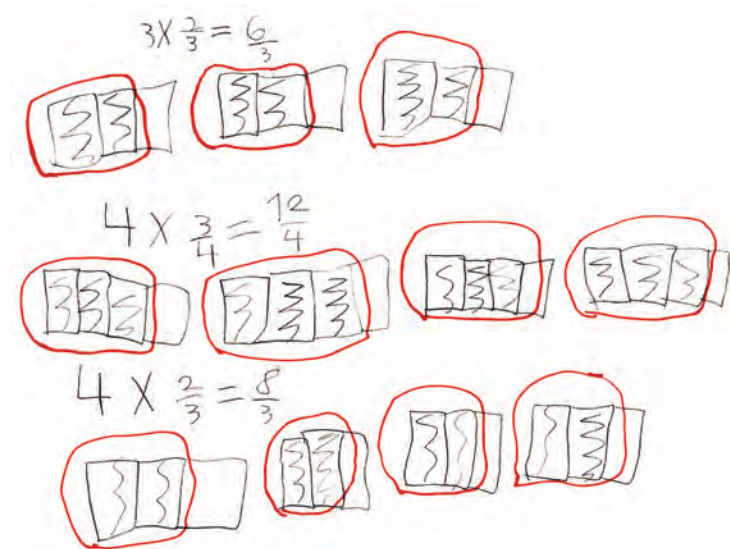


Figure 9.4 Students begin to see connections to whole-number multiplication as they see 3×2 , 4×3 , and 4×2 when determining the number of thirds, fourths, or halves in each model.

And then further observation yields discoveries as students connect the models to the equations (see Figure 9.5).



Figure 9.5 Students discover the rule for multiplying a whole number and a fraction by observing the models and equations and thinking about three $\frac{2}{3}$ as being 3×2 thirds.

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Students discover that:

When multiplying $3 \times \frac{2}{3}$, there are 3×2 thirds or 6 thirds.

When multiplying $3 \times \frac{3}{4}$, there are 3×3 fourths or 9 fourths.

When multiplying $4 \times \frac{2}{3}$, there are 4×2 thirds or 8 thirds.

Students discover they can multiply the whole number by the numerator to find out how many thirds (or fourths or sixths) they have. They discover the rule for multiplying a whole number by a fraction, and, equally important, they understand why it works.

And now that they understand both the rule and how it works, they are ready to use that understanding to multiply a fraction by a mixed number. Consider the problem:

The Billy Goat Trail was $4\frac{2}{3}$ miles long. Allison walked the trail 3 times. How far did she walk?

Students explore the problem with visual models (see Figure 9.6).

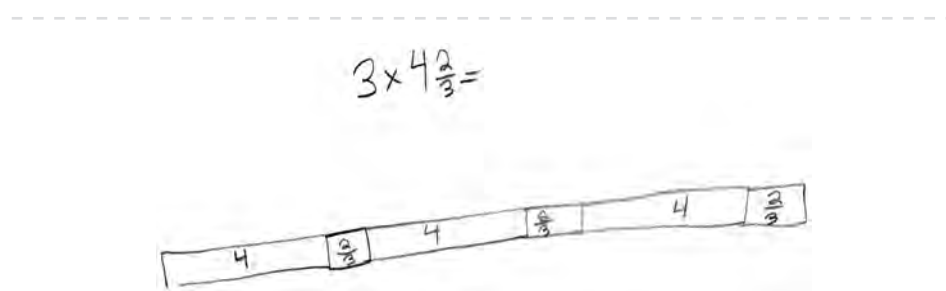


Figure 9.6 This student created a model to show $3 \times 4\frac{2}{3}$.

Students now use their understanding of multiplication to find the product by first multiplying the whole numbers (see Figure 9.7).

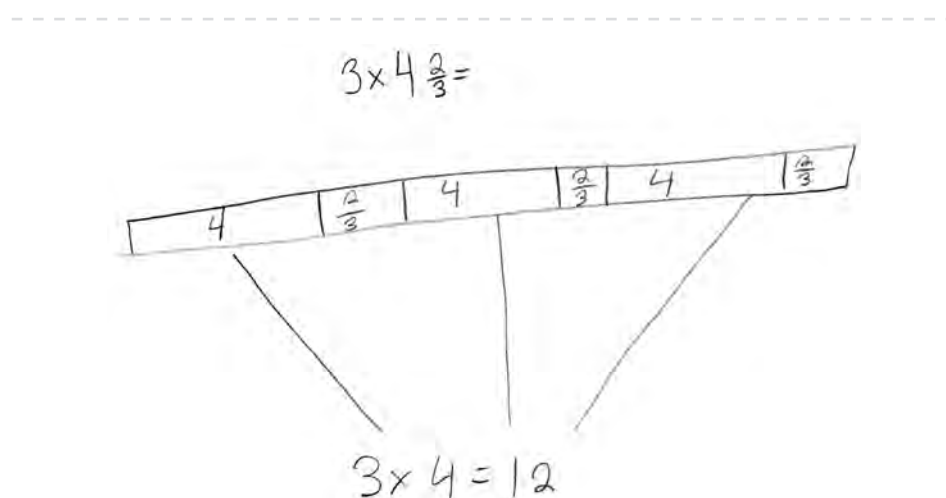


Figure 9.7 Students multiply 3×4 to find part of the answer.

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And then, students use their new understanding of multiplying whole numbers by fractions to multiply the fractions (see Figure 9.8).

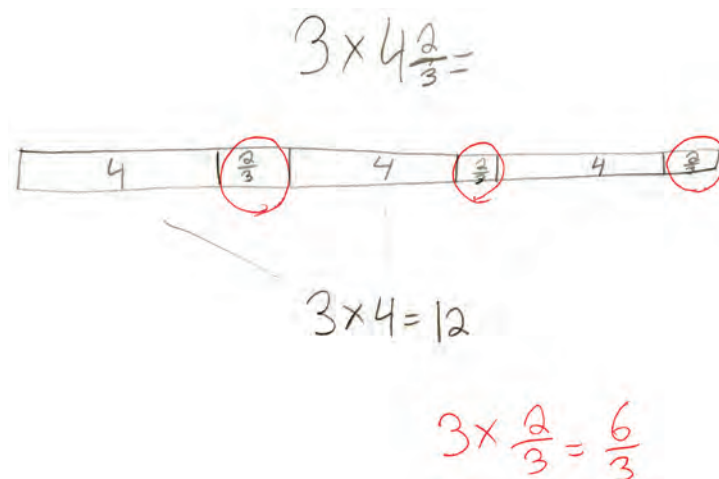


Figure 9.8 Students multiply $3 \times \frac{2}{3}$ to find another part of the answer.

Students then combine the $12 + \frac{6}{3}$ (recognizing it as 2) to find the product (14).

Allison walked 14 miles on the trail.

While students solved the problem using a visual model, they were actually using the distributive property. They separated $4\frac{2}{3}$ into $4 + \frac{2}{3}$ and multiplied 3×4 and then $3 \times \frac{2}{3}$ (see Figure 9.9). Making connections to the distributive property gives them another strategy for solving similar problems.

$$\begin{array}{r}
 3 \times 4\frac{2}{3} \\
 3 \times (4 + \frac{2}{3}) \\
 (3 \times 4) + (3 \times \frac{2}{3}) \\
 12 + \frac{6}{3} \\
 12 + 2 = 14
 \end{array}$$

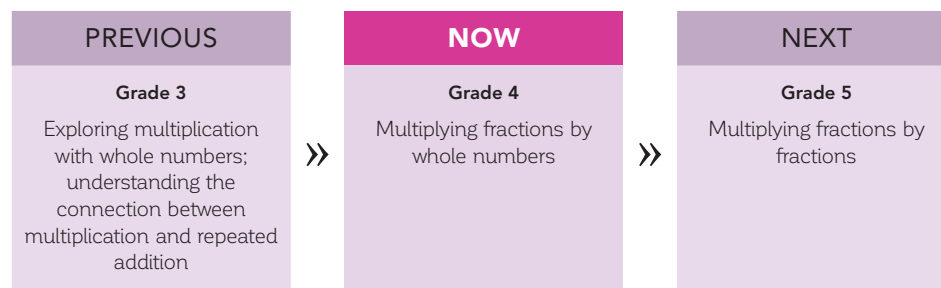
Figure 9.9 Connecting the visual model to the distributive property allows students to see the process in different ways, strengthening their understanding of the process and giving them options for solving similar problems.

Posing problems like the following provides a context for multiplying a fraction by a whole number and gives our students opportunities to apply their multiplication skills.

- Mrs. King needs $\frac{1}{3}$ cup of milk to make a lemon cake. If she makes 4 lemon cakes, how many cups of milk will she need?
- Molly uses $\frac{1}{4}$ yard of ribbon to make a hair bow. If she wants to make 5 hair bows, how much ribbon will she need?
- Kim runs $\frac{2}{3}$ mile every day. How far does she run in one week?
- Ms. Howard is making punch. The punch uses $\frac{3}{4}$ cup of orange juice for one serving. If she makes 8 servings, how many cups of orange juice does she need?

In order to effectively solve problems, our students must understand the problem and be able to build the appropriate equation to represent the situation as well as be able to perform the necessary computations. When we focus on both problem-solving and computational skills, we are providing them with the experience they need to be successful.

Exploring the Progression



Ideas for Instruction and Assessment

EXPLORING MULTIPLYING WHOLE NUMBERS BY FRACTIONS

Students explore the multiplication of a whole number by a fraction using area models.

Multiplication Connections

SHOW IT

Students use area models to explore the connection between repeated addition and multiplication of whole numbers and fractions.

Write the following on the board:

$$3 \times 4 = n$$

What does this mean? Turn and tell your partner a story that would go with 3×4 .

Learning Goals

I can multiply a whole number by a fraction using models and equations.

I can solve problems that involve multiplying a whole number by a fraction using visual models and equations.

Lessons in This Module

Exploring Multiplying Whole Numbers by Fractions

Multiplication Connections
Pattern Block Multiplication

Multiplying a Fraction by a Whole Number with Varied Models

More on Multiplying Fractions and Whole Numbers

Modeling Multiplication with a Number Line

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Have students share their stories (e.g., 3 baskets with 4 apples in each, 3 bags with 4 candies in each).

Draw a diagram to show 3×4 .

Share some diagrams.



What repeated addition equation works with this model?

Write the following on the board:

$$3 \times \frac{1}{4} = n$$

Can you think of a story that might make sense with this equation?

Brainstorm some stories together (e.g., 3 dishes of ice cream with $\frac{1}{4}$ cup in each dish, 3 plates with $\frac{1}{4}$ pizza on each plate).

Draw a diagram to show $3 \times \frac{1}{4}$.



Share some diagrams.

What repeated addition equation works with this model?

Think about 3×4 and $3 \times \frac{1}{4}$. How are these alike? How are they different? Share your ideas with your partner. (3×4 can be thought of as 3 groups of 4, and $3 \times \frac{1}{4}$ can be thought of as three groups of $\frac{1}{4}$ or 3 one-fourths.)

How does the product of 3 and 4 compare to the factors?

How does the product of 3 and $\frac{1}{4}$ compare to the factors?

That seems strange, doesn't it?

Work with your partner. Use your diagrams or a real-world example to explain why it makes sense that the product is less than one of the factors.

Common Error or Misconception

Students believe that when you multiply, your answer is always larger. This does not hold true when dealing with fractions. They may not get why $4 \times \frac{1}{3} = \frac{4}{3}$ or $1\frac{1}{3}$. Models will help them recognize this.

The idea of a “group of $\frac{1}{4}$ ” may seem strange to students, since a group with less than a whole item in it may be a new thought. Repeated use of diagrams and models can help students internalize this new idea.

SMP1; SMP3; SMP4; SMP7; SMP8

Thinking Through a Lesson

Pattern Block Multiplication

Students use pattern block pieces to visualize multiplication with fractions and whole numbers.

Distribute pattern blocks to students. Tell them that today the yellow hexagon will represent 1 whole. Ask them to turn to a partner and share

As students visualize fraction multiplication they gain insights about why the product of a whole number and a fraction less than one is less than the whole number.

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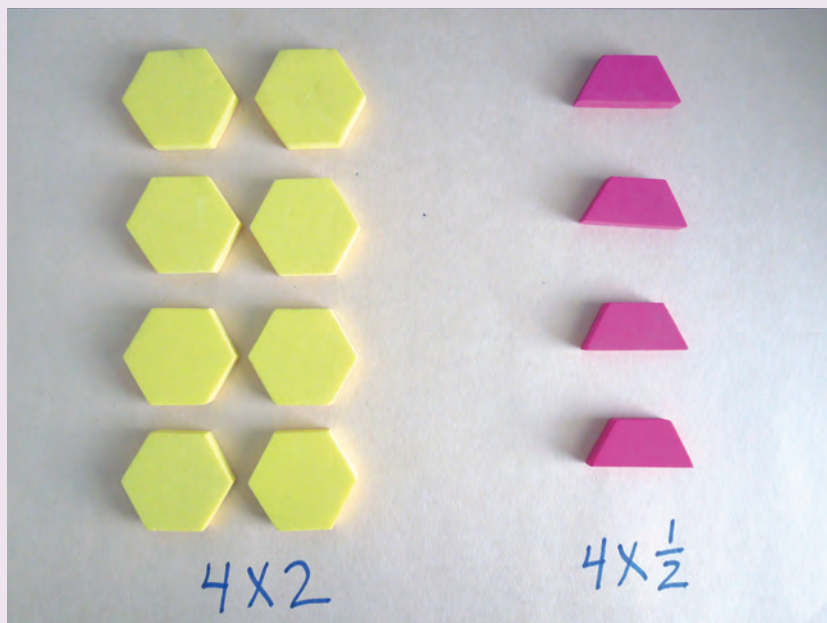
what fractional part of the whole each block represents. (The trapezoid is $\frac{1}{2}$, the rhombus is $\frac{1}{3}$, and the triangle is $\frac{1}{6}$.)

If I asked you to model 4×2 with blocks or counters, what might you do? (Make 4 groups of 2 hexagons.)

What about $4 \times \frac{1}{2}$? Work with your partner to use your blocks to model what that might look like.

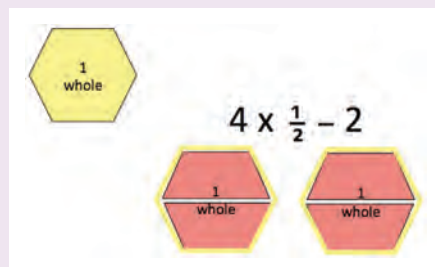
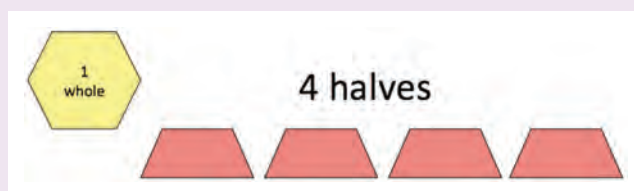
Have students share their models.

Observe as students work. If they are having difficulty starting, ask them to show you $\frac{1}{2}$ using the blocks. Then ask, "How would you show 4 halves?"



What is the product of $4 \times \frac{1}{2}$?

How many wholes would be equivalent to the blocks in your model? How could you prove that? (2; we can put them together to make shapes the same size as the hexagon.)



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Differentiation

While some students quickly see that it makes sense, others may still be confused that the product is less than one of the factors. Continue to provide visual examples to support their understanding.

Allowing students to move the fraction pieces to create 1 whole confirms that $\frac{5}{3} = 1\frac{2}{3}$.

Students may not articulate their reasoning this clearly at first. Encourage them to use pictures or models to support their explanations.

Differentiation

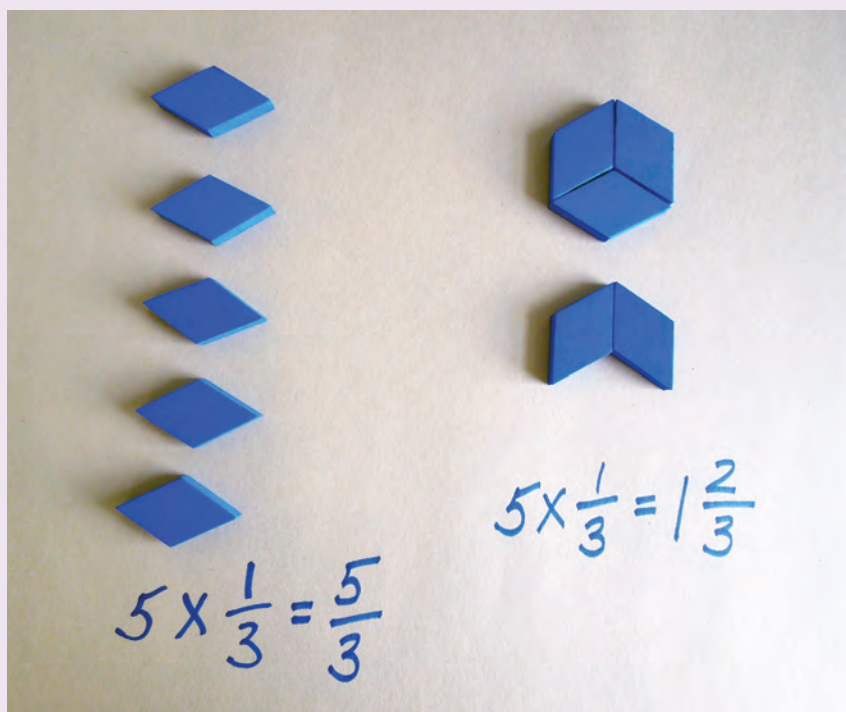
As most students work on their own, pull a small group of students who might benefit from additional guidance, talking them through the problems.

Does $4 \times \frac{1}{2} = 2$ sound right? Why or why not? (Yes; we know that two halves equals 1, so it makes sense that four halves would equal 2.)

Record $4 \times \frac{1}{2} = 2$ on the board.

Use your pattern blocks to show $5 \times \frac{1}{3}$. What is the product? ($\frac{5}{3}$.)

Is there another way we could write that? ($1\frac{2}{3}$.) Explain. (We put out 5 rhombuses. We made 3 of them into 1 whole, and that left 2 more so it is $1\frac{2}{3}$.)



Record $5 \times \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$ on the board.

Have students model $5 \times \frac{1}{6}$, and record the corresponding equation ($5 \times \frac{1}{6} = \frac{5}{6}$) on the board.

Talk to your partner. What do you notice about the relationship between the factors and the products? (The product is always less than the whole number, but greater than the fraction.)

Why does that happen? (We are making groups of pieces that are less than 1 whole, so no matter how many of them we put together the total will still be less than the number of groups.)

On your own, use the pattern blocks to solve these two problems.

Gabe uses $\frac{1}{2}$ cup of vinegar in his salad dressing recipe. How much vinegar would Gabe use to make 5 batches of his salad dressing?

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Diana collected $\frac{1}{6}$ of a bin of glass bottles to recycle. Henry collected 4 times as many bins as Diana. How many bins of bottles did Henry collect?

As students share their solutions, record the equations on the board with the other equations.

We already noticed that the products are less than one factor. What else do you notice about the relationship between the products and the factors? (The product is always a fraction with the same denominator as the fraction factor but with the numerator changed to the whole number.)

Why would that be? Work with your partner to use pictures or equations to think about why this makes sense. (It's just like adding that many fraction pieces like $4 \times \frac{1}{5}$ is 4 fifths.)

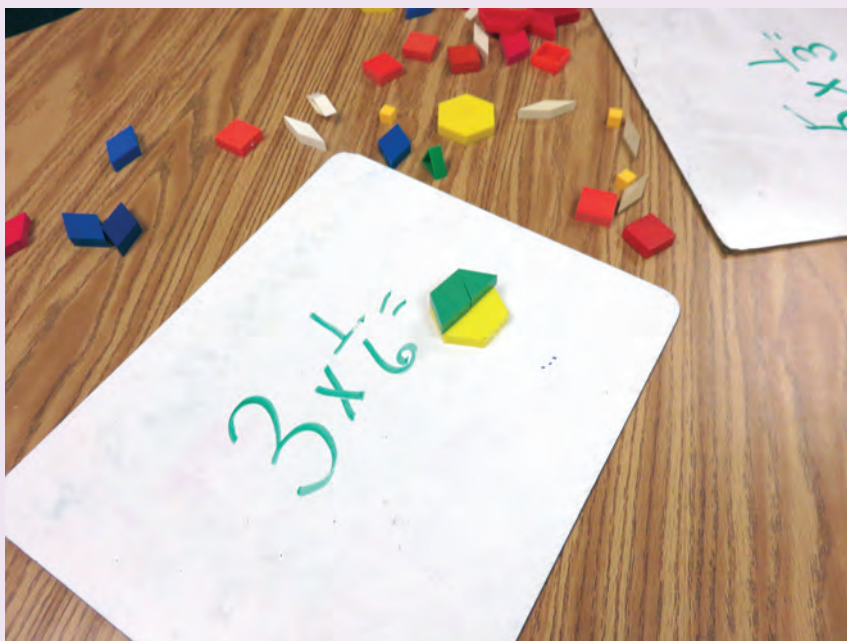
Provide additional practice tasks:

$$3 \times \frac{1}{6}$$

$$7 \times \frac{1}{2}$$

$$6 \times \frac{1}{3}$$

$$8 \times \frac{1}{6}$$



The student is showing that 3 groups of $\frac{1}{6} = \frac{3}{6}$ or $\frac{1}{2}$. By placing the $\frac{3}{6}$ on the whole hexagon, the student can see it is equal to $\frac{1}{2}$.

This problem provides a nice review of multiplicative comparison.

As students work, circulate to see if they are able to connect this idea with their experiences building fractions from unit fractions ($\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$ or $3 \times \frac{1}{6} = \frac{3}{6}$).

Students continue to use their understanding of equivalence as they explore other fraction concepts.

Multiplying a Fraction by a Whole Number with Varied Models

SHOW IT!

Students make connections between models they used for addition of fractions to represent multiplication.

Give students individual dry-erase boards.

Pose the following: $4 \times \frac{1}{8}$

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For more on the importance of partner talk, see Math in Practice: A Guide for Teachers, Chapter 4.

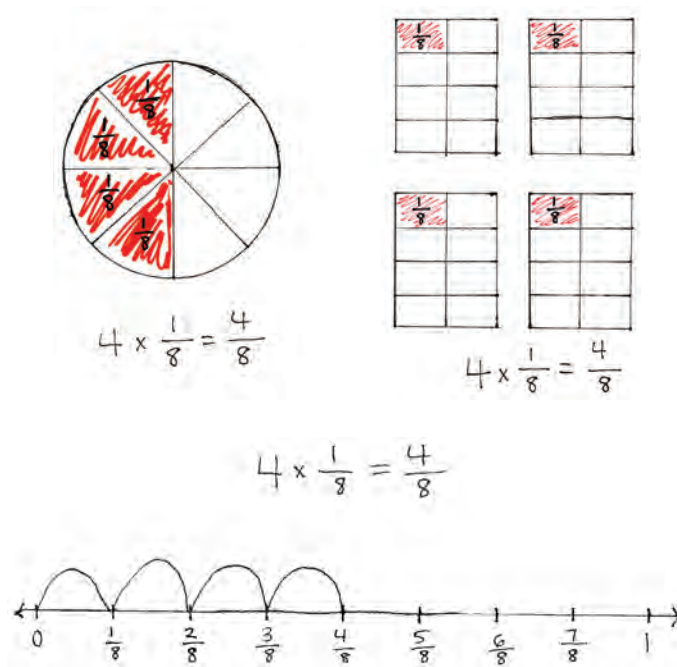
Talk with your partners about how you could create a model to show this equation.

Think about the models you used to show addition. Create a model for $4 \times \frac{1}{8}$ on your board.

Watch as students work with partners. Select a few students with different types of models (e.g., number lines or area models) to share their ideas.

What model did you choose? Explain how it shows $4 \times \frac{1}{8}$.

Draw some of the models on the board as students describe them.



Have students discuss how these multiplication models are similar to the addition models they used previously, making the connection to repeated addition.

Pose a few more expressions, having students model them in any way they choose.

$$5 \times \frac{1}{5} \quad 6 \times \frac{1}{10} \quad 5 \times \frac{1}{12} \quad 7 \times \frac{1}{8}$$

FORMATIVE ASSESSMENT

Option 1

Ask students to write a repeated addition expression that is equivalent to $5 \times \frac{1}{6}$.

If students struggle to come up with an equivalent expression, review the strategy with whole numbers. For some students, thinking about

repeated addition may be a more accessible strategy in the early stages of multiplying a whole number by a fraction.

Option 2

Ask students to model and solve the following:

$$4 \times \frac{1}{8} = n$$

Students can use any model that is appropriate to show the equation and should be able to find the product $\frac{4}{8}$ or $\frac{1}{2}$, but they do not need to simplify $\frac{4}{8}$ to $\frac{1}{2}$.

More on Multiplying Fractions and Whole Numbers

Students extend their understanding of multiplying fractions and whole numbers using nonunit fractions.

Pose the following:

Greta used $\frac{2}{5}$ cup of milk to make a batch of cookies. How much milk did she need for 3 batches?

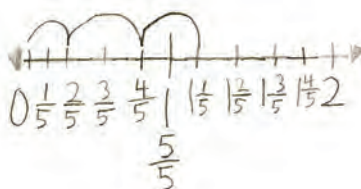
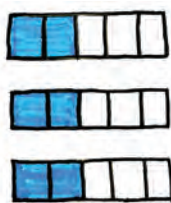
What operation would you use? Why?

What equation would show this problem?

What is different about this fraction than the others we have been working with?

Use any model you'd like to represent this problem. Be ready to share your model with the class.

Have students share their models, asking them to clarify the way they represented $\frac{2}{5}$.



$$3 \times \frac{2}{5} = \frac{6}{5}$$

$$3 \times \frac{2}{5} = 1\frac{1}{5}$$

Some students are likely to have gotten the product $\frac{6}{5}$ while others name the product $1\frac{1}{5}$.

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Which product is correct? Talk with your partner, and be ready to justify your answer.

Have students defend why $\frac{6}{5} = 1\frac{1}{5}$ using models or reasoning about $\frac{5}{5}$ being equal to 1 whole.

Record on the board: $3 \times \frac{2}{5} = \frac{6}{5}$

Pose the following for students to model and solve:

$$4 \times \frac{2}{3} \qquad 2 \times \frac{3}{5} \qquad 5 \times \frac{3}{10}$$

Have students share their models and solutions.

Record their solutions on the board:

$$\begin{aligned} 3 \times \frac{2}{5} &= \frac{6}{5} \\ 4 \times \frac{2}{3} &= \frac{8}{3} \\ 2 \times \frac{3}{5} &= \frac{6}{5} \\ 5 \times \frac{3}{10} &= \frac{15}{10} \end{aligned}$$

Talk with your partners:

What do you notice about the relationship between the factors and the product? Does anything seem to be repeating?

Based on what you see, can you find a rule for multiplying fractions by whole numbers?

Have students share their insights, which should include that the denominator stays the same since we are referring to fifths or thirds or tenths, and that the numerator is the product of the whole number and the fraction factor's numerator.

Does that make sense? Can you explain it?

Refer back to the models to help students visualize that $3 \times \frac{2}{5}$ was 3×2 fifths.

Post on the board.

$$3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$$

Have students check to see if that works for the other equations on the board.

Pose: $6 \times \frac{3}{8}$.

Turn and tell your partner what you predict the product will be. Then model it, and check your prediction.

Were you right?

Have students share their predictions and their solutions.

Provide a few more for students to predict and then model:

$$7 \times \frac{2}{5} \qquad 6 \times \frac{2}{3} \qquad 5 \times \frac{5}{5}$$

Differentiation

Observe the way students are solving the expressions. Pull a small group of students who are not seeing the relationship, and use pattern blocks or fraction circles to visually represent the problems. Starting with a problem such as $\frac{1}{3} \times 3 = 1$ with the blocks helps students see that we are talking about 3 one-thirds.

Differentiation

If students are unable to predict the product, provide more examples with models, discussing the connection to the rule.

Have students respond to the following in their math journals: What is the rule for multiplying a whole number by a fraction? Explain how it works.

Modeling Multiplication with a Number Line SHOW IT!

Students use number lines to model multiplication of nonunit fractions and whole numbers and make comparisons to unit fractions.

Solve this problem:

Anita is making muffins. The recipe calls for $\frac{2}{3}$ cup flour. She wants to triple the recipe. How much flour does she need?

Retell this problem to your partner.

What operation would you use to solve it? Why?

Solve this problem using a number line model.

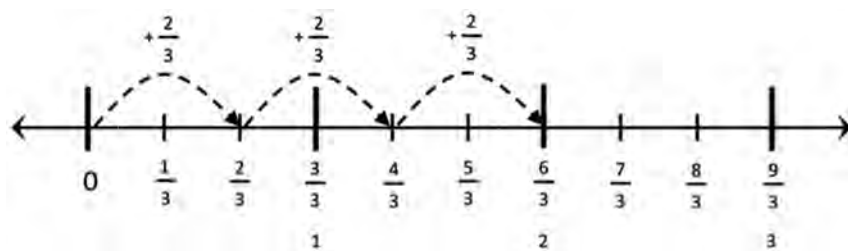
How will you partition your number line? Explain.

Will the product be greater than 1? How does that affect how you make your number line?

Circulate as students work.

Have pairs share their strategies.

How did you show the problem? Explain.



What equation matches your diagram?

What is the answer to the problem?

How many $\frac{1}{3}$ s are in your diagram?

What expression could your number line show if we were thinking about each third?

Is the product the same?

Is this true: $3 \times \frac{2}{3} = 6 \times \frac{1}{3}$? Explain your thinking.

These questions are extending students' thinking about what the fractions represent and the connection between one-third and two-thirds.

Have students model the following problem on a number line.

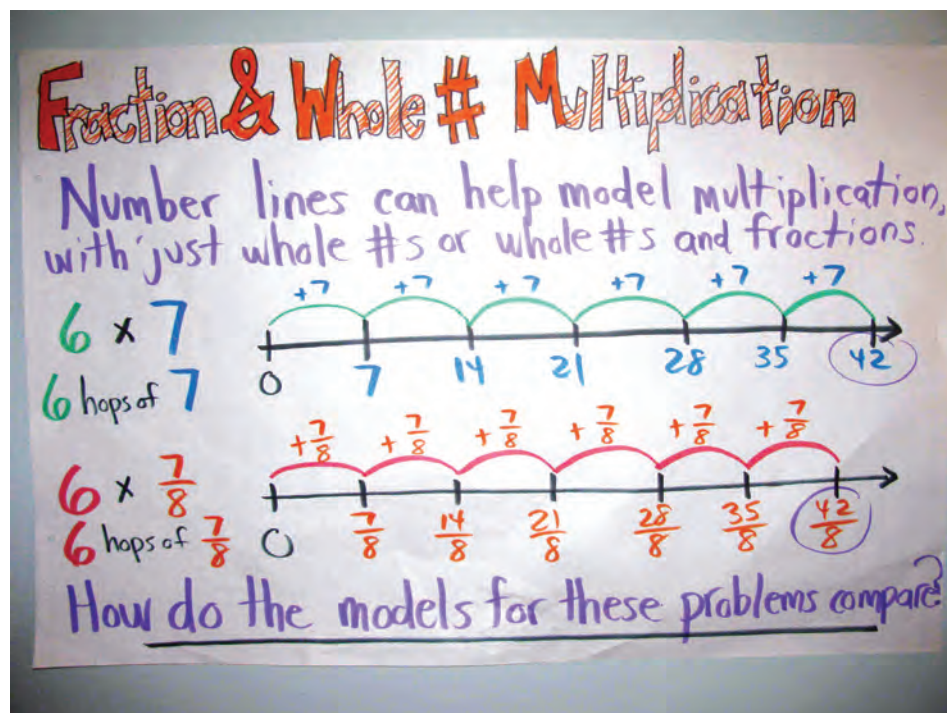
The elephants at the Washington zoo are fed $\frac{4}{5}$ of a barrel of corn each day. The bison are fed 2 times as much corn as the elephants. How many barrels of corn are the bison fed each day?

After students have time to work, have them share their strategies and the corresponding equation.

Create, or add to, the class anchor chart for multiplying a whole number by a fraction.

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**FORMATIVE ASSESSMENT**

Pose the following:

Carlos is making granola. He uses $\frac{2}{3}$ cup of oats in each batch. He is making 5 batches.

How many cups of oats does he need?

$$\frac{2}{3} \times 5 = \frac{10}{15}$$

Doesn't make sense.

This student is multiplying $\frac{2}{3} \times 5$ and getting $\frac{10}{15}$, noting that it doesn't make sense. Ask him to explain his comment. He may have realized that the answer is unreasonable but may be unsure how to find the accurate solution. This thinking about reasonableness should be praised. To build his understanding of the multiplication, return to concrete materials or visual models, solving similar problems and recording the equations. Have him think about repeated addition to check his answer.

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Additional Ideas for Support and Practice

The following ideas extend students' understanding of multiplying whole numbers and fractions and provide meaningful practice.

TALK ABOUT IT/WRITE ABOUT IT

When multiplying a whole number and a fraction, why is the product less than the original whole number factor?

Explain how to find the product of $\frac{1}{3} \times 5$ without using a model.

Nick said that $5 \times \frac{2}{3}$ is the same as $10 \times \frac{1}{3}$. Do you agree or disagree? Explain your thinking.

SPIN A FRACTION

Two players each pick a number card (1–10) and spin a fraction spinner ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$). (See the online resources.)

Each student multiplies the whole number by the fraction spun and then records the equation (e.g., $3 \times \frac{1}{5} = \frac{3}{5}$).

After ten rounds, each player finds the greatest product in their list of equations.

Players then compare them to see which is greater.

The player with the greater product wins.

Vocabulary

estimate	repeated
factors	addition
product	

You can increase the challenge by replacing the spinner with nonunit fraction cards. Write fractions on index cards (e.g., $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{2}{5}$, and $\frac{1}{10}$). Students pick a number card and a fraction card and find the product.

General Resources

Additional Problems
Fraction Spinner
Number Cards 1–10

ONLINE RESOURCES

These resources are
available at 




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