

PEARSON

Foundations and Pre-calculus Mathematics 10

MANITOBA
CURRICULUM COMPANION

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Manitoba Curriculum Companion

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Manitoba Curriculum Companion for Pearson Foundations and Pre-calculus Mathematics 10

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Lessons to Address Manitoba-Specific Content

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About This Module

The Manitoba Curriculum Framework departs from the WNCP Common Curriculum Framework in a few specific areas, often reflected in the Achievement Indicators. Use the planning charts to get general background on such curricular discrepancies. Use the Teaching Notes and Masters for support as you implement the material unique to Manitoba.

Chapter 1 Measurement

Lesson	Curriculum Note
1.1 Imperial Measures of Length	The material involving unit analysis is not required by the Manitoba Grade 10 curriculum. Exercise question 11b can be omitted.
1.2 Math Lab: Measuring Length and Distance	Include trundle wheels and micrometers in the list of measuring tools available for <i>Try This</i> .
1.3 Relating SI and Imperial Units	Question 3 of Discuss the Ideas and Exercise question 12b involve unit analysis. These questions are not required.
Checkpoint 1	
1.4 Surface Areas of Right Pyramids and Right Cones	
1.5 Volumes of Right Pyramids and Right Cones	
1.6 Surface Area and Volume of a Sphere	The Manitoba Grade 10 curriculum states that surface area and volume may be expressed with scientific notation. Encourage students to express their answers to questions 13 and 14 with scientific notation.
Checkpoint 2	
1.7 Solving Problems Involving Objects	
Study Guide and Review	
Practice Test	

Note: Allow students to use calculators as they work through the Exercise questions in this chapter.

Chapter 2 Trigonometry

Lesson	Curriculum Note
2.1 The Tangent Ratio	
2.2 Using the Tangent Ratio to Calculate Lengths	
2.3 Math Lab: Measuring an Inaccessible Height	
Checkpoint 1	
2.4 The Sine and Cosine Ratios	
2.5 Using the Sine and Cosine Ratios to Calculate Lengths	
Checkpoint 2	
2.6 Applying the Trigonometric Ratios	The Manitoba Grade 10 curriculum does not require students to solve right triangles. So, Example 1, Example 2, and Exercise question 6 may be omitted.
2.7 Solving Problems Involving More than One Right Triangle	
Study Guide and Review	Question 18 involves solving right triangles and is not required.
Practice Test	Question 5 involves solving right triangles and is not required.
Project: Measurement <i>Ramp It Up!</i>	
Cumulative Review: Chapters 1 and 2	Question 20 involves solving right triangles and is not required.

Chapter 4 Roots and Powers

Lesson	Curriculum Note
4.1 Math Lab: Estimating Roots	
4.2 Irrational Numbers	The Manitoba Grade 10 curriculum states that both horizontal and vertical number lines should be used to locate or order irrational numbers. In the Exercises, suggest students use horizontal number lines for questions 9 and 10 and vertical number lines for questions 11 and 12.
4.3 Mixed and Entire Radicals	
Checkpoint 1	
4.4 Fractional Exponents and Radicals	
4.5 Negative Exponents and Reciprocals	
Checkpoint 2	
4.6 Applying the Exponent Laws	
Study Guide and Review	
Practice Test	
Project: Algebra and Number <i>Human Calculators</i>	
Cumulative Review: Chapters 1–4	

Chapter 6 Linear Functions

Lesson	Curriculum Note
6.1 Slope of a Line	
6.2 Slopes of Parallel and Perpendicular Lines	
Checkpoint 1	
6.3 Math Lab: Investigating Graphs of Linear Functions	
6.4 Slope-Intercept Form of the Equation for a Linear Function	
6.5 Slope-Point Form of the Equation for a Linear Function	The Manitoba Grade 10 curriculum does not require students rewrite the equation of a linear relation from one form to another. However, students need this skill to use technology to graph an equation. So, it is recommended that students review Example 2b and complete Exercise questions 9b and 11.
Checkpoint 2	
6.6 General Form of the Equation for a Linear Relation	The Manitoba Grade 10 curriculum does not require students rewrite the equation of a linear relation from one form to another. However, students need this skill to use technology to graph an equation, or to identify equivalent linear relations. Example 1 and Exercise questions 6, 8b, and 18 are optional and may be omitted. The Manitoba Grade 10 curriculum does not require students to compare the graphs of linear relations in different forms. So, Exercise question 19 may be omitted.
6.7 The Length of a Line Segment	Use Master 6.14, Lesson 6.7, to cover content required in Manitoba only, for RF10: Solve problems that involve the distance between two points AI 10.1: Determine the distance between two points on a Cartesian plane using a variety of strategies. AI 10.4: Solve a contextual problem involving distance between two points ...
6.8 The Midpoint of a Line Segment	Use Master 6.15, Lesson 6.8, to cover content required in Manitoba only, for RF10: Solve problems that involve ... the midpoint of a line segment. AI 10.2: Determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies. AI 10.3: Determine an endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies. AI 10.4: Solve a contextual problem involving ... midpoint of a line segment.
6.9 Math Lab: Scatterplots and Lines of Best Fit	Use Master 6.16, Lesson 6.9, to cover content required in Manitoba only, for RF7: Determine the equation of a linear relation, given a scatterplot. AI 7.6: Determine the equation of the line of best fit from a scatterplot using technology and discuss the correlation.

6.10 Math Lab: Equation of the Line of Best Fit	Use Master 6.17, Lesson 6.10, to cover content required in Manitoba only, for RF7: Determine the equation of a linear relation, given a scatterplot. AI 7.6: Determine the equation of the line of best fit from a scatterplot using technology and discuss the correlation.
Study Guide and Review	Question 21b involves rewriting an equation in general form and is not required.
Practice Test	In question 3c, students are not required to rewrite the equation in general form.

Chapter 7 Systems of Linear Equations

Lesson	Curriculum Note
7.1 Developing Systems of Linear Equations	
7.2 Solving a System of Linear Equations Graphically	
7.3 Math Lab: Using Graphing Technology to Solve a System of Linear Equations	
Checkpoint 1	
7.4 Using a Substitution Strategy to Solve a System of Linear Equations	
7.5 Using an Elimination Strategy to Solve a System of Linear Equations	
Checkpoint 2	
7.6 Properties of Systems of Linear Equations	
Study Guide and Review	
Practice Test	
Project: Relations and Functions <i>Exercise Mind and Body</i>	
Cumulative Review: Chapters 1–7	Question 24 involves rewriting an equation in general form and is not required.

6.7

The Length of a Line Segment

Lesson Organizer

60 - 75 min

Key Math Learnings

The Pythagorean Theorem can be used to develop a formula to determine the distance between two points on a grid.

Curriculum Focus

Determine the distance between two points on a Cartesian plane.

SO	AI (MB)
RF10	10.1, 10.4

Processes

C, CN, PS, T, V

Student Materials

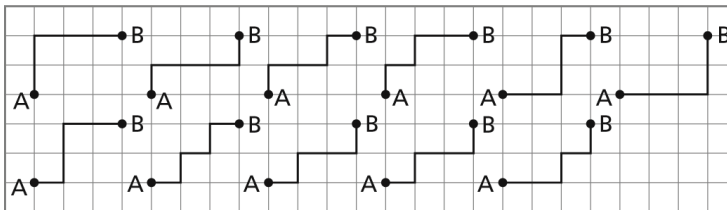
- Master 6.14
- ruler
- grid paper
- scientific calculator

Vocabulary

distance formula

Make Connections, page 19

Students should realize that Anoki can take many different routes. To determine the number of different routes, encourage students to use an organized list. Remind students that the route taken should be the shortest route along the sidewalks.



Each of these routes has length 1000 m, or 1 km. The length of the straight line from A to B is about 721 m. This can be determined by drawing a right triangle with hypotenuse AB, then using the Pythagorean Theorem.

Construct Understanding, page 19

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- How did you determine the length of AB? (*AB is a horizontal line, so I counted grid squares. AB has length 4 units.*)
- How did you determine the length of BC? (*BC is a vertical line, so I counted grid squares. BC has length 5 units.*)
- How did you determine the length of AC? (*AC is not a vertical line or a horizontal line, so I could not count grid squares. Since AC is the hypotenuse of a right triangle, I used the Pythagorean Theorem: $4^2 + 5^2 = AC^2$, $AC = \sqrt{41}$ units, or about 6.4 units.*)

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty understanding how to apply the Pythagorean Theorem to determine the length of DE.

- Have the student plot the points on 1-cm grid paper. Tell the student to draw a right triangle, $\triangle DEF$, with DE as its hypotenuse. The student can then count squares to determine the lengths of DF and EF, then use the Pythagorean Theorem to determine the length of DE. The student can measure the length of DE with a ruler to check.

DI Extending Thinking

Have students plot these points on a grid: A(3, 8), B(3, -1), C(-2, -1), D(-2, 8), then join the points to form rectangle ABCD. Students find the lengths of the diagonals, and the area and perimeter of the rectangle. (*Answer: $AC = BD \doteq 10.3$ units; Perimeter = 28 units; Area = 45 units²*)

Assessment for Learning

What to look and listen for:

- Students can use the Pythagorean Theorem to determine the length of a line segment on a grid.
- Students can count squares to determine the length of a horizontal or vertical line segment.
- Students can use the distance formula to determine the distance between two points.
- Students can solve a problem involving the distance between two points.

Debrief Try This: Assessment for Learning

Have students share their strategies for determining the length of any line segment on a grid. Ask questions such as:

- How did you determine the length of DE? (*I drew a right triangle with DE as its hypotenuse, then used the Pythagorean Theorem; DE is about 10.8 units long.*)
- How could you determine the length of any line segment on a grid? (*I could draw a right triangle with the line segment as its hypotenuse, then use the Pythagorean Theorem.*)
- Did anyone discover a different strategy? Explain. (*I discovered that I can square the difference in the x-coordinates of the points, then square the difference in the y-coordinates of the points. I add the results, then take the square root to determine the length of the line segment.*)

As you present the distance formula, some students may be confused by the use of subscripts in the formula; students may think the subscripts are powers. Emphasize the difference between subscripts and superscripts. It may be helpful to think of $P(x_1, y_1)$ as the first point chosen and $Q(x_2, y_2)$ as the second point chosen.

As you present *Example 3*, review the properties of isosceles, equilateral, and scalene triangles.

DI Identifying Common Difficulties: Possible Intervention

The student subtracts an x -coordinate from a y -coordinate, or a y -coordinate from an x -coordinate when using the distance formula.

- Relate the distance formula to a general diagram. Tell the student that we subtract two x -coordinates to determine a horizontal distance and two y -coordinates to determine a vertical distance.

DI Extending Thinking

Use the question prompts on Master 6.14.

p. 19: How could you verify this length?	<i>I could plot the points on 1-cm grid paper, then use a ruler to measure CD.</i>
p. 20, Example 1: What other ways are there to determine the lengths of FH and GH?	<i>Because each leg is horizontal or vertical, I could count grid squares to determine their lengths.</i>
p. 22, Example 3: Why do we write the lengths as radicals?	<i>When we want an exact answer for the length of a line segment, we leave it as a radical. It is one less calculation to make, and I avoid differences caused by rounding.</i>
p. 22, Example 3: In the solution for the length of VW, the expression $x_2 - x_1$ is negative. Why did this happen? Why does it not affect the answer for the length of VW?	<i>This happened because we subtracted a positive number from a negative number. The answer was not affected because when we square a negative number, we get a positive number.</i>

Assessing Understanding: Discuss the Ideas, page 23

Sample Responses

1. This triangle could be drawn in 2 different ways. For example, in Example 1, we could have drawn a right triangle by choosing the point $E(3, 1)$ above FG . The calculations would be the same because EF and GH are congruent horizontal segments and have the same length, and EG and FH are congruent vertical segments and have the same length.
2. The Pythagorean Theorem states that for any right triangle with side lengths a and b , and hypotenuse c , $c^2 = a^2 + b^2$. In the distance formula, $x_2 - x_1$ represents a , $y_2 - y_1$ represents b , and PQ represents c . The distance formula is like the Pythagorean Theorem solved for c . They are different in that $x_2 - x_1$ and $y_2 - y_1$ are differences, while a and b are lengths.
3. The formula would become $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. The length of the line segment would not change because once you square the difference between the coordinates, you get the same result whether you square a number or its opposite.

Exercises, page 23

Assessment of Learning – selected questions

AI (MB)	10.1	10.4
Question number	9	18

Sample Response: Reflect, page 24

To calculate the length of a line segment, I can:

- Draw a right triangle with the line segment as the hypotenuse, then use the Pythagorean Theorem.
- Count squares if the line segment is vertical or horizontal.
- Use the distance formula. When I use the formula, it doesn't matter which point I call P and which point I call Q because when I square the differences, I will always get positive numbers. I also know that when I want an exact answer, I leave the answer as a radical unless the radicand is a perfect square.

6.8

The Midpoint of a Line Segment

Lesson Organizer

60 - 75 min

Key Math Learnings

The midpoint of a line segment can be determined by calculating the mean of the x -coordinates and the mean of the y -coordinates of its endpoints.

Curriculum Focus

Determine the midpoint of a line segment.

SO	AI (MB)
RF10	10.2, 10.3, 10.4

Processes

C, CN, PS, T, V

Student Materials

- Master 6.15
- grid paper
- ruler
- Mira
- scientific calculator

Vocabulary

midpoint, mean, midpoint formula

Make Connections, page 25

Ask how many students have played number-guessing games. Invite a volunteer to describe a strategy Stephanie might have used.

What you might hear:

- Stephanie guesses the median number from the available numbers each time, depending on Vince's response. For example:

From 1 to 100:	50	Less
From 1 to 50:	25	Less
From 1 to 25:	12	Less
From 1 to 12:	6	Less
From 1 to 6:	3	Less
From 1 to 3:	2	Less
Less than 2:	1	Correct

Have students play the guessing games in pairs for numbers 1 to 1000. Students should discover that the number can be guessed in no more than 10 tries, using the same strategy as above.

Construct Understanding, page 25

Ensure students understand that the midpoint of a line segment is a point on the line segment that is the same distance from each endpoint.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- How did you determine the coordinates of the midpoint of PQ ? (*I counted grid squares. PQ has length 3 units, and one-half of 3 is 1.5. So, I counted 1.5 squares to the right of P . The midpoint of PQ has coordinates $(0.5, 3)$.)*
- How did you determine the coordinates of the midpoint of QR ? (*I counted grid squares. QR has length 6 units, and one-half of 6 is 3. So, I counted 3 squares down from Q . The midpoint of QR has coordinates $(2, 1)$.)*
- How did you determine the coordinates of the midpoint of PR ? (*I traced the triangle, then folded the tracing so that P coincided with R . The fold line intersected PR at $(0.5, 1)$. This is the midpoint of PR .)*
- What other strategy could you use? (*I could use a ruler to measure the length of PR , divide its length in half, then mark this length on the line segment. The mark would be at $(0.5, 1)$.)*

D1 Identifying Common Difficulties: Possible Intervention

The student thinks that one-half the length of a horizontal line segment is the x -coordinate of its midpoint.

Assessment for Learning

What to look and listen for:

- Students can count squares to determine the midpoint of a horizontal or vertical line segment.
- Students can use a Mira, a ruler, paper folding, or other strategies to determine the midpoint of a line segment.
- Students can use the midpoint formula to determine the midpoint of a line segment, given the endpoints of the segment.
- Students can use different strategies to determine an endpoint of a line segment, given the other endpoint and the midpoint.
- Students can solve a problem involving the midpoint of a line segment.

- Have the student copy $\triangle PQR$ on grid paper. The length of PQ is 3 units. The length divided by 2 is 1.5 units. Have the student plot the point $M(1.5, 3)$, then use a ruler to measure PM and MQ . The student should discover that M is not the midpoint of PQ . Point out that the midpoint is 1.5 units from either endpoint.

DI Extending Thinking

Have students determine the coordinates of the other endpoint, B , of line segment AB given that point $A(2, 3)$ is one endpoint and $M(4, 5)$ is its midpoint.

(Answer: $B(4, 5)$)

Debrief Try This: Assessment for Learning

Have students share their strategies for determining the midpoint of any line segment on a grid. Ask questions such as:

- How could you determine the midpoint of any line segment on a grid? (*I could draw a right triangle with horizontal and vertical legs and with the line segment as its hypotenuse, then determine the midpoints of the vertical and horizontal sides. I would then draw a perpendicular from the hypotenuse to each midpoint. Where the perpendiculars meet is the midpoint of the line segment.*)
- Did anyone use a different strategy? Explain. (*I discovered that the x -coordinate of the midpoint of any line segment is the mean of the x -coordinates of the endpoints, and its y -coordinate is the mean of the y -coordinates of the endpoints.*)

Before you present the midpoint formula, you might want to review how to find the mean of two numbers; for example 12 and 54.

Add the numbers: $12 + 54 = 66$

Divide their sum by 2: $66 \div 2 = 33$

The mean of 12 and 54 is 33.

DI Identifying Common Difficulties: Possible Intervention

The student confuses $x_1 + x_2$ and $y_1 + y_2$ in the midpoint formula with $x_2 - x_1$ and $y_2 - y_1$ in the distance formula.

- Encourage the student to think of a mean. The midpoint of a line segment is a mean, and the first step in calculating a mean is to add.

DI Extending Thinking

Use the question prompts on Master 6.15.

p. 27, Example 2: Why did we use the formula and not draw a graph?

The coordinates of the points are very large. My graph would have to be very big or the scales on the axes would have to be very large. It is more efficient to use the formula.

Assessing Understanding: Discuss the Ideas, page 29

Sample Responses

- I could place the Mira so that it passes through the midpoint and is perpendicular to the segment. I would look through the Mira and mark a point where I see the reflection image of the given endpoint. This is the other endpoint.*

I could use a ruler to extend the segment to the other side of the midpoint. I would measure the distance between the midpoint and the endpoint, then mark a point the same distance on the other side of the midpoint. This is the other endpoint.
- From a diagram, the midpoint can be located visually or by measuring, but it is not accurate unless the coordinates are integers. If the endpoints are far apart, graphing is not practical; if the scales on the axes are adjusted to fit the line segment on the grid, the accuracy of visually locating the midpoint is reduced. Using the midpoint formula, no graphing is required and it is an accurate method of determining the coordinates of the midpoint. However, errors may occur in calculations.*
- I can calculate the distance from the midpoint to each of the endpoints. If the coordinates are correct, the distances should be equal. For example, to verify that $M(2, 3)$ is the midpoint of the line segment with endpoints $A(-1, -3)$ and $B(5, 9)$:*

$$AM = \sqrt{(2 - (-1))^2 + (3 - (-3))^2} = \sqrt{9 + 36} = \sqrt{45}$$
$$MB = \sqrt{(5 - 2)^2 + (9 - 3)^2} = \sqrt{9 + 36} = \sqrt{45}$$

Since $AM = MB$, the coordinates of the midpoint are correct.

Exercises, page 29

Assessment of Learning – selected questions

AI (MB)	10.2	10.3	10.4
Question number	7	13	17

Sample Response: Reflect, SB page 30

To determine the midpoint of a line segment, I can:

- Count squares when the line segment is horizontal or vertical.*
- Measure the segment with a ruler, divide its length in half, then mark this length on the line segment.*
- Fold the paper so that one end of the segment coincides with the other end, then mark the point where the crease intersects the segment.*
- Place a Mira on the segment so that one end of the segment coincides with the other end, then mark the point where the Mira intersects the line segment.*
- Use the midpoint formula.*

Lesson Organizer

60 - 75 min

Key Math Learnings

The line of best fit drawn on a scatterplot suggests a possible relationship between the two sets of data.

Curriculum Focus

Draw the line of best fit for a set of data.

SO	AI (MB)
RF7	7.6

Processes

C, CN, PS, R, V

Student Materials

- Master 6.16
- tape measure or metre stick
- grid paper
- ruler

Vocabulary

scatterplot, line of best fit

Make Connections, page 31

Invite students to look at the graph of the cost of McIntosh apples. Have volunteers explain how they would estimate. What you might hear:

- To estimate the cost of 2.5 kg of apples, I would begin at 2.5 on the *Mass* axis, draw a vertical line to the graph, then draw a horizontal line from the graph to the *Cost* axis. This line intersects the *Cost* axis at about 3.25. So, the cost of 2.5 kg of apples is about \$3.25.
- To estimate the cost of 3.5 kg of apples, I would extend the grid so the *Mass* axis shows at least 3.5 kg, then use a ruler to extend the graph. I would begin at 3.5 on the *Mass* axis, draw a vertical line to the graph, then draw a horizontal line from the graph to the *Cost* axis. This line intersects the *Cost* axis at about 4.5. So, the cost of 3.5 kg of apples is about \$4.50.

These costs are approximate because their points do not lie where grid lines intersect.

Construct Understanding, page 31

Have one pair of students model what to measure for the arm span; the arms should be stretched out to the left and right, parallel to the floor. The measurement is from the tip of the longest finger on one hand to the tip of the longest finger on the other hand.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- From the table, does there appear to be a relationship between arm span and height? Explain. (*Yes, a person's arm span is close to his or her height.*)
- From the graph, what relationship do you see? (*The relationship appears to be close to linear. As the arm span increases, so does the height.*)

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty measuring the group members' heights accurately.

- Suggest that each person be measured in the same way. Since the soles and heels of shoes can vary, ensure students remove their shoes. Then, to locate the top of a person's head, place a book horizontally on her or his head, then measure to the bottom of the book.

Assessment for Learning

What to look and listen for:

- Students can draw a scatterplot for a set of data.
- Students can describe any trends in the data.
- Students can draw a line of best fit for a scatterplot.
- Students can use the line of best fit to make predictions.

D1 Extending Thinking

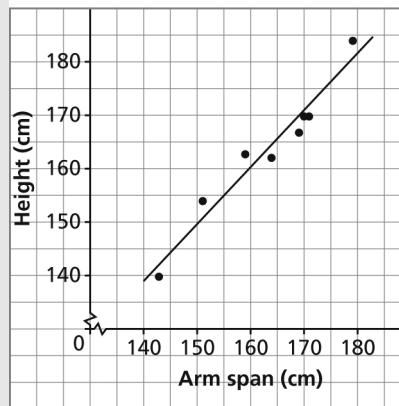
Based on the data from the table, students could determine who has the shape of a square (arm span = height) and who has the shape of a rectangle (arm span \neq height). Students could then take the same measurements on family members to determine if all members of the same family have approximately the same “shape.”

Debrief Try This: Assessment for Learning

Invite students to share their tables and graphs. What you might see:

Student's name	Arm span (cm)	Height (cm)	Student's name	Arm span (cm)	Height (cm)
A	171	170	E	159	163
B	169	167	F	179	184
C	164	162	G	143	140
D	170	170	H	151	154

Arm Span and Height for Eight People



Ask questions such as:

- What do you notice about the line of best fit? (*There are 3 points above the line and 5 points below the line, but all points are very close to the line.*)
- What does this tell you? (*The relationship between arm span and height is very close to linear.*)
- How did you predict the height of someone whose arm span is 1.4 m? (*I began at 140 on the Arm span axis, drew a vertical line to the line of best fit, then drew a horizontal line from the line to the Height axis. This line intersects the Height axis at about 138. So, I predicted the person's height would be about 138 cm.*)
- How did you predict the arm span of someone whose height is 1.8 m? (*I began at 180 on the Height axis, drew a horizontal line to the line of best fit, then drew a vertical line from the line to the Arm span axis. This line intersects the Arm span axis at about 178. So, I predicted the person's arm span would be about 178 cm.*)

DI Identifying Common Difficulties: Possible Intervention

The student always starts each axis at 0, and this often results in the line of best fit being very imprecise.

- Remind the student that he or she can use zigzag marks on the axes so that the data points are more spread out, and not all located in the top right corner of the graph. This way, the line of best fit will be more accurate.

DI Extending Thinking

Use the question prompts on Master 6.16.

p. 31: Suppose you had graphed heights horizontally and arm spans vertically. How would your graph change? Would this affect your predictions?

Because arm span and height are close in value, the graph would not change significantly. My predictions would not change; for example, to predict the height of someone whose arm span is 1.4 m, I would now begin at 140 on the Arm span axis, draw a horizontal line to the line of best fit, then draw a vertical line from the line to the Height axis. The result would not change.

Assess Your Understanding, page 32

Assessment of Learning – selected questions

AI (MB)	7.6
Question number	2

Math Lab: Equation of the Line of Best Fit

Lesson Organizer

75 - 90 min

Key Math Learnings

The correlation coefficient is a measure of how well the line of best fit fits the data.

Curriculum Focus

Determine the equation of the line of best fit for a scatterplot.

SO	AI (MB)
RF7	7.6

Processes

C, CN, PS, R, T, V

Student Materials

- Masters 6.17 and 6.18
- graphing calculator or computer with graphing software

Vocabulary

equation of the line of best fit, correlation coefficient

Make Connections, page 33

Invite students to look at the graph of the annual cost of a car-sharing plan. Have volunteers answer the questions. What you might hear:

- The slope of the graph is 7. It represents the additional cost of \$7 per hour driven.
- The C -intercept is 80. It represents the annual membership fee of \$80, which includes the application fee.
- An equation that describes the graph is: $C = 7h + 80$

Construct Understanding, page 33

Pair a student who has good graphing calculator skills with a student who has difficulties with a graphing calculator. Distribute copies of Master 6.18 to students who are using a TI-Nspire calculator.

Assessing Understanding: Observe and Listen

As students work, ask questions such as:

- Does there appear to be a linear relation between the two sets of data? (*As the year increases, the winning time tends to decrease. The relation between year and time is fairly linear.*)
- What is the equation of the line of best fit? (*The equation is: $y = -0.666x + 1449.39$*)
- How well does the line fit the data? (*The line fits the data very closely. Five data points are above the line and 4 data points are below the line; all points are close to the line.*)

DI Identifying Common Difficulties: Possible Intervention

The student expects the equation to be written in the form $y = mx + b$ and is confused when the calculator displays values for a and b .

- Tell the student that the calculator displays an equation in slope-intercept form as $y = ax + b$, where a represents the slope of the line. The student should relate the value of a to the value of m in the equation $y = mx + b$.

DI Extending Thinking

Have students research the winning times for the men's 1500-m speed-skating event since 1976. Have them graph the data and determine the equation of the line of best fit. Students explain the similarities and differences between the graphs and the lines of best fit for the men's and women's winning times.

Assessment for Learning

What to look and listen for:

- Students can use technology to draw a scatterplot and the line of best fit for a set of data.
- Students can determine the equation of the line of best fit for a set of data.
- Students can determine the correlation coefficient for a line of best fit and describe how well the line fits the data.
- Students can use the line of best fit to make predictions.

Debrief Try This: Assessment for Learning

Invite students to share their predictions for the winning time in 2016.

Ask questions such as:

- What is your prediction for the winning time in 2016?
(*About 106.37 s*)
- What other strategy could you use to determine the winning time?
(*I could substitute $x = 2016$ into the equation of the line of best fit, then solve for y . OR On my TI-83 calculator, I could press $\boxed{2nd}$ \boxed{TRACE} $\boxed{1}$, then enter $X = 2016$. The calculator shows $Y = 106.374$.)*)

As you introduce the correlation coefficient, tell students that not all data sets are necessarily linear, or close to linear. When r is close to 0, the points on the scatterplot are more random and do not show a linear trend.

Some students may be tempted to calculate the correlation coefficient for a set of data without graphing the data. For example, suppose a student calculated $r = 0.542$. This tells the student that the line of best fit does not fit the data very well, and so the points do not lie on or near a line. If the student had graphed the data, he or she might have discovered that only one point does not fit the trend of the graph, maybe as the result of a recording error. If this point were ignored, the correlation coefficient would be much closer to 1. For this reason, encourage students to start by graphing the data.

DI Identifying Common Difficulties: Possible Intervention

The student has difficulty tracing along the line of best fit to make a prediction because the cursor will not move beyond the window boundaries.

- Remind the student that when he or she wants to go to the left or right of the plotted points, it may be necessary to change the window settings to ensure that the required value is included in the domain or range.

DI Extending Thinking

Use the question prompts on Master 6.17.

p. 34: What do a and b represent?

a represents the slope of the line of best fit and b represents its vertical intercept.

Technology Note: Students who are not using a TI-83, TI-84, or TI-Nspire calculator may have to consult their calculators' handbooks for instructions on how to construct a scatterplot and determine the line of best fit.

Assess Your Understanding, page 35

Assessment of Learning – selected questions

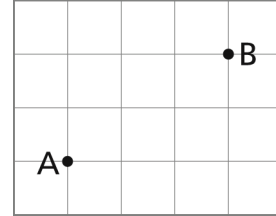
AI (MB)	7.6
Question number	3

6.7 The Length of a Line Segment

Lesson Focus: Determine the distance between two points on a Cartesian plane.

Make Connections

Anoki is at his home, A. He wants to walk to his friend's house, B. Each block in the diagram has dimensions 200 m by 200 m. Anoki uses the sidewalks and takes the shortest route. How many different routes can Anoki take? What is the length of each route? Suppose Anoki were able to travel in a straight line from A to B. What is the length of this route? Explain how you know.



Construct Understanding

Try This

Work with a partner.

You will need grid paper and a ruler.

- On a coordinate grid, plot these points:
 $A(-1, 3)$, $B(3, 3)$, $C(3, -2)$
 Join the points to form $\triangle ABC$.
 Determine the lengths of AB and BC .
 Determine the length of AC .
 What strategy did you use each time?
- On a coordinate grid, plot these points:
 $D(2, 4)$, $E(-4, -5)$
 Join the points.
 Determine the length of DE .
- Describe a strategy you could use to determine the length of any line segment on a grid.

We can use the Pythagorean Theorem to determine the length of a line segment on a grid.

To determine the length of CD , draw horizontal and vertical line segments to form right $\triangle CDE$.

Count squares to determine the lengths of the legs:

CE : 5 units

DE : 3 units

Use the Pythagorean Theorem:

$$CD^2 = CE^2 + DE^2$$

$$CD^2 = 5^2 + 3^2$$

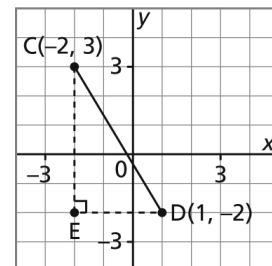
$$CD^2 = 25 + 9$$

$$CD^2 = 34$$

$$CD = \sqrt{34}$$

$$CD \doteq 5.8$$

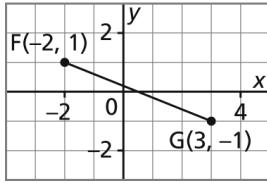
CD is approximately 5.8 units long.



How could you verify this length?

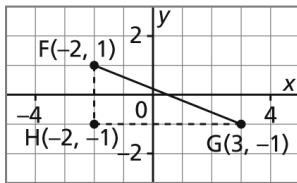
EXAMPLE 1 Determining the Length of a Line Segment

Calculate the length of line segment FG.



Solution

Draw a right $\triangle FGH$ for which FG is the hypotenuse.
Use the coordinates of the points to determine the lengths of the legs.



The length of FH is the difference in the y -coordinates: $1 - (-1) = 2$
The length of GH is the difference in the x -coordinates: $3 - (-2) = 5$

Use the Pythagorean Theorem.

$$FG^2 = FH^2 + GH^2$$

$$FG^2 = 2^2 + 5^2$$

$$FG^2 = 4 + 25$$

$$FG^2 = 29$$

$$FG = \sqrt{29} \quad \text{Use a calculator.}$$

$$FG \doteq 5.4$$

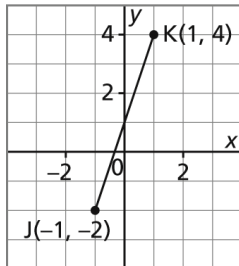
FG is approximately 5.4 units long.

What other ways are there to determine the lengths of FH and GH?

When we want an exact answer for the length of the segment, we write it as a radical.

CHECK YOUR UNDERSTANDING

Calculate the length of line segment JK.



[Answer: approximately 6.3 units]

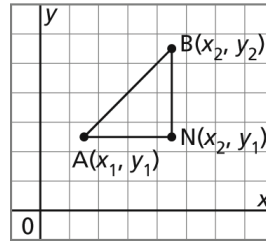
The strategy in *Example 1* can be used to develop a formula for the distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ on a grid.

$\triangle ANB$ is a right triangle.

The coordinates of N are (x_2, y_1) .

The length of AN is: $x_2 - x_1$

The length of NB is: $y_2 - y_1$



Use the Pythagorean Theorem:

$$AB^2 = AN^2 + NB^2$$

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

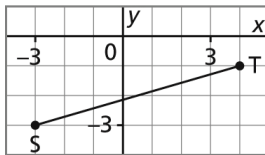
Distance Formula

The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on a coordinate grid is given by the

formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLE 2 Using the Distance Formula to Determine the Length of a Line Segment

Calculate the length of line segment ST .



Solution

The coordinates of S are: $(x_1, y_1) = (-3, -3)$

The coordinates of T are: $(x_2, y_2) = (4, -1)$

Use the formula.

$$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = 4, x_1 = -3, y_2 = -1, y_1 = -3$$

$$ST = \sqrt{(4 - (-3))^2 + (-1 - (-3))^2}$$

$$ST = \sqrt{7^2 + 2^2}$$

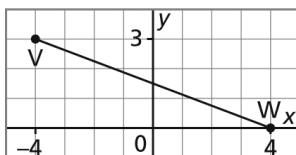
$$ST = \sqrt{53}$$

$$ST \doteq 7.3$$

ST is approximately 7.3 units long.

CHECK YOUR UNDERSTANDING

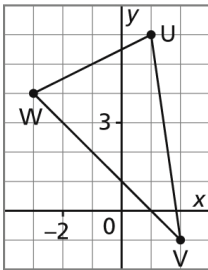
Calculate the length of line segment VW .



[Answer: approximately 8.5 units]

EXAMPLE 3 Solving a Problem Involving the Length of a Line Segment

Determine whether $\triangle UVW$ is isosceles.

**Solution**

Sides UV and VW appear to be the same length.

That would make $\triangle UVW$ isosceles.

Use the distance formula to calculate the length of sides UV and VW.

$$UV = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = 2, x_1 = 1, y_2 = -1, y_1 = 6$$

$$UV = \sqrt{(2 - 1)^2 + (-1 - 6)^2}$$

$$UV = \sqrt{(1)^2 + (-7)^2}$$

$$UV = \sqrt{50}$$

$$VW = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = -3, x_1 = 2, y_2 = 4, y_1 = -1$$

$$VW = \sqrt{(-3 - 2)^2 + (4 - (-1))^2}$$

$$VW = \sqrt{(-5)^2 + (5)^2}$$

$$VW = \sqrt{50}$$

Why do we write the lengths as radicals?

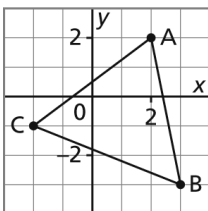
$$UV = VW = \sqrt{50}$$

Since two sides of $\triangle UVW$ have the same length, it is an isosceles triangle.

In the solution for the length of VW, the expression $x_2 - x_1$ is negative. Why did this happen? Why does it not affect the answer for the length of VW?

CHECK YOUR UNDERSTANDING

Determine whether $\triangle ABC$ is isosceles.



[Answer: No; $AB = \sqrt{26}$, $BC = \sqrt{29}$, $AC = 5$]

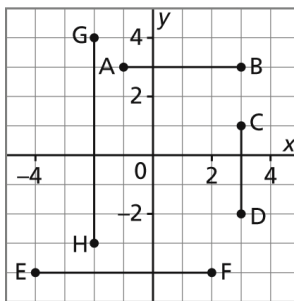
Discuss the Ideas

- To determine the length of a line segment, a right triangle was drawn with the segment as its hypotenuse. How many different ways could this triangle be drawn with one horizontal and one vertical leg? Use an example to explain.
- How is the distance formula like the Pythagorean Theorem? How is it different?
- We wrote the distance formula as $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. How would the formula change if we used $(x_1 - x_2)$ instead of $(x_2 - x_1)$? Would the length of the line segment change? Use an example to explain.

Exercises

A

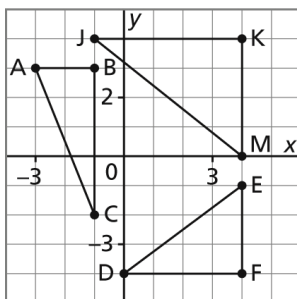
- Determine the length of each line segment on the grid below.



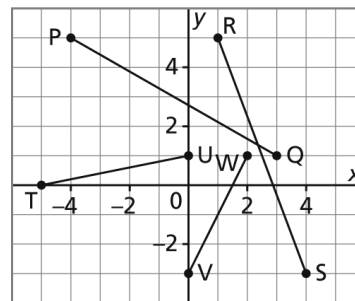
- Determine the length of the line segment with each pair of endpoints.
 - $J(5, 9), K(5, -4)$
 - $M(-2, 7), N(6, 7)$
 - $P(-4, -3), Q(-4, 11)$
 - $R(-1, 0), S(8, 0)$

B

- Calculate the lengths of the sides of each right triangle on the grid below.



- Calculate the length of each line segment on the grid. What strategy did you use? Why did you choose that strategy?



- Plot each pair of points on a grid. Calculate the distance between them.
 - $A(6, 4), B(2, 3)$
 - $C(-1, 4), D(5, 0)$
 - $E(-2, 0), F(9, -3)$
 - $G(0, 0), H(-4, 3)$
 - Choose one distance in part a. Summarize the steps you took to get the answer.
- Calculate the length of the line segment with each pair of endpoints.
 - $Q(6, 10), R(-1, 3)$
 - $S(2, -5), T(6, 3)$
 - $U(-2, 8), V(6, 3)$
 - $W(-8, -2), X(-2, 5)$
 - Plot the points $A(5, -2)$ and $B(-3, 4)$. Calculate the distance between them using:
 - the Pythagorean Theorem
 - the distance formula
 Compare these two methods.

- Why does the line segment that joins $C(5, 8)$ and $D(-3, 2)$ have length 10 units?

12. Determine the distances between A(4, -2) and B(-5, 1) and between A(4, -2) and H(5, -1). Give examples of two mistakes a student might make when calculating these distances.
13. On a grid, draw the triangle with each set of vertices. Classify each triangle as scalene, isosceles, or equilateral. Explain how you classified each triangle.
- a) B(-4, 3), C(-2, -4), D(3, 5)
 b) X(-2, 3), Y(2, 1), Z(-3, -1)
 c) E(1, 5), F(-3, 1), G(6, -4)
14. On a grid, plot these points: A(-3, 3), B(0, -6), C(3, -5), D(0, 4). Join the points to form quadrilateral ABCD.
- a) Determine the lengths of the sides of the quadrilateral. Write each length as a mixed radical where possible.
 b) Determine the perimeter and area of the quadrilateral.
 c) Determine the lengths of its diagonals.
 d) What type of quadrilateral is it? How do you know?
15. A circle has centre C(1, 1) and radius 5. Which of the points below are on the circle? How do you know?
- a) J(5, 4) b) K(2, 6)
 c) M(-3, -2) d) N(5, -1)
16. a) Calculate the length of the line segment that joins C(2, 4) to each point.
- i) D(2, 0) ii) E(3, 0)
 iii) F(4, 0) iv) G(5, 0)
- b) A line segment joins point H(x, 0) to C. It has the same length as CF. Determine the value of x.
- c) A line segment joins point J(-2, y) to C. It has the same length as CG. Determine the value of y. How many different values can you find?
17. a) For each given length of a line segment, determine possible coordinates of the endpoints of a line segment with that length.
- i) 1 unit ii) 5 units
 iii) $\sqrt{13}$ units iv) $\sqrt{34}$ units
- b) Compare your answers with those of a classmate. Did you get the same answers? Explain.
18. Chico and Cheri take part in a scavenger hunt at the Lower Fort Garry National Historic Site in Manitoba. They begin the hunt at the same starting point. Chico walks 25 m north, then 40 m east to find his first treasure. Cheri walks 60 m south, then 35 m west and stops.
- a) How far is each person from the starting point? How do you know your answers are reasonable?
 b) How far are they from each other?
 c) Suppose Chico looks directly at Cheri. Will the starting point be in his line of sight? Explain.
 Give all answers to the nearest metre.
- C**
19. Which points below are the same distance from Y(0, -4) and Z(-2, 0)?
- a) A(3, 0) b) B(7, 3)
 c) C(-7, -5) d) D(-1, -2)
20. Determine the coordinates of the points on the x-axis that are 10 units from M(6, 8).
21. Determine the coordinates of the point on the y-axis that is equidistant from each pair of points.
- a) A(3, 0) and B(3, 6)
 b) P(4, 0) and Q(2, 6)
 c) S(5, 0) and T(1, 6)
 Describe the strategy you used to determine the coordinates.

Reflect

Summarize the important ideas in calculating the length of a line segment. Include examples in your summary.

6.8 The Midpoint of a Line Segment

Lesson Focus: Determine the midpoint of a line segment.

Make Connections

Vince thinks of a number from 1 to 100.

Stephanie says she can guess his number in no more than 7 tries.

With each guess, Vince either says “greater” or “less.”

What strategy might Stephanie use?

How many tries would it take to guess

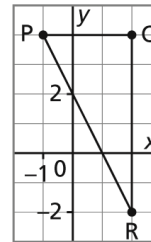
a number from 1 to 1000?

Construct Understanding

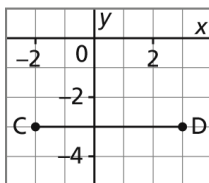
Try This

Work with a partner.

- Determine the coordinates of the midpoint of PQ and the midpoint of QR.
What method did you use?
- Determine the coordinates of the midpoint of PR.
Did you use the same method as you used in step A?
If not, describe how you determined the coordinates of this midpoint.
- Explain the steps you could use to determine the midpoint of any line segment on a grid.
Draw an oblique line segment on a grid.
Use your strategy to determine the midpoint of the line segment.



We can count squares to determine the midpoint of a horizontal or vertical line segment on a grid when its endpoints are on lattice points.

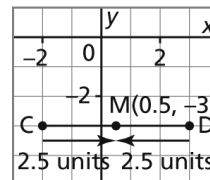


The length of CD is 5 units.

One-half of 5 is 2.5.

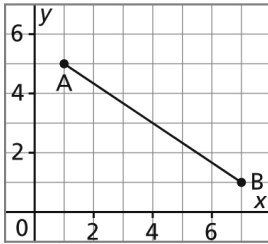
So, the midpoint of CD is 2.5 units to the right of C, or 2.5 units to the left of D.

The midpoint of CD is M, with coordinates $(0.5, -3)$.



EXAMPLE 1 Determining the Midpoint of a Line Segment

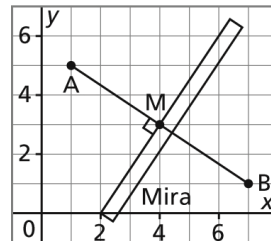
Determine the coordinates of the midpoint, M, of line segment AB.



Solution

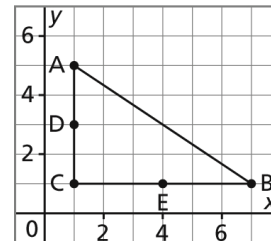
Method 1

Place a Mira perpendicular to AB.
 Move the Mira until A coincides with B.
 Mark the point where the Mira intersects the line segment.
 Label the point M.
 This is the midpoint of segment AB.
 It appears to have coordinates (4, 3).

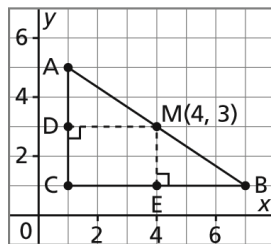


Method 2

Draw a right $\triangle ABC$ for which AB is the hypotenuse.
 Determine the midpoint of AC.
 AC is 4 units long.
 One-half of 4 is 2.
 So, the midpoint of AC is 2 units down from A.
 Label the point D. It has coordinates (1, 3).



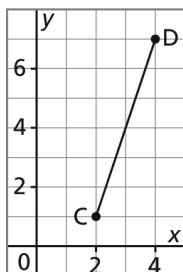
Determine the midpoint of BC.
 BC is 6 units long.
 One-half of 6 is 3.
 So, the midpoint of BC is 3 units to the left of B.
 Label the point E. It has coordinates (4, 1).



Draw the perpendicular to AC at D.
 Draw the perpendicular to BC at E.
 The perpendiculars meet at M, the midpoint of AB.
 The coordinates of M are (4, 3).
 So, the midpoint M of segment AC is M(4, 3).

CHECK YOUR UNDERSTANDING

Determine the coordinates of the midpoint, M, of line segment CD.



[Answer: M(3, 4)]

The strategy in Method 2 of *Example 1* can be used to develop a formula for the midpoint of a line segment on a grid.

In the diagram above, A has coordinates (1, 5) and B has coordinates (7, 1).

The coordinates of M are the means of the coordinates of A and B.

$$\begin{aligned}x\text{-coordinate of } M &= \frac{1+7}{2} \\ &= 4\end{aligned}$$

$$\begin{aligned}y\text{-coordinate of } M &= \frac{5+1}{2} \\ &= 3\end{aligned}$$

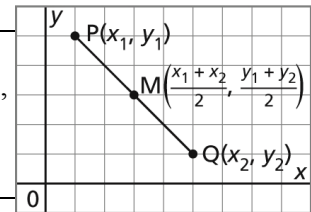
The coordinates of M are (4, 3).

The *mean* of 2 numbers is the sum of the numbers divided by 2.

Midpoint Formula

When M is the midpoint of a line segment with endpoints P(x_1, y_1) and Q(x_2, y_2),

$$\text{the coordinates of M are: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



EXAMPLE 2 Solving a Problem Using the Midpoint of a Line Segment

An engineer has to design a highway so that it is the same distance from Port Whitby, located at P(6826, 2014), and Comet, located at C(6792, 1996). Determine the coordinates of one point on the highway.

Solution

One point on the highway is the midpoint, M, of the line segment PC.

Use the formula.

$$\begin{aligned}x\text{-coordinate of } M &= \frac{x_1 + x_2}{2} \\ &= \frac{6826 + 6792}{2} \\ &= \frac{13\,618}{2} \\ &= 6809\end{aligned}$$

Substitute: $x_1 = 6826$
and $x_2 = 6792$

$$\begin{aligned}y\text{-coordinate of } M &= \frac{y_1 + y_2}{2} \\ &= \frac{2014 + 1996}{2} \\ &= \frac{4010}{2} \\ &= 2005\end{aligned}$$

Substitute: $y_1 = 2014$
and $y_2 = 1996$

Why did we use the formula and not draw a graph?

The coordinates of M are (6809, 2005).

One point on the highway has coordinates (6809, 2005).

CHECK YOUR UNDERSTANDING

An engineer has to design a highway so that it is the same distance from Red Soil, located at R(5312, 4308), and White Springs, located at W(5368, 4344).

Determine the coordinates of one point on the highway. [Answer: (5340, 4326)]

EXAMPLE 3 Determining an Endpoint of a Line Segment Given the Other Endpoint and the Midpoint

Point $J(-3, 4)$ is one endpoint of line segment JK . Point $M(2, 1)$ is its midpoint. Determine the coordinates of the other endpoint, K .

Solution**Method 1**

Plot J and M on a grid. Join the points.

Count squares.

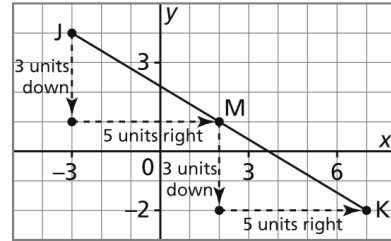
To get from J to M , move 3 units down and 5 units right.

Since M is the midpoint of JK ,

to get from M to K , move 3 units down and 5 units right.

Plot point K . Join K and M .

K has coordinates $(7, -2)$.

**Method 2**

Let the endpoint K have coordinates (x_2, y_2) .

One endpoint is $J(-3, 4)$.

The midpoint is $M(2, 1)$.

Use the midpoint formula to determine the coordinates of K .

The x -coordinate of M is:

$$2 = \frac{x_1 + x_2}{2} \quad \text{Substitute: } x_1 = -3$$

$$2 = \frac{-3 + x_2}{2} \quad \text{Multiply each side by 2.}$$

$$2(2) = 2\left(\frac{-3 + x_2}{2}\right)$$

$$4 = -3 + x_2 \quad \text{Add 3 to both sides to isolate } x_2.$$

$$3 + 4 = -3 + 3 + x_2$$

$$x_2 = 7$$

The y -coordinate of M is:

$$1 = \frac{y_1 + y_2}{2} \quad \text{Substitute: } y_1 = 4$$

$$1 = \frac{4 + y_2}{2} \quad \text{Multiply each side by 2.}$$

$$2(1) = 2\left(\frac{4 + y_2}{2}\right)$$

$$2 = 4 + y_2 \quad \text{Subtract 4 from both sides to isolate } y_2.$$

$$2 - 4 = 4 - 4 + y_2$$

$$y_2 = -2$$

The coordinates of K are $(7, -2)$.

CHECK YOUR UNDERSTANDING

Point $P(-1, 5)$ is one endpoint of line segment PQ . Point $M(3, 2)$ is its midpoint.

Determine the coordinates of the other endpoint, Q .

[Answer: $Q(7, -1)$]

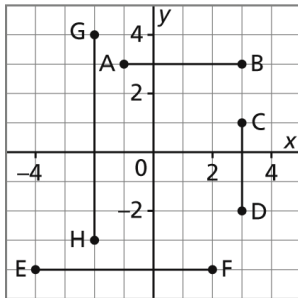
Discuss the Ideas

- How could you use a Mira to determine one endpoint of a line segment, given the other endpoint and the midpoint? What other strategy could you use?
- You can determine the coordinates of the midpoint of a line segment from a diagram and by using the midpoint formula. What are the advantages and disadvantages of each strategy?
- How can you verify that the coordinates of a midpoint are correct? Give an example.

Exercises

A

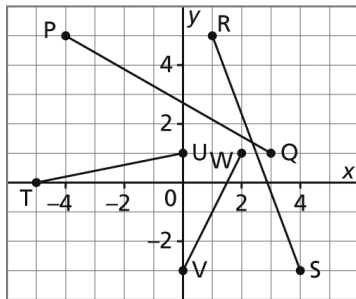
- Determine the coordinates of the midpoint of each line segment on the grid below.



- Draw a line segment with each pair of endpoints. Determine the coordinates of the midpoint of each line segment. What strategy did you use?
 - $J(5, 9)$, $K(5, -4)$
 - $M(-2, 7)$, $N(6, 7)$
 - $P(-4, -3)$, $Q(-4, 11)$
 - $R(-1, 0)$, $S(8, 0)$

B

- Determine the coordinates of the midpoint of each line segment on the grid below. What strategy did you use?



- Determine the coordinates of the midpoint of the line segment with each pair of endpoints.
 - $A(7, 4)$, $B(1, 3)$
 - $C(-3, 4)$, $D(6, 0)$
 - $E(-5, 0)$, $F(8, -3)$
 - $G(0, 0)$, $H(-4, 5)$

- Choose one segment from part a. Explain the steps you took to get the answer.

- On a grid, draw the triangle with vertices $A(0, 4)$, $B(8, 2)$, and $C(6, 10)$.
 - Determine the coordinates of the midpoint of each side. Use a different strategy to check your answer.
 - Determine the length of each side of the triangle.
 - Join each pair of midpoints. Determine the length of each line segment.
 - How are the lengths in parts b and c related?
 - What happens to the answer to part d if the coordinates of A , B , and C are doubled? Justify your answer.

- Draw a line segment on a blank sheet of paper.
 - Describe how you can use a ruler to determine the midpoint of the line segment.
 - Describe how you can use paper folding to determine the midpoint.
 - Describe a different method to solve this problem.

- The endpoints of the diameter of a circle are $E(6, 6)$ and $F(-2, -4)$.
 - Determine the coordinates of the centre of the circle.
 - Determine the length of the radius.

- On a grid, plot the points $Q(-2, -4)$, $R(5, -2)$, $S(8, 3)$, and $T(1, 1)$. Join the points to form parallelogram $QRST$. Determine the coordinates of the midpoint of each diagonal. What do you notice? Explain.

12. On a grid, plot the points $D(-2, 4)$, $E(3, 0)$, $F(1, -3)$, and $G(-5, 1)$. Join the points to form quadrilateral $DEFG$. Determine the coordinates of the midpoint of each diagonal. Is $DEFG$ a parallelogram? How do you know?
13. a) Each point Q is one endpoint and M the midpoint of a line segment. Determine the coordinates of the other endpoint of each segment.
- $Q(8, -1)$, $M(5, -5)$
 - $Q(-5, 2)$, $M(0, 5)$
 - $Q(-6, -4)$, $M(-3, -2)$
- b) Explain the steps you took to get the answer to part i.
14. How could you use a rotation to determine the coordinates of the other endpoints in question 13a?
15. The numerical coordinates of a map are in kilometres. The town of Saddleville is at $(3.5, 6.9)$ and Pedal Place is at $(10.7, 4.1)$. A bicycle path is to be built in a straight line joining the two towns. Each town is responsible for the construction of the path to the midpoint. The cost of construction is \$47 000 per kilometre.
- Determine the coordinates of the midpoint of the bicycle path.
 - Determine the cost of the construction for Saddleville, to the nearest dollar.
 - What is the total cost of the bicycle path, to the nearest dollar?
- Think of another way you could solve this problem.
16. Many people say that Red Deer is half-way between Edmonton and Calgary. When a coordinate grid is superimposed on a map, the coordinates for the cities are: Edmonton $(4887, 7824)$, Calgary $(5438, 7811)$, and Red Deer $(5168, 7820)$. Is Red Deer really half-way between Edmonton and Calgary? Justify your answer.
17. Mia and an old school friend decide to meet at a location midway between their homes. Mia used a website to find the midpoint of the direct line between their two addresses. The numerical coordinates of the map are in kilometres. Mia's home is at $(2.2, 17.9)$ and her friend's home is at $(11.3, 1.4)$. The computer suggested they meet at $(7.8, 10.7)$. Is this location midway between their homes? If yes, how far does each person have to travel? If not, what are the coordinates of the point midway between their homes?
18. A line segment has endpoints $A(-2, -6)$ and $B(8, 4)$. Determine the coordinates of the 3 points that divide AB into 4 equal parts. Describe the strategy you used.
- C**
19. A line segment has endpoints $E(x_1, y_1)$ and $F(x_2, y_2)$. Determine expressions for the coordinates of the points that divide EF into 4 equal parts.
20. A line segment has endpoints $S(4, 8)$ and $T(-2, -1)$. Determine the coordinates of the points that divide ST into 3 equal parts. Explain the steps you used.
21. A line segment has endpoints $C(x_1, y_1)$ and $D(x_2, y_2)$. Determine expressions for the coordinates of the points that divide CD into 3 equal parts.
22. A square has vertices $S(3, 1)$, $T(10, 4)$, $U(7, 11)$, and $V(0, 8)$.
- Calculate the mean of the x -coordinates and the mean of the y -coordinates of the vertices.
 - Use the results of part a as the coordinates of a point W . How is point W related to the square? Why do you think this result occurs?

Reflect

Describe different methods you can use to determine the midpoint of a line segment.

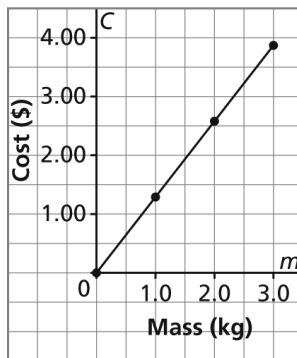
6.9 Math Lab: Scatterplots and Lines of Best Fit

Lesson Focus: Draw the line of best fit for a set of data.

Make Connections

This graph shows the costs of different masses of McIntosh apples at St. Norbert Farmers' Market in Winnipeg.

Cost of McIntosh Apples



The points lie on a straight line.

How could we use the graph to determine:

- the cost of 2.5 kg of apples?
- the cost of 3.5 kg of apples?

Are these costs approximate or exact?

Construct Understanding

Try This

Work in a group of 4. You will need a tape measure or a metre stick, and grid paper.

- A.** Measure and record your arm spans with your arms stretched out as much as possible.

Measure and record your heights.

Combine your data with those of another group.

Record the data in a table.

Student's name	Arm span (cm)	Height (cm)

- B.** Plot the data on a grid. Represent *Arm span* on the horizontal axis and *Height* on the vertical axis. Describe any trends in the data.

What appears to be the relationship between arm span and height?

- C.** Place a ruler so that it passes as close as possible to all the plotted points (use a transparent ruler, if possible). Draw a straight line along the ruler.

- D.** Use the line you drew.

- Predict the height of someone whose arm span is 1.4 m.
- Predict the arm span of someone whose height is 1.8 m.

Suppose you had graphed heights horizontally and arm spans vertically. How would your graph change? Would this affect your predictions?

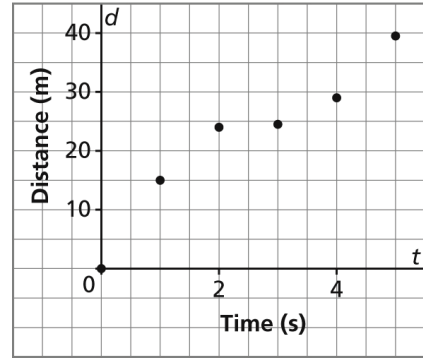
The graph you drew in Part B of *Try This* is a **scatterplot**.
 A scatterplot is any graph that consists of a set of points.

The straight line you drew is a **line of best fit**.
 The line suggests a possible relationship between the two sets of data.

Assess Your Understanding

1. Look at this scatterplot.

- a) Should the line of best fit pass through the origin? Explain.
- b) Should we always construct a line of best fit by joining the first and last points? Explain.



2. This table shows the heights of several seedlings after various growing times.

Time (days)	1	3	2	3	3	2	1	2
Height (mm)	6	13	11	14	15	10	4	7

- a) What relationship would you expect to see between growing time and height?
- b) Draw a scatterplot for the data above.
- c) Describe any trends in the data. What appears to be the relationship between growing time and height?
 How does this compare with your prediction in part a?
- d) Draw a line of best fit.
- e) Predict the height of a seedling after 5 days.
 How did you do this?

3. Use the data from the table.

- a) Draw a scatterplot for this set of data.
- b) Construct a line of best fit for the scatterplot.
- c) Estimate the number of people who were 90 years and over in 1999.

Manitoba Population Data

Year	Number of people 90 years and over (nearest hundred)
1992	5400
1994	5700
1996	6100
1998	6400
2000	6900
2002	7300
2004	7900
2006	8700
2008	9200

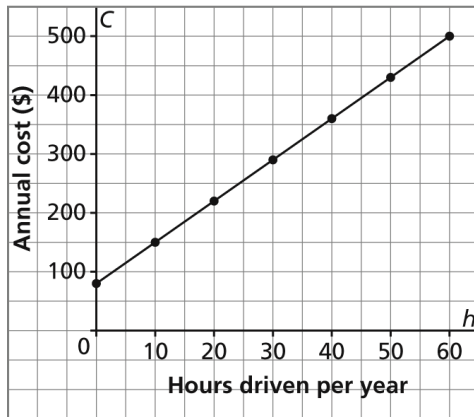
6.10 Math Lab: Equation of the Line of Best Fit

Lesson Focus: Determine the equation of the line of best fit for a scatterplot.

Make Connections

Winnipeg is studying the idea of bringing a car-sharing co-operative to the city. Car sharing has been successful in Vancouver since 1997. This graph shows the annual rates for a member on an occasional driver's plan in Vancouver, where the annual membership fee, including the application fee, is \$80.

Annual Cost of Car-Sharing Plan



Determine the slope and C-intercept of the graph.

What do they represent?

Write an equation to describe the graph.

Construct Understanding

Try This

Work with a partner. You will need a graphing calculator.

In the 2006 Turin Winter Olympics, Cindy Klassen of Winnipeg, Manitoba became the first Canadian to win five medals in one Olympic Games. IOC President Jacques Rogge called her the “woman of the games.” This table shows the winning times for the women’s 1500-m speed skating event since 1976.

Year	Gold Medalist	Time (s)
1976	Galina Stepankaya, URS	136.58
1980	Annie Borckink, NED	130.95
1984	Karin Enke, GDR	123.42
1988	Yvonne van Gennip, NED	120.68
1992	Jacqueline Börner, GER	125.87
1994	Emese Hunyady, AUT	122.19
1998	Marianne Timmer, NED	117.58
2002	Anna Friesinger, GER	114.02
2006	Cindy Klassen, CAN	115.30

To graph these data using a TI-83 or TI-84 calculator:

A. To enter the data, press **[STAT]** **[1]**.

Enter the years in L1 and the times in L2.

L1	L2	L3
1976	136.58	
1980	130.95	
1984	123.42	
1988	120.68	
1992	125.87	
1994	122.19	
1998	117.58	

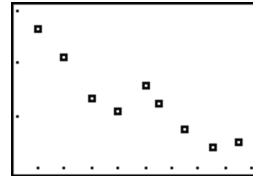
B. To set up the scatterplot, press **[2nd]** **[Y=]**.

Press **[1]** to select Plot 1.

Make sure your screen looks like this.



C. To graph the data, press **[ZOOM]** **[9]**.

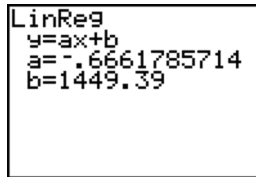


Does there appear to be a linear relation between the two sets of data? Explain.

To graph the line of best fit:

D. To determine the equation of the line of best fit, press **[STAT]** **[>]** to get the CALC menu.

Press **[4]** to select LinReg(ax + b). Press **[ENTER]**.



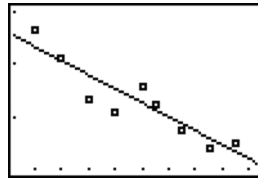
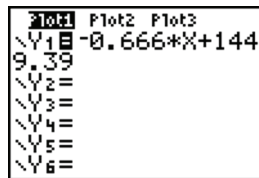
What do a and b represent?

The equation of the line of best fit has the form $y = ax + b$, with the values of a and b shown. Write the equation. Write the value of a to 3 decimal places.

E. To graph the equation, press **[Y=]** and enter the equation.

Alternatively, press: **[Y=]** **[VARS]** **[5]** **[>]** **[>]** **[1]**

Press **[GRAPH]**.

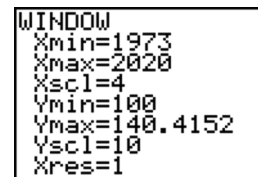


To use the graph to predict the winning time in 2016:

Adjust the window settings to show years up to at least 2016 and times from at least 105 s.

Press **[TRACE]**, then use the arrow keys to move the cursor to the line. Use the arrow keys again to move the cursor along the line until the screen shows $X = 2016$.

Record the corresponding Y -value.



In *Try This*, you used a graphing calculator to determine the equation of the line of best fit. We can “measure” how well a line fits the two sets of data.

To do this, press $\boxed{2\text{nd}} \boxed{0}$ to obtain the CATALOG.

Scroll down to DiagnosticOn, then press $\boxed{\text{ENTER}} \boxed{\text{ENTER}}$.

Repeat step E. This screen should appear.

```

LinReg
y=ax+b
a=-.6661785714
b=1449.39
r2=.8322072653
r=-.9122539478
  
```

r is a measure of how well the line fits the data.

r is called the **correlation coefficient**.

When $r = 1$ or -1 , the fit is perfect. This means that all the points lie on the line.

The closer r is to 1 or -1 , the closer the line fits the data.

In this case, $r \doteq -0.912$; since this is fairly close to -1 , the fit is reasonable.

Assess Your Understanding

1. For each set of data below:

- Graph the data and its line of best fit.
- Determine the value of r , the correlation coefficient.
- Use one of these terms to describe how well the line fits the data: “exactly,” “very closely,” “somewhat closely”

a)

x	1	2	4	5	8	8	10
y	2	2	3	5	6	8	10

b)

x	1.0	1.5	2.0	2.5	4.0	4.5	5.0
y	2	3	4	5	8	9	10

c)

x	1	1	3	4	7	8	10
y	5	8	2	6	5	2	1

d)

x	1	2	4	5	5	8	8
y	1	6	5	7	2	5	8

e)

x	0	1	2	3	6	8	10
y	8.0	7.5	7.0	6.5	5.0	4.0	3.0

2. Which lines in question 1 have a negative slope, and which have a positive slope?

How is the sign of r related to the slope of the line?

3. Tom Longboat, from the Six Nations of the Grand River First Nation, won the Boston Marathon in 1907. This table shows some of the winning men's times for the Boston Marathon since 1947.

Year	Winner	Time (min)
1947	Suh Yun-Bok, ROK	145.39
1957	John J. Kelley, US	140.05
1967	David McKenzie, NZ	135.45
1977	Jerome Drayton, CA	134.46
1987	Toshihiko Seko, JP	131.50
1997	Lameck Aguta, KE	130.34
2007	Robert Kipkoech Cheruiyot, KE	134.13

- a) Graph the data.
 b) Determine the equation of the line of best fit. How well does the line fit these data? Explain.
 c) Predict the winning time in 2017. What strategy did you use?
4. The temperature at which water boils depends on the altitude. This table shows approximate boiling points for water at the tops of different mountains.

Mountain	Altitude (m)	Temperature (°C)
Mount Everest, NP	8848	68
Mount McKinley, US	6194	78
Mount Logan, CA	5959	79
Mount Alvernia, BS	63	100
Sugarloaf Mountain, BR	396	99
Nilkantha Mountain, IN	6597	77
Mount Feathertop, AU	1922	94
Finsteraarhorn Mountain, CH	4274	86


- a) Graph the data.
 b) Determine the equation of the line of best fit.
 How well does the line fit these data? Explain.
 c) Predict the boiling point of water at an altitude of 5100 m.
 Describe the strategy you used.
5. Chantal Van Landeghem of Winnipeg swam for Canada in international meets in 2009. In 2008, at the age of 14, she broke 20 provincial age group records in Manitoba, including the record for the 50-m freestyle. This table shows the record times for 13- and 14-year-old female freestyle swimmers in Manitoba in 2009.

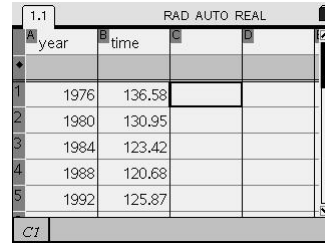
Distance (m)	Time (s)
50	26.30
100	57.37
200	127.16
400	267.26
800	550.50
1500	1034.24

- a) Graph the data.
 b) Determine the equation of the line of best fit.
 How well does the line fit these data? Explain.
 c) Suppose there was a 1000-m freestyle race. Predict what the record time for this race might be. Describe your strategy.




TI-Nspire Instructions for *Try This Lesson 6.10*

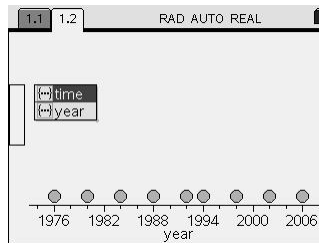
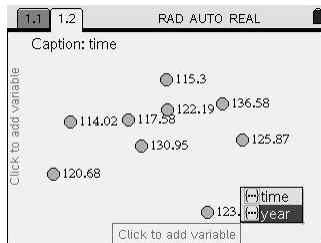
To graph these data:

- A. Press , then select **3: Lists and Spreadsheets** to insert a Lists and Spreadsheets page.
Enter the years in column A and the times in column B.
Enter *year* and *time* as headings.





	A	B	C	D
1	year	time		
1	1976	136.58		
2	1980	130.95		
3	1984	123.42		
4	1988	120.68		
5	1992	125.87		

- B. To set up the scatterplot, press , then select **5: Data and Statistics** to add a Data and Statistics page.
Move the cursor to the bottom of the screen.
When a box appears, click and select *year*, then press .
Move the cursor to the left of the screen.
When a box appears, click and select *time*, then press .

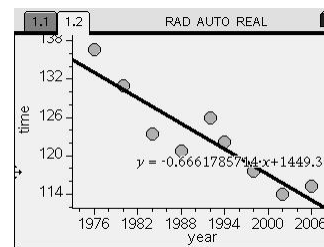


Is there a linear relation between the two sets of data? Explain.

To graph the line of best fit:

- C. Press , then select **4: Analyze > 6: Regression > 1: Show Linear (mx + b)**, then press .

The line of best fit and its equation appears on the scatterplot.





Use the equation or the graph.


Predict the winning time in 2016.

If you use the graph, adjust the window settings:

Press , then select **5: Window/Zoom > 1: Window Settings**.

Adjust the settings to show Xmax as 2016 and Ymin as 105. Press .

To trace along the graph, press , then select **4: Analyze > A: Graph Trace**.

Press .

Move the cursor along the line until the first coordinate of a point is 2016.

If the cursor does not show 2016, choose values of time on either side of 2016, then calculate the mean of these times.

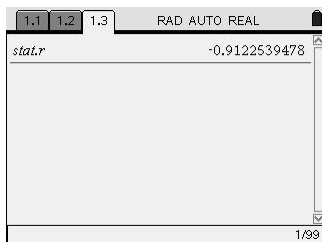
In *Try This*, you used a graphing calculator to determine the equation of the line of best fit.

We can “measure” how well a line fits the two sets of data.

To do this, Press $\left(\frac{\text{stat}}{\text{var}}\right)$, then select **1: Calculator** to insert a Calculator page.

Press $\left(\frac{\text{stat}}{\text{var}}\right)$, then select **stat.r**.

Press $\left(\frac{\approx}{\text{enter}}\right)$ $\left(\frac{\approx}{\text{enter}}\right)$.



r is a measure of how well the line fits the data.

When $r = 1$ or -1 , the fit is perfect. This means that all the points lie on the line.

The closer r is to 1 or -1 , the closer the line fits the data.

In this case, $r \doteq -0.912$; since this is fairly close to -1 , the fit is reasonable.

Answers for Masters 6.14 to 6.17

Master 6.14, for Manitoba Lesson 6.7

4. AB: 4 units; CD: 3 units; EF: 6 units; GH: 7 units

5. a) 13 units b) 8 units c) 14 units d) 9 units

6. AB: 2 units; BC: 5 units; AC: $\sqrt{29}$ units

JK: 5 units; KM: 4 units; JM: $\sqrt{41}$ units

DE: 5 units; EF: 3 units; DF: 4 units

7. PQ: $\sqrt{65}$ units; RS: $\sqrt{73}$ units; TU: $\sqrt{26}$ units; VW: $\sqrt{20}$ units

For example, I used the Pythagorean Theorem because I could count squares.

8. a) i) $\sqrt{17}$ units ii) $\sqrt{52}$ units iii) $\sqrt{130}$ units iv) 5 units

b) Answers may vary.

9. a) $\sqrt{98}$ units b) $\sqrt{80}$ units c) $\sqrt{89}$ units d) $\sqrt{85}$ units

10. a) 10 units b) 10 units

I got the same answer using both methods. To use the Pythagorean Theorem, I counted squares. To use the distance formula, I subtracted the coordinates.

11. The line segment has length 10 units because the horizontal distance between the points is 8 units, the vertical distance is 6 units, and $\sqrt{(8)^2 + (6)^2} = \sqrt{100}$, or 10.

12. AB: $\sqrt{90}$ units; AH: $\sqrt{2}$ units

A student might add the coordinates instead of subtracting them:

$$\begin{aligned} AB &= \sqrt{(4 + (-5))^2 + (-2 + 1)^2} \\ &= \sqrt{(-1)^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

A student might think AB and AH have the same length because the coordinates of B and H are opposite integers.

13. a) BC: $\sqrt{53}$ units; CD: $\sqrt{106}$ units; BD: $\sqrt{53}$ units

$\triangle BCD$ is isosceles because it has 2 equal sides.

b) XY: $\sqrt{20}$ units; YZ: $\sqrt{29}$ units; XZ: $\sqrt{17}$ units

$\triangle XYZ$ is scalene because no sides are equal.

c) EF: $\sqrt{32}$ units; FG: $\sqrt{106}$ units; EG: $\sqrt{106}$ units

$\triangle EFG$ is isosceles because it has 2 equal sides.

14. a) AB: $3\sqrt{10}$ units; BC: $\sqrt{10}$ units; CD: $3\sqrt{10}$ units; AD: $\sqrt{10}$ units

b) $8\sqrt{10}$ units; 30 units²

c) 10 units; 10 units

d) Rectangle; opposite sides are equal and the diagonals are equal.

15. J(5, 4) and M(-3, -2) are both on the circle because the distance from each of these points to the centre of the circle is 5 units.

16. a) i) 4 units ii) $\sqrt{17}$ units iii) $\sqrt{20}$ units iv) 5 units

b) $x = 0$

c) $y = 1$ or $y = 7$

Answers continued

17. Answers may vary. For example:
a) i) (1, 0), (1, 1) **ii)** (0, 5), (0, 0) **iii)** (0, 0), (2, 3) **iv)** (5, 4), (0, 1)
b) No, there are many different line segments with each length.
18. **a)** Chico: approximately 47 m; Cheri: approximately 69 m; for each person, the distance I calculated is greater than the two distances given, but less than their sum.
b) Approximately 113 m
c) No, $47\text{ m} + 69\text{ m} \neq 113\text{ m}$
19. a, c, d
20. (0, 0) and (12, 0)
21. **a)** (0, 3) **b)** (0, 2) **c)** (0, 1)

Master 6.15, for Manitoba Lesson 6.8

4. AB: (1, 3); CD: (3, -0.5); EF: (-1, -4); GH: (-2, 0.5)
5. **a)** (5, 2.5) **b)** (2, 7) **c)** (-4, 4) **d)** (3.5, 0)
 For example, because the lines are horizontal or vertical, I counted squares to determine the midpoints.
6. PQ: (-0.5, 3); RS: (2.5, 1); TU: (-2.5, 0.5); VW: (1, -1)
 For example, I used a Mira to find the midpoints.
7. **a) i)** (4, 3.5) **ii)** (1.5, 2) **iii)** (1.5, -1.5) **iv)** (-2, 2.5)
b) Answers may vary.
8. **a)** AB: (4, 3); BC: (7, 6); AC: (3, 7)
b) AB: $\sqrt{68}$ units; BC: $\sqrt{68}$ units; AC: $\sqrt{72}$ units
c) $\sqrt{17}$ units; $\sqrt{17}$ units; $\sqrt{18}$ units
d) The radicands of the lengths in part b are 4 times the radicands in part c. So, the lengths in part b are $\sqrt{4} = 2$ times as long as the lengths in part c.
e) The lengths of the sides of the triangle and the lengths of the line segments joining the midpoints are double their original lengths. But, the lengths in part b are still twice as long as the lengths in part c.
9. **a)** I could measure the segment with a ruler, divide its length in half, then mark this length on the line segment.
b) I could fold the paper so that one end of the segment coincides with the other end, then mark the point where the crease intersects the segment.
c) I could place a Mira on the segment so that one end of the segment coincides with the other end, then mark the point where the Mira intersects the line segment.
10. **a)** (2, 1) **b)** $\sqrt{41}$ units
11. QS: (3, -0.5); RT: (3, -0.5)
 The diagonals have the same midpoints.
12. DF: (-0.5, 0.5); EG: (-1, 0.5)
 DEFG is not a parallelogram because the midpoints of the diagonals are different.
13. **a) i)** (2, -9) **ii)** (5, 8) **iii)** (0, 0)
b) Answers may vary.
14. I could trace each line segment, then rotate point Q 180° about point M to locate the other endpoint.

Answers continued

15. a) (7.1, 5.5) b) \$181 544 c) \$363 088

I could determine the length of the bicycle path and its total cost of construction, then divide the cost by 2 to get the cost for Saddleville.

16. Red Deer is very close to half-way between Edmonton and Calgary. The coordinates of the point halfway between the two cities are (5162.5, 7817.5).

17. No; (6.75, 9.65)

18. (0.5, -3.5), (3, -1), (5.5, 1.5)

First I found the midpoint, M, of AB. Then, I found the midpoints of AM and BM.

19. $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right), \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right), \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$

20. (2, 5), (0, 2)

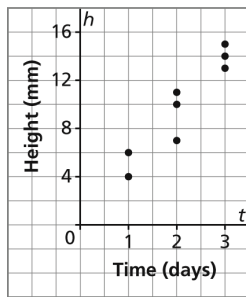
21. $\left(\frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3}\right), \left(\frac{2x_2 + x_1}{3}, \frac{2y_2 + y_1}{3}\right)$

22. a) 5; 6

b) W(5, 6) is the centre of the square.

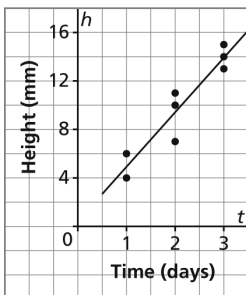
Master 6.16, for Manitoba Lesson 6.9

1. a) No; a line that is as close as possible to all the plotted points does not pass through the origin.
 b) No, by joining the first and last points, all other points may not be as close as possible to the line.
2. a) Linear
 b) **Seedling Heights**



- c) As the number of days increases, the height of the seedlings increases. The relationship between growing time and height is linear as predicted in part a.

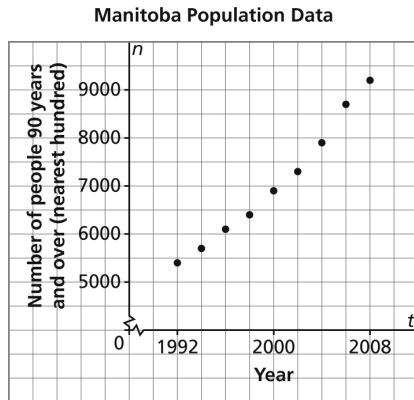
- d) **Seedling Heights**



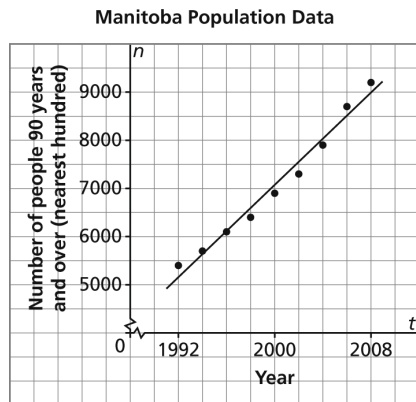
- e) About 23 mm; I extended the line of best fit to the right.

Answers continued

3. a)



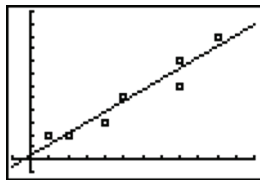
b)



c) About 6800 people

Master 6.17, for Manitoba Lesson 6.10

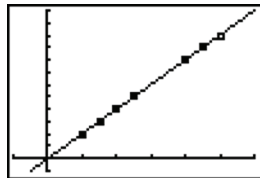
1. a) i)



ii) $r \doteq 0.9601$

iii) Very closely

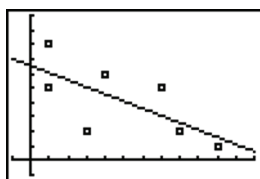
b) i)



ii) $r = 1$

iii) Exactly

c) i)

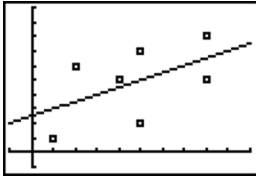


ii) $r \doteq -0.6834$

iii) Somewhat closely

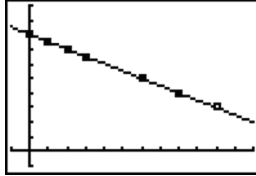
Answers continued

d) i)

ii) $r \doteq 0.5286$

iii) Somewhat closely

e) i)

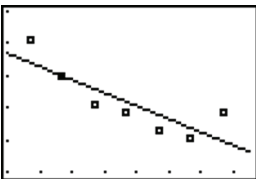
ii) $r = -1$

iii) Exactly

2. The lines in parts a, b, and d have a negative slope and the lines in parts c and e have a positive slope.

The sign of r is the same as the sign of the slope of the line.

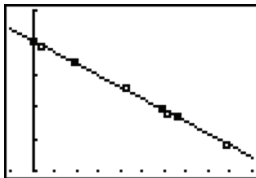
3. a) b)



The equation of the line of best fit is approximately: $y = -0.204x + 539.423$
 $r \doteq -0.8460$; the line fits the data quite closely.

c) Approximately 127.74 min; I used the TRACE feature on the graphing calculator.

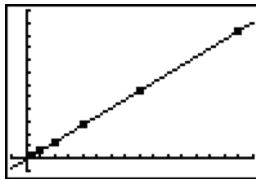
4. a) b)



The equation of the line of best fit is approximately: $y = -0.0036x + 100.683$
 $r \doteq -0.9992$; the line fits the data almost exactly.

c) Approximately 82°C; I used the TRACE feature on the graphing calculator.

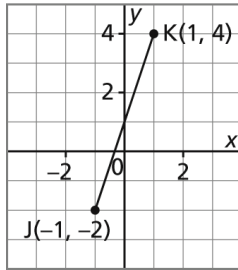
5. a) b)



The equation of the line of best fit is approximately: $y = 0.697x - 10.746$
 $r \doteq 1$; the line fits the data almost exactly.

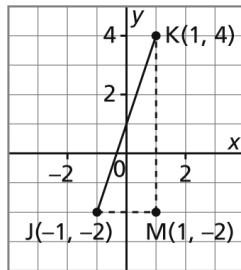
c) About 686.73 s; I used the TRACE feature on the graphing calculator.

1. Calculate the length of line segment JK.



Solution

Draw a right $\triangle JKM$ for which JK is the hypotenuse.
Use the coordinates of the points to determine the lengths of the legs.



The length of KM is the difference in the y -coordinates: $4 - (-2) = 6$

The length of MJ is the difference in the x -coordinates: $1 - (-1) = 2$

Use the Pythagorean Theorem.

$$JK^2 = KM^2 + MJ^2$$

$$JK^2 = 6^2 + 2^2$$

$$JK^2 = 36 + 4$$

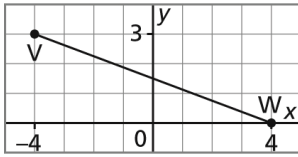
$$JK^2 = 40$$

$$JK = \sqrt{40} \quad \text{Use a calculator.}$$

$$JK \doteq 6.3$$

JK is approximately 6.3 units long.

2. Calculate the length of line segment VW.



Solution

The coordinates of V are: $(-4, 3)$

The coordinates of W are: $(4, 0)$

Use the formula.

$$VW = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = 4, x_1 = -4, y_2 = 0, y_1 = 3$$

$$VW = \sqrt{(4 - (-4))^2 + (0 - 3)^2}$$

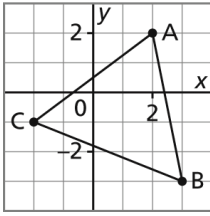
$$VW = \sqrt{8^2 + (-3)^2}$$

$$VW = \sqrt{73}$$

$$VW \doteq 8.5$$

VW is approximately 8.5 units long.

3. Determine whether $\triangle ABC$ is isosceles.



Solution

The coordinates of the vertices of the triangle are:

$$A(2, 2), B(3, -3), C(-2, -1)$$

Use the distance formula to calculate the length of each side.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = 3, x_1 = 2, y_2 = -3, y_1 = 2$$

$$AB = \sqrt{(3 - 2)^2 + (-3 - 2)^2}$$

$$AB = \sqrt{(1)^2 + (-5)^2}$$

$$AB = \sqrt{26}$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = -2, x_1 = 3, y_2 = -1, y_1 = -3$$

$$BC = \sqrt{(-2 - 3)^2 + (-1 - (-3))^2}$$

$$BC = \sqrt{(-5)^2 + (2)^2}$$

$$BC = \sqrt{29}$$

$$CA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Substitute: } x_2 = 2, x_1 = -2, y_2 = 2, y_1 = -1$$

$$CA = \sqrt{(2 - (-2))^2 + (2 - (-1))^2}$$

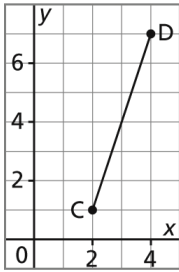
$$CA = \sqrt{(4)^2 + (3)^2}$$

$$CA = \sqrt{25}$$

$$CA = 5$$

Since no two sides of $\triangle ABC$ have the same length, it is not an isosceles triangle.

1. Determine the coordinates of the midpoint, M , of line segment CD .



Solution

Method 1

Place a Mira perpendicular to CD .

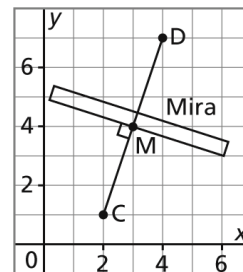
Move the Mira until C coincides with D .

Mark the point where the Mira intersects the line segment.

Label the point M .

This is the midpoint of segment CD .

It appears to have coordinates $(3, 4)$.



Method 2

Draw a right $\triangle CDE$ for which CD is the hypotenuse.

Determine the midpoint of DE .

DE is 6 units long.

One-half of 6 is 3.

So, the midpoint of DE is 3 units down from D .

Label the point F . It has coordinates $(4, 4)$.

Determine the midpoint of CE .

CE is 2 units long.

One-half of 2 is 1.

So, the midpoint of CE is 1 unit to the right of C .

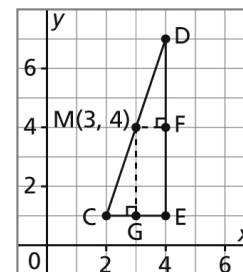
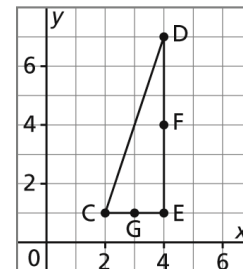
Label the point G . It has coordinates $(3, 1)$.

Draw the perpendicular to DE at F .

Draw the perpendicular to CE at G .

The perpendiculars meet at M , the midpoint of CD .

The coordinates of M are $(3, 4)$.



2. An engineer has to design a highway so that it is the same distance from Red Soil, located at R(5312, 4308), and White Springs, located at W(5368, 4344). Determine the coordinates of one point on the highway.

Solution

One point on the highway is the midpoint, M, of the line segment RW.

Use the formula.

$$\begin{aligned}x\text{-coordinate of } M &= \frac{x_1 + x_2}{2} && \text{Substitute: } x_1 = 5312 \\ & && \text{and } x_2 = 5368 \\ &= \frac{5312 + 5368}{2} \\ &= \frac{10\,680}{2} \\ &= 5340\end{aligned}$$

$$\begin{aligned}y\text{-coordinate of } M &= \frac{y_1 + y_2}{2} && \text{Substitute: } y_1 = 4308 \\ & && \text{and } y_2 = 4344 \\ &= \frac{4308 + 4344}{2} \\ &= \frac{8652}{2} \\ &= 4326\end{aligned}$$

The coordinates of M are (5340, 4326).

One point on the highway has coordinates (5340, 4326).

Lesson 6.8 Check Your Understanding

3. Point P(-1, 5) is one endpoint of line segment PQ. Point M(3, 2) is its midpoint.
Determine the coordinates of the other endpoint, Q.

Solution**Method 1**

Plot P and M on a grid. Join the points.

Count squares.

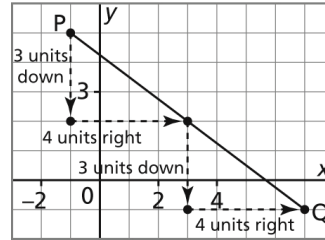
To get from P to M, move 3 units down and 4 units right.

Since M is the midpoint of PQ,

to get from M to Q, move 3 units down
and 4 units right.

Plot point Q. Join P and Q.

Q has coordinates (7, -1).

**Method 2**

Let the endpoint Q have coordinates (x_2, y_2) .

One endpoint is P(-1, 5).

The midpoint is M(3, 2).

Use the midpoint formula to determine the coordinates of Q.

The x-coordinate of M is:

$$3 = \frac{x_1 + x_2}{2} \quad \text{Substitute: } x_1 = -1$$

$$3 = \frac{-1 + x_2}{2} \quad \text{Multiply each side by 2.}$$

$$2(3) = 2\left(\frac{-1 + x_2}{2}\right)$$

$$6 = -1 + x_2 \quad \text{Add 1 to both sides to isolate } x_2.$$

$$1 + 6 = -1 + 1 + x_2$$

$$x_2 = 7$$

The y-coordinate of M is:

$$2 = \frac{y_1 + y_2}{2} \quad \text{Substitute: } y_1 = 5$$

$$2 = \frac{5 + y_2}{2} \quad \text{Multiply each side by 2.}$$

$$2(2) = 2\left(\frac{5 + y_2}{2}\right)$$

$$4 = 5 + y_2 \quad \text{Subtract 5 from both sides to isolate } y_2.$$

$$4 - 5 = 5 - 5 + y_2$$

$$y_2 = -1$$

The coordinates of Q are (7, -1).

PEARSON

Foundations and Pre-calculus Mathematics 10

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